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THEORY OF THE DISPERSION OF MAGNETIC PERMEABILITY IN FERROMAGNETIC MATERIALS AT MICROWAVE FREQUENCIES

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I. INTRODUCTION

The initial magnetic permeability of iron and nickel is known to decrease sharply with increasing frequency in the range from the d.c. permeability of the order of 100, which obtains up to frequencies near 100 mc/sec, to the low value of permeability of one found by Hagen and Rubens[1] at 10^7 mc/sec.

At frequencies higher than 100 mc/sec the skin depth for the penetration of the electromagnetic field into the specimen is smaller generally than the dimensions of the elementary ferromagnetic domains. Domain dimensions are of the order of magnitude 10^{-2} - 10^{-4} cm. It is plausible to expect at such frequencies that the usual concepts of the domain mechanisms used in the theory of saturation will break down, since the applied magnetic field is no longer effective over the entire volume of a domain. The present paper represents an attempt to construct a theory of domain dynamics applicable to such a situation. The theory is found to account satisfactorily for the general trends of the existing experimental data on magnetic dispersion in ferromagnetic materials at microwave frequencies.

Ferromagnetic materials are composed of small regions called domains.[2] In general the volume of a domain is smaller than that of the microcrystalline constituents of the material. The atomic spins in a domain are nearly all lined up in the same direction, but the principal direction of the domain spins varies from one domain to its neighbors. In the demagnetized state the domain directions are distributed at random, giving zero macroscopic magnetization.

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[1] E. Hagen and H. Rubens, Ann. d. Phys. [4] 11, 873 (1903).

[2] For an account of modern theories of ferromagnetism see F. Bitter, Introduction to Ferromagnetism (McGraw Hill Book Co., New York, 1937), and R. Becker and W. Doering Ferromagnetismus, (Edwards Bros., Ann Arbor, Michigan. 1943).

[illegible]) [illegible] of the [illegible] energy [illegible] of [illegible] fields.

b) [illegible] Landau [illegible] and [illegible] of [illegible] to the [illegible] field [illegible] magnetic [illegible] possible [illegible] placement [illegible].

c) [illegible] the [illegible] of [illegible] as a whole [illegible] field. These fields [illegible] the important [illegible] dependence [illegible] forces responsible for anisotropy [illegible] the [illegible] forces are equivalent to ~10^7 [illegible].

[illegible] restricted at high frequencies [illegible].

e) [illegible] incomplete penetration of surface [illegible] the [illegible] effect.

f) [illegible] and [illegible] local [illegible] and Kittel[3]; and Becker[4].

g) [illegible] microscopic [illegible] which [illegible] Landau and Lifshitz[5] [illegible] effects [illegible].

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[3] [illegible]

[4] R. Becker, [illegible]

[5] L. Landau and E. Lifshitz, [illegible]

[illegible]

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[5] [illegible]

[7] [illegible] Zo[illegible]

[illegible]

II. Experimental Results

A comprehensive review of the experimental data on the permeability of ferromagnetic materials (including alloys and dusts) at frequencies between 100 kc/sec and 10,000 mc/sec has been published by Allanson[8]. It is proposed here for the purpose of comparison with the theory developed below to summarize briefly the results of measurements on iron, nickel and cobalt at frequencies above 100 mc/sec, including some data not available to Allanson.

A. Iron

Experimental values for iron and steel are plotted in Fig. 1, as determined by Arkadiew[9], Hoag and Jones[10], Potapenko and Sänger[11], Lindman[12], Hoag and Gottlieb[13], Glathart[14], and E. Maxwell[15]. The curve labeled μ_R is drawn through the points for the experimental permeabilities deducted from <u>resistive losses</u> in a circuit element containing the ferromagnetic specimen, while the curve labeled μ_L is deduced from the <u>reactance</u> of the circuit element. The relationship between μ_L and μ_R is discussed in the next section.

The measurements of Lindman have not been taken into account in drawing the μ_L curve, since his values are far out of line with those of Hoag and Jones, Hoag and Gottlieb, Glathart, and Potapenko and Sänger. The apparent discrepancy here may be due to real differences in the dimensions of surface domains or in the electrical conductivity of the surface layer of the specimen. Maxwell has studied the effect of surface finish on microwave attenuation in wave guides and finds considerable

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[8] J. T. Allanson, Journ. Inst. Elect. Eng. 92 [pt. III], 247 (1945); see also, W. Arkadiew, Jour. of Phys. USSR 9, 373 (1945).

[9] See reference 8.

[10] J. B. Hoag and H. Jones, Phys. Rev. 42, 571 (1932).

[11] C. Potapenko and R. Sänger, Naturwiss. 21, 818 (1933); Zeit. f. Phys. 104, 779 (1937).

[12] K. F. Lindman, Zeit. f. Tech. Phys. 19, 159 (1938).

[13] J. B. Hoag and N. Gottlieb, Phys. Rev. 55, 410L (1939).

[14] J. L. Glathart, Phys. Rev. 55, 833 (1939).

[15] E. Maxwell, M.I.T. R.L. Report 854 (1946), unclassified. The values of the permeabilities credited to Maxwell were calculated from his values for the attenuation of 1.25 cm radiation in rectangular wave guides.

[illegible] 241 m/sec [illegible] 12.1 [illegible]

[illegible]

[illegible]

[illegible]

III. Definitions of Permeability at High Frequencies

There is a great deal of confusion in the literature with regard to the connection between the permeability μ_R determined from resistive losses (as by measuring the "Q" of a cavity or the attenuation of energy along a coax line) and the permeability μ_L determined from reactance measurements (as by measuring the resonant frequency of a cavity or the wave length of standing waves along a coax line). It does not appear to have been realized in the literature that the two types of measurements inherently reveal different aspects of the same fundamental physical phenomena. In this section the fundamental philosophy will be developed which underlies the interpretation, in terms of permeability, of r.f. measurements in a dispersive region. The ideas were stimulated by a paper of Rayleigh's[19] in which the concept of an out-of-phase component of magnetization is introduced in connection with hysteresis losses.

The usual definition of initial permeability as the ratio of B to H for weak fields does not correspond at high frequencies to the quantities actually observed in experiments. In a dispersive region the value of the ratio B/H may vary from point to point in the radiation field within the specimen in both amplitude and phase. Detailed knowledge of this "permeability field", supposing it could be determined, would be less useful (for most purposes) than a convenient summing up of the magnetic behavior of the material in terms of an effective permeability.

The natural and logical method of defining the effective permeability is as follows: The impedance

$$Z_{calc}(\mu,\omega) = R_{calc}(\mu,\omega) + j X_{calc}(\mu,\omega) \qquad (1)$$

of a circuit element containing the ferromagnetic material can in principle be calculated from Maxwell's equations in the usual form, given μ and ω. This calculation can actually be carried out in closed form for the important experimental geometries, such as a rectangular wave guide or coaxial line resonator. Suppose that the result of a series of measurements on the circuit element gives us experimental values

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[19] Lord Rayleigh, Phil. Mag. 23, 225 (1887); Scientific Papers, 2, 579 (Cambridge University Press); cf. W. Arkadiew, Zeit. f. Phys. 27, 37 (1924) and F. Hinze, Ann. d. Phys. [5] 19, 143 (1934).

$$[illegible]$$

The effective permeability [illegible] which satisfies [illegible]

$$[illegible]$$

In general μ defined in [illegible] is a function of [illegible] and will be complex.

In [illegible] permeability, [illegible] the impedance of [illegible] experiments performed in [illegible] part of the reactive part was measured [illegible] a serious deficiency [illegible] R(ω) or X(ω) for all frequencies, [illegible] theorem.

If [illegible] μ_{ap} [illegible] the permeability is [illegible] taken to be

$$[illegible]$$

This relation determines the real function $\mu_1(\omega)$ [illegible] only $X_{exp}(\omega)$ [illegible] permeability [illegible]

$$X_{calc} [illegible]$$

thus determining the real function $\mu_2(\omega)$.

There is no [illegible] direct connection between μ_1, μ_2 and the complex μ. [illegible] that μ_2 has some of the behavior of the real part of μ and μ_1 has some of the behavior of the imaginary part of μ.

The preceding [illegible], which [illegible] the introduction of a complex permeability, [illegible] the introduction of a complex dielectric constant in the [illegible] case. There are two reasons for this [illegible]. Because of the skin effect the magnetic flux distribution in the [illegible] is a complicated function of the space of definition, and because of the domain or [illegible] structure of ferromagnetic material it is possible to have a microscopic permeability which varies from point to point in the material.

In the literature μ_H is sometimes called the "outer permeability" and may be denoted also by μ_a or μ_K; μ_L is sometimes called the "inner permeability" and may be denoted by μ_i or μ_r.

IV Theory of Domain Dynamics

In the microwave region the skin depth for field penetration is comparable to or smaller than the dimensions of a domain. For iron with $\mu = 100$ we have $\delta \cong 1.6 \times 10^{-2}/\sqrt{f}$, where δ is the skin depth in cm and f is the frequency in mc/sec. This gives the following values:

f (mc/sec)	10^2	10^4	10^6
δ (cm)	1.6×10^{-4}	1.6×10^{-5}	1.6×10^{-6}

whereas domain dimensions are estimated at 10^{-3} to 10^{-4} cm. It is therefore necessary to reconsider the conventional application of Maxwell's equations to the skin effect problem.

In the limiting case in which the surface energy of the domain boundaries is greater than the magnetization energy, the domain wall shifts as a whole in the direction of the applied field even if, due to the skin effect, the magnetic field only penetrates a short distance into the domain. If H is constant across the domain boundary (low frequency case) the average macroscopic magnetization M for weak fields is given by

$$M = \chi H \qquad (6)$$

by the usual definition of the initial susceptibility χ. If now H varies across the boundary we suppose that the effective magnetization is given by the susceptibility times the average value of H across the boundary:

$$M = \chi (1/2d) \int_{-d}^{d} H(y)\, dy \qquad (7)$$

This assumption is based on the physical concept that the boundary pressure $2 M_s h$ is integrated over the area of the boundary to give a force which shifts the boundary in the direction of the applied field up to a point where this force is balanced by the force exerted on the boundary by the internal stresses in the material. Here M_s is the saturation [illegible] in a domain. The distance the wall is [illegible] to the macroscopic magnetization M observed in an experiment. [illegible] frequencies the following happens: [illegible] depth [illegible] domain, [illegible] of the domain thickness, so that the applied magnetic field only penetrates a little way down the domain boundary. The force on this boundary associated with the surface value of this field strength is correspondingly less than the force obtaining at low frequencies, so that the boundary is shifted by a reduced amount. To the field this reduced shift looks like a reduced permeability.

It is of interest to consider as a simplified model a film one domain in thickness (Fig. 3), since in this case Maxwell's equations can be solved to give what is essentially the equation of motion of a domain boundary. The longitudinal extent of a domain along the surface of the film is supposed to be small in comparison with the thickness of the film. The domains are considered for simplicity to have only "180°" walls — that is, the domains are either magnetized in the direction of the applied field or in the opposite direction. The applied field is parallel to the surface of the film and is symmetric about the central plane of the film.

From Maxwell's equations

$$\operatorname{curl} \underset{\sim}{H} = 4\pi\sigma \underset{\sim}{E}/c \qquad (8)$$

$$\operatorname{curl} \underset{\sim}{E} = -(\dot{\underset{\sim}{H}} + 4\pi \dot{\underset{\sim}{M}})/c \qquad (9)$$

we get

$$\operatorname{curl}\operatorname{curl} \underset{\sim}{H} = -i(4\pi\omega\sigma/c^2)(\underset{\sim}{H} + 4\pi\underset{\sim}{M}) \qquad (10)$$

for time dependence of the form $\underline{\exp[i\omega t]}$. By the symmetry of the problem we have

$$\operatorname{curl}\operatorname{curl} \underset{\sim}{H} = -\underset{\sim}{\vec{i}}\,(\partial^2 H_x/\partial y^2) \qquad (11)$$

so that

$$\partial^2 H_x/\partial y^2 = i(4\pi\sigma\omega/c^2)(H_x + 4\pi M_x) \ . \qquad (12)$$

Since $4\pi \underline{M}_x$ is constant with respect to $\underline{y}$ we can add this quantity to $\underline{H}_x$ on the left side without altering the value of the derivative there. From the definition $\underline{B}_x = \underline{H}_x + 4\pi \underline{M}_x$, giving

$$\partial^2 B_x/\partial y^2 = i(4\pi\sigma\omega/c^2)B_x \qquad (13)$$

which can be compared with the usual skin effect equation:

$$\partial^2 E_x/\partial y^2 = i(4\pi\omega\sigma\mu/c^2)\, E_x \qquad (14)$$

For convenience in working with Eq. (13) we shall hereafter omit the subscript *x* on B_x, M_x, and H_x. We write

$$p^2 = i(4\pi\sigma\omega/c^2) = 2i/D^2 \qquad (15)$$

where *D* is the skin depth for permeability unity. Eq. (13) becomes

$$\partial^2 B/\partial y^2 = p^2 B. \qquad (13a)$$

A symmetric solution of this equation is

$$B = C \cosh py \qquad (16)$$

where *C* is a constant to be determined in terms of *H(d)*, the magnetic field at the surface of the film. By the definition of *B* we have $H = B - 4\pi M$, or using Eq. (7),

$$H(y) = B(y) - (4\pi/2d)\chi \int_{-d}^{d} H(y)\, dy \;; \qquad (17)$$

where *B(y)* is given by Eq. (16). The solution of this equation is found to be

$$H(y) = C[\cosh py - (4\pi\chi/\mu_0 pd) \sinh pd] \qquad (18)$$

where μ_0 is defined as $1 + 4\pi\chi$.

Thus the constant *C* is given in terms of *H(d)* by

$$C = H(d)[\cosh pd - (1 - 1/\mu_0)(\sinh pd/pd)]^{-1} \;. \qquad (19)$$

The solution of the ordinary eddy current equation [illegible] above for permeability μ is

$$B(y) = \mu H(d) \cosh qy/\cosh qd = \mu H(y) \qquad (20)$$

where

$$q^2 = i(4\pi\sigma\omega\mu/c^2) \;. \qquad (21)$$

Now that we have the formal solutions of both the ordinary eddy current equation and the domain eddy current equation, we can go on to calculate μ, μ_R and μ_L by following the procedure outlined in Section III. In calculating the impedances it is not necessary to specialize the calculation for a particular cavity or line; we can work with the <u>intrinsic surface impedance</u> of the film, which is defined by[20]

$$Z = E_{\text{tang.}}/H_{\text{tang.}} = E_z/H_x \tag{22}$$

From Eq. (8),

$$E_z = -(c/4\pi\sigma)\,\partial H_x/\partial y \tag{23}$$

so that

$$Z = -(c/4\pi\sigma)(\partial H_x/H_x\partial y)_{y=d} \tag{22a}$$

For the ordinary eddy current equation we have, using Eq. (20):

$$Z_{\text{ord}} = -(c/4\pi\sigma)\,\frac{p\tanh pd}{1-(1-1/\mu_0)(\tanh pd/pd)}. \tag{25}$$

The two expressions for <u>Z</u> are equal if <u>q</u> is chosen so that

$$qd\tanh qd = pd\,\frac{\tanh pd}{1-(1-1/\mu_0)(\tanh pd/pd)}; \tag{26}$$

that is, if the <u>effective permeability</u> μ is chosen so that

$$\sqrt{\mu}\tanh pd\sqrt{\mu} = \frac{\tanh pd}{1-(1-1/\mu_0)(\tanh pd/pd)}. \tag{27}$$

Since p involves $\sqrt{i}$ the value of μ satisfying this equation will be complex.

Let us consider limiting cases of Eq. (27).

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[20] S. A. Schelkunoff, <u>Electromagnetic Waves</u> (D. Van Nostrand Co., New York, 1943); J. A. Stratton, <u>Electromagnetic Theory</u> (McGraw-Hill, New York, 1941) p. 282; J. C. Slater, <u>Microwave Transmission</u> (McGraw-Hill, New York, 1942) p. 95.

a) Low frequency

Here $|pd| \ll 1$, so that we can replace $\tanh \phi/\phi$ by unity. This gives $\mu = \mu_o$, the correct low frequency value.

b) Very high frequency

Here $|pd| \gg 1$, so that $\tanh \phi/\phi$ approaches zero. This gives $\mu = 1$ in agreement with the measurements of Hagen and Rubens.

Values of μ satisfying Eq. (27) for various values of pd are given in Table 1. These values were calculated by cut-and-try methods with assistance from Kennelly's tables[21] and the tables prepared by Lowan, Morse, Feshbach, and Haurwitz[22].

Theoretical values of μ_R and μ_L are also given in Table 1 according to the definitions, Eqs. (4) and (5), where we identify

$$R_{exp} \leftrightarrow R_{dom}; \qquad X_{exp} \leftrightarrow X_{dom};$$

$$R_{calc} \leftrightarrow R_{ord}; \qquad X_{calc} \leftrightarrow X_{ord};$$

so that μ_R is the real number which satisfies the real part of Eq. (27).

$$\mathrm{Re}\left\{\sqrt{\mu_R}\ \tanh pd\sqrt{\mu_R}\right. = \mathrm{Re}\left\{\frac{\tanh pd}{1 - (1 - 1/\mu_o)(\tanh pd/pd)}\right\} \qquad (27a)$$

and μ_L is the real number which satisfies the imaginary part:

$$\mathrm{Im}\left\{\sqrt{\mu_L}\ \tanh pd\sqrt{\mu_L}\right\} = \mathrm{Im}\left\{\frac{\tanh pd}{1 - (1 - 1/\mu_o)(\tanh pd/pd)}\right\}. \qquad (27b)$$

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[21] A. E. Kennelly, Tables of Complex Hyperbolic and Circular Functions (Harvard University Press, Cambridge, 1914).

[22] A. N. Lowan, P. M. Morse, H. Feshbach, and E. Haurwitz, Tables for Solutions of the Wave Equation for Rectangular Boundaries Having Finite Impedance, Applied Mathematics Panel Note No. 18; Section No. 5.1 - sr1046 - 2043 (June, 1945); unclassified.

Table 1

Theoretical Permeability vs. Frequency

Film Thickness 2.5×10^{-4} cm

Frequency	Parameter	Permeability μ			
mc/sec	$(d/D)\sqrt{2}$	Amplitude	Phase	μ_R	μ_L
83.	0.1	104	0°	102	100
334.	0.2	127	- 18°	157	58
750.	0.3	39	- 53°	160	11.5
2080.	0.5	35	- 73°	70	1.7
4670.	0.75	16	- 79°	32	0.5
8300.	1.0	9	- 80°	18	0.3
33000.	2.0	2.6	- 62°	4.5	0.2
75000.	3.0	1.6	- 38°	2.6	0.6
210000.	5.0	1.0	- 18°	1.7	0.9

In Fig. 4 μ_R and μ_L are plotted together with the smoothed experimental curves for iron taken from Fig. 1. The arbitrary constant 2d, which is the thickness of the domain film model, has been taken to be 2.48×10^{-4} cm; this value was chosen to make the half-value points on the experimental and theoretical curves coincide.

It is seen that the theory predicts the order of magnitude of the spacing between the μ_R and μ_L curves correctly. The general nature of the theoretical permeability change is in accordance with the experimental data, but the slopes of the theoretical curves are steeper than the experimental. The thickness of the film is within the limits of reasonable estimates of domain dimensions, although the thickness is somewhat on the small side.

The discrepancy in the slopes is most likely to be accounted for by local variations in domain dimensions and d.c. permeability, since these variations will act to smear out the dispersive region. The absence of the hump predicted for the μ_R curve on the low frequency side may be du in part to these causes and in part to the oversimplification of the present model.

In the ordinary microwave radio range of frequencies from 3,000 to 30,000 mc/sec the permeability μ is chiefly imaginary, according to Table 1.

7 Ferromagnetic Resonance

Several predictions have been made that resonance effects or peaks in the permeability vs. frequency curve would be found at high frequencies; see for example the paper of Landau and Lifschitz[23] in which magnetic resonance is predicted in nickel at ~2500 mc/sec.

Such effects have not been found experimentally, and it is possible to see one of the reasons why from the argument of the preceding section. The predictions have all neglected completely the effect of skin depth and eddy currents, yet in the frequency range considered we have shown that such effects are of predominant importance.

It is possible, however, that magnetic resonance effects may be detected in the magnetic oxides and sulfides of iron[24]. These are ferromagnetic but have low electrical conductivity, so that the skin depth will be much greater than in the ferromagnetic metals. The skin depth in magnetite (Fe_3O_4) is ~5 x 10^{-3} cm at 10^4 mc/sec, as compared with 1.6 x 10^{-5} cm in iron at the same frequency. The d.c. initial permeability[25] of magnetite is ~17. Measurements on films of ferromagnetic materials should also be pertinent when the film thickness is less than the calculated skin depth.

The resonance phenomenon may be understood as occurring when the frequency of the applied field is equal to the Larmor frequency of the atomic spins in the internal anisotropy field. This is the field due to spin-orbit interactions and distinguishes energetically different directions of magnetization in the crystal lattice. Since the anisotropy field is of the order of 1000 gauss, the corresponding Larmor frequency is in the microwave range.

It is interesting to consider a classical model in which the atoms are replaced by non-gyroscopic bar magnets pivoted at the lattice points of the crystal. With zero applied field each magnet is attracted

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[23] Reference 5; see also R. Gans and R. G. Loyarte, Ann.d.Phys. [IV] 64, 209 (1921); L. Pare, Phys. Rev. 21, 456 (1923); J. Dorfmann, Zeit. f. Phys. 17, 98 (1923); K. Kartschagin, Ann. d. Phys. [IV] 67, 325 (1922). The experiments by Kartschagin and others in which resonance phenomena were reported are now discredited.

[24] The interesting possibilities of the ferromagnetic semiconductors were pointed out to the writer by Prof. A. v. Hippel, who is planning to investigate them experimentally.

[25] International Critical Tables, vol. VI, p. 374

in a definite direction relative to the lattice by means of individual coiled springs representing the spin-orbit interaction, and the magnets will oscillate in an applied field with a component perpendicular to the rest position of the magnets. The sine of the angle of oscillation is proportional to the macroscopic magnetization. Resonance occurs when the applied frequency is equal to the free period of a magnet + spring unit.

The bar magnet analogy supposes that the relaxation time of the spins is sufficiently short so that gyroscopic effects may be neglected. It is not usually recognized that this assumption is being made. If this assumption is not true, the spins will precess about the field direction without lining up. It is indeed a prerequisite for any type of magnetization that the magnetic moments have time in which to line up in the instantaneous local field to which they are subjected. The calculation of the relevant relaxation time is a problem in the kinetics of thermodynamic equilibrium.

The time-dependent processes can be described by assuming the existence of a relaxation time, as was done by Gorter and Kronig for paramagnetic relaxation, and by Landau and Lifschitz for the ferromagnetic case. The quantum mechanical calculation of the relaxation times, starting from the detailed interactions of spins with the lattice, is extremely difficult and uncertain. Calculations for the paramagnetic case have been made by Waller[26] and others. No calculations have been carried out for the ferromagnetic case, so far as the author is aware. It seems plausible to suppose that the strong spin-dependent coupling in ferromagnetic materials will assure that the relaxation frequency will occur above the microwave range. This question should be looked into more closely.

It should be noted in passing that the collision frequency of the lattice phonons at room temperature is $\sim 10^{12}$ collisions per second, as estimated from values of the thermal conductivity of non-metallic crystals. This figure determines an approximate upper limit to the order of magnitude of the spin relaxation frequency. The actual spin relaxation frequency may be lower depending on the strength of the coupling between the spins and the lattice. In metals the relaxation frequency of the lattice phonons is estimated to be of the order of 10^{13} $\sec^{-1}$ at room temperature, based on electrical conductivities.

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[26] See, for example, I. Waller, Zeit. f. Phys. 79, 370 (1932). The writer is indebted to Prof. L. Tisza for several discussions of the paramagnetic relaxation problem.

VI. Discussion

Arkadiew[27] first suggested that eddy current effects might be important in ferromagnetic dispersion. This approach was developed further by Becker[28], who pointed out that the local microscopic eddy currents associated with the movement of domain boundaries and the rotation of domains set up a magnetic field which opposes the applied field. This back field adds a term to the equations of motion which is proportional to the velocity of boundary movement or spin rotation; that is, the eddy currents behave like a viscous force. Becker's treatment gives a good qualitative account of the damping of irreversible displacements characterizing magnetization in medium fields, at frequencies below the microwave range, although an apparent difficulty in reconciling these results with the measurements of Sixtus and Tonks on the velocity of boundary propagation has been suggested by Miss van Leeuwen[29]. It should be pointed out that the local eddy current effects considered by Becker have no direct connection with the use made of the eddy current equation in the present paper, according to which the incomplete penetration of the surface domains by the applied field is the major cause of dispersion.

Becker also has given a calculation for the dispersion of the initial permeability, and this calculation leads to results in some respects similar to those of the present paper. The "back field" is calculated as in the medium field strength case just mentioned. The basis of Becker's theory supposes that the skin depth is greater than the domain dimensions, so that the calculation is not applicable to the microwave range, where the skin depth is less than the domain dimensions. At 3×10^3 mc/sec the skin depth is only ~ 0.1 of the domain thickness.

The present theory probably could be improved by working with a more complicated model than that of a film one domain thick. If the film is backed on one side by a mass of ferromagnetic material the motion of the domain boundaries in the film will induce eddy currents in the backing

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27. W. Arkadiew, C. R. Acad. Sci. URSS (Doklady) 2, 204 (1935); see also reference 3.
28. See reference 4.
29. H. J. van Leeuwen, Physica 11, 35 (1944).

material. The eddy currents will of course also lead to additional damping. However, the correction to the permeabilities calculated in this paper is expected to be small: since on the low frequency side of the dispersive region the eddy current damping is small anyway, while on the high frequency side the failure of the applied field to penetrate the film leads to greatly restricted domain movement and hence the additional eddy field is unimportant.

It does not seem worthwhile at this time to attempt to calculate the permeability with a more elaborate model. The present model gives results in reasonable agreement with experiment, and the dispersive mechanism proposed here appears to correspond to the physics of the situation. The most important direction in which the model should be extended would seem to be in a treatment of the case in which the surface energy of the domain wall is small in comparison with the magnetization energy, so that the domain wall yields locally to the field. The dispersion would be due to the magnetic inhomogeneity of the material. The model treated in the present paper supposes that the domain wall moves rigidly under the influence of the applied field. There are reasons for believing that both cases may occur in different actual materials.

It should be pointed out that the information regarding domain behavior obtained from dispersion measurements on metals pertain only to the domains in the surface layers of the material. With this qualification, dispersion measurements may prove to be an important method for studying domain mechanisms.

Acknowledgments

It is a pleasure to thank Prof. [illegible] for his encouragement and interest. Miss Patricia Boland has been of great assistance in the numerical work.

Appendix [illegible]

Relation of Intrinsic Surface Impedance to Resistive Losses and Inductance of Film

It can be shown that the resistive losses in the film considered in Section IV and the contribution of the film to the inductance of a circuit element are related directly to the intrinsic surface impedance which is defined according to Eq. (22) by $Z = E_z/H_x$, evaluated at $y = d$.

The average rate of energy loss per unit area normal to the y direction is given by the average value of the Poynting vector

$$S = - \mathrm{Re}[c/4\pi)\, E_z(d)\, H_x^*(d)],$$

when it is recalled that the film has two surfaces. Now $E_z = Z\, H_x$, so that

$$S = - (c/4\pi)\, H_x(d)\, H_x^*(d)\, \mathrm{Re}[Z],$$

a well-known result.

The contribution of the film to the inductance of the magnetizing circuit is given by the quotient of the magnetic flux through the film by the current in the magnetizing circuit:

$$L = \mathrm{Re}[\mu \int_{-d}^{d} H_x\, dy/J].$$

Now $J = (c/4\pi)H_x(d)$ and $\mu H_x = + j(c/\omega)(\partial E_z/\partial y)$, so that

$$\omega L = 8\pi\, \mathrm{Re}[jZ] = - 8\pi\, \mathrm{Im}[Z].$$

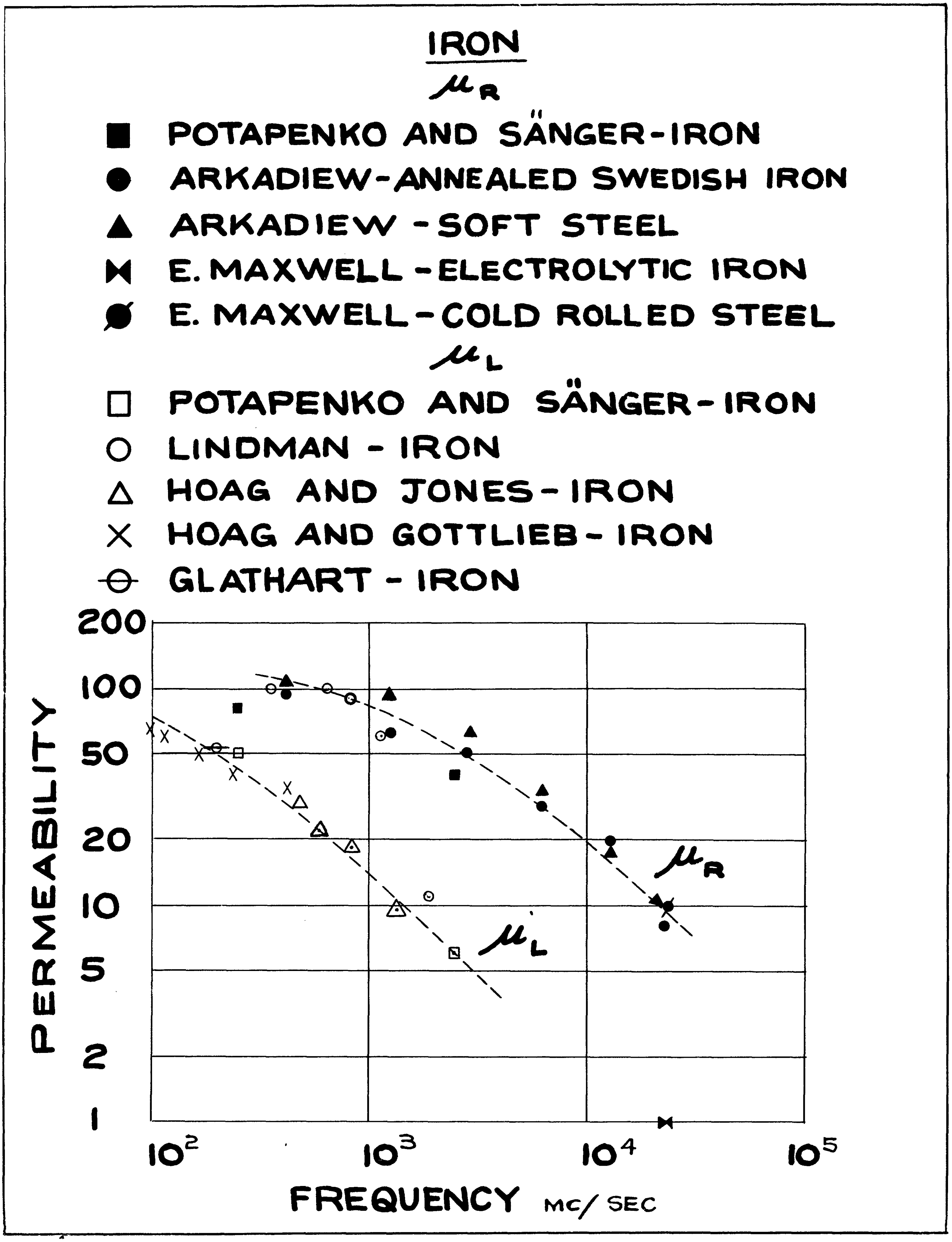
[illegible]

Figure 1 Permeability measurements for iron

Figure 2 Permeability measurements for nickel

Figure 3 Model of film one domain in thickness for theoretical calculation of permeability

Figure 4 Comparison of smoothed experimental values for iron with theoretical calculations using $d_i = 2.5 \times 10^{-4}$ cm


IRON
μ_R
■ POTAPENKO AND SÄNGER - IRON
● ARKADIEW - ANNEALED SWEDISH IRON
▲ ARKADIEW - SOFT STEEL
⧓ E. MAXWELL - ELECTROLYTIC IRON
● E. MAXWELL - COLD ROLLED STEEL
μ_L
□ POTAPENKO AND SÄNGER - IRON
○ LINDMAN - IRON
△ HOAG AND JONES - IRON
× HOAG AND GOTTLIEB - IRON
⊖ GLATHART - IRON
PERMEABILITY
200
100
50
20
10
5
2
1
μ_R
μ_L
10^2
10^3
10^4
10^5
FREQUENCY MC/SEC

NICKEL

μ_R

■ ARKADIEW

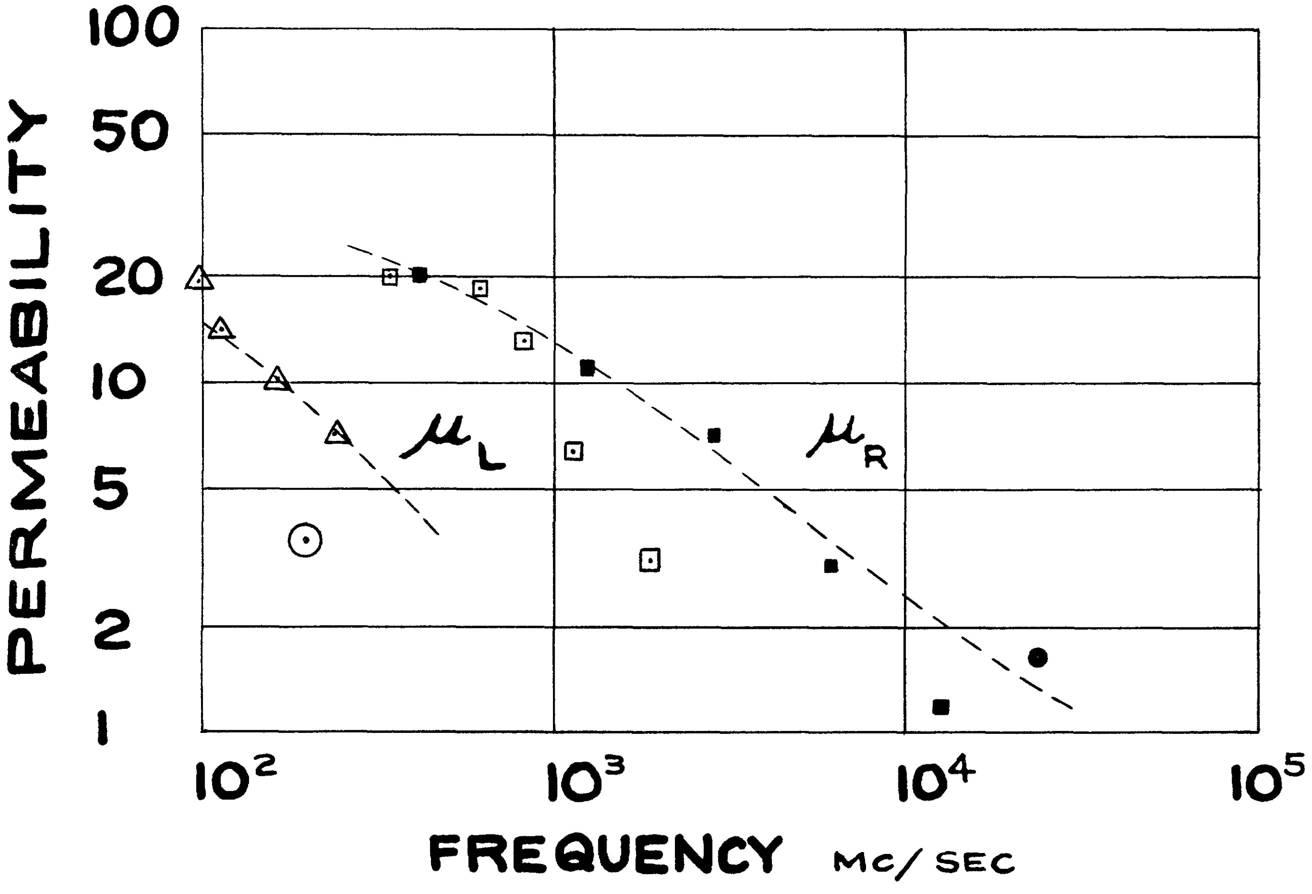
● E. MAXWELL

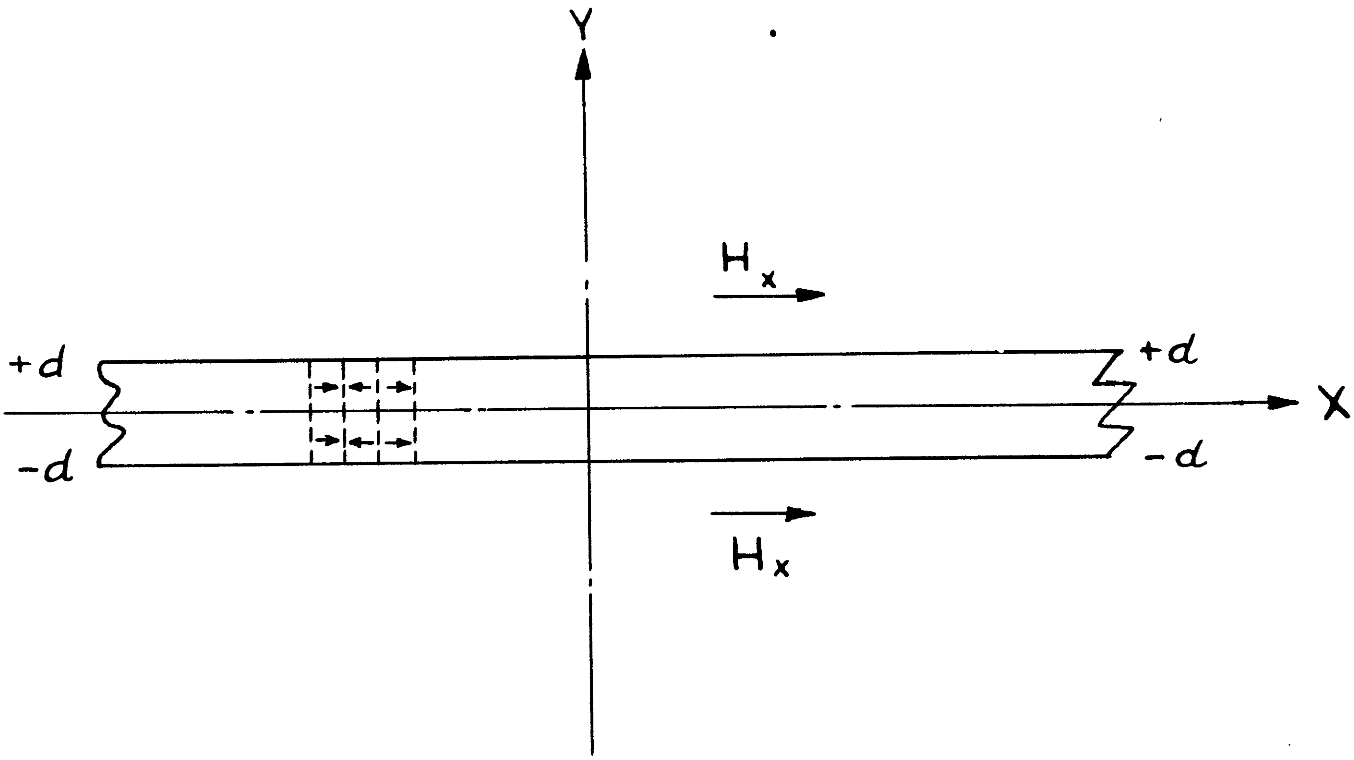
μ_L

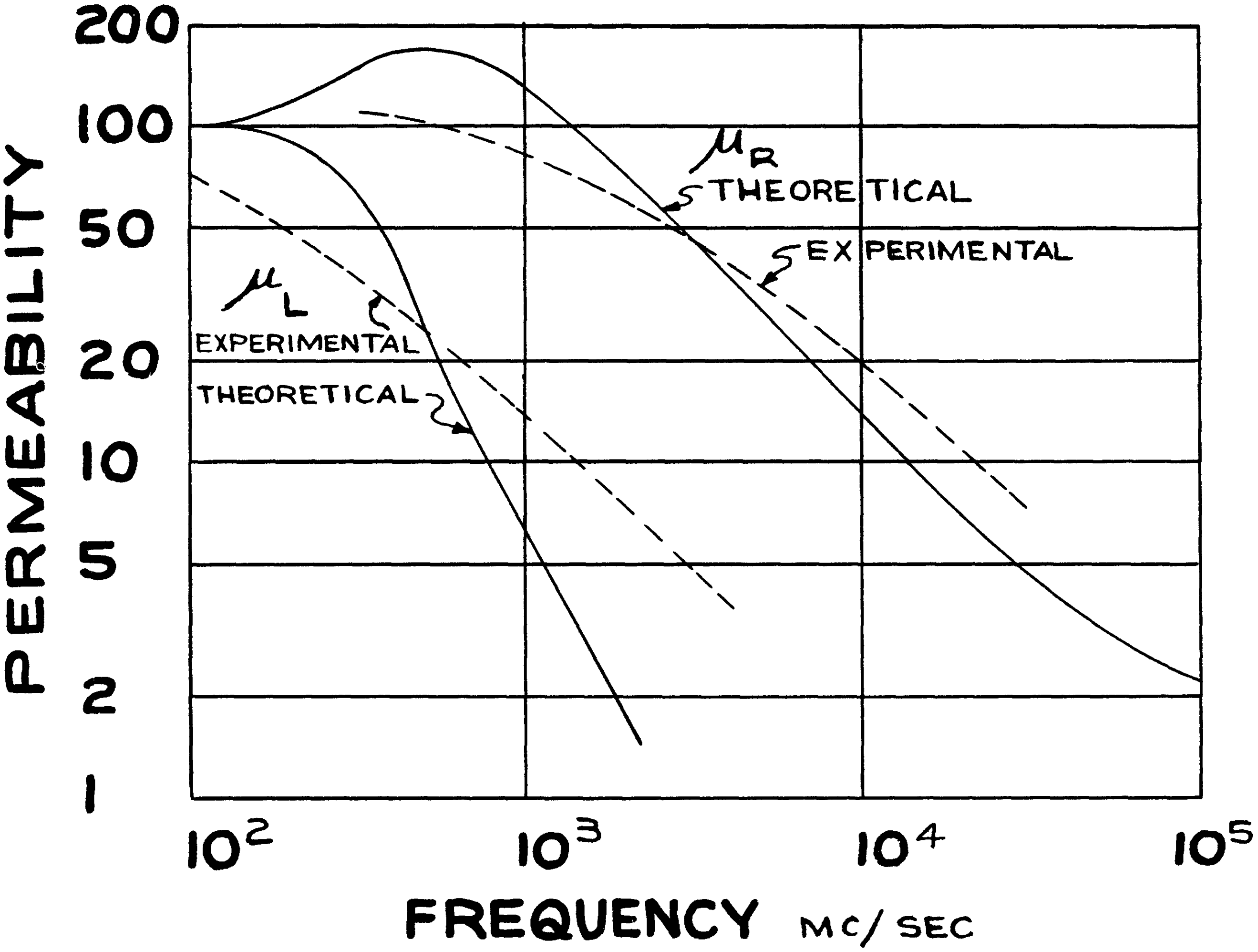
□ LINDMAN

△ HOAG AND GOTTLIEB

○ GLATHART




Y
H
x
+d
+d
X
-d
-d
H
x


200
100
50
20
10
5
2
1
PERMEABILITY
μ_R
THEORETICAL
EXPERIMENTAL
μ_L
EXPERIMENTAL
THEORETICAL
10^2
10^3
10^4
10^5
FREQUENCY MC/SEC