**Irreversibility and Uncertainty in Species Valuation**

by

**Eileen L. Brooks**

**MIT-CEEPR 94-007WP** **September 1994**

# Irreversibility and Uncertainty in Species Valuation*

by

Eileen L. Brooks

This draft: September 8, 1994

**Abstract**: This paper incorporates an option value into deforestation policy analysis. Similar to an option value in finance, the option value here reflects the advantage to delaying irreversible species extinction until more information about the uncertain value of species is known. The return from species is modeled as a stochastic flow of benefits which ceases if policy makers choose to deforest. Deforestation produces known profits from wood, and agriculture or ranching. Model variations include using geometric Brownian motion and a Poisson jump process to model the variation in species values, and the effect of considering whether "harvesting" of species can occur during deforestation. The model demonstrates that uncertainty over the value of species should encourage forest protection beyond what present discounted value comparisons (traditional cost-benefit analysis) would imply. Data from studies of Brazil's Amazonia region are used to provide numerical examples of the differences between traditional cost-benefit methods and the option-theoretic approach described here.

*This paper is a rewritten version of a B.S. thesis completed in the spring of 1994 at the Massachusetts Institute of Technology. The author thanks the Center for Energy and Environmental Policy Research, the National Science Foundation, and the Undergraduate Research Opportunities Program at MIT for financial support. My thanks to Dr. Robert Pindyck for extensive advice as well as Alex Chisholm, Raul Acevedo, Amy Weaver, and Alex Pfaff for their comments and assistance.

# 1 Introduction

At the forefront of our social agenda, the issue of tropical deforestation has produced a plethora of books asking how much tropical forests are worth, what society risks by cutting them down, and what role the loss of biodiversity should play in the decision-making process. Typical analysis has involved one of two ideas: either cost-benefit analysis (comparing expected costs and benefits), or SMS (safe minimum standard), which weighs the value of species very highly, almost infinitely, because of the uncertain and non-economic role which species play in our own society (Wilson, 310). The pages that follow offer an estimate in between these two dichotomies; they analyze the value of temporarily preserving species until more information is obtained.

More specifically, the issue addressed here is how one should decide whether to preserve portions of the Amazonia tropical forest region. Current cost-benefit analysis compares the present discounted flow of expected costs and expected benefits from forest preservation. If the costs of preserving a section of forest are expected to outweigh the benefits over the long run, then that section would be deforested. This analysis ignores three important characteristics: the opportunity to delay, the irreversibility of species extinction, and the uncertainty involved in the decision. In other words, when costs and benefits are compared, one implicitly compares deforesting now with never cutting the forest. This paper analyzes the implication of a third possibility: to delay until a later time, rather than acting now. The "option" value of waiting is essentially the value of behaving cautiously in a world of irreversible decisions and uncertain outcomes.

This option value exists solely due to the irreversibility and uncertainty involved in deforestation. With complete reversibility, society would do what appeared optimal in each time period, knowing its decisions could be reversed in the future. With complete certainty, society would be indifferent to the irreversibility of deforestation because it is certain that the correct policy decision is being made. However, because species may be irreversibly destroyed by deforestation (even millenia of evolution could not replace them) and because society is uncertain about the decision to deforest (due to lack of good measurement ability, scientific knowledge, social understanding, and ability to predict future value) there is a clear advantage to delaying policy action. Hence, the option value in this paper will reflect the difference between the actual value of preserving the forest and the expected benefits from preservation—the model will implicitly calculate the advantage of waiting to decide. In other words, preserving the forest has benefits stemming not only from the gains to society from having forest, but also from the option to wait and decide to deforest later. Deforestation, on the other hand, only has the benefit of the economic gain from cutting the forest down.

Before weighing the decision to invest in preserving species, it is important to ask why society should value species in the first place. There are three main reasons to preserve species.[1]

- **Economic value.** Species may provide a potential tangible future return to society. A species may be an important source of medications, food, or scientific information. Preserving species maintains genetic information which could be useful in the future. Moreover, some species may contribute to the economy by attracting tourism or by providing production materials (e.g., flowers, rubber).

- **Ecological value.** In addition, species may be essential to other portions of nature. As members of the food chain, some plants and animals are vital to the existence of others. More significantly, some species could be vital to the sustainability of our ecosystem.

- **Existence value.** Society also gains from species preservation by simply knowing that they exist. We value them for aesthetic reasons—we like knowing variety exists and we like knowing that other people can appreciate species. In addition, some would argue that we gain moral integrity by preserving species because the human species has no moral right to destroy another species.

It should be noted that while one can fairly easily classify the motivations for preserving species, it is much more difficult to calculate that value or even understand the nature

[1]See Wilson (1992) or Randall (1991) for a more extensive discussion.

of the uncertainty. Valuing species is problematic because species resemble public goods and are associated with many externalities. As a result, most all-inclusive attempts at valuing species have resorted to contingent valuation, because market valuation techniques would ignore the positive externalities associated with species. Unfortunately, contingent valuation is usually inaccurate—simply asking people how much species are worth neither reflects actual individual preferences nor produces reasonably consistent estimates. In the end, determining the benefits from species directly is hindered by the subjective nature of the aesthetic value of species and the uncertainty over economic and ecological returns from species.

Given these difficulties, this paper does not attempt to value species. Instead, numerical examples illustrate the impact of using option value analysis upon the threshold value of species at which society should deforest. Other parameters (such as the value of the timber) determine the range of species values which should lead to species preservation.

The focus of the model will be a variable, $\theta^*$, which is the threshold value of species at which the species should be preserved. In other words, the policy maker should deforest only if the current value of species falls below the necessary threshold value, $\theta^*$. These models will demonstrate that uncertainty lowers the optimal $\theta^*$, or that uncertainty over the value of species, combined with the irreversibility of species destruction, makes it optimal to keep seemingly less valued species around until it is certain that the correct choice is to cut the forest down. The point at which society is certain that the correct choice is to cut the forest down will be the point where the value of species falls below $\theta^*$, which is also the point where the value of holding the forest combined with the option value of waiting is still less than the profit from deforesting.

After developing the basic model, this paper will experiment with different assumptions about the stochastic nature of species value. One question underlying these experiments is: what type of benefit flow does society receive from holding species? The model will assume that society receives a stochastic flow of benefits or a "dividend" by preserving species. Within this theory, each additional year that society keeps species alive it receives a "dividend" which includes economic value, ecological value, and existence value. It is assumed that while one cannot purchase or replace a moral value of species, the relative annual value of the morality of preserving species must be determined to analyze the tradeoff between human economic welfare and the existence value of species.[2]

After developing a framework with which to evaluate the costs and benefits of deforestation, the model is compared with the current policy method of cost-benefit analysis (present discounted value comparisons) to determine the effect uncertainty should have upon deforestation decision-making analysis. After initial conclusions, variations on the modelling process are considered, including changing the nature of uncertainty over the value of species, and considering different ways of capturing the value of species.

# 2 Modelling Assumptions

This section discusses the basic parameter estimates used in the option models below. All variations of the model will weigh the value of keeping species against the return from harvesting wood and making farmland/ranchland. Analyzing the different options requires some basic aggregate data, including the number of species in the Amazonia region, the area of the forest, and the economic return from forest harvesting.

The number of species in the Amazonia region has been hotly contested. A reasonable estimate is $s$ = number of species = 10 million.[3] There are, however, many different ways to

[2]There is an extensive body of literature discussing the morality of placing a value upon species and other natural ecosystems, including Randall (1991) and Schulze (1994). It is the belief of the author, however, that while it appears arbitrary to place a dollar value upon the existence value of species, it is important to decide where species stand in policy maker's hierarchy of goals and what tradeoff society is making when it preserves species. While it is possible that the moral value of species is too high to warrant their extinction, the tradeoff being made for species should be determined; and then once the value of species is known to exceed the threshold necessary to warrant preservation, they should be preserved.

[3]While this number is easily contested, the 10 million species estimate was made for a few reasons. See Mahar (1989), Ehrlich (1991), Wilson (1992), Ryan (1992), and Lovejoy (1985) for more information. It is estimated in Ehrlich (1991) as well as other sources that the world's biodiversity level is between 20 million and 100 million species, although only 1.4 million species have scientific classifications. In addition, Lovejoy (1985) refers to estimates that the Amazonia rain forest region has well over 10% of the world's biodiversity, and the remaining sources refer to the extreme uncertainty over the number of species as well as the large number of arthropods and microorganisms which are not yet discovered. Studies about Amazonia have little agreement on the number of species in the region. All of these predictions depend upon assumptions

estimate the number of species as well as debates on how to distinguish or classify species.[4]

For the parameter A, the area of the Amazonia region, this paper will use an estimate of 5.8 million square kilometers which was calculated using LANDSAT photographs in the late 1980's.[5] While some of this region could not be deforested due to rivers and marshes, this paper assumes for simplicity that the entire region can be deforested.[6]

The economic return from deforesting is hard to assess. Clearly, the value of the forest would depend upon government subsidies, property rights laws, harvesting method, transportation costs for exporting wood, and other factors. In addition, there is some uncertainty as to the value of the deforested area in the future. If the land became city, for example, deforestation would have created a much higher economic value than if the land became rural area. The estimate used for this parameter is actually the predicted value of land to speculators who cut the trees, farm for two years, and then sell the property rights. It has been estimated by the FAO/World Bank Cooperative Program that speculators pay U.S. $9,000 for 14 hectares of land; this is the equivalent of U.S. $64,286 per km$^2$.[7] In using this estimate, it is assumed that speculators are not profiting from land purchase.

It is interesting to note, however, that deforestation typically occurs without harvesting the timber. Deforested land is more valuable than forest area because it is easier to convert to farmland. As a result, simply burning the forest can be a profitable endeavor. Burning also increases the carbon content in the soil which encourages plant growth. In addition, the causes of deforestation are not always forest or agricultural returns, but rather government subsidies and policies which are intended to redistribute wealth yet consequently encourage homesteading in the tropical forest region (Repetto, 270).

---

about the correlation between the quantity of different arthropod species and the quantity of plant species, as well as assumptions that the distribution of species diversity (such as the ratio of animals to plants to microorganisms) contained in one forest (usually in Panama) can be applied to the Amazon.

[4]For more information, see Wilson (1992).

[5]For a description of the techniques and the data, see Barbier (1990).

[6]If this seems unreasonable, consider the rivers and marshes to be a fixed portion of the preserved area. In other words, since the model emphasizes partial deforestation, assume that the rivers are always part of the preserved portion of the Amazonia Region.

[7]Mahar (1989), p. 38. A hectare is 10,000 square meters or 0.01 square kilometers.

After discussing all of the difficulties in determining the value of land, it is important to note the "ideal policy scenario." Ideally, an analyst would be evaluating preservation for a specific plot of land by estimating how many species deforestation would destroy, and assessing the profitability of deforesting the plot.

The complexities mentioned about the data will not be discussed in the remainder of this paper. The intention is to use the data to provide an example of how uncertainty impacts the decision to deforest. The data mentioned above would be vital to the cost-benefit analysis solution as well as decisions using the models below. The models below will show how uncertainty should bias the cost-benefit decision. The total number of species, the total land area, and the payoff will be irrelevant to assessing the difference between cost benefit analysis and using option values. In other words, errors in the above data would bias the cost-benefit analysis solution and the option model solution equally. In essence, the above data is irrelevant to the message and conclusions of this paper: that uncertainty should encourage caution when a completely irreversible decision is being made.

# 3 A Two-Period Model

Evaluating the value of species in an option model requires a few assumptions. Because this model is answering the question of maximizing the potential return from forests, it is assumed that a central planner possesses the rainforest in the Amazonia Region in Latin America and is deciding how much of the forest to preserve. The planner can then choose to deforest it, keep it around until time T, or keep it forever. Any enforcement difficulties are ignored. Admittedly, this social planner approach ignores the role that individual wealth plays in the deforestation process. Normally, speculators gain most of the profit from deforestation, yet suffer only a small amount of costs from environmental degradation. As a result, deforestation is driven by the individual's incentive to deforest. Despite the individual, however, the social planner approach does answer the question of what is socially optimal, or what the goal of policy makers should be. Hence, the question to be pursued is, how does uncertainty affect whether it is optimal for society to wait until time T to see what the species are worth?

This first example considers the economic benefits of deforestation and the cost of species lost in a two-period model. A central planner is deciding whether to cut down five percent of the trees in Amazonia. Cutting down the forest would irreversibly destroy the unique species that live solely within that area. An estimate of the number of unique species which will go extinct as a result of deforestation is necessary. This is calculated using a model which originated from the field of island biogeography relating habitat size to the sustainability of species. This model requires a number of species, $S$, an area, $A$, a constant, $c$, and a parameter, $z$, which implicitly assesses the average adaptability of species. The parameter estimate $z = 0.30$ is considered by ecologists to be a reasonable average across species.[8] The relationship between habitat destruction and species destruction is characterized by:

$$S = cA^z \tag{1}$$

Taking the above model as given, cutting down five percent of the forest will leave 9.85 million species remaining, or kill 152,700 species.[9] Clearly, this is a rough estimate—there are regional differences within the Amazonia region, as well as different methods of deforestation.

For simplicity, this model assumes that species are homogeneous. One difficulty is that different animal types have different z-values—certain varieties of orchids will be the first to go extinct because they are immobile, while birds and insects are much more likely to be able to relocate. Along the same lines, society does not attach the same aesthetic value to each species. A seal, for example, is worth much more to society than a roach, simply because it is more aesthetically pleasing. The assumption that species are homogeneous does not,

[8]More sources about species survival models as well as the foundations of the species-land area model are outlined in simple terms by Wilson (1986). Wilson also contains a bibliography of more complex readings about modelling species destruction.

[9]The area of the Amazonia region, A, and the constant c are irrelevant in determining how many species are destroyed. Applying Equation 1, one knows that surviving species, $s_{new} = 10,000,000(0.95)^z =$ 9.85 million with z = 0.30.

however, affect any of the conclusions of the model.

Each of the estimated 152,700 species which would be destroyed by deforestation currently provides society with a dividend. Assume the current expected flow value of the species is $\hat{\theta}$ = \$4500/species/year. The planner can either destroy the forest now, knowing only a guess of the value of species, or she can wait until a time T when she will have completed a study and know the precise value of the species.

To begin the analysis, consider two cases: the expected welfare from preserving forests, $W_p$, and the expected welfare from cutting forests now, $W_c$. The expected welfare of cutting the forest now is the value of the forest, F. Applying the above parameters for the speculative value of the forest and the area of the Amazon, the value received from harvesting five percent of the forest would be \$18.6 billion.[10] Since timber and land value are the only welfare gains from deforesting, the welfare from cutting the portion of forest is $W_c$ = \$18.6 billion. The expected welfare from preserving forests is the number of species held and the expected value of their return. The expected value of their return is:

$$\mathcal{E}_0 W_p = \int_0^\infty s\hat{\theta}e^{-\rho t}dt = \frac{s\hat{\theta}}{\rho} \tag{2}$$

Assuming a discount rate of $\rho = 0.04$, the estimated value of preserving species forever is $W_p$ = \$17.2 billion.

Under either perfect certainty or traditional cost-benefit analysis, the central planner would elect to deforest, because cutting produces a larger expected welfare than preserving species forever. Now consider the option value of preserving until a future decision opportunity. First, the nature of the uncertainty of $\theta$ must be determined. For the sake of simplicity, assume that with one-half probability $\theta = \overline{\theta}$ = \$6000 and otherwise $\theta = \underline{\theta}$ = \$3000. After waiting, the planner will be able to diagnose whether or not policy adoption is optimal and then will adopt only if $\theta = \overline{\theta}$.

[10]This value assumes that the market equilibriums for land and timber would not change as a result of deforestation.

Suppose now that the planner can wait until a time $T$ to adopt the policy. Then, the value of the policy to wait until time $T$ is denoted by:

$$W_T = W_p(1 - e^{-\rho T}) + \left(\frac{s\overline{\theta}}{2\rho} + \frac{W_c}{2}\right) e^{-\rho T} \tag{3}$$

Note that at time $T$, the central planner deforests only if $\theta = \underline{\theta}$ which should occur one half of the time. Hence, the above equation could be read as follows: "The total expected welfare from policy adoption at time T is equal to the expected value of preserving the forest up until time T plus the expected return from policy adoption (preservation if species have a high value, deforestation otherwise) at time T." Using the above equation with $T = 10$, $W_T =$ \$19.6 billion. The value of having the option to change policy at a later date made preservation desirable even though its expected return was lower than that of deforestation.

You may have noticed that there is also a disadvantage to waiting—the value of the forest is discounted because society must wait to obtain it. This is why under normal conditions (without irreversibility and uncertainty) a policy maker has no advantage (in fact a disadvantage) to waiting. Hence, the "option value of waiting" in this example actually contains two components: the positive ability to decide with complete information, and the negative effect from discounted forest values if deforestation would have been optimal.

It is important to realize how the degree of uncertainty affects the value of waiting to decide. Under perfect certainty, for example, deforestation is always desirable. Similarly, under a lower variance it may still be desirable to deforest. For example, if one used $\theta = \overline{\theta} =$ \$5000 and $\theta = \underline{\theta} =$ \$4000, the welfare of acting in ten years would be $W_T =$ \$18.3 billion and it would still be prudent to deforest now. This analysis can be used to criticize SMS (safe minimum standard) policy—the mere existence of uncertainty does not imply that one should protect all species, rather, it must be considered along with costs and benefits of species preservation in deciding whether the uncertainty is of large enough magnitude to justify delay.

Another issue related to uncertainty is not only the variance, but also the likelihood and magnitude of a high return from species. It is unlikely that there is a fifty-fifty relationship between a high return on species and a low return. Rather, we think that there is some small risk of species having a great return. Consider a scenario in which $\theta = \overline{\theta} = \$10,000$ and $\theta = \underline{\theta} = \$4444$ but $Pr(\overline{\theta}) = 0.01$. While the expected value of $\theta$ remains the same, $\hat{\theta}$=\$4500, deforesting risks a greater loss of species value. After adjusting for the new probabilities in Equation 3, $W_t$ = \$24.7 billion. Hence, having the same value for $\hat{\theta}$, there is still a significantly greater option value associated with preserving species. The understanding behind this result is that when deforestation occurs, the potentially high benefit of species in the second case is irreversibly lost. This example reinforces a fundamental idea discussed in the safe minimum standard literature. The SMS literature argues that there is a potentially extreme value of species to society, so we should be certain to maintain as many as possible to keep a minimum level of ecological insurance. Under this example, the small likelihood that species may be saving us from ecological disaster would suggest a much higher level of caution than a case where $\theta$ has a similar variance, but a much smaller disparity in the possible outcomes (e.g., the possibility of the value of species going up or down by 5%).

This section has illustrated two fundamental aspects to modelling species preservation as an option: first, there is a value to being able to decide later, and, second, the higher the uncertainty, the better it is to wait. While these two end results will remain the same, the next section contains more realistic and detailed continuous models of valuing species as options.

# 4 The Continuous Model

This section adds two dimensions to the discussion. First, it operates under continuous time. Second, it offers different versions of the model to analyze various perspectives about how value is obtained from preserving biodiversity. Variations on the model include different stochastic types for $\theta$, the potential to harvest the economic value of some species during deforestation, and the potential for ecological problems when deforestation occurs.

## 4.1 The Basic Model: Species Yielding a Continuous Dividend

The two-period model provided the intuition behind applying option values to deforestation analysis: uncertainties about the future and the irreversibility of species extinction combine to form an advantage to delaying. Even when cost-benefit analysis suggests that species should be destroyed because their value is less than the value of deforesting the land, holding species could be a better policy approach because species may be worth more than their estimate, and society can decide to deforest at a later date. In the two-period model, the value of the species was allowed to fluctuate only once. This section analyzes the impact of uncertainty when the value of species varies continuously, following a stochastic path that fluctuates each increment of time. Causes of these fluctuations could include scientific discoveries or the development of new production techniques, either of which may increase or decrease the value of species.

Like the two-period example, deforestation irreversibly kills species with a payoff made up of timber profits and agricultural potential. Unlike the discrete scenario, however, the value of species $\theta$ will fluctuate in continuous time. $\theta$ is assumed to follow geometric Brownian motion as in Equation 4 with constant parameters $\alpha$ and $\sigma$:

$$d\theta = \alpha\theta dt + \sigma\theta dz \tag{4}$$

For some intuition as to what these stochastic paths can look like, there are some sample plots in Figure 3. Geometric Brownian motion was chosen for three main reasons: first, it is a Markov process, which is useful when applying dynamic programming technique because one knows the expected change in $\theta$ at any given time. Second, it is log-normally distributed. This prevents the value of species from becoming negative, as is much more likely in a random walk process. Finally, it contains a drift parameter $\alpha$ so that the expected value of species is increasing over time. It is intuitive that the value of species would increase over time—we will learn more about them, there are more people to appreciate them, and other species on the planet are being killed off, making each remaining creature more valuable.

Like the previous example, a central planner controls the Amazonia forest and is deciding whether or not to clear cut 5% of the forest. The planner's first step in determining the optimal value at which to deforest is determining the payoff, $\Omega_0$, from cutting the forest now. The benefits from cutting the forest are the wood and the income from agriculture received each year thereafter. As in the two-period example, $W_c = \Omega_0 =$ U.S. \$18.6 billion is used as an estimate for this payoff. This estimate is derived from the value of land to a prospector discussed in Section 2: "Modelling Assumptions." It is assumed that the prospectors have negligible profits and that this value reflects the expected return to deforesting.

Also like the previous example, there are two benefits to holding species. First, keeping species returns an annual dividend. Second, in the case of uncertainty, holding species prevents them from being irreversibly destroyed until enough is known about them to justify their destruction. The optimal solution to this model is found by maximizing the following welfare equation with respect to $T$:

$$W(T) = \mathcal{E}_0 \int_0^T s\theta e^{-\rho t} dt + \Omega_T \tag{5}$$

An interesting exercise is to determine what the result of this calculation is using present discounted value comparisons (tradional cost-benefit analysis). Without uncertainty, the minimum value, $\theta^*$, at which one would still want to cut down the forest is where benefits equal costs, or:[11]

$$\frac{s\theta^*}{\rho - \alpha} = \Omega_0 \tag{6}$$

Using simple algebra, one can solve for the threshold level, $\theta^*$, the minimum value of species at which it is still optimal to preserve the forest under complete certainty ($\sigma$=0):

$$\theta^* = \frac{\Omega_0(\rho - \alpha)}{s} \tag{7}$$

[11] This is found by taking Equation 5 and substituting $\hat{\theta} = \theta_0 e^{\alpha t}$ in for $\theta$. The cost-benefit analysis solution is found by comparing $W(T = \infty)$ (never adopting) and $W(T = 0)$ (adopting now). The threshold $\theta^*$ is where $W(T = \infty) = W(T = 0)$.

As an aside, note that Equation 7 would also be the optimal solution in an option value case with irreversibility yet no uncertainty. Unlike Pindyck (1994) where the existence of irreversibilities can increase the option value to delay, here the effect of irreversibility alone is unnoticable. In Pindyck's article, the advantage to delaying under certainty yet irreversibility stemmed from two effects: the opportunity to discount the sunk costs of global warming policy adoption, and the small relative cost of emissions in early stages of policy analysis. In this scenario, however, the discounting effect creates an advantage to acting now rather than delaying the sunk benefit of deforestation. Only under uncertainty is there a positive option value to waiting.

When uncertainty is added into the model, the traditional net present value comparisons begin to fail. As was elaborated in the two-period example, the opportunity to delay a completely irreversible action carries the advantage of having more knowledge about the issue in the future before the irreversible action is taken. In order to maximize the welfare equation (Equation 5) with uncertainty, dynamic programming techniques will be used to find a threshold, $\theta^*$, below which deforestation is optimal. Let the value of the option of holding the forest be $Y(\theta)$. The Bellman equation is:[12]

$$Y(\theta) = \max\{\Omega(\theta),\ s\theta + (1-\rho dt)\mathcal{E}_0[Y(\theta) + dY(\theta)]\} \tag{8}$$

The solution to this equation must be broken down into two parts, the region where deforestation occurs, and the region where species are preserved. The case where deforestation occurs is simple because the payoff to deforestation, $\Omega_0$, is independent of the value of species, $\theta$. The case where the forest is preserved is as follows:

$$Y(\theta) = (1-\rho dt)\mathcal{E}_0[s\theta + Y(\theta) + dY(\theta)] \tag{9}$$

[12]For a more in-depth look at applying dynamic programming to option values in stopping models, see Dixit and Pindyck(1994), Chapters 3-6.

This equation can be simplified to:

$$\rho Y dt = \mathcal{E}_0[dY] + s\theta dt \tag{10}$$

Applying Ito's lemma and geometric Brownian motion for $\theta$,[13] one obtains the following second order homogeneous differential equation for $Y(\theta)$:

$$\frac{1}{2}\sigma^2\theta^2 Y''(\theta) + \alpha\theta Y'(\theta) - \rho Y(\theta) + s\theta = 0 \tag{11}$$

The solution to this equation for $Y(\theta)$ is:

$$Y(\theta) = \beta\theta^\gamma + \frac{s\theta}{\rho - \alpha} \tag{12}$$

$$\gamma = \frac{(\sigma^2/2) - \alpha - \sqrt{\alpha^2 - \sigma^2\alpha + \sigma^4/4 + 2\sigma^2\rho}}{\sigma^2} \tag{13}$$

In order to solve for $\beta$ and $\theta^*$, however, the following boundary conditions are necessary:[14]

$$\lim_{\theta \to \infty} Y(\theta) = 0 \tag{14}$$

$$Y(\theta^*) = \Omega_0 \tag{15}$$

$$Y'(\theta^*) = 0 \tag{16}$$

The first boundary condition should be intuitive: as the value of species becomes larger and larger, one would hold the species no matter what, and the option value is low. The second two boundary conditions are the value matching and smooth pasting conditions, in

[13]Defined in Equation 4.

[14]The negative root of $\gamma$ in Equation 13 was selected because of the limit in Equation 14.

which the two scenarios (the option functions for deforesting and preserving) should meet at the minimum $\theta^*$ under which one still preserves the forest, and with the same tradeoff value (a.k.a. derivative).[15]

The above boundary conditions lead to the following solution:

$$\beta = \frac{\Omega_0}{1-\gamma}\left(\frac{s(\gamma-1)}{(\rho-\alpha)\gamma\Omega_0}\right)^{\gamma} \tag{17}$$

$$\theta^* = \frac{\gamma(\rho-\alpha)\Omega_0}{(\gamma-1)s} \tag{18}$$

On first glance, one can already diagnose that there is a dramatic difference in the solution of this model (Equation 18) when compared to the net present value solution in Equation 7. Using the above information about Brazil's Amazonia region, with $\sigma = 0.1$, $\alpha = 0.005$, and $\rho = 0.04$, the impact of this optimizing method can be better understood. Substituting those parameters into Equation 7, the present discounted value (cost-benefit analysis) threshold is $\theta^* = \$4263$. Applying the option value to preserve the forest, the threshold value $\theta^* = \$3150$. As in the two-period example, the value of preserving the option to hold species yields a rather high margin of caution. The nature of the function Y (Equation 12) is illustrated in Figure 1.

It is important to note that uncertainty is vital to the interpretation of the model. With an extremely high value of $\sigma$, deforestation is inadvisable. Likewise, a very low $\sigma$ would justify cost-benefit analysis, because it would be analogous to the case under certainty discussed previously. The continuous model completes the spectrum with the threshold values of when to deforest as a function of sigma. The result is plotted in Figure 2. As uncertainty increases, the threshold at which to deforest asymptotically approaches $\theta^* = 0$. Under complete certainty, the threshold value is \$4,263. The forest should definitely be preserved if species are worth more than \$4,263. Beneath this threshold, however, one can see that

[15]For more information on smooth pasting and value matching conditions see Dixit and Pindyck (1994), chapter 4.

the level of uncertainty directly affects the optimal outcome. The forest should be destroyed when the option value resulting from uncertainty is not enough to justify saving species which initially do not appear worthwhile. Hence, our beliefs about uncertainty directly govern the decision making process when the value of species is below the cost-benefit analysis threshold.

An interesting application of the model is to calculate a SMS, or safe minimum standard, such that policy makers can develop a threshold to use in uncertain policy decisions. A reasonable value for $\alpha$ and a conservative value of $\sigma$ can be approximated. One could estimate $\alpha$ by asking how much people expect the value of species to increase in the next fifty years. The previous examples predicted that the value of species will increase by 25% over the next fifty years by setting $\alpha$=0.005. It is much more difficult to assert a level of uncertainty. While a precise level of $\sigma$ and therefore $\theta^*$ probably cannot be calculated, policy makers could consider using an extraordinarily cautious value. A reasonable estimate is $\sigma$=0.05. This would be the same as claiming society is 99% certain that the value of species will not increase or decrease by more than 525% of the expectation over the next 50 years. Using this value of $\sigma$, one can approximate a safe minimum standard, or a reasonable level, $\theta^*$, below which deforestation is clearly advisable. Between the cost benefit analysis solution in Equation 7, and the value of $\theta^*$ corresponding with $\sigma$=0.05 (a form of safe minimum standard), there is a "critical range" at which we need further observation of the value of species and greater attention to the level and nature of uncertainty involved. This critical range can be estimated from Figure 2 to be between \$3,000 and \$4,263. At values below \$3,000 in this example, it would be optimal to elect to deforest.

## 4.2 Species With Dividend As a Poisson Process

One of the economic aspects to valuing species is that they are likely to create innovations, such as a new medication. These innovations may not occur as continuous improvements, but rather "jumps" in the average value of species. Hence, there is an alternative stochastic path which the value of species might follow—either a Poisson process, or geometric Brownian motion with a jumping factor. To better understand the nature of the different paths to be discussed, sample paths of the different types of motion can be found in Figures 3, 4, and 5. The difference between a Poisson process (percent jumps at random intervals of time) and geometric Brownian motion stem from two basic intuitions: whether the value of species can decrease, and the distribution of possible changes in the value of species. In a Poisson process, the value of species is fixed over time unless an "event" occurs. An event occurs each time period with probability $\lambda$. In this example the event will be to increase the value $\theta$ by a fraction $\phi\theta$. These jumps or "events" would represent some newly discovered appreciation of species. For example, if some plant were found which cured AIDS, there would be a jump in the level at which society values species. Different than innovation, geometric Brownian motion would model changing popular tastes toward species, slowly improving understandings about species, or new discoveries (good and bad) about species. Geometric Brownian motion can have a positive or negative change in each period, while a Poisson process only increases. Although the Poisson process probably provides a good explanation of how species could gain economic value over time, geometric Brownian motion offers a better explanation of the multifaceted valuation of species, including the existence, ecological, and economic value.

More theoretically, this section looks at the impact which the nature of the stochastic uncertainty has upon the decision-making process. Comparing two stochastic processes offers insight into how the fluctuations in $\theta$ affect the option value analysis.

Before contrasting geometric Brownian motion with the Poisson process, the combined model (geometric Brownian motion with a jumping factor) will be solved algebraically. Geometric Brownian motion with a jumping factor is defined as follows:

$$d\theta = \alpha\theta dt + \sigma\theta dz + \phi\theta dq \tag{19}$$

Note that $\alpha$, $\sigma$, and $\phi$ are all constant parameters. In addition, $dq = 1$ with probability $\lambda dt$ and $dq = 0$ with probability $1 - \lambda dt$. These parameters describe a Poisson process with arrival rate $\lambda$ and jump size equal to $\phi\theta$.

Applying dynamic programming yields:

$$\rho Y = s\theta + \frac{1}{2}\sigma^2\theta^2 Y''(\theta) + a\theta Y'(\theta) + \lambda Y((\phi+1)\theta) - \lambda Y(\theta) \tag{20}$$

The solutions for $Y(\theta)$ and $\theta^*$ are:

$$Y(\theta) = \beta\theta^\gamma + \frac{s\theta}{\rho - \lambda\phi - \alpha} \tag{21}$$

$$\frac{1}{2}\sigma^2\gamma^2 + \left(\alpha - \frac{\sigma^2}{2}\right)\gamma + \lambda(\phi+1)^\gamma - \lambda - \rho = 0 \tag{22}$$

$$\theta^* = \frac{\gamma}{\gamma - 1}\Omega_0\left(\frac{\rho - a - \lambda\phi}{s}\right) \tag{23}$$

Unfortunately, solving Equation 22 requires computational complexity beyond the scope of this paper.[16] However, one can solve for $\gamma$ fairly easily if one assumes that $\theta$ follows a Poisson process, or $\alpha$, $\sigma = 0$. An interesting comparison to make is, with the same expected value,[17] how the parameter $\lambda$ affects the decision to deforest. In Figure 6,[18] one can see that as $\lambda$ increases, the advantage to delay decreases, and $\theta^*$ increases. In other words, species must be worth more with a high $\lambda$ because the chances of having a big fluctuation in the future value of species are less. This is the previously obtained intuitive result: as $\lambda$ approaches 1, the actual value of $\theta$ is less uncertain ("better understood"), and cost-benefit analysis is more applicable.

In addition, one may be curious how this would compare to the initial continuous model where $\theta$ followed geometric Brownian motion. Comparisons are drawn for $\alpha = 0.005$ and equal variances in Figures 7 and 8. The graphs relied upon the following two equations to

---

[16]One problem is that the iterative solutions for $\gamma$ are no longer continuous over sigma. In addition, the iterative solutions, due to the nature of the equation, are very difficult to solve for—there are even cases when $\gamma$ has multiple negative roots.

[17]Having the same expected value implies $\phi_1\lambda_1 = \phi_2\lambda_2$. Hence, lower probabilities of jumping correspond with a bigger change when the jump occurs.

[18]Lambda was bound at 0.3 in order for gamma to be negative and satisfy the boundary constraints.

equalize the expected value and variance of $\theta$ between cases:

$$\alpha = \lambda\phi \tag{24}$$

$$\sigma = \sqrt{\lambda(1-\lambda)\phi^2} \tag{25}$$

Equation 24 is derived by setting the expected value of the Poisson jump process, $e^{\lambda\phi t}$, equal to the expected value of the geometric Brownian motion process, $e^{\alpha t}$. Equation 25 is derived by setting the variance of $d\theta$ equal in each period. The variance of the geometric Brownian motion process, $\sigma^2\theta^2$, is set equal to the variance of the Poisson jump process, $\lambda(1-\lambda)\phi^2\theta^2$.[19]

In looking at Figure 7, one realizes that for typical values of $\alpha$ and $\sigma$ as discussed earlier, the relative difference is actually very small. This is interesting in that it implies using an approximate stochastic process would not significantly change the safe minimum standard ($\sigma$=.05) as discussed above. However, the result corresponding with higher, very extreme, values of $\sigma$, as in Figure 8, diverges significantly. This is likely the result of such different-natured distributions. When uncertainty is high in a normal distribution, the likelihood of a good outcome is still 50%. In the Poisson jump case, keeping the same expected value meant that each more positive outcome corresponded with a less likely occurrence. Although they have identical variances (one measure of their uncertainty), the two processes are clearly of a very different nature. At the same time, in the range of parameters which policy makers should concern themselves with ($\sigma \leq 0.05$) the final results are quite similar.

## 4.3 Species Which Can Be "Harvested" Upon Deforestation

A difficulty in modelling the value of species is exactly how society receives benefits from biodiversity. The next two sections involve variations as to how society may benefit from

[19]This variance was derived by using Var(X)=$\mathcal{E}[X^2] - (\mathcal{E}[X])^2$.

preserving species: by allowing them to be "harvested," and by taking a different look at ecological uncertainty.

"Harvesting" species refers to what happens when deforestation occurs. As the forests are removed, one possibility is to harvest the species which have been discovered. In other words, the value of these species can be partially transferred out of the forest once they are identified. To incorporate this into the model, I will model species as an investment with a stock value as well as a dividend return. Each year, the return on the investment will be $s\theta$. There will also be a value, $ks\theta$, which represents the "harvest" value of species at a given time. In other words, in addition to the dividend which society continuously receives from species existence, society also can harvest some of the value of species when deforestation occurs. The parameter $k$ reflects the relative value of the dividend and the "harvest." A higher $k$ would imply a higher ability to extract wealth from species during a "harvest."

Examples of "harvests" would include relocating species to zoos, industry, etc. or capturing species and using their economic value. Under this new method of benefitting from species, the welfare equation is:

$$W = \mathcal{E}_0[ks\theta e^{-\rho T} + \int_0^T s\theta e^{-\rho t}dt] + \Omega_T \tag{26}$$

This is similar to the initial model except the reward for deforestation also fluctuates with an increase in the value of $\theta$. Because the reward to preservation has not changed, the Bellman equation and differential equation for the option value are unchanged:

$$Y(\theta) = \max\{\Omega(\theta),\ s\theta + (1 - \rho dt)\mathcal{E}_0[Y(\theta) + dY(\theta)]\} \tag{27}$$

$$\frac{1}{2}\sigma^2\theta^2 Y''(\theta) + \alpha\theta Y'(\theta) - \rho Y(\theta) + s\theta = 0 \tag{28}$$

(These equations are identical to Equations 12 and 13.) The opportunity to harvest the value of species changes the boundary conditions because the payoff from deforesting is now larger. The new boundary conditions are the limit in Equation (14) and the following value matching and smooth pasting conditions:

$$Y(\theta^*) = \Omega_0 - ks\theta \tag{29}$$

$$Y'(\theta^*) = -ks \tag{30}$$

The solution to this model only slightly differs from the previous one. Labelling the $\theta^*$ from the original basic model (Equation 18) as $\theta^*_B$, the $\theta^*$ when species can be "harvested" is:

$$\theta^* = \frac{\gamma(\rho - alpha)\Omega_0}{(\gamma - 1)s(1 - k\rho - k\alpha)} \tag{31}$$

$$\theta^* = \frac{\theta^*_B}{1 - k\rho - k\alpha} \tag{32}$$

The impact of $k$, or the value of the return when species are sold, is shown in Figure 6. Before solving the model, there are two effects which $k$ could possibly have had in this scenario. On the one hand, one might expect that because the value of species will increase for a greater harvest amount in the future, a higher level in $k$ would make it more attractive to hold species. On the other hand, a higher $k$ increases the value of deforesting in a given time period. The differential of the equation for $\theta^*$ (Equation 32) with respect to $k$ combined with Figure 9 illustrates that the latter effect is prevalent: by calculating a return from species which can be harvested or captured without the species in the natural habitat, deforestation becomes even more attractive.

This diversion illustrates two main ideas about modelling the valuation of species. First, it illustrates that a small change in modelling assumptions can have relatively large effects on the results, so it is important to decide exactly how society derives benefits from species. Second, this diversion illustrates impact which habitat-dependence should have upon deforestation policy. In deciding whether to deforest, a policy maker must also ask, can a species be harvested? In the case of the spotted owl in the Northwest United States, the answer is believed to be no. This attachment of species to their environment makes the pleas of environmentalists more convincing. On the other hand, many innovations which occur as a result of species can be "harvested" from that day forward. Molds and plants could easily be grown under similar environmental conditions. This separability undermines the case to preserve the forest.

The example of molds which can be harvested after society discovers usefulness illustrates a fundamental difficulty in determining how to model the benefit which society receives from species preservation. Speaking purely economically, species preservation could resemble a form of R & D. Without species, innovations would never be discovered. The destruction of species does not reverse past discoveries, however. The economic value of some species, therefore, could be modelled as a peak in which before discovery, the creature is worthless to society, at discovery, the creature offers society a high return, and after discovery, the creature (at least in its natural habitat) returns to being worthless to society. Modelling a "peaked" stochastic path of some species, however, would be very difficult because the mathematical solutions would not be analytic. In addition, it would ignore the ecological and existence values which species also provide to society.

## 4.4 Species as Essential Elements of the Ecosystem

Another justification for holding species is that they may be a vital component of the ecosystem. Destroying them risks ecological consequences yet unknown. Unfortunately, the above models do not accommodate for this possibility very well. Ecological sustainability could, in a way, be thought of as part of the benefit obtained by preserving species, and hence part of the dividend. In reality, however, what deforestation risks is actually an uncertain negative payoff. Hence, one could think of $\Omega_0$ as containing a random element which on small occasions is very negative as a result of possible ecological risks. Under complete risk neutrality, the impact of this alteration would be to simply lower the expected value of $\Omega_0$. The impact under risk aversion is difficult to solve convincingly because the coefficient of risk aversion is subjective. However, it is readily clear that ecological uncertainty would lower the threshold value at which to preserve species (and discourage deforestation) even more.

# 5 Incremental Deforestation

The previous sections have asked the question, how does one decide whether to deforest 5% of the Amazonia region? This section illustrates how one would decide which level to deforest at. It will ask what the $\theta^*$ is at any given level of forest cover.

The relationship between species lost and area deforested, as was assumed in Equation 1, indicates that there are increasing marginal damages to deforestation—each additional kilometer of forest destroyed causes the extinction of more and more species. Hence, when it is optimal to destroy 5% of the forest, it is not necessarily optimal to destroy 10% of the forest.

The question is whether each individual square kilometer is optimal to deforest. The economic return from deforesting one square kilometer of forest was estimated in the modelling assumptions to be $\Omega_{dA} = \$64,286$. The number of the species destroyed is simply the derivative of the species function (Equation 1), or:

$$dS = czA^{z-1}dA \tag{33}$$

The threshold value, $\theta^*$, is then calculated for each increment of deforestation, applying the solution to Equation 18 in a more general form:

$$\theta^* = \frac{\gamma(\rho - \alpha)\Omega_{dA}}{(\gamma - 1)dS} \tag{34}$$

The solution to the case of incremental change for $\sigma = 0.01$ and $\sigma = 0.1$ is plotted in Figure 10. It illustrates that different sigmas correspond with a "caution coefficient." In other words, at any level of deforestation and a given uncertainty level, $\sigma$, the optimal threshold value at which to deforest is a fixed percentage of the cost-benefit analysis solution. The fraction of the cost-benefit analysis threshold at which the planner ought to deforest is independent of the quantity of species being destroyed.

# 6 Summary and Conclusions

Applying an option-theoretic approach to the question of when deforestation is optimal offers some significant conclusions about species preservation and policy analysis. They are:

- **Rephrasing the policy question to avoid uncertain estimates can be useful.** In this case, because the value of species is so uncertain, it would have been nearly impossible to make a reasonable estimate of what society receives as a dividend from species. It is advantageous to ask the question "What is the minimum threshold value of species which would justify their preservation?" Using this method, one can decide whether the value of species is worth debating, or if it is confidently on one side of the threshold or the other.

- **Uncertainty should encourage caution, but not necessarily inaction.** Cost-benefit analysis fails in situations with irreversibility and uncertainty because it ignores the benefit to be able to decide later. In this case, the possibility of preserving species is eliminated upon deforestation. At the same time, the inaction suggested by some advocates of the safe minimum standard is equally inadequate. Uncertainty should encourage policy delay, but only in those cases when the advantages to policy delay outweigh the losses from further discounting the value of acting now. The value of the advantage to delay is calculated implicitly as the option value of this paper.

- **Reasonable safe minimum standards can be calculated.** The option value methodology enables policy makers to calculate a reasonable safe minimum standard. By making estimations of maximum possible uncertainty such as $\sigma$=0.05, policy makers can be reasonably certain that a non-optimal, irreversible action would be avoided. This method allows policy makers to determine a "critical range" in between the normal cost-benefit analysis solution and the $\sigma$=0.05 safe minimum standard. When species are valued above that range, preservation is always warranted. Below that range, deforestation is warranted, because species are not worth enough to outweigh the benefits of deforestions and that situation is unlikely to change. Within that range, further debate over the level of uncertainty as well as a thorough study of the value of species is justified.

- **$\theta$ can follow an approximated process.** The stochastic process affects the outcome, however a similar process will probably approximate the optimal solution adequately. In the range of important values of $\sigma$ (0-0.05), there was not a very significant difference between the Poisson process solution and a geometric Brownian motion process solution. While any process should be modelled as best as possible, it is not unreasonable to use similar processes to estimate a safe-minimum standard.

- **There is a large impact to how the benefits from species are modelled.** The separability of species from the forest vitally changes the value of the forest. The more inseparable an ecosystem is, the more beneficial habitat preservation is. As was illustrated by the "harvest" example, the ability of humans to benefit from species after their habitats are destroyed undermines one of their key motivations for habitat preservation: preserving known species. An inseparable ecosystem is much more likely to be preserved simply because one or two nostalgic creatures rely upon it, such as the Panda in China, or the Bengal tiger in parts of India, etc.

While there are many worthwhile insights in applying option theoretic analysis to deforestation, there is also room for improvement. One difficulty in using this model is that it ignores risk aversion. The option value method selects the policy which maximizes the return to society. It ignores the level of risk associated with that policy. Some advocates of the SMS argue that species may be too valuable to risk losing–that irrespective of the expected net return to environmental policy, the costs of environmental destruction are too great to risk at all. It may be the case that a policy which gambles less is favored by society despite its lower expected value. This is especially the case in ecological uncertainty.

Perhaps society values knowing their ecosystem is fully intact, whether or not the combined expected damage to the ecosystem is less than the return from forests.

Another fallback to this model is the role of information in the stochastic variables. While a Markov process makes the math easier and allows the application of Ito's Lemma, it also creates somewhat of a fallacy. At any given time t, the expected change in the estimated value of species is a constant, and the variance of the value of species is increasing. This is philosophically different than the two-period example in this paper. In the two-period example, one of the reasons for delay is in order to obtain more information about the species; however, in the continuous model, the uncertainty (the level at which the future value of $\theta$ can be predicted) is actually increasing as a function of time. Hence, there is no definite increase in information by waiting in the continuous model.

This paper poses some interesting additional avenues for research. One avenue would be to look more concisely at the impact of potential ecological costs as discussed earlier. Another would be to create uncertainty over the number of species which will die. Some environmentalists believe that there are a significant number of species which have fallen below a minimum population level and are headed toward extinction. This type of modelling would require a delay parameter and uncertainty over the amount of damage analogous to Pindyck (1994) where the effects of greenhouse gas release are uncertain and delayed over time. Finally, attempting to integrate risk aversion or some direct increases in the level of information in the model by using a different stochastic process could greatly improve the model.

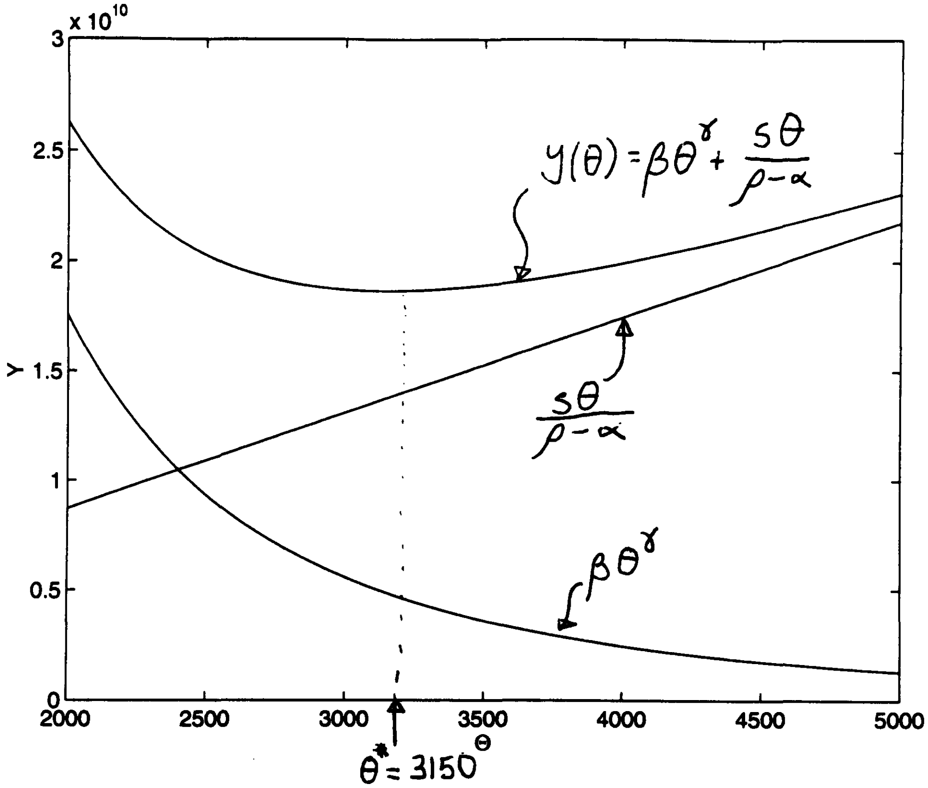


Figure 1: The Graphical Solution to the Basic Model

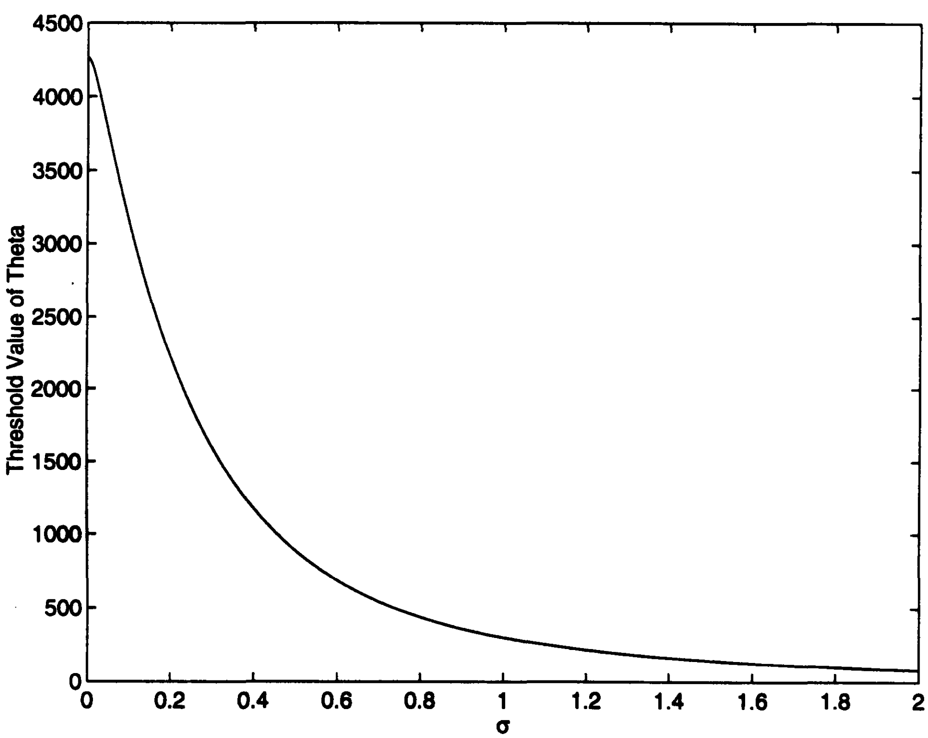


Figure 2: The Impact of Uncertainty Upon the Option to Deforest

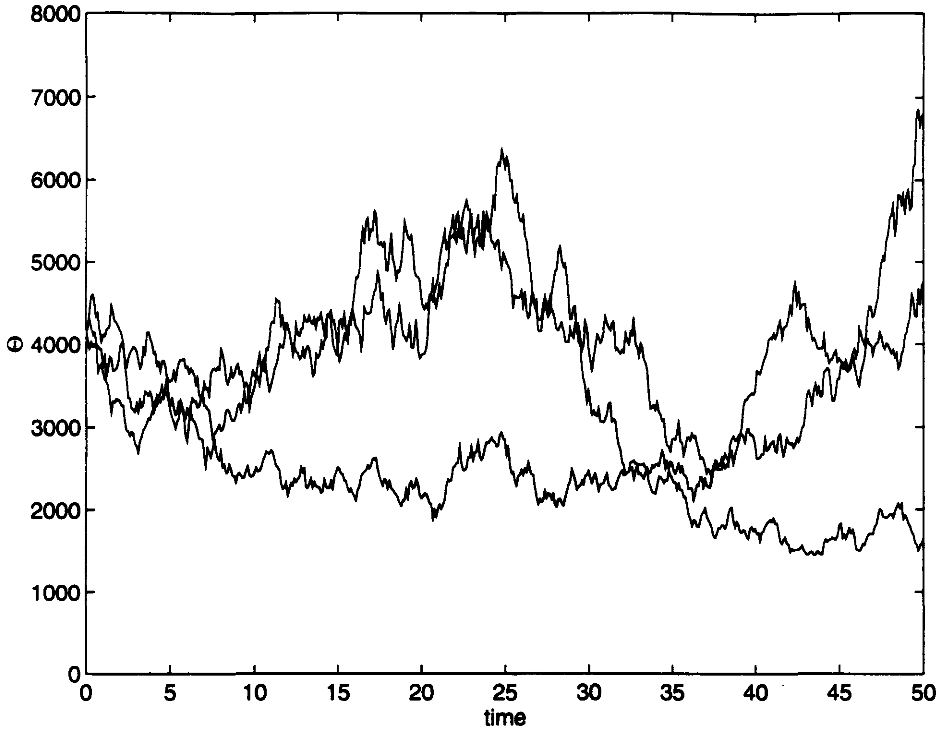


Figure 3: Sample Brownian Motion Paths

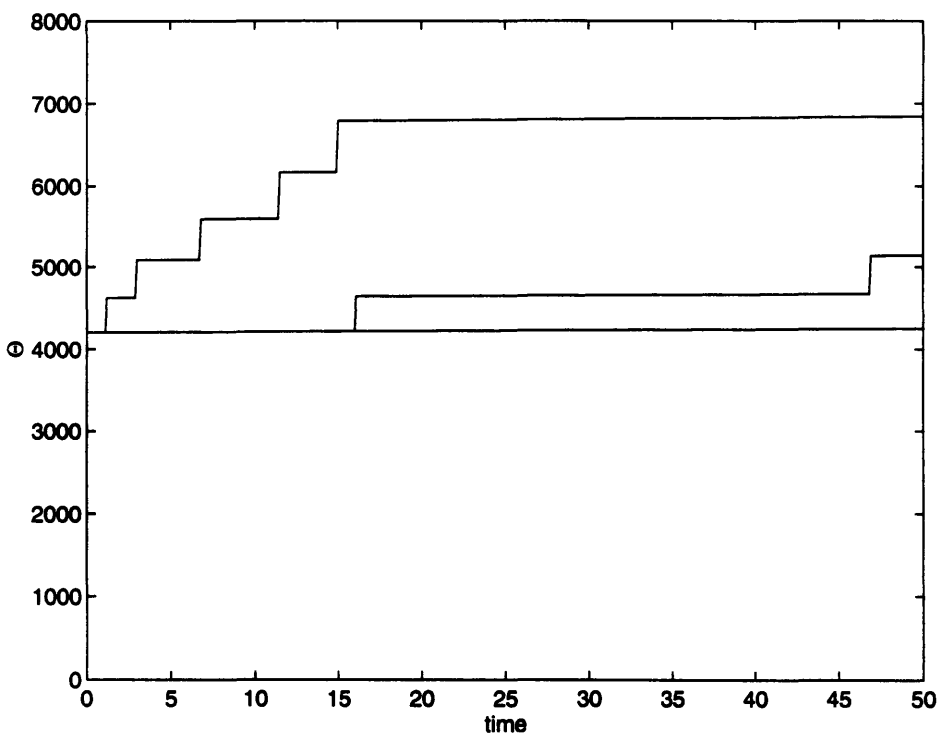


Figure 4: Sample Poisson Processes

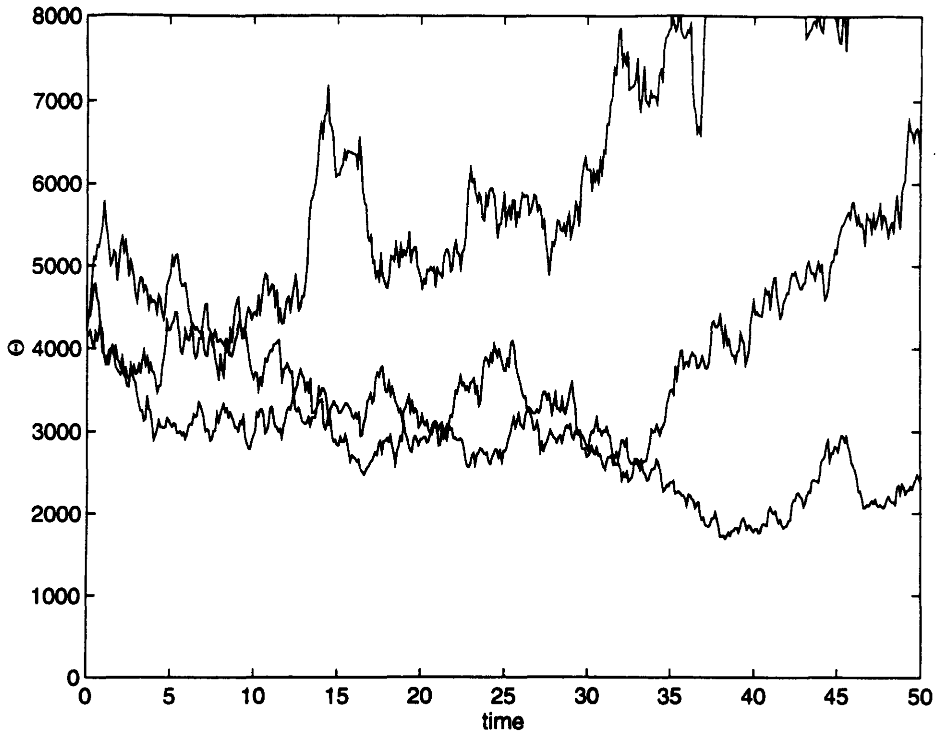


Figure 5: Sample Brownian Motion Paths With Random Jumps

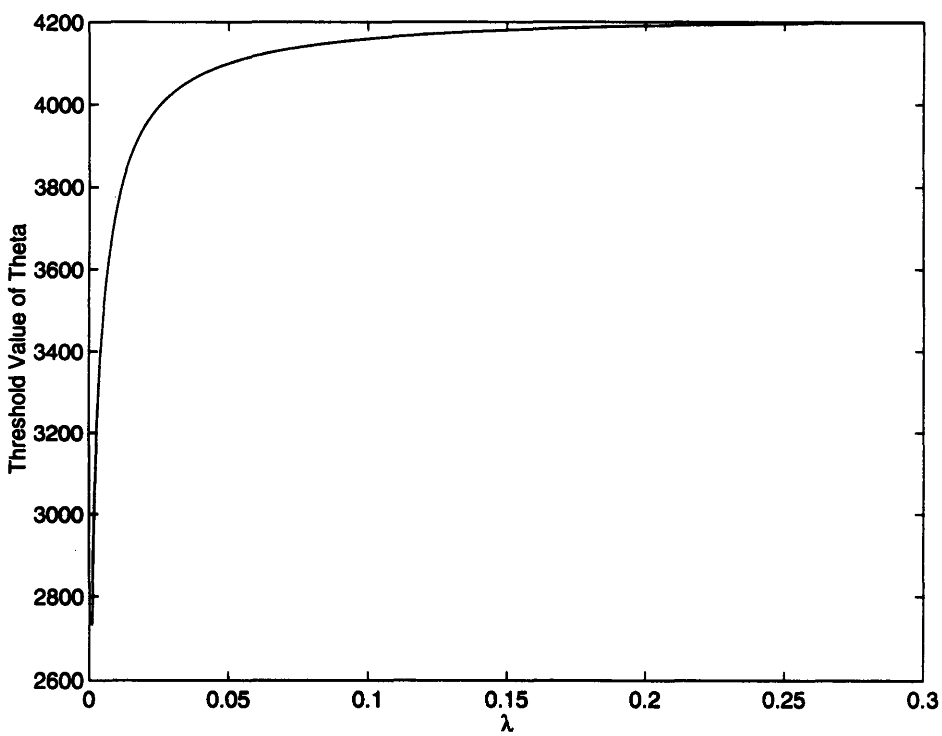


Figure 6: The Impact of Lambda Upon the Threshold Value

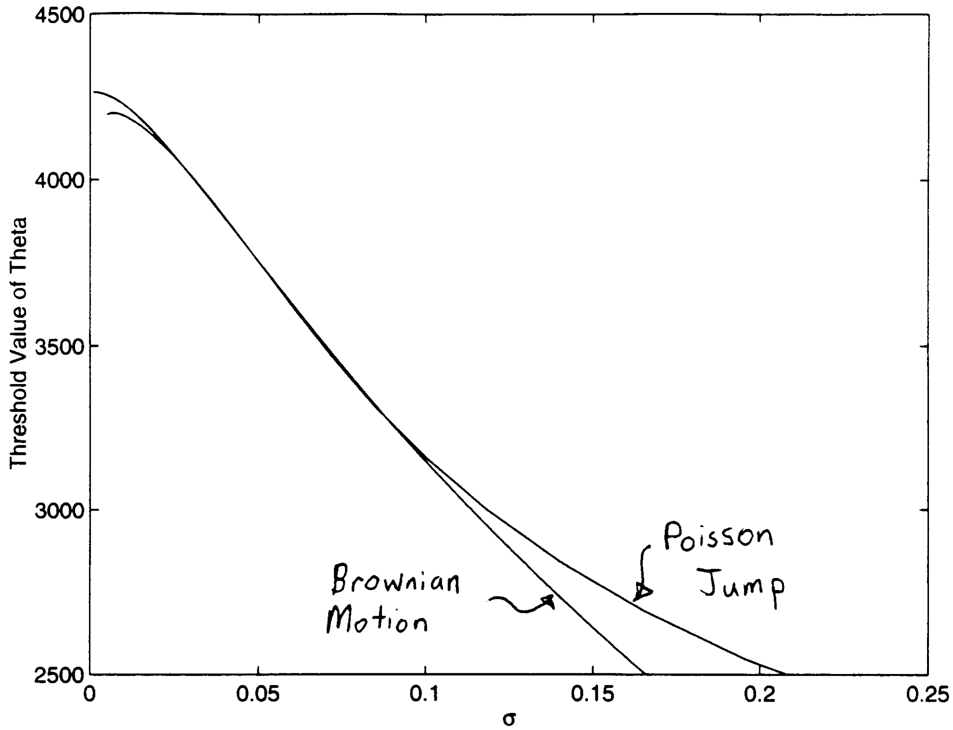


Figure 7: Comparing the Poisson Jumps with Brownian Motion

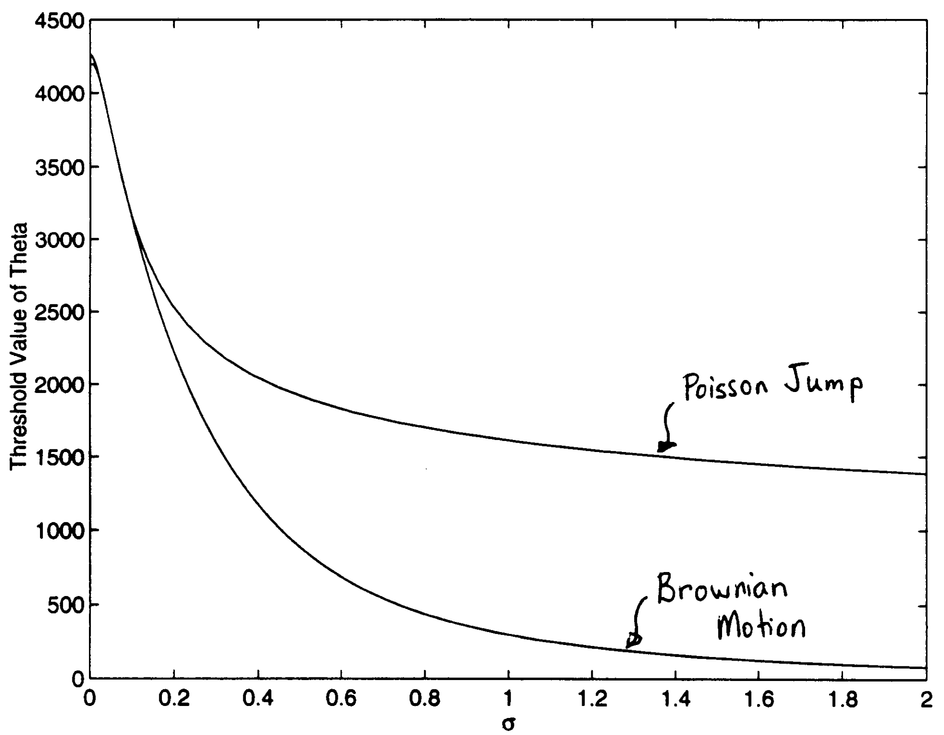


Figure 8: Comparing Poisson Jumps with Brownian Motion

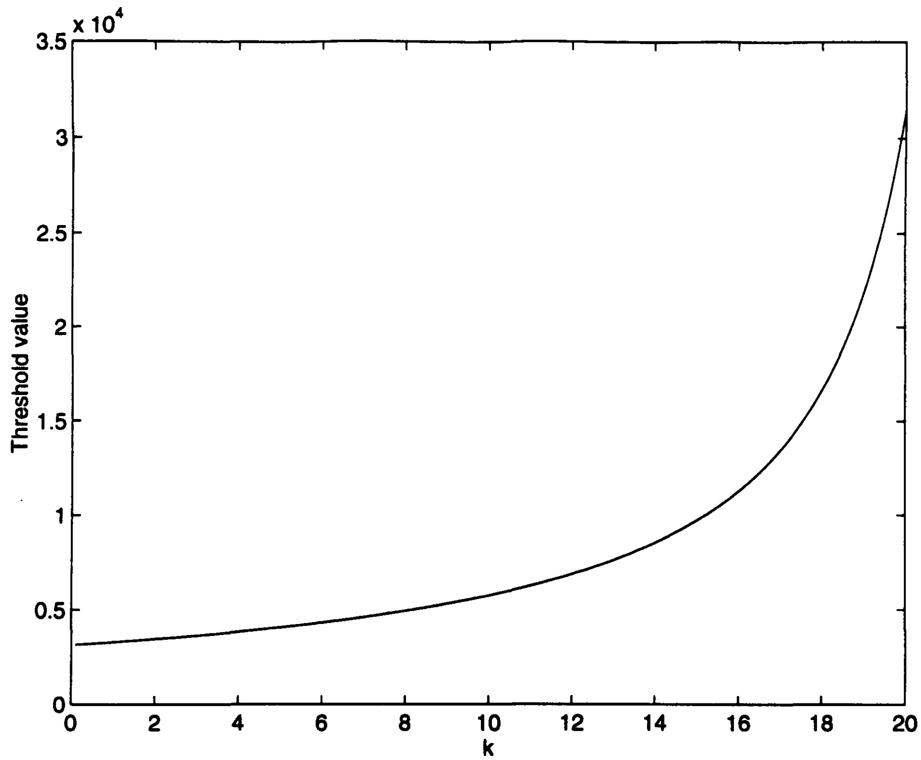


Figure 9: The Impact of Harvesting Potential Upon Option Analysis

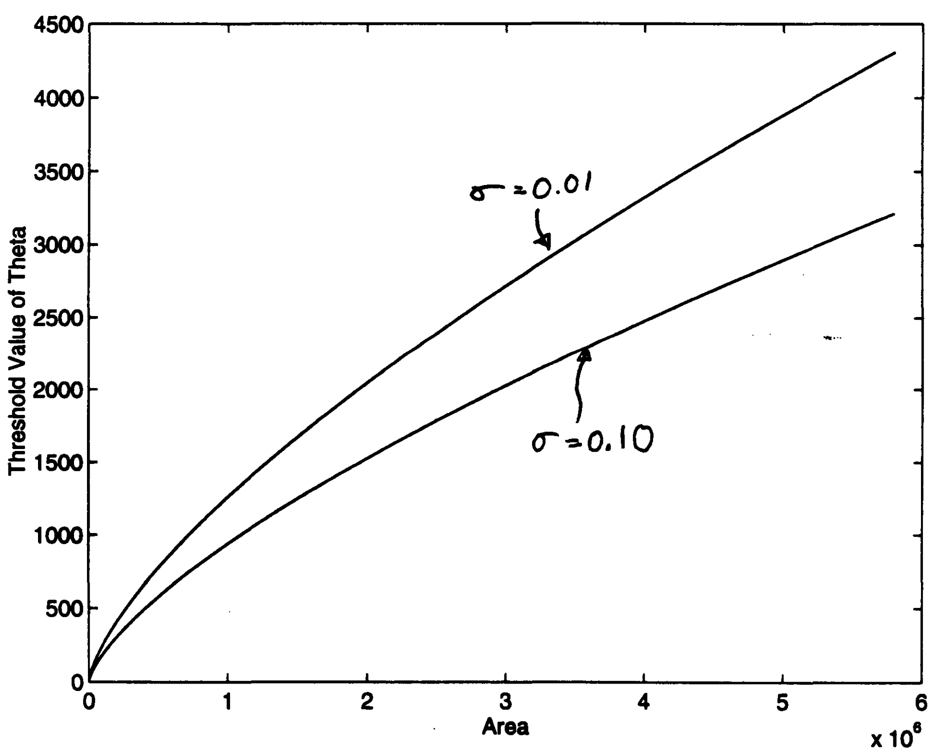


Figure 10: The Incremental Policy Solution

# References

[1] Barbier, Edward, Anil Markandya, and David Pierce. *Sustainable Development: Economics and Environment in the Third World.* London Environmental Economics Centre, 1990.

[2] Browder, John O. "Public Policy and Deforestation in the Brazilian Amazon," in Malcolm Gillis and Robert Repetto eds., *Public Policies and the Misuse of Forest Resources.* Cambridge University Press, 1988.

[3] Dixit, Avinash and Robert Pindyck. *Investment Under Uncertainty.* Princeton University Press, Princeton, 1994.

[4] Ehrlich, Paul R. "Biodiversity Studies: Science and Policy." *Science.* August 16, 1991. 758-761.

[5] Lovejoy, Thomas E. and Ghillean T. Prance. eds. *Amazonia.* New York: Pergamon Press, 1985.

[6] Mahar, Dennis J. *Government Policies and Deforestation in Brazil's Amazon Region.* The World Bank, Washington D.C., 1989.

[7] McNeill, John R. "Deforestation in the Araucaria Zone of Southern Brazil, 1900-1983." in John F. Richards and Richard P. Tucker eds., *World Deforestation in the Twentieth Century.* Duke University Press, London, 1988.

[8] Pindyck, Robert S. "Sunk Costs and Sunk Benefits in Environmental Policy: I. Basic Theory." Massachusetts Institute of Technology, Working Draft, April 3, 1994.

[9] Randall, Alan. "The Value of Biodiversity." *Ambio*, 20(2), 64-68, 1991.

[10] Ryan, John C. *Life Support: Conserving Biological Diversity*, Worldwatch Institute, Washington D.C., 1992.

[11] Schulze, Peter C. "Cost-benefit analyses and environmental policy." *Ecological Economics*, 9, 197-199, 1994.

[12] Wilson, Edward O. *The Diversity of Life.* W.W. Norton and Company, New York, 1992.