Introduction to Modeling and Simulation, Particle Methods

Monte Carlo Methods II: Fractal Patterns

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Figure removed for copyright reasons.
See Fig. 1 in Bazant, Martin Z., Jaehyuk Choi, and Benny Davidovitch. "Dynamics of Conformal Maps for a Class of Non-Laplacian Growth Phenomena." *Phys Rev Lett* 91, no. 045503 (2003).

Advection-Diffusion-Limited Aggregation

The Sierpinski Carpet: A Non-random Fractal

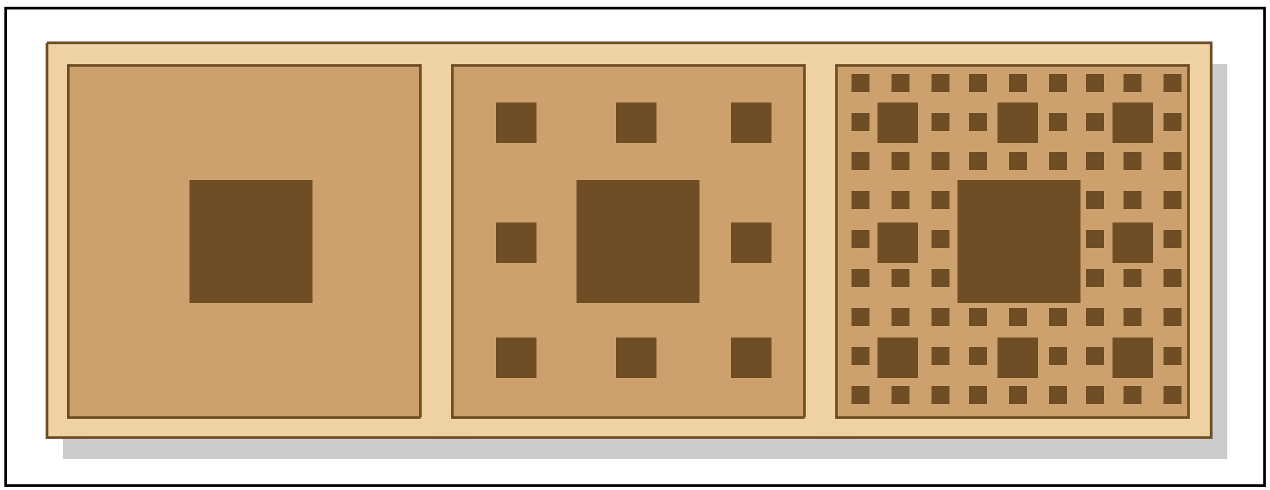

Figure by MIT OCW.

Fractal dimension: $D = \log 8 / \log 3 = 1.8928$

Perfectly self-similar

Critical Percolation Clusters: Random Fractals

Figure removed for copyright reasons.
See: Bunde, A., and S. Havlin, eds.
Fractals and Disordered Systems. New York, NY:
Springer, 1996.

Fractal dimension:

$D = 91/48 = 1.896$

Statistically self-similar

(Bunde & Havlin)

Fractal Geometry: Linear vs. Chemical Distance

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See: in Bunde, A., and S. Havlin, eds. *Fractals and Disordered Systems*. New York, NY: Springer, 1996.

(Bunde & Havlin)

Fractal Geometry: Backbone and "Red Bonds"

Figure removed for copyright reasons.
See: in Bunde, A., and S. Havlin, eds. *Fractals and Disordered Systems*. New York, NY: Springer, 1996.

(Bunde & Havlin)

Diffusion-Limited Aggregation

T. Witten and L. M. Sander (1981)

Fractal growth by “sticky” random walkers

Try Bob Sumner’s
Mod/Sim demo:
dla

Figure removed due to copyright reasons.

Off-lattice DLA cluster of
1,000,000 random walkers,
colored by time of arrival
(L. M. Sander)

$D = 1.71$

Java applets for DLA and many other Monte Carlo simulations: **http://ccl.northwestern.edu/netlogo**

Some DLA-like patterns in nature

- Electrodeposits

  (CuSO4 deposit, J. R. Melrose)

- Thin-film surface deposits

  (GeSe2/C/Cu film, T. Vicsek)

- Snowflakes (Nittman, Stanley)

Figures removed due to copyright reasons.

DLA simulation

Experiment

Variants of the DLA Model

To obtain patterns resembling snowflakes, viscous fingers, electrodeposits, etc. you can play with the model in various ways:

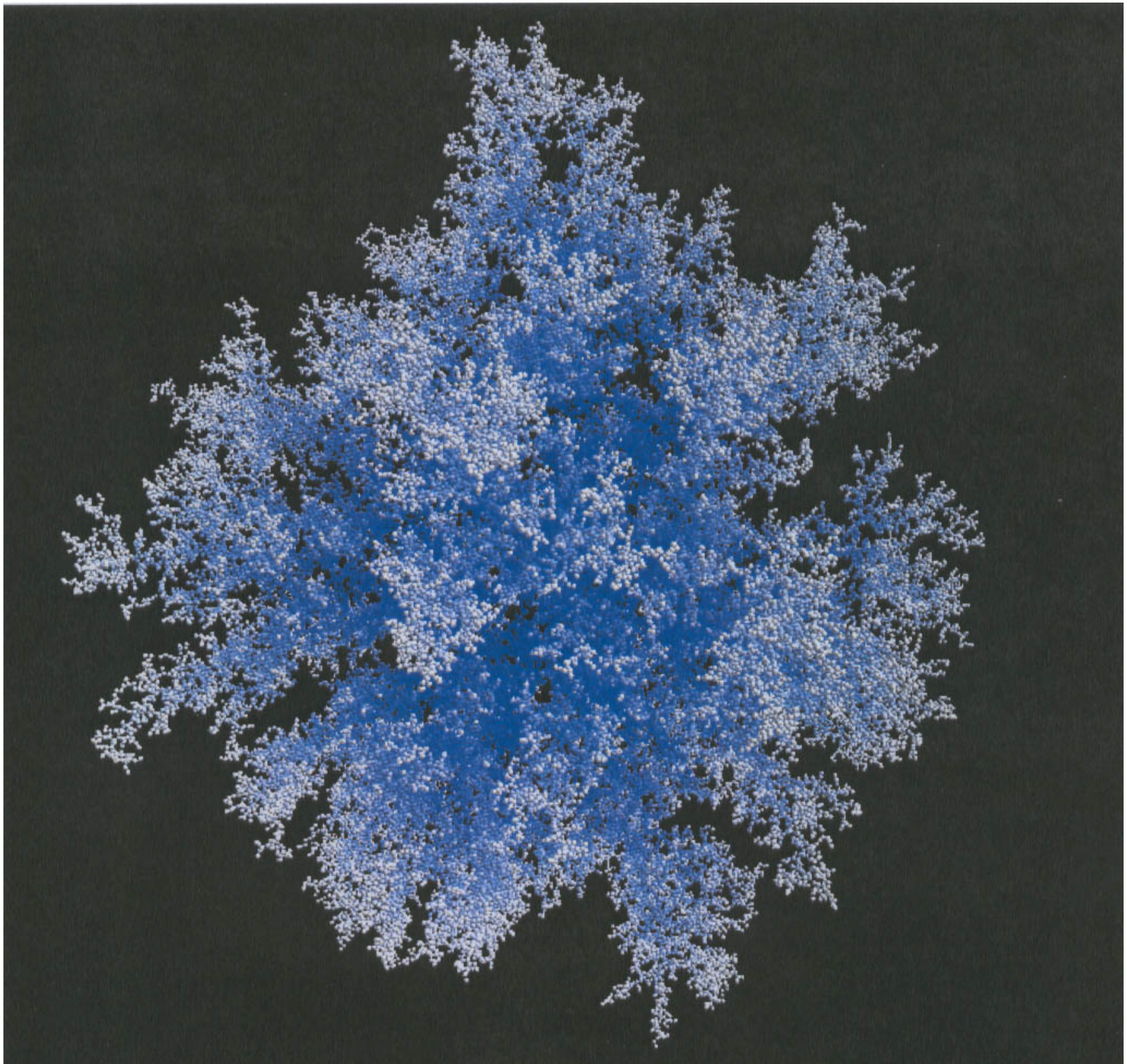
(i) Non-spherical particle shape to make the fingers more or less "pointy"

(i) Anisotropy attachment (or walk on a lattice) to establish preferred growth directions

(ii) Sticking probability < 1 ("noise control") to fill in the fjords more and lower the fractal dimension

Try the demo "dla" by Bob Sumner to play with (iii).

DLA in Three Dimensions



100,000 spheres

Simulation by Jacquie Yen for a Mod/Sim project in 2004

Must use analytical tricks to accelerate the random walks in 3D

Advection-Diffusion-Limited Aggregation

M. Z. Bazant, J. Choi, B. Davidovitch (2003)

“DLA in a fluid flow”

Figures removed for copyright reasons.
See Fig. 1 in Bazant, Martin Z., Jaehyuk Choi, and Benny Davidovitch. "Dynamics of Conformal Maps for a Class of Non-Laplacian Growth Phenomena." *Phys Rev Lett* 91, no. 045503 (2003).

(Simulation by stochastic conformal mapping, NOT sticky random walkers)

DLA on a Sphere

(J. Choi, D. Crowdy, M. Z. Bazant)

Figure removed for copyright reasons.

See Fig. 12 in Bazant, Martin, and D. Crowdy. "Conformal mapping methods for interfacial dynamics." In *The Handbook of Materials Modeling*. Edited by S. Yip, et al. Vol. 1. Article 4.10. New York, NY: Springer, 2005.

DLA on Constant Curvature Surfaces

“Heaven and Hell” M.C. Escher (www.mcescher.com) “Circle Limit IV”

Figure removed for copyright reasons.
See
http://www.mcescher.com/Gallery/symmetry-bmp/E45.jpg

Figure removed for copyright reasons.
See "Circle Limit IV," artwork by M.C. Escher.

Figures removed for copyright reasons.

Source: Choi, J., M. Z. Bazant, and D. Crowdy. *Diffusion-limited Aggregation on Curved Surfaces*. Preprint.