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# Feature Based Variation Modeling of Preliminary Injection Molded Parts


by

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B.S. Mechanical Engineering (1996)

New Jersey Institute of Technology


Submitted to the Department of Mechanical Engineering
In Partial Fulfillment of the Requirements for the Degree of
Master Of Science In Mechanical Engineering

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May 8, 1998

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ABSTRACT

This thesis presents a physics-based method for predicting the variation of injection molded part features during the preliminary design phase. Understanding part feature variation is often necessary when attempting to determine how an assembly of parts will fit and perform in their operating environment. For each feature of interest, a physical mechanism for variation is conceived and modeled. Past production data is used to validate the models. Design teams can use these variational models for preliminary design studies such as tolerance analyses. The end goal is to develop a handbook design guide containing models which can be used to predict variation. Such a guide will be constructed for several different features of interest and for various material types. This thesis discusses both features located on a plane, and hole diameters. The model of features moving on a plane compares favorably to the industry standard SPI tolerancing charts in accuracy. Also, the variational model for hole diameter has been decomposed into variation within a cavity, and variation between cavities. These models have been favorably compared to production data in accuracy.

Thesis Supervisor: Kevin N. Otto

Title: Associate Professor of Mechanical Engineering

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Thomas Albrecht has considerably helped me to understand the basic principles of tolerances, variations and, a tolerance analysis. Without which, this thesis would be falsely grounded. Chris Muir has also helped to shape my knowledge of variational analysis, through the use of properly formatted mathematics, and valid assumptions. Pat McAuliffe, and Mike Insalaco have provided an excellent forum for discussing new methods that can be used to store and manage measurement data. Ed Ostrowski has shown a considerable amount of long term vision, by being brave enough to aid in the development of this project, regardless of its immediate benefits

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# Chapter 1

## Introduction

One of the main drivers for design decisions regarding injection molded parts is anticipated system variation. Unfortunately, tools for part variation analysis are limited. A tolerance analysis can be used to understand preliminary design stackup variations, however, this requires knowledge about how parts will typically vary in production. Both heuristics and process simulation are currently used to develop this knowledge. For use in statistical tolerance analysis, feature variations must be expressed as a statistical distribution with corresponding parameters, such as a normal distribution with a mean and standard deviation. Predicting manufacturing variation in terms of those quantities which are known during preliminary design is a non-trivial task, and is the focus of this thesis. A physical modeling approach is presented here, based on design parameter values known during the conceptual design phase, and then validated using production data. With the models presented in this thesis, the manufacturing variation of an injection molded feature can be predicted successfully, and tolerance analysis can be used as a preliminary design tool.

Tolerance analysis is a valuable tool that can help predict possible problems with an assembly before tooling has been constructed. The methods presented in this thesis improve the tolerance analysis process, by defining a set of variational models that help to increase the repeatability of a tolerance analysis' results. This allows for comparisons of alternative designs and concepts at a faster rate than is currently possible. The results obtained from a tolerance analysis will then be more consistent from user to user. This consistency can be achieved because all input information is generated by a single model, rather than being drawn from many different experts, handbooks and rules of thumb.

Over time the variational models will become more accurate as more sources of production data are used to improve the model. These variational rules will help to provide faster turnaround times and to simplify a tolerance analysis. Also, the commitment to conducting a tolerance analysis on the preliminary design of a product or subassembly can provide far-reaching results. The frantic redesign stage, aimed at correcting functional faults, can often be avoided when a tolerance analysis is performed on a conceptual design.

## 1.1 Guidelines for Modeling Variation

Throughout the development of this thesis, certain guidelines were used to focus the scope of the project. These guidelines are listed below so that the reader has the correct context in which to view the models developed in this thesis. These guidelines define the applicability and limitations of the set of variational rules developed by this project. The variational models presented in this thesis should be:

- limited to plastic injection molded parts produced only within a specific organization at a specific industrial site. Different companies will most certainly have different standards and best practices. It is also possible that different manufacturing plants within the same company will have different injection molding process capabilities due to cultural or climatic reasons. However these variation models could be robustly designed so as to be eventually expandable to other manufacturers and geographical locations.
- limited to a small number of plastic materials, determined during the project, but could be expandable to include other plastic materials.
- flexible enough to calculate the parameters that are required as inputs to a tolerance analysis on several conceptual designs. These inputs include the variation types, distributions, and values that will be applied to each feature of importance. "Values" are defined here as the statistical descriptive parameters required for the chosen distribution placed on a feature. For example, if a normal distribution is used, then the variational model must have access to a standard deviation and mean.
- predictive of process capability variations. A significant difference exists between a feature's variation (how a part or feature is expected to vary) and a functional-driven tolerance (how much a part or feature is allowed to vary and yet still satisfy a functional requirement).

- predictive of both common variations and smaller more difficult to obtain variations. Industry standard variations are easily obtainable through the use of conventional mold making and molding techniques, where smaller more difficult to obtain variations are achievable by unusual tool design, maintenance, and process control approaches.
- reasonably accurate. Due to the preliminary design application of these variational models, they need only be as accurate as is required when choosing between different conceptual designs. The accuracy of these variational rules should be determined by using current production parts to verify the model.
- dependent only on input obtainable from:
    - a solid modeling CAD file.
    - a tolerance analysis model.
    - a database that stores conceptual design information, e.g. materials, and material properties.
    - the user. It is desirable for the analyst to play only a small role when implementing the variational rules. If the amount of time required to complete a tolerance analysis is reduced due to input automation, the user can spend more time analyzing conceptual designs.

## 1.2 Steps for Achieving Robust Variational Models

Several different preliminary tasks were performed throughout the course of this project and the main tasks are listed below for reference.

- Identify which types of features are typically included in a tolerance analysis
- Choose which types of variation to predict for each feature or set of features
- Suggest a set of variational models that result in those variation predictions
- Compare the variational models to current production data and iterate to improve the model's accuracy
- Verify that the variational models can be implemented in a computational environment, thus ensuring that the models can be utilized by industry

Inspiration for developing the variation prediction models presented in this thesis has originated from several different sources including:

- examining parts and their measurements while looking for trends and dependencies
- learning about tool design, tool making, and the molding processes. Knowledge in these areas has been useful when determining underlying causes of variation.
- interviewing experts to capture some of their knowledge about how quick predictions of manufacturing variation can be made.
- conducting a literature search to learn about other efforts aimed at predicting injection molded plastic part variation.
- evaluating current handbook and rule-of-thumb based variational models to determine what the current state of the art is at predicting plastic part feature variation. Also an investigation was conducted concerning whether or not any of these current rule-of-thumb based methodologies have been implemented into an automated system.

# Chapter 2

## Using Tolerance Analysis as a Preliminary Design Tool

Traditionally tolerances are assigned to a dimension after a part has already been designed. These tolerance values are often obtained from handbooks, past design experiences, and expert opinions. It is possible that a quick one-dimensional, or even sometimes two-dimensional, worst case or root sum squares analysis may be used to determine if a robustness issue existed. This last minute tolerance assignment process has several flaws. First, the tolerances have been assigned at the end of the design process, when it is too late to change the fundamental design of the part. Second, the quick preliminary analysis may not realistically reflect the three-dimensional assembly geometry of the part. Third, the manufacturing, assembly, and quality assurance engineers have not "signed-on" to accept these tolerances as actually achievable.

[Craig, 1996] uses a term called *Dimensional Management* when describing a methodology that contains that basic principles of concurrent engineering, and also one that uses a tolerance analysis as a fundamental building block. The first steps include clearly defining the dimensional requirements of the product early on in the product development process. These dimensional requirements should be accepted and agreed upon by members of the design, manufacturing, assembly, and quality assurance terms. Craig also suggests that a tolerance analysis be used to establish these dimensional requirements, thus simulating customer, manufacturing, and assembly variations.

## 2.0 Managing Variation in the Current Market

In today's rapidly changing consumer market, product dominance is often achieved by producing high quality products at a faster rate, and at a lower cost, than one's competitors. The rate at which a product can be developed for a market is often regulated by how efficiently a product's functional requirements and design parameters can be communicated between marketing, design, manufacturing and assembly.

Miscommunication of a product's specifications between any of these large areas can lead to increased product costs, resulting from redesign, rework, or scrapped parts.

Manufacturing and assembly variations also vastly affect the part's overall quality, cost, and time to market. If a part could be manufactured at its specified nominal value at all times there would be no variation in part quality and hence no need for dimensional tolerances. This is not the case, however, since manufacturing and assembly variations do exist. It is in a manufacturer's best interest to successfully manage and budget the variations, possibly using a tolerance analysis to maximize product quality, minimize production costs, and reduce time to market. This will enable the product to successfully compete in today's global market.

## 2.1 Tolerance Analysis

There are several different types of analysis that can be used to determine whether an assembly criterion will be met. These methods vary in accuracy, modeling time, and complexity, and will each serve a specific purpose. A simpler model would require less information and time to build, but would also yield less precise results than a more detailed simulation.

The easiest type of analysis to consider is called a "worst case" analysis. In this situation, the maximum or minimum values of the tolerances are either added or subtracted to determine if an assembly specification is being met. An example of a worst case analysis is shown in Figure 2.1. In this case we wish to determine the tolerance on the length of an assembly composed of three blocks. Each block has a dimension of 0.4 and tolerance values of ±0.002. We can say that the worst case tolerance of the assembly is ±0.006. This type of analysis does not reflect the true nature of the manufacturing process producing parts A, B and C shown below, because it is quite unlikely that all three parts would be at either their maximum or minimum tolerances.

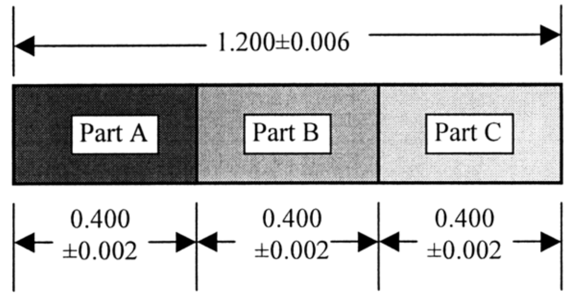



**Figure 2.1: A Worse Case Analysis**

To obtain a better estimate of assembly variation, one must consider the variation of the manufacturing processes used to create each of the parts. For the simple example shown in Figure 2.1, we presume that all three parts have been made with the same process. Furthermore, assume that the length of the parts produced can be described by a random variable following a normal distribution with a mean equal to the nominal value. The standard deviation of the process has been found to equal 0.0005, and the process is capable of making parts at a $\pm 4\sigma_{part}$ level.

A method that takes into account the statistical distribution of the manufacturing process is called a Root Sum Squared (RSS) analysis. The method derives its name from the way by which an assembly tolerance is calculated:

$$\sigma_{assembly} = \sqrt{\sigma_{part1}^2 + \sigma_{part2}^2 + \sigma_{part3}^2} \,. \tag{2.1}$$

Using this formula one can calculate either the resulting assembly tolerance achievable given the three part tolerances, or calculate the allowable part tolerances given the assembly tolerance.

Using the example above, we will first determine when the assembly tolerance is being met, given the specified manufacturing variation. The standard deviation for each part is the same, and has a value of 0.0005. Substituting this value into Equation 2.1 yields the following achievable assembly standard deviation: $\sigma_{assembly} = .00087 = \sqrt{(0.0005)^2 + (0.0005)^2 + (0.0005)^2}$. If the assembly tolerance is set at a $\pm 4\sigma_{assembly}$ level, just as the part tolerances were set at a $\pm 4\sigma_{part}$ level, the resulting assembly tolerance will be $4 * 0.00087 = .00346$. This assembly tolerance (0.00346) is far below the worst case assembly tolerance (0.006). This large difference may lead one to wonder what the allowable part standard deviation would be if Equation 2.1 were used only knowing the assembly tolerance. Since each part is manufactured using the same process it can be assumed that the standard deviation for part 1 is the same as part 2 which is the same for part 3. Substituting this into Equation 2.1 and solving for the individual part standard deviation yields: $\sigma_{part} = \frac{\left(\sigma_{assembky}/4\right)}{\sqrt{3}} = .00087$.

Because the process has been determined to be at a $\pm 4\sigma_{part}$ level, the individual part tolerances are set to be $4\sigma_{part} = 4 * 0.00087 = 0.0035$, which is higher than the original 0.002 specification determined using a worst case analysis. From this analysis it is possible that the original part tolerances could be widened to be 0.0035, perhaps allowing for the process manufacturing the part to be changed to save cost.

A numerical analysis approach to variation prediction is called a Monte Carlo simulation. This method is a further improvement on both the worst case and RSS methods described. It is described in the following section.

## 2.2 Monte Carlo Based Tolerance Analysis: Definition and Example

[Avallone and Baumeister, 1987] have described the Monte Carlo technique as follows:

> "The Mote Carlo technique is used when the list of possible conditions in which the activity under investigation can find itself is too large or too complex to be easily stated. As its name implies, the Monte Carlo technique uses random numbers (which are easily made available through the computer) to determine statistically what conditions exist or what changes will take place. A large number of solutions is then run, and statistical inferences are then drawn."

Several different definitions have been established for what a tolerance analysis is and what it can establish. [T. Albrecht, 1998], an expert tolerance analyst, has defined a tolerance analysis as, "A technique that is used to understand how variations affect the quality of a product." He continues to say that, "it can foster the generation of high quality product designs which are very robust to variations that are inherent in the components and processes used to create them." Two terms which must be understood and not mixed, are variation and tolerance. The distinction between these terms will be discussed next.

### 2.2.1 Variation vs. Tolerance

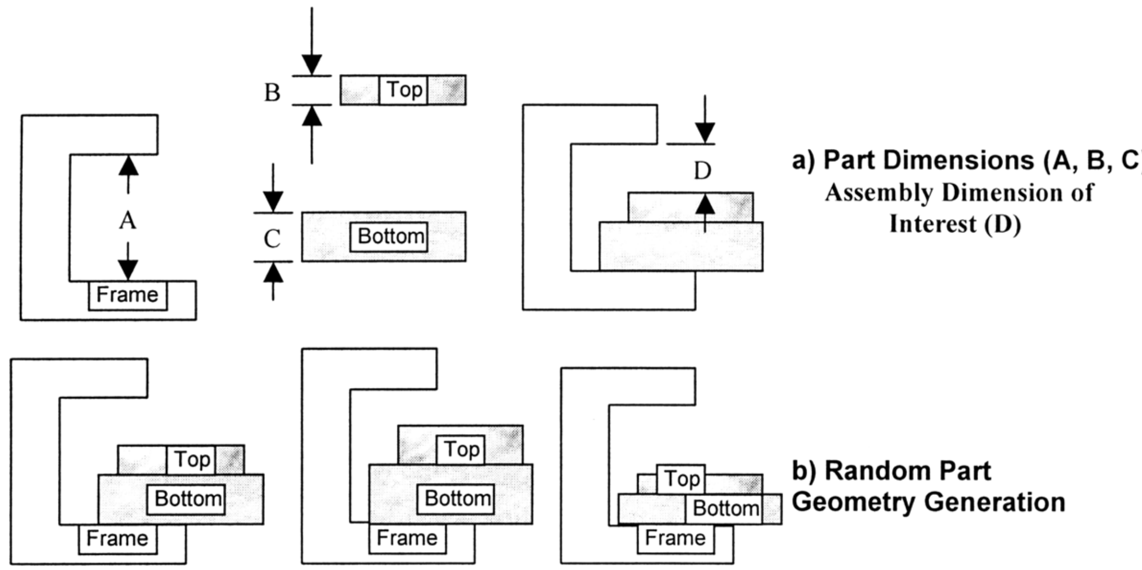
The terms variation and tolerance are often interchanged but do not have the same definition [ASME, 1994], [ASME, 1994(2)]. Simply stated, a tolerance is the amount a dimension *is allowed* to differ from its nominal value to minimally satisfy a functional requirement. Furthermore tolerances are commonly expressed as a number, ±0.005", and are typically placed on engineering drawings. Variation, on the other hand, can be defined as the amount a feature *will actually* differ from its nominal value during production. Variation is expressed as a statistical distribution with its appropriate descriptive properties (*e.g.* a normal distribution with a mean and standard deviation).

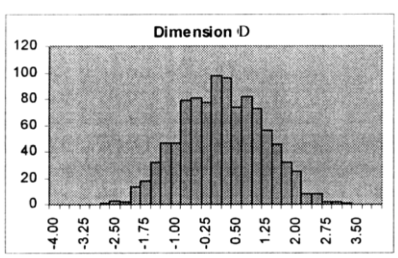
The current method of transferring functional requirements from design to manufacturing is through CAD files and part prints. Most commonly these media contain tolerance and not variation information. This implies that any part produced which meets the tolerance specifications will be accepted, thus providing no direct incentive for process improvement. Although binary acceptance is contrary to the current practices of robust design and Taguchi methods [Taguchi, 1990], it is being encouraged by current practices used to communicate product information and specifications.

The adverse effects resulting from binary acceptance could be easily corrected in several different ways. First, a note could be placed next to a dimension's tolerance limits stating that these limits reflect a certain multiple of a standard deviation and its associated statistical distribution for all cavities used to manufacture this part.

Another method that can be used to counteract the binary nature of tolerancing is specific to the injection molding process. This method would include specifying the allowable distribution of the means for each cavity with respect to nominal, in conjunction with the allowable distribution of the spread of each cavity with respect to its mean value. The reason for this type of specification is because often a part will be made with multiple cavities, and it is possible that the mean of one cavity will not be equal to the means of the other cavities or even the intended nominal.

Regardless of the method used to specify a product's variation requirements, important design information will be preserved when statistical distributions are used to express dimensional requirements.




| | |
|---|---|
| Mean | 0.025064118 |
| Standard Error | 0.032289451 |
| Median | 0.017136017 |
| Mode | 1.733133104 |
| Standard Deviation | 1.021082097 |
| Sample Variance | 1.042608649 |
| Kurtosis | -0.388877776 |
| Skewness | 0.048003657 |
| Minimum | -2.769447747 |
| Maximum | 3.015738912 |

**Figure 2.3: A Simple Tolerance Analysis**

## 2.2.2 Description of Steps for Running a Tolerance Analysis

Generally a statistical tolerance analysis will begin by placing ranges on certain key dimensions. The computer will then create many different cases of the assembly, randomly sampling the value of each part dimension (dimensions A, B, and C in Figure 2.3a). Finally the program gives the statistics of the value of the dimension of interest (dimension D in Figure 2.3a). These steps are shown in Figures 2.3a, 2.3b, and 2.3c. The example shown above is relatively simple in nature and is ideal for illustration purposes. The process used to complete the tolerance analysis for these parts can be generalized and a form of this generalization is listed below. The following list has been derived from a training course taught at Eastman Kodak Company entitled "Statistics and Principles of Tolerance Analysis" [Hackert-Kroot and Nelson 1995].

1.0 *Verify the overall functionality of the design at its nominal conditions.* Whenever a tolerance analysis is performed on a product or subassembly with a large number of parts, this step becomes critical. It is quite possible that a small change made in one part may result in a faulty nominal design caused by not propagating this change through the design loop.

2.0 I*dentify critical interfaces and choose those that might cause the most trouble.* This is one of the most important steps in a tolerance analysis and is often phrased as a question. A well-phrased tolerance question begins with the functional requirements of the part. There are often multiple relevant functional requirements, each with different costs associated with them if geometric change becomes necessary. Tolerance questions must not be vague, and they must model what is highly critical to the successful functioning of the part. Finally the tolerance question must be phrased such that the results obtained from the statistical analysis can easily answer the posed question. The obtained results should also be able to be physically verified when inspecting the hardware being modeled.

3.0 *For each critical interface, determine the acceptable design limits.* There are often hard limits on a particular design. For example, a lever cannot go beyond a certain point because it will be stopped by a protruding post a certain percentage of the time. Or a post cannot be twisted with more that a certain torque, before it is sheared from its supporting base a certain percentage of the time. Design limits can also be set in a Taguchi framework [Taguchi, 1990], by determining how much loss the manufacturer is willing to incur before rejection.

4.0 *Determine which parts affect the interface and how they will mate with one another.* The field of how different parts mate with one another is a non-trivial one and can be very complicated when many parts are involved. For more traditional assembly cases, computer aided tolerance analysis programs can be used. These programs allow the user to graphically interact with the parts to easily specify the assembly's mating conditions. For example, the user can click on a post on part A, click on a hole on part B, and then specify that these two features are to mate.

5.0 *Determine which tolerances affect the part features used for assembly or in the interface.* It is seldom the case that all dimensions on the part are required in the tolerance analysis. It is therefore prudent, to save computational time, that the analysis use only those tolerances that are required.

6.0 *Model the variational nature of the part in the chosen tolerance analysis tool.* The application of this step tests the analyst's knowledge of the definitions of variation and tolerance and also of the overall goal of the tolerance analysis. If the tolerance analysis is being used to aid in the diagnosis of a part that is already in production and is experiencing problems, then the dimension should have a statistical distribution that is similar to one encountered in production, and not what the dimension was intended to be. If the goal of the tolerance analysis is to evaluate a candidate design, then each dimension should have a statistical distribution associated with the tolerance placed on that dimension. In some cases the design tolerances are used as guidelines when modeling manufacturing variation, but measurements of these parts are required to ultimately verify the initial assumptions.

7.0 *If the result from the tolerance analysis is not within acceptable limits, then several options are available and are listed below in random orde*r.

- Determine the largest contributing tolerances, through a sensitivity analysis, and then revisit these tolerances to verify that the assumptions made were correct and that the model was properly constructed and constrained.
- Consider redesign. It is possible that the proposed design will never successfully answer the posed tolerance question, unless the applied statistical variations are unachievably small.
- Consider tightening the applied variations through a process change. Tightening the variations applied to the part often appears to be a favorable solution, but this alternative often backfires when the manufacturing process used to create the part cannot produce the variations assumed in the tolerance analysis. This is why the variations applied to the part should only be

tightened when a different process is widely known to produce parts possessing smaller variations. It is also possible to make trade-offs between certain dimensions. For example perhaps the results of a tolerance analysis may be changed if dimensions A and B are manipulated to reflect the following statement: "I can live with more of dimension A if dimension B is reduced accordingly."

- Consider changing the design limits. If the tolerance analysis does not produce a favorable result because of a hard design limit, then possibly altering the limit may yield positive results. The effect of changing the design limit must be fully investigated before it is implemented in the analysis.
- Evaluate alternatives, using tolerance analysis, life cycle cost modeling, and other weighting factors.

8.0 *If the result is well within the product's specification limits, consider loosening the part's tolerances to reduce the part's cost.* There is a clear relationship between the specifications applied to a dimension, and the cost associated with manufacturing the part. For example, the costs associated with creating a hole in a block of steel vary depending on the manufacturing process used to create that hole. Drilling is most likely the least expensive and quickest process available to create a hole, but drilling also has poor dimensional accuracy associated with it. Jig boring, is a considerably more expensive process, but can yield holes with more accurate dimensional accuracy. If the dimensional requirements on the hole can be widened, then faster, and less expensive processes could be used to create the hole.

## 2.3 Versatility of the Tolerance Analysis Process

The application of a tolerance analysis is not limited to parts created using the same manufacturing processes. Parts created using net-shape manufacturing processes (defined in Section 4.1.1), such as casting and injection molding, can be combined with sheet metal or machined parts to create a functional assembly, provided the variation for each feature, created using that specific process, is known. It would be expected that an

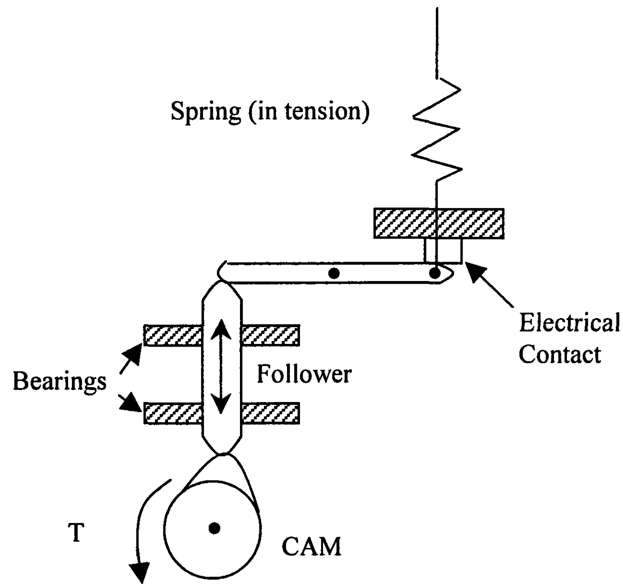


**Figure 2.4: Electrical Switching Mechanism**

injection molded post would not have a variation similar to that of a post that has been die cast.

A tolerance analysis is versatile and flexible enough to model variations in the part's material properties. These material property variations may then affect other assembly models that depend on these material properties. For example, consider the mechanism shown in Figure 2.4. An important question to consider would be, "Is the torque, T, large enough to turn the cam, which then activates the mechanism, thus disconnecting the electrical contact?" In order to answer this question, a model would be developed to represent the frictional interaction between the cam and the follower. The tolerance analysis model would be built and constrained, followed by applying variations to each individual feature, and also to the material properties of each part. Next a Monte Carlo simulation would be initiated, using anywhere from $10^3$ to $10^5$ trials. From this simulation, and also using classical mechanics, one would be able to determine if the provided torque will be able to be overcome the resistive friction, thus answering the posed question.

If the first question concerning having enough torque to activate the system had been successfully answered, then emphasis could be placed on the point of electrical contact in the mechanism. The next question asked could be, "Does the electrical contact open and close at the right times?" To answer this question, a similar approach as before would be taken. The tolerance chain would be defined and then variations would be applied to the system.

It is quite possible for the proposed system not to meet a product specification stating that the electrical switch must open and close at the correct time 99.99% of the time. If this is indeed the case, then one can turn to a sensitivity analysis that demonstrates which feature variations most directly impact the assembly's final quality level. These features might then be targeted for a possible redesign.

# Chapter 3

# Metrology

There are several different types of inspection schemes to be considered today when first specifying a measurement plan for a production part. Ideally this plan should be developed concurrently with the actual design of the part. However this is not the prevalent method in industry today, and therefore the task of the inspector is not as ideal as it could be.

[Miller, 1962] has defined metrology, as "the science of precise measurement, the term 'precise' implying an attempt to determine the true value of a magnitude. It is however, a simple scientific truth that the true value of any magnitude is only that which we agree to regard as such, the value, that is, beyond which we have not investigated, or cannot at any particular time investigate further." Miller points out one of the basic flaws of measurements and quality inspections; they are only approximations of what the parts really are.

Section 3.1 discusses the two main types of automated measurement systems encountered throughout the development of this thesis, coordinate measuring machines and vision systems. Some of the data used in this thesis was taken using manual techniques, but these techniques are not discussed in detail here. [Farago, 1968], and [Beckwith, *et al.*, 1982], provide a comprehensive listing of gaging techniques and their different uses for measuring part characteristics. Also the mathematics of proper metrology techniques are not discussed here, but [Wright, 1995], provides for such. Section 3.1 goes on to describe a technique that can be used to take measurement variability into account when determining a part's quality.

Section 3.2 describes several common methods used to report a part's quality. These methods include short-term and longer term reporting methods. These methods can also be used to determine the capability of the process. Two measures of process capability are also discussed in Section 3.2.

Section 3.3 describes how to calculate the overall variance and mean of an entire data set consisting of multiple cavities. Furthermore this technique is accomplished by using only the individual cavity means, variances, and sample sizes. This method can therefore be used when only the summary information about a cavity is known, or the individual cavity information is only in print, and would take a considerable amount of time to re-enter into a spreadsheet format.

Section 3.4 discusses the methods that were used throughout this thesis to organize and manage the gathered measurement data. Web technology was used so that this data was available on multiple platforms, and accessible regardless of geographic location.

## 3.1 Types of Measurement Equipment

Production parts can be measured in several different ways, but these different types of measurement schemes are most commonly broken down into two fields: automated and non-automated. Among the automated techniques for measuring parts are coordinate measuring machines (CMM), and vision systems. The non-automated or manual techniques include using dial indicators, gauge blocks, and optical comparators. These traditional methods are still used regularly to measure production parts, but as parts with tighter dimensional requirements are being manufactured, these methods are slowly being replaced by automated techniques as the measurement method of choice, although there will still be opportunities for inspectors to measure parts using manual techniques.

Coordinate measuring machines are the most common type of automated measuring equipment in industry today because of several reasons, including their versatility in measuring many different part geometries with features not necessarily located on the same side of the part. CMMs are also directly linked with a computer-controlled system and are consequently sometimes called DCCs for Directly Computer Controlled. Figure 3.1 shows a typical CMM. A CMM uses a probe to determine the location of the edges of a part by sensing the amount of resistance the probe encounters, or by sensing a deflection in the probe caused by contact with the part. When an edge has been detected, the CMM records the X, Y, and Z location of the probe, thus recording the location of the desired point.

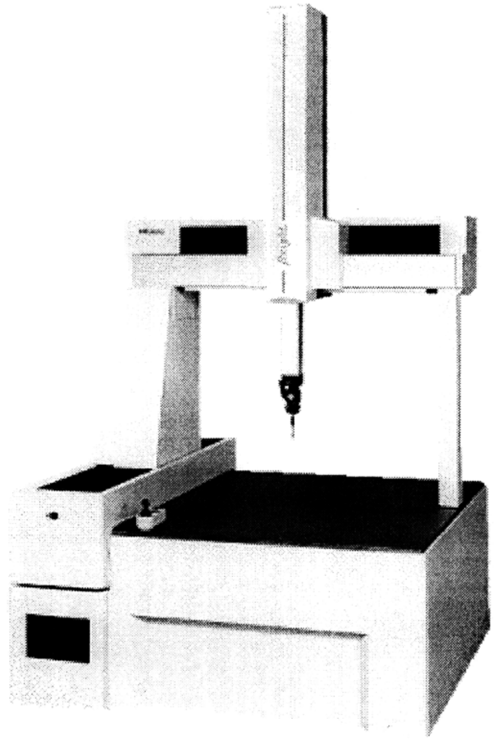


**Figure 3.1: Typical Coordinate Measuring Machine**

There are several different factors that can introduce variation when measuring parts with a CMM. If a probe tip senses the amount of force it encounters, then the type of force sensor used must be calibrated properly, and also specified for the particular part being measured. The amount of probe deflection should also be taken into account. [Nawara and Kowalski, 1985] Also the diameter of the probe, which is typically spherical must also be considered. If the probe tip moves at a rather rapid pace, when measuring the part, the dynamic effects of the entire CMM must also be considered. [Singhose, *et al.*, 1996] and [Yuhai, *et al.* 1996] have considered these effects and developed methods to model the dynamic effects of a CMM. [Zhang, *et al.* 1985] have also modeled some of the different sources of error in a CMM. Changes in ambient temperature have also been known to effect the quality of a measurement.

A vision system is another type of automated measurement equipment that is frequently used to determine a part's dimensional quality. A vision system focuses different sources of light on the fixtured part, and when directed properly, will illuminate the desired feature being measured. This contrast of light and dark can then be used to determine the boundaries of the part. Using light sensors poses some significant limitations on the types of parts and features that can be measured using a vision system. Because a vision system uses a light and dark contrast to determine a feature's boundaries, the part must be opaque enough to produce enough contrast for the vision system to detect. This almost completely eliminates parts that are transparent, although focusing light at different angles and positions around the part can sometimes permit the measurement of transparent parts for the trained and well-skilled inspector. Blind holes are also often difficult to measure for the same reason, and features placed in locations where the vision system cannot see are obviously not measurable.

Again the use of light as a non-mechanical method to measure a dimension on a part, can be affected by any surface irregularities in the feature being measured. For example, any residual molding grease or dirt remaining on the part will be also detected and will therefore distort the resulting measurements. Small imperfections, such as small indentations, or scratches will also be detected.

Both CMMs and vision systems allow for the inspector to rapidly measure several different parts in a short period of time, by using a stored sequence of automated commands. Also these types of automated inspection techniques allow for the calculation of statistical parameters that are essential when determining the dimensional stability of the process.

### 3.1.1 Measures of Measurement Capability

The aim of measuring parts is to determine their dimensional quality. Inherent in this goal is the reality that the measurement process is not completely precise. Therefore the degree to which a measurement machine can inspect parts must also be measured. Before this measure is discussed, several terms must be clearly defined, so as to prevent confusion. [Kalpakjian, 1995], lists several of these terms and they are listed below for reference.

*Accuracy* - The degree of agreement of the measured dimension with its true magnitude.

*Calibration* - Adjusting or setting an instrument to give readings that are accurate within a reference standard.

*Precision* - Degree to which an instrument gives repeated measurement of the same standard.

*Resolution* - Smallest dimension that can be read on an instrument.

*Rule of 10 (Gage Maker's Rule)* - An instrument or gage should be 10 times more accurate than the dimensional tolerances of the part being measured. Similarly, a factor of 4 is known as the *Mil Standard rule.*

*Stability* - An instrument's capability to maintain its calibration over a period of time (also called *drift*).

Often the measure of how well a measurement machine inspects parts is precision. This is accomplished by measuring the same part multiple times, and comparing these measurements. This measure is called a Tool Capability Index (TCI) and is defined using Equation 3.1:

$$TCI = \frac{(USL - LSL)}{6\sigma_{measurement}} \tag{3.1}$$

where USL is the upper specification limit or the dimension's nominal + upper tolerance, LSL is the lower specification limit or the dimension's nominal - lower tolerance, and $\sigma_{measurement}$ is the standard deviation of the measurements taken on the dimension of interest.

To conduct a TCI evaluation, first a sample size is chosen, typically about 20, and then the same part is measured many times. If a fixture is used to secure the part to the worktable, then the part is removed and replaced into the fixture after the part is measured each time. This also introduces another source of variation into the measurement scheme, often encountered when real measurements are taken. Removing the part from the fixture after each iteration in a TCI study reflects the actual case when the operator must remove the part just measured, and replace it with another part yet to be measured.

Fixturing error can significantly affect the dimensional quality of a part especially when measuring features that are easily deflected. Therefore the fixture must be designed to help minimize these variations, and also to assure that the part is placed into the fixture in the same position for each iteration.

### 3.1.2 Combining Measurement Variability with Dimensional Variability

The goal of inspecting a part is to determine the part's “true” dimensions. Determining the exact dimensions of a part is an impossible task, but determining the approximate dimensions of a part is not. The Rule of 10, as stated in section 3.3.1, was intended to guide the inspector to choose a measurement device that is at least 10 times more accurate than the dimensional tolerances of the part being measured. If the measurement tool being used was exact, there would be no need for this guideline. Because measurements are not exact and are only approximations, the presence of measurement variability must be recognized and addressed.

The proper technique that should be used to take measurement variability into account is described by Equation 3.2.

$$\sigma^2_{\substack{Actual\\Dimensional\\Variability}} = \sigma^2_{\substack{Observed\\During\\Measurement}} - \sigma^2_{Measurement} \tag{3.2}$$

where, the measurement variance is subtracted from the dimensional variation observed during measurement yielding what the dimension's actual variance is [Caffrey, *et al.* 1995]. This relationship assumes that the variation incurred through measurement is linearly independent from the variation of the actual dimensional. Linear independence appears to be valid when considering injection molded parts, because the molding of the part has no direct first order relationships with the measurement equipment.

## 3.2 Standard Measurement Reporting Measures

Determining the accuracy of a mold and a part is one of the first steps taken when certifying a tool for full-scale production. This is first accomplished with a procedure that can be called a Tool Inspection Report (TIR). A TIR consists of measuring each specified dimension on a single part only once. This is not a method of statistical sampling and can only determine if the particular sampled part is within the specified tolerance band. Other measures are often used to report dimensional measurements including Process Capability Indices, and a procedure that can be called a Statistical Tool Inspection Report (STIR). The explanation of a STIR is given in the following section.

### 3.2.1 Statistical Tool Inspection Report (STIR)

A statistical method used to determine the quality of a mold could be called a Statistical Tool Inspection Report (STIR). This consists of sampling 20-40 parts, per cavity, from an injection molding machine and determining their dimensional quality by using a CMM, vision system, or other inspection process. Most commonly, automated procedures are used to measure the parts, but manual methods are still sometimes used.

The data taken from a STIR is often separated according to cavity and typically lists the individual part measurements for each dimension, in conjunction with the data's average and standard deviation.

STIRs are not only conducted when initially certifying a mold for production. A STIR can be included in a part's typical quality inspection plan aimed at monitoring the part's dimensional stability over time. A STIR can also be used as a diagnostic tool when the assembly stage of production indicates that the parts being produced are not acceptable.

The fact that a STIR can be used as a trouble-shooting tool is one of considerable importance. If the assembly of a group of parts is experiencing problems, and the particular part that is the source of the problems can be identified as being dimensionally unstable, a STIR could be used as a diagnostic tool to determine the source of the undesired variation. The STIR would then produce data that is only characteristic of problematic parts, and not parts that are run consistently without any dimensional instability.

This consideration may very well eliminate the possibility of using STIR data to build a variational model. However the amount of data obtained from different STIR reports is often the most plentiful, compared to all others, and should therefore not be excluded. Also, the parts used in a STIR are most commonly taken sequentially, shot after shot, and only represent short-term variation. Longer-term studies capture other sources of variation like tool wear, cavity rework, or a change of material supplier.

Another disadvantage of conducting a STIR is that often the results obtained from a STIR are reported back to the STIR's requestor and then discarded. The trend in the past was for a STIR to be printed out and passed along to the STIR's requestor and the source datafiles for the STIR would be deleted. When automated measurement equipment is used to conduct a STIR, the data is organized in an easy to retrieve and convenient format. This convenience of obtaining the information is lost when the measurements are printed and the source files are deleted. The author has found this to be the case in several instances, and there are two possible alternatives to recover the part's dimensional information in an easy to retrieve format.

First the data could be re-entered into spreadsheet format point by point because the original data points are often printed on the STIR report. This is a time consuming process and is not ideal. The second option is aimed at obtaining the standard deviation and average of all the cavities in the STIR. Often the measurement requestor suggests that the data taken for a particular part be organized according to cavity and mold. This separation will aid in the determination of which cavity is the troublesome one if such is the goal. Expressions have been derived, and are presented in section 3.3, that describe the mathematics that relate individual cavity averages and standard deviations to the combined average and standard deviation over all cavities.

### 3.2.2 Measures of Process Capability

Besides reporting the direct measurements of the dimensions being considered, measures of process capability are also becoming increasingly popular. Equation 3.3 shows one way by which dimensional variation can be related to the specifications of the dimensions.

$$C_p = \frac{USL - LSL}{6\sigma} \tag{3.3}$$

Cp relates the product's tolerances and the standard deviation of the dimension to yield a dimensionless number. One of the main criticisms of using Cp is that it does not take the mean of the process into account. It is quite possible to have a high value of Cp, for example 2.0, although the process is still producing vast quantities of poor quality parts. This has led to the development of another measure of process capability, called Cpk, given in Equation 3.4.

$$C_{pk} = \frac{Min(USL - \bar{x}, \bar{x} - LSL)}{6\sigma} \tag{3.4}$$

Cpk is again, like Cp, a dimensionless number that measures process capability, but Cpk takes the mean of the process into account. Figure 3.2 illustrates the differences between Cp and Cpk. Three different cavities (A, B and C) have been previously determined to produce parts according to the distributions shown in Figure 3.2. Figure 3.2 illustrates that in order for the values of Cp and Cpk to be identical, the process must be centered about its nominal value, otherwise Cpk will be less than Cp.

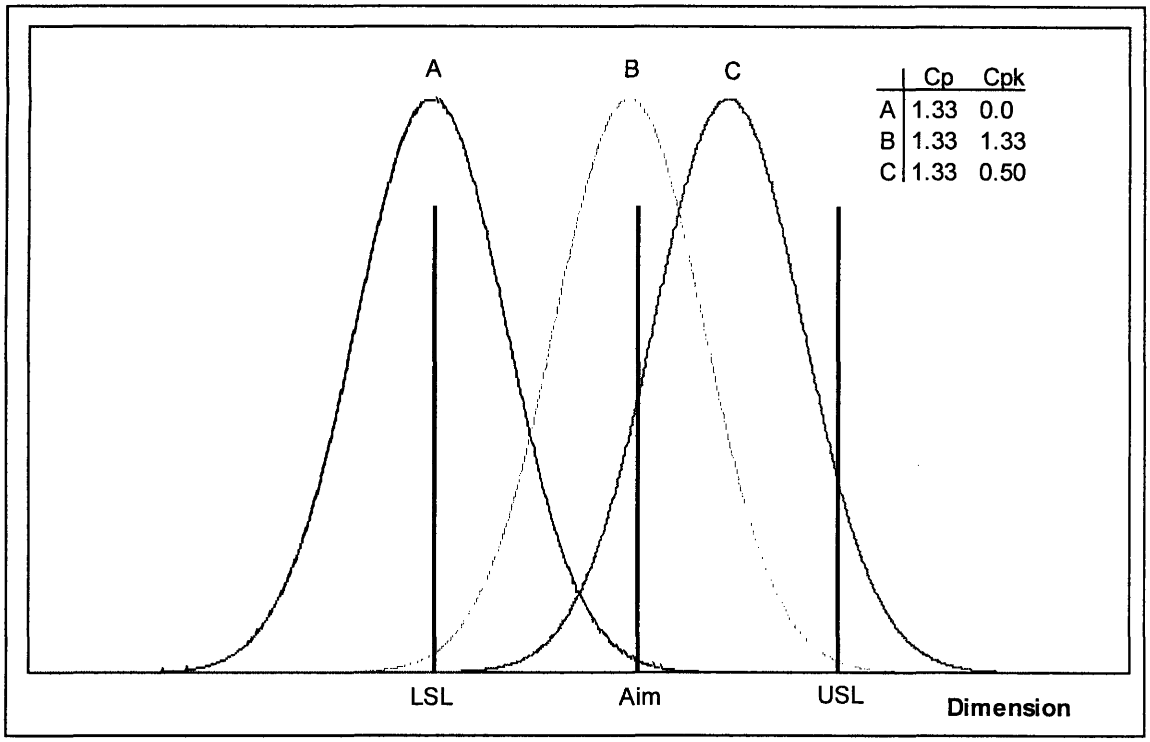



**Figure 3.2: Effect or Process Centering on Cp and Cpk**

## 3.3 Calculating Overall Standard Deviation and Average

When a product is being assembled, it is quite possible that a part can originate from multiple sources. Examples of this are parts made by outside suppliers, and parts being molded using different cavities, molds and different presses. In the case of injection molded parts, the assembly station completing the assembly task does not care which mold or cavity the part originated from, but only cares that the part meets its specifications. This is why the quality of a dimension is reported separately for each cavity and also collectively for the entire part. The data is reported separately according to cavity so that the cavity producing unacceptable parts can be identified, and possibly reworked.

As was stated in section 3.2.1 the output from a quality study is often printed, followed by a deletion of the associated measurement computer files. The deletion of these files significantly reduces the amount of analysis that can be conducted on the data because the printed measurements are sorted according to cavity and rarely give the total statistical properties of the sample. This is the reason that the relationship between individual cavity standard deviation and mean and overall standard deviation and mean has been investigated. This relationship is discussed next beginning with the development of the relationship for overall mean.

The mean of a set of data is defined by the ratio of the sum of all data points to the total number of data points, and is expressed by Equation 3.5.

$$\bar{x}_j = \frac{\sum_{i=1}^{nj} x_i}{n_j} \tag{3.5}$$

where n is the total number of samples in the data set, $x_i$ is an individual data point, and j can either represent an entire cavity or the entire data set. Equation 3.5 holds true for either an individual cavity or for all cavities combined. If all the data points were known then Equation 3.5 would be the only formula required to determine the average of the entire data set. However if only the cavity averages, and the cavity sample sizes were available then further analysis is required, because Equation 3.5 requires that the individual data points be known.

If both sides of Equation 3.5 are multiplied by $n_j$, then the following expression will be found:

$$\sum_{i=1}^{nj} x_i = \bar{x}_j n_j, \tag{3.6}$$

thus resulting in the sum of all data points. The extrapolation of this simple concept to the derivation of an overall mean will be discussed in an example considering only two cavities.

First assume that two cavities have averages $\bar{x}_1$, and $\bar{x}_2$, and sample sizes $n_1$, and $n_2$ for cavities one and two respectively. The mean for both cavities can be expressed as the sum of all data points for cavity 1 and cavity 2 divided by the sum of the samples in cavity 1 and cavity 2. This is shown in Equation 3.7.

$$\bar{x}_{total} = \frac{\sum_{i=1}^{n1} x_i + \sum_{i=1}^{n2} x_i}{n_1 + n_2}. \tag{3.7}$$

Substitution of Equation 3.6 into Equation 3.7 yields:

$$\bar{x}_{total} = \frac{\bar{x}_1 n_1 + \bar{x}_2 n_2}{n_1 + n_2}. \tag{3.8}$$

This expression for the mean of two cavities as a function of the individual cavity averages and sample sizes can be generalized to determine the average of *m* cavities. This generalized expression for a total average is given in Equation 3.9.

$$\bar{x}_{total} = \frac{\sum_{j=1}^{m} \left(\bar{x}_j n_j\right)}{\sum_{j=1}^{m} n_j}. \tag{3.9}$$

where j represents an individual cavity.

Now that an expression for the total average has been developed, the task of determining the overall variance is investigated. The definition of a sample variance is the sum of the squared deviations of each data point with the sample average, divided by the sample size minus one. This relationship is shown in Equation 3.10.

$$\sigma^2 = \frac{\sum_{i=1}^{n} \left((x_i - \bar{x})^2\right)}{n-1}, \tag{3.10}$$

where $\sigma^2$ is the variance of a particular data set with a sample size *n* and an average $\bar{x}$. The numerator of Equation 3.10 is expanded and yields Equation 3.11.

$$\sigma^2 = \frac{\sum_{i=1}^{n} \left(x_i^2 - 2x_i\bar{x} + \bar{x}^2\right)}{n-1}. \tag{3.11}$$

Distributing the summation, and remembering the identity that $\sum_{i=1}^{n} (2x_i) = 2\sum_{i=1}^{n} (x_i)$, Equation 3.11 is reduced to Equation 3.12.

$$\sigma^2 = \frac{\sum_{i=1}^{n} \left(x_i^2\right) - 2\bar{x}\sum_{i=1}^{n} (x_i) + \sum_{i=1}^{n} \left(\bar{x}^2\right)}{n-1}. \tag{3.12}$$

The $\bar{x}$ term in Equation 3.12 can be taken outside the summation because $\bar{x}$ is a constant. Using the identity that the summation of a constant over all n, is n times that constant $\sum_{i=1}^{n} (\bar{x}) = n\bar{x}$, Equation 3.12 reduces to Equation 3.13.

$$\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i^2\right) - 2\bar{x}\sum_{i=1}^{n}\left(x_i\right) + n\bar{x}^2}{n-1}. \qquad (3.13)$$

Next the substitution of the definition of $\bar{x}$ (Equation 3.5) into Equation 3.13 yields Equation 3.14.

$$\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i^2\right) - 2\frac{\sum_{i=1}^{n}\left(x_i\right)}{n}\sum_{i=1}^{n}\left(x_i\right) + n\left(\frac{\sum_{i=1}^{n}\left(x_i\right)}{n}\right)^2}{n-1}. \qquad (3.14)$$

Rearranging terms in Equation 3.14 yields Equation 3.15.

$$\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i^2\right) - \frac{2}{n}\left(\sum_{i=1}^{n}\left(x_i\right)\right)^2 + \frac{1}{n}\left(\sum_{i=1}^{n}\left(x_i\right)\right)^2}{n-1}. \qquad (3.15)$$

Equation 3.15 can the be simplified to obtain,

$$\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i^2\right) - \frac{1}{n}\left(\sum_{i=1}^{n}\left(x_i\right)\right)^2}{n-1}. \qquad (3.16)$$

Although Equation 3.16 was developed to represent the individual variance of a single cavity, it can also be used to derive the variance of the entire data set if the following extensions to Equation 3.16 are made. First, $n$ represents the sample size of the entire data set. Second, $\sum_{i=1}^{n}\left(x_i^2\right)$ represents the sum of each data point squared for the entire data set. Finally, $\sum_{i=1}^{n}\left(x_i\right)$ represents the sum of each data point, also over the entire data set.

All the terms in Equation 3.16 are derivable from the individual cavity averages, and sample sizes, although in a semi-clouded form, except for $\sum_{i=1}^{n}\left(x_i^2\right)$. This quantity cannot be derived without the further manipulation of Equation 3.16 and furthermore, requires the variances of the individual cavities. Solving Equation 3.16 for $\sum_{i=1}^{n}\left(x_i^2\right)$ yields Equation 3.17.

$$\sum_{i=1}^{n}\left(x_i^2\right)=\sigma^2(n-1)+\frac{1}{n}\left(\sum_{i=1}^{n}(x_i)\right)^2. \tag{3.17}$$

Therefore, if the variance of each cavity, the cavity averages, and the cavity sample sizes are known, the total variance of all cavities can be determined. This will be illustrated through an example consisting of only two cavities.

Again assume that two cavities have averages $\bar{x}_1$, and $\bar{x}_2$, sample sizes $n_1$, and $n_2$, and variances $\sigma_1^2$, and $\sigma_2^2$ for cavities one and two respectively. First the quantities $\left.\sum_{i=1}^{n}\left(x_i^2\right)\right|_{cavity1}$, and $\left.\sum_{i=1}^{n}\left(x_i^2\right)\right|_{cavity2}$ are calculated using Equation 3.17 for each individual cavity. Next the second term in the numerator of Equation 3.16 can be found by manipulation of the definition of cavity averages to obtain Equation 3.18.

$$\sum_{i=1}^{n}(x_i)=\bar{x}_1 n_1+\bar{x}_2 n_2 \tag{3.18}$$

Substituting $\left.\sum_{i=1}^{n}\left(x_i^2\right)\right|_{cavity1}$, $\left.\sum_{i=1}^{n}\left(x_i^2\right)\right|_{cavity2}$, Equation 3.18 and also the relationship for the total sample size into Equation 3.16 yields Equation 3.19.

$$\sigma_{total}^2=\frac{\left(\left.\sum_{i=1}^{n1}\left(x_i^2\right)\right|_{cavity1}+\left.\sum_{i=1}^{n2}\left(x_i^2\right)\right|_{cavity2}\right)-\frac{\left(\bar{x}_1 n_1+\bar{x}_2 n_2\right)^2}{\left(n_1+n_2\right)}}{\left(n_1+n_2\right)-1} \tag{3.19}$$

Substituting Equation 3.17 for $\left.\sum_{i=1}^{n}\left(x_i^2\right)\right|_{cavity1}$, and $\left.\sum_{i=1}^{n}\left(x_i^2\right)\right|_{cavity2}$ yields Equation 3.20.

$$\sigma_{total}^2=\frac{\left[\left(\sigma_1^2\left(n_1-1\right)+\bar{x}_1^2 n_1\right)+\left(\sigma_2^2\left(n_2-1\right)+\bar{x}_2^2 n_2\right)\right]-\frac{\left(\bar{x}_1 n_1+\bar{x}_2 n_2\right)^2}{n_1+n_2}}{\left(n_1+n_2\right)-1} \tag{3.20}$$

Equation 3.20 represents the relationship that can be used to describe the standard deviation of two cavities when only given their individual sample sizes, means and variances. Equation 3.20 can be generalized for a sample including $m$ cavities. This expression is shown in Equation 3.21.

$$\sigma_{total}^2 = \frac{\sum_{j=1}^{m}\left(\sigma_j^2(n_j-1)+\bar{x}_j^2 n_j\right)-\frac{\left(\sum_{j=1}^{m}\bar{x}_j n_j\right)^2}{\sum_{j=1}^{m} n_j}}{\sum_{j=1}^{m} n_j - 1} \tag{3.21}$$

where j represents a particular cavity.

To summarize, an analytical expression has been derived that relates individual cavity variances, means, and sample sizes, to the overall mean and variance. This is an important relationship to consider because an assembly station receives parts from the entire population of cavities, and not just a single cavity. This method of sampling usually leads to the reporting of both individual cavity and overall statistical characteristics.

## 3.4 Measurement Management throughout this Thesis

In order to validate the variational models developed in this thesis, production data has been used as a statistical verification tool. Verification of these rules is only possible when looking at the parts on a feature level. For example the data associated with a plane cannot be used as an input to the variational model describing the variation of a hole. Therefore one of the first tasks required after a data set was received, was to classify each part and each feature on a part according to several categories.

To aid in this process, an MS-Access database was created to store and manage this part and feature information. The data was classified into several categories including mold, measurement, feature, and general information, and there was also one record for each dimension for every part. The records were divided in this manner so as to ease later classification according to a number of different categories. A complete listing of these categories and their groups is shown in Table 3.1. Information for every field and every dimension was not known, but all the data that was obtained was entered into the database.

Several of the fields in the database are actually hyperlinks to other files such as spreadsheets and image files that are scanned in print drawings. Hyperlinks were found

| General Information | Mold Information | Measurement Information | Feature Information |
|---|---|---|---|
| Date Data Entered | Press# | Meas. Validation | Units |
| Part Number | Press Make | Meas. Fixture Available | Feature # |
| Part Name | Press Tonnage | Meas. Inspector | Feature Type |
| Part Program | Gate Type | Meas. Date | Image of Feature (link) |
| Revision # | Runner Type | Meas. Tool | Feature location on Print |
| Part Contact | Balance Runner | Meas. Data (link) | Across Parting Line (Y/N) |
| CAD File Location (link) | Velocity Control | Meas. Explanation (link) | Tool Number(s) |
| Material Type | Process Window | Meas. Video (link) | Cavity Number(s) |
| Regrind (Y/N) | | Meas. Program (link) | |
| Yearly Production Volume | | | |
| Production Location | | | |
| # of Cavities in Database | | | |
| # of Tools in Database | | | |
| Current Variational Model Using Data | | | |
| Other | | | |

**Table 3.1: Listing of All Database Fields Categorized to Group**

to be especially well suited for storing measurement information, because the process of entering all the required statistical parameters into the database can be a time consuming process, and subject to change. However hyperlinks only store the location of the file being linked to. The contents of the source link can change, but the location of the file being linked to will not change, unless the object is moved to a different location, and the link not updated. Using a hyperlink to retrieve information, as opposed to a single cell storing a number, gives the user more flexibility to use the data. The user can manipulate the data to serve a particular purpose by downloading a local copy of the source spreadsheet, use it to serve a particular purpose, and then upload the file, back to a central location.

A hyperlink also allows the user to view an engineering drawing of the dimension being investigated. The appropriate part prints were classified according to their orthographic and section views and were scanned individually using a scanner at a resolution of 75 DPI. Next each image file was associated with its appropriate dimension, and a hyperlink was created for each dimension.

In conjunction with part links to spreadsheets and part prints, the database has been designed to be flexible enough to rapidly change, while still remaining stable. The database also draws information from external tables, such as material type and feature type. The flexibility of a database also allows for the possibility to link this database with a web browser through the use of Active Server Page technology.

Presenting the collected dimensional measurement information via a web interface provides several advantages. First, using the web allows users located on multiple platforms to view the same data. Second, a web interface allows a central database to be stored on one computer, so multiple copies are not necessary. Therefore outdated paper copies are never an issue. Third, using web technology does not restrict the geographic location of a user. Appropriate passwords and permissions can also be applied to proprietary pages so that an unauthorized visitor will be locked out. Figure 3.3 shows how a database and web system was used throughout this thesis.

Also the presentation of part information can take any form imaginable when using a web interface. Sorting, searching, and editing the data contained in the central database can all be done directly from a web browser. The information's format can be changed to incorporate new fields or groups as it becomes necessary. As it stands today it appears as if using web technology to distribute information is the most efficient, and least expensive method, providing that the appropriate security precautions are taken.

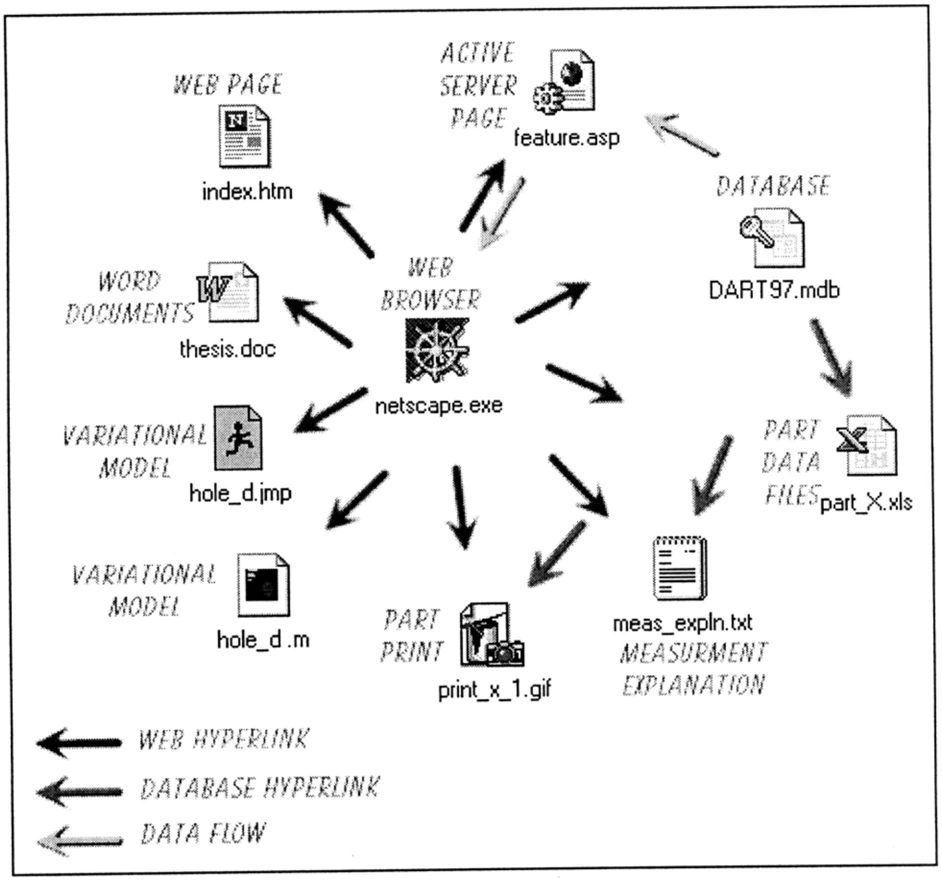



**Figure 3.3: Schematic of Measurement Data Management Using Web Technology**

# Chapter 4

## Design and Modeling of Injection Molded Parts

The design of several interacting injection molded parts is not a trivial process and often involves engineers in multiple disciplines, including structural analysis, manufacturing tool design, cost analysis, and variation analysis. Each of these areas can be decomposed further into sub-disciplines each requiring much skill and knowledge to complete effectively. Only when all of these different fields seamlessly interact will a successful product be produced.

Section 4.1 describes several reasons why designing a successful product composed of injection molded parts is difficult, including large part count, structural part modeling, and long lead times associated with constructing and debugging a mold. Plastic parts are more difficult to model than traditional metal parts, because their material properties are very complex and non-linear. In addition to structural issues, the long time delay required for mold manufacturing between the design and actual manufacturing of a part reduces the number of possible design iterations.

Section 4.2 discusses three different methods commonly used to design an assembly of injection molded parts: design based on experience, prototyping and analysis. When designing a part, the requirements for the product are usually specified, and it is then left for the designer to creatively satisfy these functional requirements. Often an older design will be adapted to fit the current challenge, and the design team will use their past experiences when designing a part. Prototyping is another tool that is commonly used throughout the design stages of a part, to better understand the esthetics and mechanics of the part. The most traditional and sound practice to use when designing a part is to use different types of Engineering analysis to verify that the design in question will indeed meet the part's functional requirements. In practice a combination of all three methods are used when designing a part.

Section 4.3 discusses different sources of variation that influence the final quality of a part. Expressions have been derived that relate the overall mean and variance of parts produced using multiple cavities to the variances, means, sample sizes, and cavity mean offsets of each cavity. These equations can be used to determine the effect that nominalizing a cavity will have on the total variance of a part across multiple cavities.

Section 4.4 describes the most common types of methods available today for modeling, and managing a part's variation. These methods are classified according to two main categories: models developed before the complete geometry of the part is known, and models developed after. The last method presented in this section involves redesigning the part and/or mold using a trial and error approach where the goal is to decrease the part's variation.

Section 4.5 gives an overview of the author's approach to modeling variation in injection molded parts. These methods are based on quantities that should be known during the preliminary design of a part, thus providing a large benefit for the amount invested in modeling.

## 4.1 Difficulties of Designing Injection Molded Parts

There are several different reasons why designing an assembly of Injection Molded parts is so difficult (Appendix C). First, traditional theories using formulas derived from structural mechanics can be used as first order approximations, but at higher levels of stress and strain these relationships become non-linear. The second factor that makes an injection molded product so difficult to design is the large lead times associated with tooling creation. Molding simulations are often used to aid the tool designer when designing cavity geometry, because the design of a mold is rarely a simple “scale-up” procedure. Designing an assembly of injection molded parts also becomes more complex as the number of parts in the assembly increases. As the number of plastic parts increases, the interactions between the manufacturing and design teams becomes more critical if the target manufacturing and assembly delivery dates are to be met.

### 4.1.1 Large Part Count

Injection Molded parts are extremely common today and can be found in numerous products with a part count ranging from one to several hundred different functional parts. Products containing several hundred parts are obviously more difficult to design and build because the numerous design teams must seamlessly interact. Similar problems also exist with non-plastic parts, but one difference between the two is that injection molding is a net-shape manufacturing process. [Kalpakjian, 1995] has defined a net-shape or near-net-shape manufacturing process to be one, "in which the part is made as close to the final desired dimensions, tolerances, and specifications as possible." Machining, on the other hand, uses many different operations to create a part and is not a net-shape manufacturing process.

Net-shape manufacturing processes provide the designer with several options that other processes can not provide, but also suffer from several disadvantages. These include the high costs associated with tooling and the long tooling times required to create and then debug the mold. Consequently tool development for net-shape manufacturing processes is a costly process, and should not be lightly interrupted. These disadvantages could deter a design team from using a net-shape manufacturing process to create a part. However the advantages of cost savings associated with a large production run of parts often negates these disadvantages. Another advantage of the injection molding process is the short cycle times required to create the parts. Depending on the volume and overall geometry of the part, the part's cycle time could be as low as a few seconds. This would result in the rapid production of a large number of parts, once the mold is built and debugged.

As the number of planned production parts increases, the benefits gained by using a net-shape manufacturing process increases because the expensive tooling costs can be divided by a larger number of parts thus reducing the individual piece part cost. This reduction in piece part cost is possible because the operating cost of a net-shape manufacturing process is relatively negligible compared to the initial tooling creation costs. Once the planned number of parts increases past a break-even point, a net shape manufacturing process becomes more favorable, due to economies of scale. Figure 4.1 shows this relationship for the injection molding and machining processes.

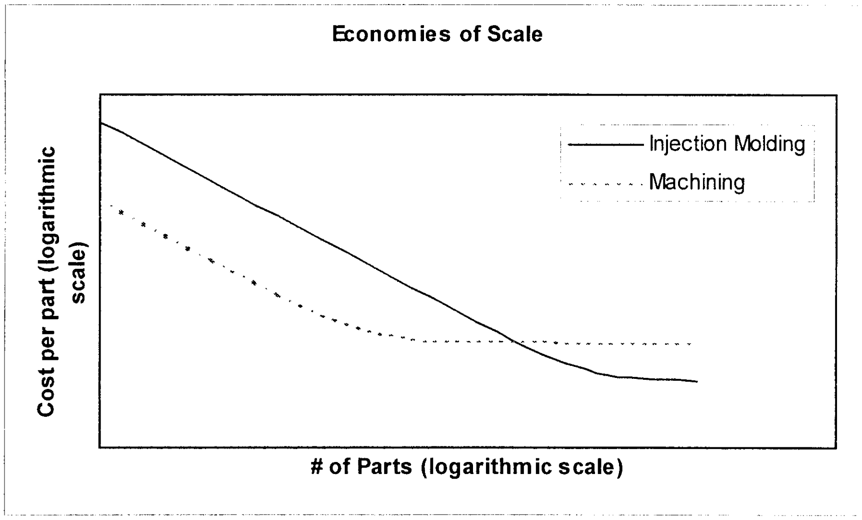



**Figure 4.1: Economies of Scale for two manufacturing processes**

To most effectively use Figure 4.1, an engineer would pick the process that gives the lowest cost for a particular number of parts. Because the initial cost required to create the mold for injection molded parts is so high, the injection molding curve is initially much higher than the machining curve. However, as the number of estimated parts increases, the difference between the two processes decreases because the per part cost for injection molding is much lower than the corresponding price for machining. Therefore, if the estimated number of manufactured parts is not predicted to be larger than a "break even" point, it does not make economic sense to manufacture the part using the injection molding process. Figure 4.1 was generated assuming that the initial one-time costs for the injection molding and machining processes were \$200,000 and \$2,000 respectively. Also the injection molding and machining processes were assumed to cost \$0.05/part and \$0.50/part respectively at full production.

## 4.1.2 Structural Part Modeling

Traditional theories involving structural mechanics can often be used as first order solutions when questioning the structural integrity of a part subject to certain loading conditions. However, it must also be understood that these approximations were originally developed to approximate the structural behavior of metal, and not plastic materials. Consequently, structural models have been specifically designed to model plastics, but these approximations are considerably more complex. When the amount of stress or strain in a part is expected to be at a high level, the relationship between stress and strain becomes nonlinear, and higher order approximations should be used to accurately describe the behavior of the part.

Plastic injection molded parts are also heavily dependent on the processing conditions at the time of molding. This is true for all net-shape manufacturing processes. Specifics to the injection molding process are processing issues like the flow orientation of fillers in the material, and anisotropic material properties resulting from this orientation. Filler orientation can lead to differences in material properties between the parallel to flow and perpendicular (cross) to flow directions. Traditional first order approximations for structural conditions assume homogeneous material properties throughout the part and thus do not reflect this anisotropic behavior.

As [Rosato and Rosato, 1995] state, "The relationship between stress and strain, or structural response of plastics varies from viscous to elastic." It is common to expect a plastic part to display a structural response that is somewhere between the two depending on several factors. The elastic response of the part can be modeled as a Hookean (elastic) solid or a linear spring. In this case stress is proportional to strain and all energy put into the part is fully recoverable. The viscous response of the part can be modeled as a Newtonian fluid or a dashpot. In this case the stress is proportional to strain rate and very little of the energy put into the dashpot is recoverable after the load has been removed. [Nielsen, 1962]

Creep in a plastic part is modeled as the time-dependent increase in the strain of a viscous or viscoelastic material under sustained stress. Some of the energy placed into the system is recoverable after the load has been removed. Creep experiments on plastics are often conducted with high stresses resulting in possible "necking" of the part. Consequently, the applied load may be reduced during the experiment to reduce the probability of the test specimen from failing. [Rosato and Rosato, 1995]

The structural modeling of a plastic part must also include such conditions as the magnitude and duration of stress, strain and temperature. Rosato states it simply, "At a given temperature, both the magnitude and duration of stress or strain affect structural response and strength behavior. Conversely, at a given magnitude and duration of stress and strain, a shift in temperature can produce marked changes in structural response and strength behavior." [Rosato and Rosato, 1995] The environment in which a plastic part is intended to operate will also vastly affect the structural response of the part. Ultraviolet (UV) radiation, caustic chemical agents, sustained elevated temperatures, and even the amount of water present can affect the part's performance. All these factors make the structural modeling of a part difficult and a non-trivial process.

### 4.1.3 Long Lead Times

The long lead-time often encountered between the design and manufacturing of an injection molded part is primarily due to the time required to create and troubleshoot a mold. The process used to determine the geometry of the mold cavity is complex, and requires the combination of properly constrained molding simulations reinforced with past experience. The redesign and troubleshooting stages of mold development have been significantly reduced recently due to the improvement of shrinkage predictions using improved material models [Mahishi, 1998].

One technique that should be used to reduce the time required to create a mold is to use standard sized components when building the mold. This includes using standard mold and cavity sizes as well as standard interfaces between the mold and the machine. Standardization of components could improve the phasing of the mold into production, especially when the mold is designed outside of the factory.

Because the costs associated with creating an injection mold are so high, estimating these costs is an important factor in the preliminary design stage. [Boothroyd, *et al., 1994]* have developed an estimate of the costs involved, and have broken these costs down into two areas: "(a) The costs of the prefabricated mold base consisting of the required plates, pillars, guide bushings, etc. and (b) cavity and core fabrication costs." [Boothroyd, *et al.,* 1994] In addition to these two cost groups, the costs associated with debugging the mold must also be added into the total cost estimate.

The unfortunate result of a long design to manufacturing lead-time is that the design team will be reluctant to fundamentally change the part's geometry even though a more robust design may be possible. Redesign of the part may force parts of the mold to be recreated, thus increasing the necessary time until the part's final delivery. Redesign changes often cause product and delivery schedules to be missed thus costing the manufacturer money already invested in advertising, shipping agreements, and customer disappointment.

## 4.2 Methods of Designing Injection Molded Parts

Once the decision has been made to use the injection molding process to create a part, several different techniques are used to aid the design team. These methods include, but are not limited to, design using past experience, design based on experimental prototypes, and design using analytical methods. Most often, the design process is a mixture of each of these three; however, for simplicity, each of these topics will be discussed separately. The topic of variational analysis will also be more thoroughly discussed in the next section.

### 4.2.1 Design Based on Past Experience

A design team has several tools available when considering how to satisfy a product's functional requirement. One tool that is frequently discounted is the successful experience with past designs. A design team will often subconsciously influence the design of a part by remembering how previous designs were configured. Using past experience can have both advantages and disadvantages. Frequently, the process of adapting previous designs to fit the current design challenge can readily solve current design challenges. It is possible, however, that the past design being adapted caused severe problems when originally manufactured. If the product development loop was not closed by passing this information back to the design team from manufacturing, poor designs can often be reinitiated due to lack of information.

### 4.2.2 Design Based on Experimental Prototypes

Another method available to test different candidate designs is to build functional and aesthetic prototypes. These prototypes can be developed using several different methods including, building prototype molds and actually molding trial parts, creating the parts using conventional material removal processes, using rapid prototyping technology and also visual models using foam and wood.

Typically the first round of prototypes are not intended to be functional, but purely aesthetic in nature. They are most commonly created using different types of foams, soft types of wood and plastics that are painted to have the same look and texture as the real product. These types of prototypes are aimed at determining if the customer accepts the overall size, texture, and color of the product before any substantial engineering analysis is done on the product.

The development of rapid prototyping technologies in the 1980's has given the design team the ability to have working prototypes earlier on in the design cycle. "Rapid prototyping is a process by which a solid physical model of a part is made directly from a three-dimensional CAD solid model." [Kalpakjian, 1995] Rapid prototyping allows for full-scale models of the proposed design to be made in less time and at a lower cost than traditional material removal prototyping processes. These advantages can be accomplished by using various processes including stereo-lithography, powder metallurgy, selective laser sintering, and three-dimensional printing.

Actually creating a prototype mold is the best method available to evaluate an injection molded part's design, but this process is costly and time consuming, just like actual production molding. Creating a prototype mold would reveal problems that wouldn't be encountered using any other prototyping method.

Another, cheaper and quicker method used to evaluate several candidate designs would be to create the parts using traditional material removal processes such as milling, drilling, turning, etc. These material removal processes are fast and allow for several prototypes of candidate designs to be evaluated in the same time that a prototype tool can be created. Unlike using a prototype mold, this method tells nothing about the process. This method of prototyping is made even faster by the interfacing of solid modeling CAD/CAM packages with the material removal machine tools. Manufacturing simulations help aid the prototyping process by eliminating possible machining errors before they manifest themselves into bad parts. Current state-of-the-art technology also allows for these machines to be run unattended and over night, thus decreasing the conventional time required to prototype a part.

### 4.2.3 Design Using Analysis

One design tool that is widely understood as being necessary to the successful implementation of a product is proper engineering analysis. There are several different techniques available today requiring different levels of geometric knowledge about the part. Finite element simulations provide the most information about how a part will perform during operation, but require complete geometric knowledge about the part. Handbook rules of thumb, on the other hand, can also provide valuable information about how the part's performance. These rules of thumb, however, will not be as accurate as a finite element simulation, which is part specific. Their advantage is that they require less geometric information for an analysis, and require much less time. Another design analysis tool available today is a tolerance analysis which predicts how different parts will be expected to interact with one another, given a predicted amount of manufacturing variation.

## 4.3 Parameters Affecting Dimensional Variation

There are several parameters that can affect the amount of variation resulting from a manufacturing process. These parameters can be grouped into three main categories: design, material, and process parameters. For example, some design parameters that affect variation are part geometry, number of estimated cavities, mold geometry, and machine characteristics. Material variation can arise when the vendor supplying material to the process changes, or when a new batch of material is mixed from two different materials.

There are also several different process parameters that can effect the final quality of a product. [Hunkar, 1992] has compiled a list of such parameters, with the intention of using these parameters to rank the quality of an injection molding machine. Hunkar has developed a classification system by collecting data from about 1800 different injection molding machines. He states that, "...Class 1 represents the best machines in operation today and Class 9 represents the worst machines that can still produce quality on highly tolerant tooling and material combinations." Some of these parameters include cycle time, hold time, cavity pressure, oil temperature, *etc.*

One design parameter that plays a large role in the variability of the quality of a part is the displacement of a cavity from it intended nominal position. This quantity is called cavity mean offset. A mathematical relationship has been developed which relates cavity mean offset to the overall variance of a dimension. This relationship will be discussed next, and builds on the discussion provided in section 3.3 that derived relationships for overall variance and mean.

### 4.4.1 Calculating a Change in Variance Resulting from a Cavity Mean Offset

Production quantities and schedules often require multiple cavities to operate in multiple tools in multiple presses. Often all the cavities created do not have means that coincide with one another or even the nominal specification. Cavity mean offset is especially prevalent over time due to factors like tool wear. This is the reason that the effects of cavity mean offset have been investigated with respect to the overall dimensional variance.

The Equations developed in section 3.3 served as the basis for the following derivation. In the following derivation the change in the dimension's overall variance resulting from a change in the mean position of a single cavity is investigated. To make the derivation of this equation more intuitive, it will first be derived assuming that there are only two cavities present, and that only cavity one has a change in its mean. After this derivation is completed, it will be generalized for an arbitrary number of cavities, each of which can experience a mean shift.

First Equation 3.20 can be expressed as a function of six variables as follows:

$$\sigma_{total}^2 = f\left(\bar{x}_1, \bar{x}_2, n_1, n_2, \sigma_1^2, \sigma_1^2\right). \tag{4.1}$$

In order to determine how a small change in $\bar{x}$ will effect $\sigma_{total}^2$, a Taylor series approximation about $(x-a)$, as defined by Equation 4.2 [Salas and Hille, 1990], is used to approximate Equation 4.1.

$$f(x) = \sum_{k=0}^{\infty}\left(\frac{f^{(k)}(a)}{k!}(x-a)\right) \tag{4.2}$$

In this case $a$ represents the current point and $x$ is the point that the Taylor series is approximating. In the case where all cavities are at their nominal values, and the effect of displacing these cavities with respect to nominal is desired, $x$ will be the point of interest and $a$ will be the nominal. However, when each cavity is already displaced from its mean, and the cavities are being nominalized, $x$ will represent the nominal set point and $a$

will represent the point of interest. The definition of a Taylor series in $(x-a)$, shown in Equation 4.2 is applied to Equation 4.1 and yields Equation 4.3.

| Nominal | Cavity # | Mean | Variance | Sample Size |
|---|---|---|---|---|
| 55.6 | 1 | 55.58575 | 0.00016029 | 40 |
| 55.6 | 2 | 55.59836 | 0.00009772 | 40 |
| | total | 55.59205 | 0.00016760 | |

**Table 4.1: Numerical Example for Cavity Mean Offset**

(4.3)

The q … eforth be known as $\Delta\bar{x}_1$. It can be shown that only a second order Taylor series approximation about $(\bar{x}_1 - a)$ is required because the third and subsequently all following derivatives of $f(x)$ are equal to zero. Therefore a second order Taylor series expansion about $(\bar{x}_1 - a)$ will be an exact mapping of a change in $\bar{x}_1$ to a change in $\sigma^2_{total}$. When $f(a)$ is subtracted from both sides of Equation 4.3 the following result is obtained:

$$f(x) - f(a) = \frac{\partial f}{\partial \bar{x}_1} + \frac{\partial^2 f}{\partial \bar{x}_1^2}\frac{(\Delta\bar{x}_1)^2}{2!} \tag{4.4}$$

The left-hand side of Equation 4.4 is now the difference of the original variance and the variance at the new value of $\Delta\bar{x}_1$ and will now be referred to as $\Delta\sigma^2_{total}$ resulting in:

$$\Delta\sigma^2_{total} = \frac{\partial f}{\partial \bar{x}_1} + \frac{\partial^2 f}{\partial \bar{x}_1^2}\frac{(\Delta\bar{x}_1)^2}{2!}. \tag{4.5}$$

The next step is to take the appropriate derivatives of Equation 3.20, shown in Equations 4.6. Substituting these derivatives into Equation 4.5 yields Equations 4.7.

$$\frac{\partial f}{\partial \bar{x}_1} = \frac{1}{[(n_1+n_2)-1]}\left[2\bar{x}_1 n_1 - \frac{2(\bar{x}_1 n_1 + \bar{x}_2 n_2)}{n_1+n_2} n_1\right] \tag{4.6a}$$

$$\frac{\partial^2 f}{\partial \bar{x}_1^2} = \frac{1}{[(n_1+n_2)-1]}\left[2n_1 - \frac{2n_1}{n_1+n_2} n_1\right] \tag{4.6b}$$

$$\Delta\sigma^2_{total} = \frac{2n_1}{[(n_1+n_2)-1]}\left[\bar{x}_1 - \frac{(\bar{x}_1 n_1 + \bar{x}_2 n_2)}{n_1+n_2}\right]\Delta\bar{x}_1 + \frac{2n_1}{[(n_1+n_2)-1]}\left[1 - \frac{n_1}{n_1+n_2}\right]\frac{(\Delta\bar{x}_1)^2}{2!} \tag{4.7}$$

Equation 4.7 represents the exact relationship between the total variance of a multiple cavity sample, as a function of a change in one of the cavity means. The effects of this equation will be illustrated through a two-cavity example.

The two cavities in question have been previously determined to be producing parts according to a normal distribution with the statistical parameters shown in Table 4.1. A graphical representation of Table 4.1 is shown in Figure 4.2, where both cavities have been plotted as Normal distributions with the parameters shown in Table 4.1.

To illustrate Equation 4.7, cavity 1 will be shifted so that its mean is equal to the Nominal. This will be accomplished by increasing the mean of cavity 1 by 0.01425 so the quantity $(\bar{x}_1 - a)$ in Equation 4.3 or $\Delta\bar{x}_1$ in Equation 4.7 is 55.6 - 55.58575 = 0.01425. It can be expected that the total variance of both cavities will be reduced because of this mean shift. The proper numbers are substituted into Equation 4.7 to calculate the effects of cavity one being nominalized.

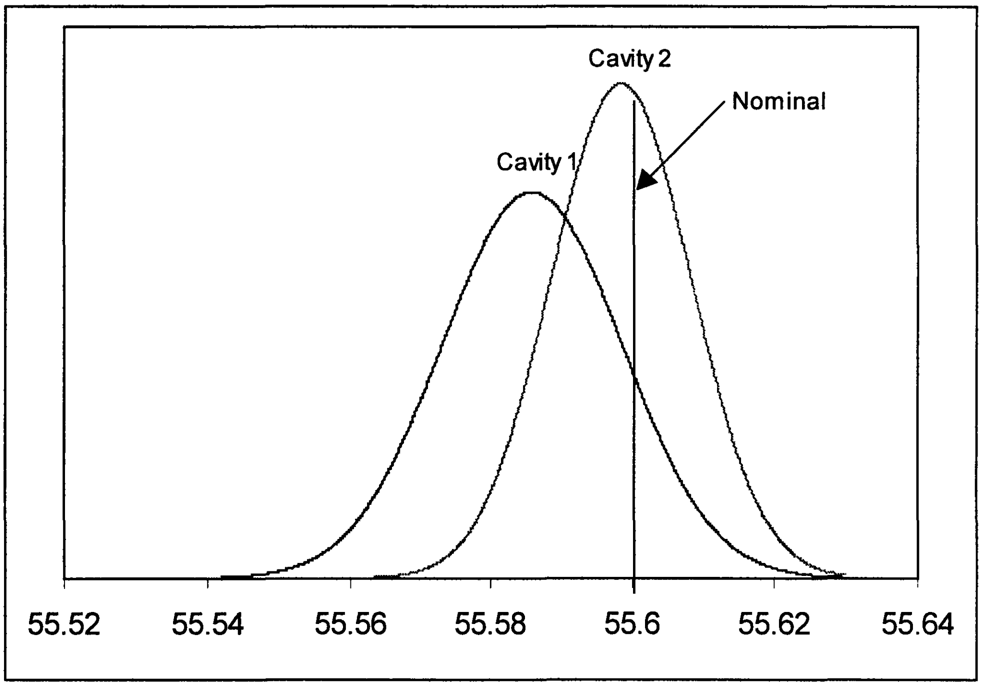



**Figure 4.2: Cavities 1 and 2 being Offset from Nominal**

$$\Delta\sigma^2_{total} = \frac{2*40}{[(40+40)-1]}\left[55.5875 - \frac{(55.5875*40+55.59836*40)}{40+40}\right](0.01425)+$$
$$\frac{2*40}{[(40+40)-1]}\left[1-\frac{40}{40+40}\right]\frac{(0.01425)^2}{2!}$$

$$\Delta\sigma^2_{total} = (-0.00638)(0.01425)+(0.050633)(0.01425)^2 = -3.95392E-05$$

When $\Delta\sigma^2_{total}$ is added to the total variance in Table 4.1 the final result is obtained.

$$\sigma^2_{total}\Big|_{\substack{Cavity1\\offset}} = \sigma^2_{total} + \Delta\sigma^2_{total} = 0.00016760 + (-3.95392E-05) = .00012806$$

These results are shown in Table 4.2. The newly calculated variance is significantly smaller than the original variances, because cavity one was offset from the nominal value by a significant distance.

| Cavity # | Mean | Variance |
|---|---|---|
| 1 | 55.6 | 0.00016030 |
| 2 | 55.59835 | 0.00009772 |
| total | 55.59917 | **0.00012806** |

**Table 4.2: Summary of Nominalization of Cavity One**

Next the process of nominalizing cavity two will be investigated. Equation 4.7 is again used to determine the effect that a shift in mean will have on the variance of both cavities. The only exception in calculating the effects of nominalizing cavity two over cavity one is that the new average for cavity one must be used in the calculation (55.6). The proper method used to determine the effect of moving the means of multiple cavities is to chain the calculations so that the current averages of each cavity are used. Using the derivation shown in this thesis, it is not possible to nominalize both cavity one and cavity two simultaneously. This will be shown in the following equation.

In this case cavity 2 only required that its mean be shifted by 0.001645 as opposed to 0.01425 in the case of cavity 1.

$$\Delta\sigma^2_{total} = \frac{2*40}{[(40+40)-1]}\left[55.59835 - \frac{(55.6*40+55.59835*40)}{40+40}\right](0.001645) +$$
$$\frac{2*40}{[(40+40)-1]}\left[1-\frac{40}{40+40}\right]\frac{(0.001645)^2}{2!}$$

$$\Delta\sigma^2_{total} = (-.000833)(0.001645) + (0.506329)(0.001645)^2 = -6.85070E-07$$

When $\Delta\sigma^2_{total}$ is added to the total variance in Table 4.2 the final result is obtained.

$$\sigma^2_{total}\Big|_{\substack{Cavity1\\offset}} = \sigma^2_{total} + \Delta\sigma^2_{total} = 0.00012806 + (-6.85070E-07) = .00012738$$

The results of nominalizing both cavities one and two are shown in Table 4.3 and in Figure 4.4. From Figure 4.3 it is easy to understand why the standard deviations, or spread, of each cavity does not change with a change in mean. Only the mean, or center point, of the cavity is shifted, and not the spread of the data.

Now that a relationship has been developed for a change in total variance as a function of a cavity mean offset for two cavities, Equation 4.7 will be generalized for $m$ cavities.

| Cavity # | Mean | Variance |
|---|---|---|
| 1 | 55.6 | 0.00016030 |
| 2 | 55.6 | 0.00009772 |
| total | 55.6 | **0.00012738** |

**Table 4.3: Final Values of Nominalizing Cavities 1 and 2**

$$\Delta\sigma_{total}^2 = \frac{2n_j}{\left[\sum_{i=1}^{m} n_i - 1\right]}\left[\bar{x}_j - \frac{\sum_{i=1}^{m}(\bar{x}_i n_i)}{\sum_{i=1}^{m} n_i}\right]\Delta\bar{x}_j + \frac{2n_j}{\left[\sum_{i=1}^{m} n_i - 1\right]}\left[1 - \frac{n_j}{\sum_{i=1}^{m} n_i}\right]\frac{(\Delta\bar{x}_j)^2}{2!} \quad (4.8)$$

Equation 4.8 is valid when there are $m$ cavities and cavity $j$ is the current cavity experiencing a mean shift. The application of Equation 4.8 to multi-cavity systems requires that only one cavity be shifted at a time, where the updated average is used in each new equation.

A mathematical method has been derived to take multi cavity systems into consideration when predicting overall mean and variance. This method also uses only the

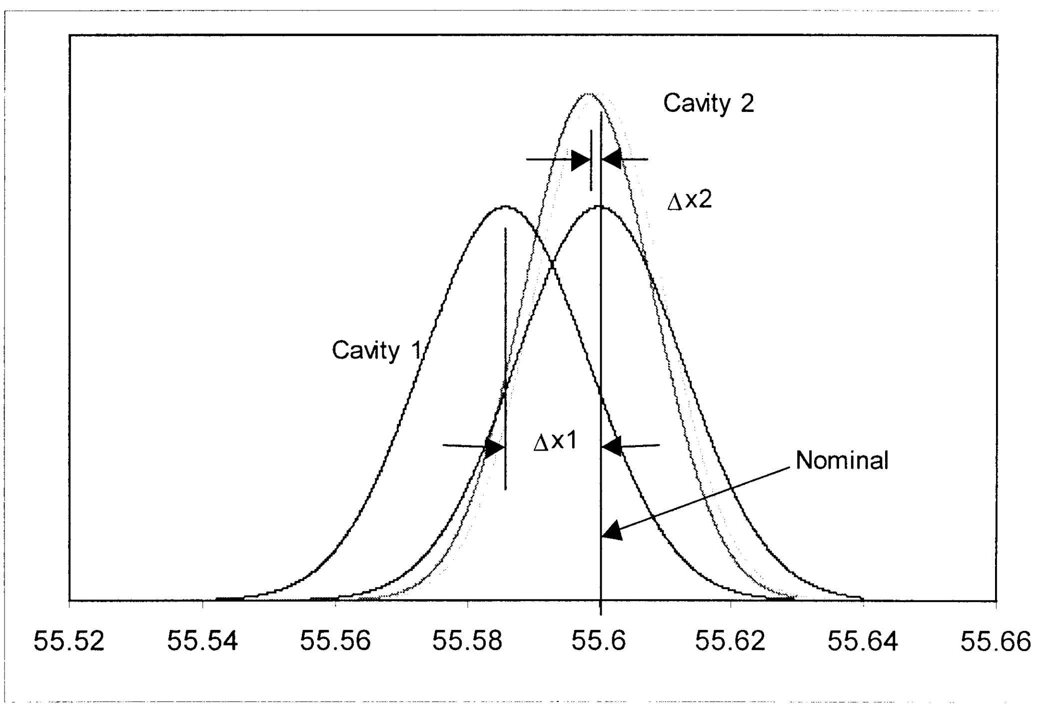



**Figure 4.3: Effects of Cavity Mean Offset for Two Cavities**

summary information of each cavity, and does not require each data point, which is convenient when this is the only information available. Using these equations it is also possible to illustrate how important the process of cavity nominalization is and how it affects the overall variation of all cavities combined.

## 4.4 Current Variational Models of Injection Molded Parts

The research on injection molded part quality can be divided into two distinct categories: investigations in part quality before the complete geometry of the part has been constructed, and investigations after. The former are related to part design, and the latter are related to manufacturing process improvement. Once the final geometry of the part is known, construction of the molds required to create that part will begin.

The time at which the tool creation process begins is important because once a tool is being built, and debugged, a design team will be reluctant to change the fundamental design of the part. Manufacturing often pressures the design community to "get it right the first time," because of the high costs associated with making changes to a mold. Even if a more robust design is discovered during the creation of an injection molding tool, it is difficult for the design team to recommend tooling changes, because of the time and money already invested in the tool.

### 4.4.1 Pre-Complete Geometry Variational Models

Some work has been published in the area of design tools for injection molded parts that help to minimize variations. [Beiter and Busick, 1995] have used dimensional analysis to make recommendations about nominal wall thickness and gating locations. [Kazmer, *et al.,* 1996] describes a methodology that can be used to assess the robustness of candidate designs at the detailed design stage. There are several quality design recommendations available for the designer to use when initially designing an assembly of injection molded parts. For example, [Bralla, 1986] and [Malloy, 1994] provide recommendations on how to design a feature with respect to several feature variables. [Rosato and Rosato, 1995] also provide comprehensive resources containing design guidelines for injection molded features. These are not necessarily quality related, but are often concerned with ease of part ejection from the mold, or methods used to reduce cycle time.

#### 4.4.1.1 Heuristics

In the preliminary phase of new product design, a void exists between handbook rules of thumb ("parts vary an extra 0.001" per added inch of length") and detailed finite element process simulation when used to analyze variation [Zemel, *et al.,* 1996], [Busick, *et al*, 1996]. As a result, a design team often utilizes abstract and frequently unrepeatable and inconsistent techniques when choosing between several different design concepts. Past projects with similar assemblies, design suggestions commonly obtained from handbooks and also the experiences of local experts serve as the tools commonly used when choosing between conceptual designs. While these approaches are considerably informative when making design decisions, they are often inconsistent depending on the information's origin. Nevertheless, these approaches are still used because they only require simple information about a part or assembly of parts.

#### 4.4.1.2 SPE/SPI

The design team can also refer to the Society of Plastics Engineer's and Society of Plastic's Industry (SPI) handbook on variation to determine tolerances to allocate on an injection molded part [SPI, 1993].

Figure 4.5 shows several design recommendations for polystyrene parts depending on the type of feature being considered. These tolerance guidelines were not obtained by a measurement study of typical and above average injection molding process capability, but were compiled based on the results of questionnaires and surveys distributed to various mold shops. For example, Figure 4.4 taken from [Zemel, *et al.,* 1996] shows that for one part studied in actual production, the SPI guidelines had relatively little correlation to the actual production data. The solid and dashed lines represent the SPI "Commercial" and "Fine" lines respectively, and the diamonds represent actual part data. Zemel has found that these guidelines were typically too conservative in their estimates.

The SPE/SPI guidelines are often used to estimate how well a dimension can be held, given only the nominal geometry of the part, and the part's material. The designer can also choose between two different estimates given the above information. One estimate is usually called "Commercial" and the other is called "Fine". The "Commercial"

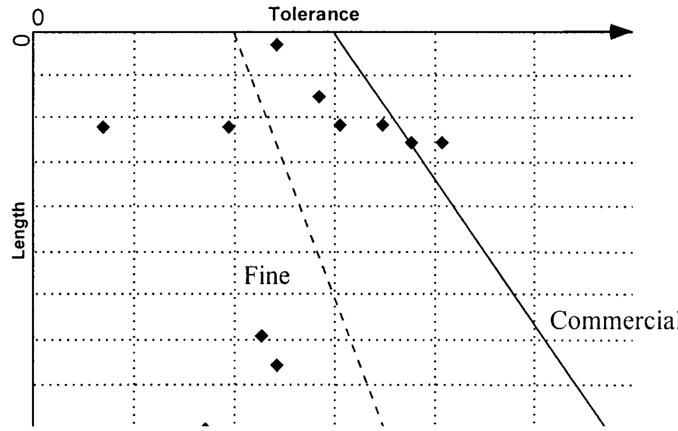



**Figure 4.4: Production Data and the SPI Prediction Model (Zemel and Otto, 1996).**

estimate is based on what is commonly accepted as being achievable given that a normal level of effort is extended to obtain a satisfactory part. The "Fine" line estimates the tolerance that a dimension can achieve given that special care, and an above average cost is spent on the development and maintenance of the mold. Such extra factors would include the centering of each cavity to its nominal value, and consistent SPC investigations monitoring those process variables which are expected to drift with time.

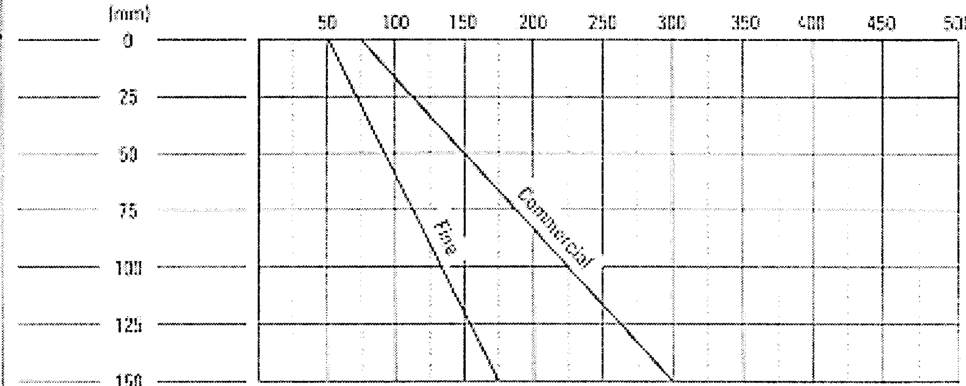
The design team must be cautious to choose an estimate based on the "Fine" guideline just because these tolerances are required to achieve proper functioning of the part. It is possible that the manufacturing capability of the injection molding tool being used to create the part may not be able to produce parts according to these specifications. This will lead to parts being out of spec because a tight tolerance was required for the part to properly function. The task of choosing either the fine or commercial line when estimating the tolerances to place on a dimension is not an exact science. Often, this decision is made through discussion with manufacturing experts who have made similar parts in the past. Using experiences obtained by manufacturing past parts is a considerable advantage when designing a new part, but these experiences must also be use cautiously in conjunction with a proper engineering analysis.

# Standards & Practices of Plastics Molders

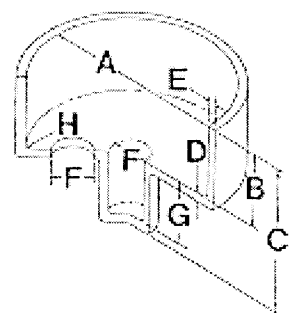
**Material**
Polystyrene
(PS)

*Note:* The *Commercial* values shown below represent common production tolerances at the most economical level. The *Fine* values represent closer tolerances that can be held but at a greater cost. Any addition of fillers will compromise physical properties and alter dimensional stability. Please consult the manufacturer.

Drawing Code: A = Diameter (See note #1); B = Depth (See note #3); C = Height (See note #3)




| Drawing Code | Dimensions (mm) | Comm. ± | Fine ± |
|---|---|---|---|
| | 150 to 300 for each additional 25 mm add (mm) | 0.100 | 0.050 |
| D = Bottom Wall | (See note #3) | 0.155 | 0.075 |
| E = Side Wall | (See note #4) | 0.150 | 0.075 |
| F = Hole Size Diameter (See note #1) | 0.000 to 3.000 | 0.055 | 0.025 |
| | 3.001 to 6.000 | 0.055 | 0.025 |
| | 6.001 to 12.000 | 0.055 | 0.025 |
| | 12.001 & over | 0.105 | 0.050 |
| G = Hole Size Depth (See note #5) | 0.000 to 6.000 | 0.105 | 0.050 |
| | 6.001 to 12.000 | 0.105 | 0.050 |
| | 12.001 to 25.000 | 0.130 | 0.075 |
| H = Corners, Ribs, Fillets | (See note #6) | 0.305 | 0.250 |
| Flatness (See note #4) | 0.000 to 75.000 | 0.180 | 0.100 |
| | 75.001 to 150.000 | 0.330 | 0.125 |
| Thread Size (Class) | Internal | 1 | 2 |
| | External | 1 | 2 |
| Concentricity | (See note #4) (F.I.M.) | 0.255 | 0.200 |
| Draft Allowance Per Side | (See note #5) | 1.0° | 0.5° |
| Surface Finish | (See note #7) | | |
| Color Stability | (See note #7) | | |




REFERENCE NOTES

1. These tolerances do not include allowance for aging characteristics of material.
2. Tolerances are based on 3.175 mm wall section.
3. Parting line must be taken into consideration.
4. Part design should maintain a wall thickness as nearly constant as possible. Complete uniformity in this dimension is sometimes impossible to achieve. Walls of non-uniform thickness should be gradually blended from thick to thin.
5. Care must be taken that the ratio of the depth of a cored hole to its diameter does not reach a point that will result in excessive pin damage.
6. These values should be increased whenever compatible with desired design and good molding techniques.
7. Customer-Molder understanding is necessary prior to tooling.

 Revised 1991

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**Figure 4.5: Sample page from the SPE/SPI Tolerance Guidelines**

#### 4.4.1.3 Tolerance Analysis

Predicted feature variations are used as an input to a tolerance analyses as described in Chapter 2. Care must be taken to select an appropriate set of inputs and ranges, to vary in the tolerance analysis. Feature based design techniques can serve as the foundations for this choice. [Salomons, *et al* 1993] provides a review of research in the area of feature based design. [Treacy, *et al*, 1991] proposed a method by which a tolerance chain could be generated automatically based on previously defined assembly constraints. Although it is possible to automate the generation of a tolerance chain, this method requires that the user input parameters that describe the statistical distribution of the individual dimension [Wang and Ozsoy, 1990]. Generation of a tolerance chain from the assembly constraints of several parts obviously follows from the assumption that a tolerance is derived from the function requirements of the part [Weill, 1988]. Generally, most of this research presumes that the variation associated with each feature has been previously determined, whereas this thesis is focused on using process physics to develop models that will predict these feature variations.

### 4.4.2 Post-Complete Geometry Variational Models

In addition to the heuristic rule-of-thumb based part design tools mentioned above, there are several manufacturing process improvement options available to help control the quality of a part after its design is complete, the mold has been created, and the part is in production. These methods include, but are not limited to: process simulation, design of experiments, and Statistical Process Control (SPC).

#### 4.4.2.1 Process Simulation

A process simulation, can more accurately predict how the part will *"behave"* during manufacturing, than an industry standard rule-of-thumb. There are several injection molding simulation packages available on the market that simulate how a mold will be filled, and also how the part will be expected to cool and shrink. Much can be gained from these types of simulations including predictions of gross defects like sink marks and warpage. However, much must also be known a priori to using them. The detailed geometry of the mold, including cooling lines and gating schemes are commonly used as inputs to the simulation. However, these inputs are often not known during the preliminary selection phase of the design process, and thus the simulations are basically relegated to detailed mold design or perhaps problem parts in redesign. Performing a rapid process simulation on several different design concepts is basically infeasible, not so much due to computational times, but rather due to problem formulation and setup difficulties. At the concept selection stage, processing conditions are difficult to estimate. The result is the delegation of process simulation to situations where the detailed part and mold geometries are known.

The research field in plastic injection molding process simulation, including both closed form analytical and finite element simulations attempts to understand the relationship between process settings and final part quality. [Titomanlio, *et al.,* 1996] uses an analytical approach to predict the shrinkage of a thin slab in length, width, and thickness directions. These are calculated as a function of the local temperature, pressure, and crystallization (or reaction) effects, which are intended to originate from a finite element process simulation that has been previously completed. Although these three inputs to the variation model are often obtainable from a molding simulation, details about the entire part and process are required. This shifts the approach to a detailed design tool. [Woll and Cooper, 1996] have also investigated the relationship between processing conditions and final part quality, and has proposed using artificial neural networks to aid in this feedback process. [Busick, *et al.,* 1994] has investigated using processing simulation to quantify dimensional errors due to processing conditions.

#### 4.4.2.2 Design of Experiments

The use of designed experiments can have two possible goals. First would be the determination of a process parameter's nominal set point while the process is still being tuned. Second, a set of designed experiments would determine how different process parameters combine to influence the final quality of the part. In both cases, baseline process parameter set points are chosen around which to vary these process parameters. The quality characteristic being measured is recorded at all points during which the process parameters are varied.

[Devor, *et al.*, 1992] provides a simple explanation of the purpose of conducting a set of designed experiments. "The purpose of most experimental work is to discover the direction(s) of change which may lead to improvements in both quality and the productivity of a product or process." As was stated above, a set of designed experiments can be conducted when a process is first being defined, when it is fully mature, or anywhere in between. However the current trend is to push the issue of quality further upstream in the design process, thus designing variation reduction strategies before the manufacturing of the part. These types of strategies lead to more robust variational models because quality is considered a product and not a manufacturing issue.

Although the use of designed experiments is common in industry, they cannot actually be conducted until the process is operating, although the trend is to specify the proposed designed experiments during part design. This relegates the real benefits achieved through using a design of experiments technique to the realm of manufacturing. If this information is then, in turn, given back to the design community, it can benefit them during a redesign of the part, or perhaps the next generation of the product.

#### 4.4.2.3 Statistical Process Control (SPC)

SPC involves the monitoring of process settings and their variation. Once the operating settings and limits for the variables on an injection molding press have been determined, a computer system will monitor these process variables to detect if a predefined limit has been exceeded. This will alert an operator of a possible problem with the machine's operation, who could then stop the production of possibly poor parts.

This method of quality control has proven effective in several case studies [Mason, 1989], [Naitove, 1995], but it relies on the assumption that only incorrect process settings will yield poor quality parts. It is often possible, though, to be producing poor quality parts while remaining within acceptable processing limits. [Sachs, *et al.,* 1995] has proposed a method by which SPC could be coupled with feedback control to monitor and change processing settings *in situ* for batch production. [Frey and Otto, 1997] extend SPC to multiple criteria and to multiple operations.

### 4.4.3 Redesign

Of the several methods available used to design an injection molded part, redesign is probably the most costly, depending on how far along the part is in the product development loop. For example, if a part is still being conceptually developed, and individual part geometry is still vague, then making major changes to several parts, so that they interact more robustly is not a problem, and can be easily accomplished. However, if a part is very far along in the product development process, *e.g.* tool steel has already been cut, then the cost of even a small design change is significant. This is especially true when a large number of cavities are involved.

One of the main points of this thesis is to provide variational models that will predict the amount of variation that can be expected on an injection molded feature so as to minimize the probability of a redesigned part. Although modeling the variability of a product does incur some cost, the cost savings associated with preventing the redesign of poor parts are large. However it is not possible to have an accurate benchmark for this cost savings because if an error is found in the design of the part through modeling, then the appropriate changes are made to the design. The mistake is never allowed to flow through the product development loop, just so that the cost savings of an analysis can be calculated.

## 4.5 A More Specific and Physics Based Variational Modeling Approach

From the perspective of conceptual design, both the abstract heuristic and the detailed process simulation approaches provide information about plastic part variation that can be used to understand how a design and its subsystems will vary in performance. Yet what would serve a design team best is to have models of intermediate complexity. These models would not require full manufacturing process details, but would provide a quantitative understanding of the process. An approach based on physical principles is developed here, where different methods of variation are conceived, models of these different types of variation are developed, and then statistically validated against past production data, nominal geometry and materials.

The next chapter presents an overview of the proposed approach to constructing preliminary design phase variational models based upon physical principles.

# Chapter 5

## Modeling Methodology

Variational models based on characteristics that are commonly known at preliminary selection time, *e.g.* material and feature geometry is the focus of this thesis. Because the application of these types of variational models are aimed at aiding the designer when choosing between different conceptual designs when limited geometric information is known, the proposed model need not be complex, although more repeatable on a more consistent basis then the current heuristic approach.

This thesis proposes that such design phase models of production variation should be constructed based upon process physics and then validated with manufacturing data. This idea is depicted in Figure 5.1. First, a list of features that are important for preliminary variation analysis must be developed. For plastic injection molded parts, this

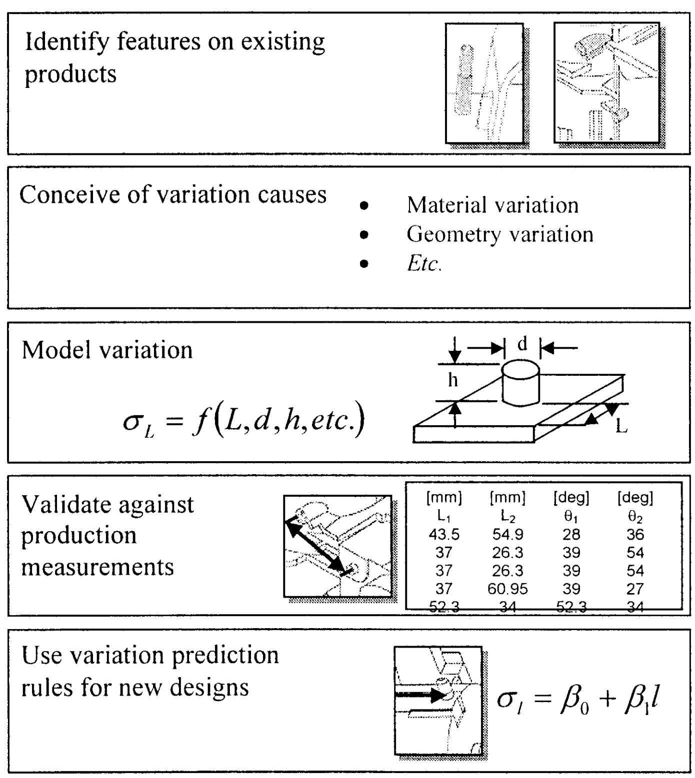



**Figure 5.1: Variation Analysis Procedure.**

includes hole dimensions, post dimensions, boss dimensions, positional variations of these features on a plane, *etc*. Appendix B contains a document [Hackert-Kroot, 1998] which shows the majority of features used in a tolerance analysis.

This document was designed to be used with a tolerance analysis package named TARGET, which stands for Tolerance Analysis for the Robust Generation of Engineering Tolerances. TARGET has been generated internally within Kodak for its proprietary tolerance analysis needs. The features vs. variation document lists how TARGET would recognize the feature, what variations TARGET would and would not generate for that feature, and also the associating GD&T (Geometric Dimensioning and Tolerancing) symbols associated with that feature. This document describes many features that are not common to a typical tolerance analysis. This thesis was never intended to predict the variations for all the features described in this document.

Next, physical causes for the feature's variation must be conceived. An extensive list of different sources of variation was generated and is listed in Appendix A. For plastic injection molded parts, the underlying physics behind the variational model originates from a volumetric decrease resulting from the phase transition from a liquid to a solid. Variation in the amount of shrinkage in this phase transition can be due to such causes as material property variation, mold and cavity differences, etc.

Next, a model of this variation must be developed. This will be discussed in Chapter 6 for features on a plane, and Chapter 7 for the diameter of a hole. A simplistic representation of these models is the concept of linear shrinkage, where the variation in shrinkage is proportional to the smallest distance between the two measured features. However, the amount of volumetric shrinkage variation is not consistent throughout the part due to differences in cooling, material flow and packing, etc. This is one reason why volumes of information have been published listing design guidelines for injection molded features. [Bralla, 1986], [Malloy, 1994] To further complicate the task of predicting how a part will shrink, multiple cavities in multiple tools placed in multiple presses are often required in order to satisfy a specific production volume and rate. Combining all these different factors results in the difficulties encountered when constructing an effective model to predict variability.

Next, after these variational models have been constructed, they must be validated against many features that fit the topology of the model. That is, data from several different configurations of the feature topology using the same production process are needed. This ensures that the validated model will scale across the feature design variables. The constancy of the production process, though possibly at different process settings, ensures that process factors can be reduced into statistical constants.

## 5.1 Shrinkage Variation

Mold designs are often created with the aid of sophisticated analytical tools aimed at predicting how a part will be expected to shrink during molding. Designing the geometry of a mold becomes even more difficult when a high shrinkage material is used. It is especially true with these high shrinkage materials that the shrinkage occurring in the mold is not always isotropic. Consequently, if shrinkage is no longer isotropic, the task of designing a mold is no longer just a simple "scale-up" procedure. Anisotropic shrinkage can also lead to warpage (out of plane distortion) or internal stresses that can significantly distort the shape of the part.

To better understand how shrinkage is predicted, one must investigate how shrinkage is defined. This requires a basic understanding of the physics of polymers, and also how the injection molding process itself is aimed at counteracting the effects of shrinkage.

### 5.1.1 Polymeric Shrinkage

Section 4.1.2 described several reasons why the modeling of plastic parts is so complex including many different environmental, design and manufacturing issues. The structural modeling of plastic parts is made even more complex due to the solid→liquid→solid phase transition, and associated volume changes that the plastic experiences during molding. If a plastic had the same volume as a solid when it is a liquid, the process of injection molding would be considerably easier. This zero volume change would result in lower injection pressures, cheaper and less bulky injection molding machines. Unfortunately this simplification can not be made, and anisotropic shrinkage does exist.

A simple method that can be used to explain the effects of shrinkage is derived from non-uniform cooling of the part. Parts cool non-uniformly because of the inherent heat conduction characteristics of thick and thin sections, where thick sections cool slower than thinner ones. As a result of these different rates of cooling, the polymer's density increases more rapidly in thin sections, than in thicker ones. This polymer density gradient causes a pressure differential that causes some of the polymer in the thick section to flow into the thin section. This flow of polymer from thick sections to thinner ones is often termed *internal flow*. If there are large changes in thickness in the part, unequal cooling rates could cause large internal flows, resulting in sink marks. [McKelvey, 1962]

### 5.1.2 Processing Steps for Injection Molded Parts

The inherent shrinkage that occurs during the liquid-solid phase change can be counteracted to a certain extent during the injection molding process. To better understand how this can be accomplished, a description of the injection molding process is given next, from [Malloy, 1994].

> "The Molding process begins with the injection of hot melt into a relatively cool mold. The polymer begins to cool, and its specific volume begins to decrease. During this time, the injection ram (screw and check valve) are packing additional material into the tool cavity, thereby compensating for the shrinkage effects. This process can continue until one of the sections between the injection unit and the cavity (usually the gate) solidifies. At this point, the central core sections of the molding remain molten, and continue to shrink without compensation. Once the part is rigid enough to keep its own shape (without distorting due to ejection related stresses, internal stress or gravity) the mold opens and the part can be ejected. After ejection, the part continues to cool and shrink until ambient temperature is reached. "

Malloy then goes on to graphically describe the injection molding process using the P-*v*-T curve shown in Figure 5.2.

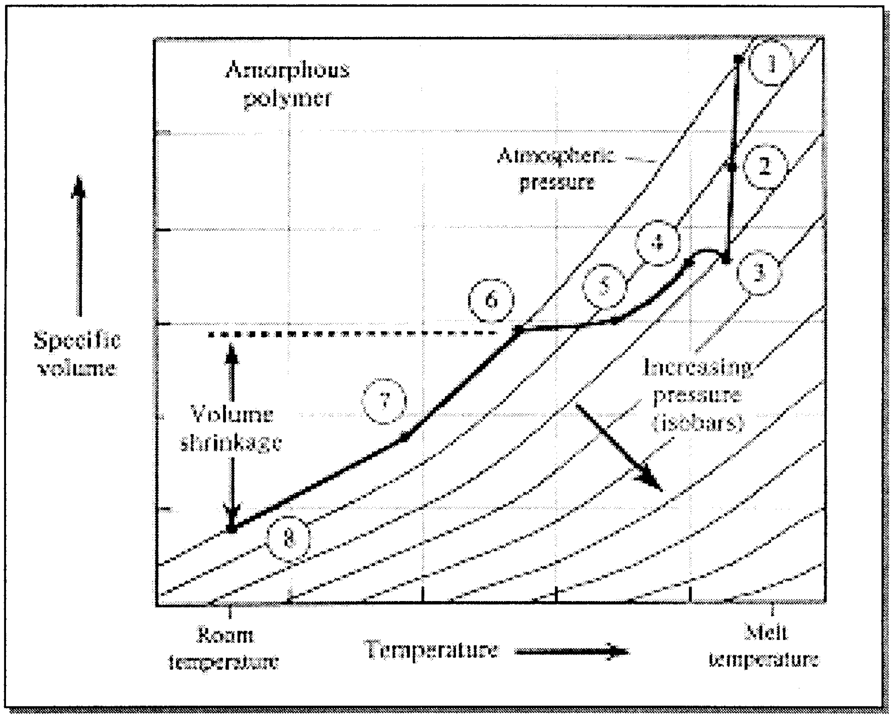



**Figure 5.2: Processing Steps in terms of Material's Pressure Temperature, and Volume [Malloy 1994]**

1. Melt pressure builds as material enters the cavity

1-2. Filling of the mold cavity
2. The instant of fill (zero pressure at the end of the flow)
2-3. Packing or compression phase
3. Peak cavity pressure is achieved and transfer to holding pressure is initiated.
3-4. Switch over to holding pressure with some pressure loss due to back flow (discharge) of material when pressure switchover occurs.
4. Holding pressure phase begins
4-5. Pressure drop due to cooling and an increase in the solid layer thickness. Material flow continues to compensate for contraction resulting in a specific volume decrease
5. Gate freeze-off (solidification of gate) preventing material flow, end of holding phase
5-6. Pressure drops as the part cools and shrinks without compensation
6. Atmospheric pressure is reached indicating that the part size equals that of the cavity, and “mold shrinkage” (as defined) begins.
6-7. Isobaric cooling in the mold
7. Mold open-part ejection
7-8. Post mold isobaric cooling
8. Thermal equilibrium - final part volume (neglecting and morphological or moisture related volume changes)

Thus defining shrinkage as the volumetric decrease occurring between steps 6 and 8 above, shrinkage can be broken down according to two different contributors: shrinkage before ejection, and shrinkage after the part has been ejected.

### 5.1.3 Pre-Ejection Shrinkage

Shrinkage occurring before the mold is opened is compensated to a certain degree by pushing as much plastic into the mold, before the gate freezes and during the packing and holding phases of molding, as possible. However once the gate is frozen, no more molten plastic can be forced into the mold and then the effects of shrinkage can no longer be counteracted. Uneven shrinkage in a part is caused by several factors, and is not limited to the following things: temperature, and pressure gradients in the mold, geometry of features, material behavior caused by injection, *etc.*

It would be ideal for every point on the part, at the same depth below the part's surface, to cool at exactly the same time. For example, points A & A', and B & B' in Figure 5.3 should be at exactly the same temperature and pressure at any particular point in time, but they frequently are not because of differences in cooling line locations, relative locations of other cavities, gate locations, and the like. These differences will all lead to points A & A', and B & B' not being at the same temperature and pressure at any particular snapshot in time. This often leads to internal stresses, warpage, defects, and dimensional inaccuracy.

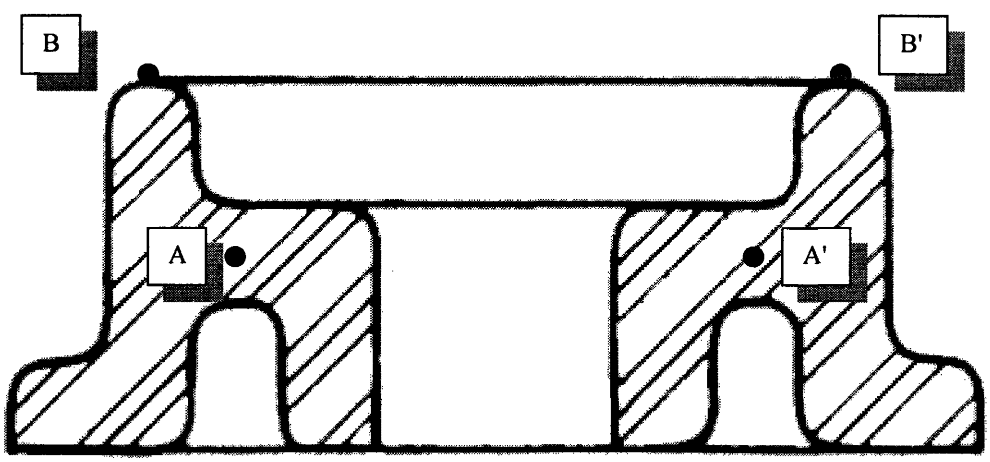



**Figure 5.3: Typical Symmetric Part**

Not all part geometries are as simple and symmetric as those shown in Figure 5.3, and controlling the relative rate at which two similar and symmetric points on a part cool becomes incredibly difficult. Current state-of-the-art injection molded part designs sometimes appear to be unmoldable, let alone dimensionally stable. With these types of intricate designs, anisotropic shrinkage is common, due to purely geometric design issues alone. Anisotropic shrinkage can also be caused by differences in the part and cavity geometry that occur in places such as internal and external corners.

As was discussed in Section 5.1.1, internal flow is one of the reasons for shrinkage variation, and internal flow is directly related to how the cavity is cooled. If one specific feature on a part is limiting the part's cycle time, due to a particularly long cooling time, then there is realistically no reason to cool the other features on the part at a significantly slower rate. Essentially the part can only be cooled as fast as its slowest feature. For example, Figure 5.4 shows a graph of feature number *vs.* time required for the feature to reach its ejection state. Feature 1, shown in this graph is the feature that takes the longest time to cool, as opposed to features 2-4. The circles in Figure 5.4 represent design nominal times, and the vertical lines around each circle represent production variation. To reduce the probability that internal flow will occur within the part, it would be ideal for each feature to reach its ejection state at the approximately the same time. This could be accomplished by either decreasing the time to ejection for feature 1, which is the ideal case, or increasing the time to ejection for the other features. It is often possible for this

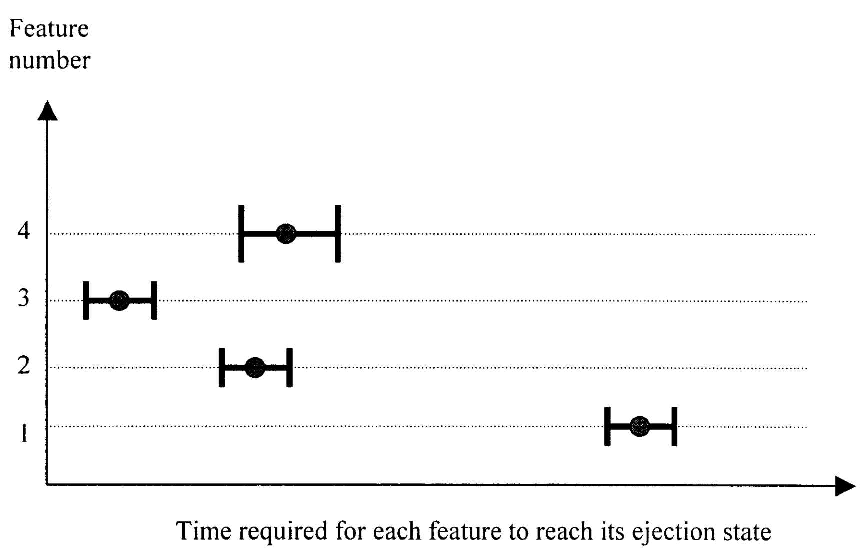



**Figure 5.4: Relative times required before features can be ejected from the mold**

type of control to be implemented through the use of independently controlled cooling lines, where the temperature for each cooling line can be set to be a specific value.

Shrinkage prediction is also made more difficult as the geometric complexity of the part grows. Features are often constrained to shrink in certain directions due to the geometry of the mold. Because the part is restrained in certain directions, an increase in shrinkage will occur in other directions to satisfy the final volumetric requirement of the part.

Besides geometric mold and part design issues complicating shrinkage prediction, the direction of plastic flow is often important when molding particular polymers. Semi-crystalline polymers, such as Polyethylene and Polypropylene, have enough regularity in their chemistry that they can form ordered, rather than random, arrangements. These ordered regions are caused as the material cools from a liquid to a solid state, and often cause anisotropic shrinkage. [Malloy 1994]

### 5.1.4 Post-Ejection Shrinkage

After the part is ejected from the mold it can be collected and stored using automated or manual techniques. These different techniques frequently depend on economies of scale resulting from the planned quantity and life span of parts in production. Although economics often drive these types of decisions, part quality is often heavily coupled with how the part will be handled and stored after molding.

Injection molding presses equipped with automated pickers that remove the parts from a multi cavity mold often place the parts into specially designed trays so that they can be easily accessed for assembly processes. Properly designed storage trays not only serve to improve assembly times, but also prevent the part from distorting in unwanted directions. They can support the part in weak areas while preventing unwanted warping and distortion frequently encountered over time. Figure 5.5 shows a part that would obviously require support to keep it from deforming over time.

The automated procedure described above works well for molds designed using hot runners, because with hot runners, there is no de-gating procedure required. However it is possible to add a de-gating station to a cold runner mold and then have the parts stored in a similar way. When a mold is not designed to use a hot runner system, the parts are often ejected and then are allowed fall into several different types of collection systems

**Figure 5.5: Feature Requiring Support During Storage**

that can consist of a neatly stacked arrangement or a large unorganized bin. This type of collection system works well for parts that are dimensionally stable after ejection (e.g. small parts), but does not work as well for large volume parts. Larger volume parts can deform under their own mass, combined with the effects of gravity, to unwontedly deform if not properly supported. Figure 5.5 shows a part that possesses a feature that resembles a cantilever beam. If the long feature is not supported in some way, the end of the feature could deform over time.

Another important factor in controlling post-ejection shrinkage is the amount of moisture the part absorbs. In order to mold certain types of plastics they are pre-dried before molding and are especially susceptible to moisture absorption over time. As the part absorbs moisture it will tend to grow in volume, depending on wall thickness, thus increasing the amount of shrinkage variation.

## 5.2 Feature Based Models

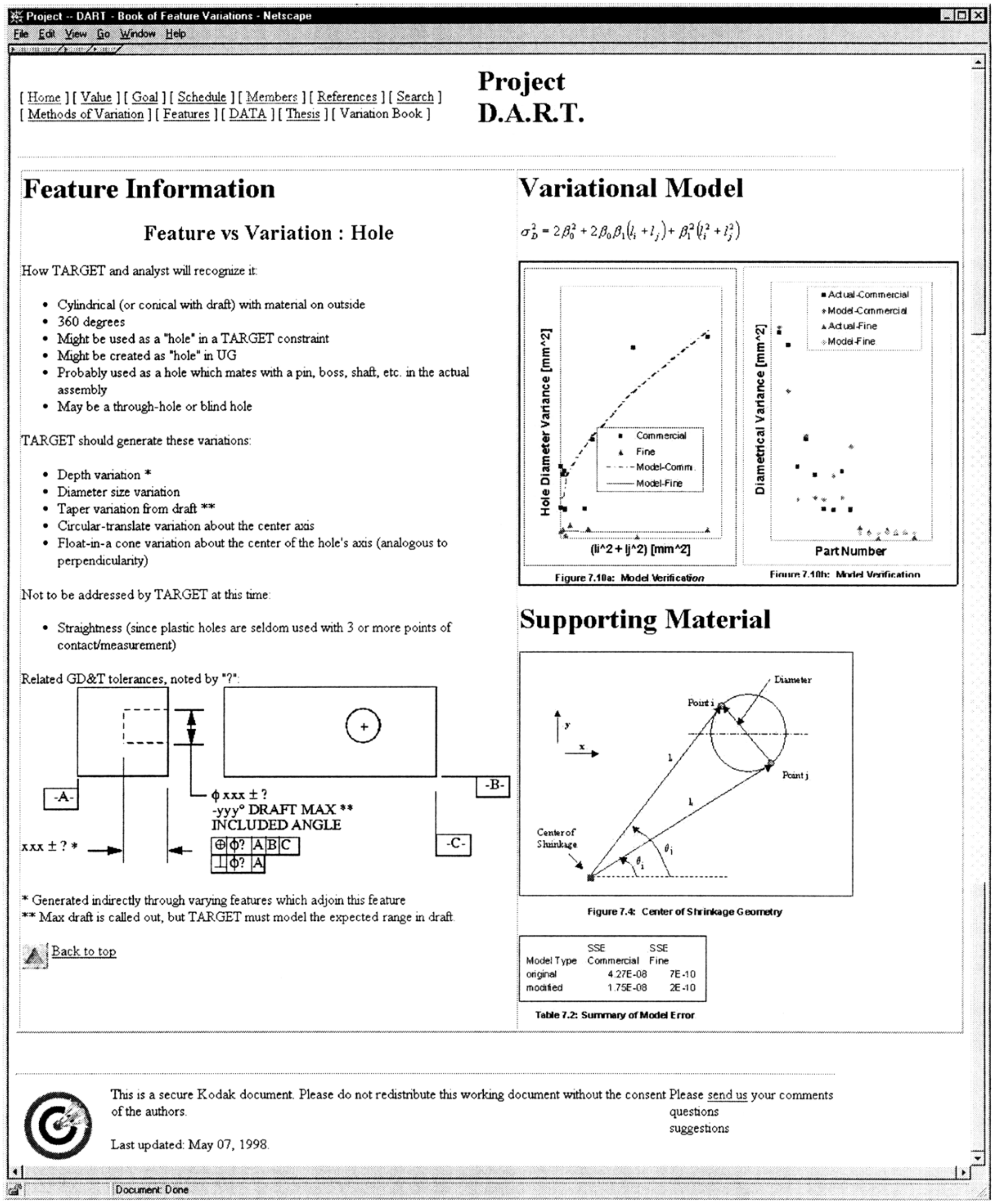
Although creating models to predict the variation in the amount that an entire part shrinks across all these variables is a complicated task, it becomes easier when analyzing the part on a feature level. Using a feature based approach a book of feature variations can be compiled which summarizes how each feature will be expected to vary in production.

Contributors to dimensional variation originate from one of three main areas: material variation, process variation, and variation introduced through the design of the product. Because the developed model is aimed at being a concept selection tool, using process variables is not ideal, because process parameters are not commonly known. Thus the model should incorporate design and manufacturing variables that are known at concept selection time while statistically reducing out variables that are not known.

This thesis details the methods that have been used to derive variational models for two types of features; hole diameter, and the position of a feature moving on a plane. These models have been combined into a format that can be considered the beginning of a book of feature variations. The first section of which contains the appropriate GD&T [ASME, 1994] information that is typically placed on a part drawing and used to identify the feature. The second section provides the variational model, and the derivation and motivation of deriving such a model. The final section lists any supporting information that has been used in the development of these models.

This book of variations was implemented as a web based multimedia tool similar to Figure 5.6, to both be easily used as a reference and to present the obtained production data in an easily retrievable format.

In the following chapters, variational models for the positions of various shaped post or hole features on a plane are developed in Chapter 6 and hole diameters in Chapter 7. In essence, the "position on a plane" is the feature of study in the following Chapter.



Project -- DART - Book of Feature Variations - Netscape

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[ Home ] [ Value ] [ Goal ] [ Schedule ] [ Members ] [ References ] [ Search ]
[ Methods of Variation ] [ Features ] [ DATA ] [ Thesis ] [ Variation Book ]

**Project D.A.R.T.**

# Feature Information

## Feature vs Variation : Hole

How TARGET and analyst will recognize it:

- Cylindrical (or conical with draft) with material on outside
- 360 degrees
- Might be used as a "hole" in a TARGET constraint
- Might be created as "hole" in UG
- Probably used as a hole which mates with a pin, boss, shaft, etc. in the actual assembly
- May be a through-hole or blind hole

TARGET should generate these variations:

- Depth variation *
- Diameter size variation
- Taper variation from draft **
- Circular-translate variation about the center axis
- Float-in-a cone variation about the center of the hole's axis (analogous to perpendicularity)

Not to be addressed by TARGET at this time:

- Straightness (since plastic holes are seldom used with 3 or more points of contact/measurement)

Related GD&T tolerances, noted by "?":



* Generated indirectly through varying features which adjoin this feature
** Max draft is called out, but TARGET must model the expected range in draft.

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# Variational Model

$$\sigma_D^2 = 2\beta_0^2 + 2\beta_0\beta_1(l_i + l_j) + \beta_1^2(l_i^2 + l_j^2)$$



Figure 7.10a: Model Verification

Figure 7.10b: Model Verification

# Supporting Material



Figure 7.4: Center of Shrinkage Geometry

| Model Type | SSE Commercial | SSE Fine |
|---|---|---|
| original | 4.27E-08 | 7E-10 |
| modified | 1.75E-08 | 2E-10 |

Table 7.2: Summary of Model Error

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Please send us your comments questions suggestions

Last updated: May 07, 1998.

Document: Done

Figure 5.6: Typical Page in the Variation Guidebook

# Chapter 6

## Plastic Feature Position Variation

Clearly, the majority of the dimensions shown on an engineering drawing are related to the relative positions of features with respect to each other and the part's Datums. [ASME, 1994] This has led the author to investigate the positional variation of features moving on a plane.

Section 6.1 begins the development of a model aimed at predicting the positional variation of features moving on a plane, such as posts, and holes of different diameters and shapes. This model begins with the underlying cause of variation that can be derived from local differences in shrinkage, and is constructed considering the positional variation of available production data.

Sections 6.2 and 6.3 discuss the issues related to the type of methods used to report the quality of a dimension, and how they relate to the development of the variational models. Typically, production quality measurements of position report the distance between two features along an arbitrary axis. Therefore, the positional variation model must incorporate this constraint of validation from two features, but which can be applied to a single feature variation. This idea was developed with the idea of a local center of shrinkage.

Section 6.4 details two different analytical methods that were used to apply the center of shrinkage concept to the positional variation of features. The first analytical approach uses a Taylor series about $(x-a)$ to derive the variational equation. The second analytical method only uses the geometry of the part and advanced probability theory. Although two different analytical methods were used to derive the positional variation of a feature moving on a plane using different techniques, their results are identical.

Section 6.5 uses production data to statistically validate the model. These results are then favorably compared to the industry standard SPE/SPI charts.

## 6.1 Center of Shrinkage

Local to any relatively small feature, one can argue that all plastic shrinks toward a common point. That is, a sphere of plastic will tend to shrink toward the center of the sphere. Non-spherical shapes whose surfaces all shrink away from a mold in cooling will similarly shrink toward a common centroid. This centroidal point that all plastic in some vicinity shrinks toward, is called the *center of shrinkage*. The center of shrinkage is defined as the point to which all features will approach, as they are able, as the part cools. It is possible that the center of shrinkage will not lie within the plastic of the part. For a part with shapes whose surfaces all shrink away from the mold in cooling, the center of shrinkage is approximated by the center of mass. For parts that are comprised of features that shrink on to the mold wall, such as with holes, undercuts and the like, the center of shrinkage is more toward that feature than from the center of mass. While the center of shrinkage is not completely defined objectively without a detailed analysis, it nonetheless can be used effectively in the preliminary design phase through rough estimation. Estimating the center of shrinkage in the following derivations has been one of approximating the center of mass using two-dimensional techniques, and not through the assistance of a CAD solid modeler. Because this approach has been designed to be used in the preliminary design stage, when limited geometric information is known, and is subject to change, this approach may still be used.

Given one physical cause, namely that locally all plastic will shrink toward a center of shrinkage, one can now develop a model predicting feature variation. First, the modeling is started with two features of interest. In this case, consider either feature $i$ or $j$ which lie on a flat surface shown in Figure 6.1. When the feature cools and experiences shrinkage variation, the positional variation of the feature, as modeled, will grow as the distance from the center of shrinkage. Other sources of variation, are not modeled to grow with this distance.

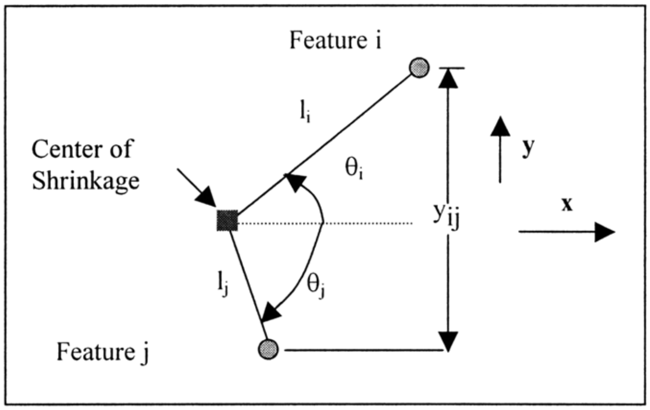



**Figure 6.1: Center of Shrinkage Geometry.**

Therefore, an effective decomposition of positional variation on a plane is to consider variation along the line from the feature toward the center of shrinkage, and perpendicular to this direction. With this observation, one can say that the position of a feature can be described as varying parallel and perpendicular to $l$, where $l$ and $\theta$ fully describe the position of a feature relative to the center of shrinkage.

## 6.2 Relative Variation

If a feature is nominally located at a given position on a plane, indicated by the circle in Figure 6.2, both the parallel and perpendicular variational components with respect to the center of shrinkage will cause the feature to be displaced from its nominal position. Models for these two components of variation are now developed.

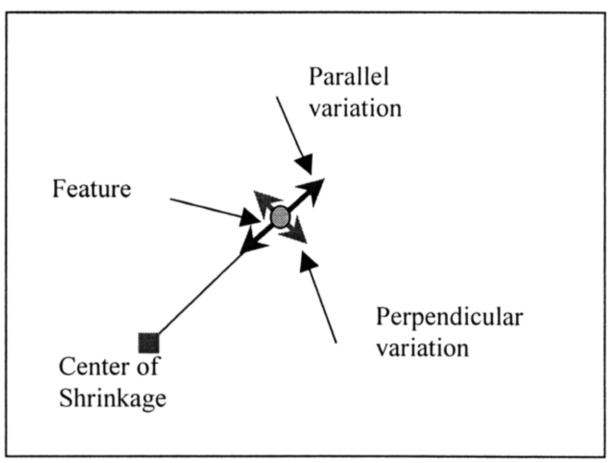



**Figure 6.2: Feature variation components.**

Given the argument that variation in shrinkage is due to local effects, then with added distance from the center of shrinkage, the variation ought to increase. That is,

$$\sigma_{\parallel} = \beta_1 l, \tag{6.1}$$

where $\sigma_{\parallel}$ is the standard deviation of the variation of the shortest distance from the feature to the center of shrinkage, and $l$ is length.

The standard deviation of the perpendicular component ($\sigma_{\perp}$) is basically unexplored in our model, and so fit to a constant, i.e.,

$$\sigma_{\perp} = \beta_0. \tag{6.2}$$

Such other contributors are causes such as mold machining variation, cooling variation, etc. Both the constants, $\beta_0$ and $\beta_1$, are material dependent as the shrink rate of each material is significantly different.

## 6.3 Measurements of the Distance between Features

There are several methods currently used in industry today that measure the quality of an injection molded part. The most common metric used involves sampling a part at a specific rate and measuring several critical dimensions between two features. These measurements are often taken on an automated coordinate measuring machine that is programmed to output the distances between features in a particular reference frame. The measurements provided are the distances between features, and not the absolute positions of these features with respect to a common reference coordinate system. This is shown in Figure 6.1 as the measured length $y_{ij}$, relative to the center of shrinkage. The center of shrinkage can be described with lengths $l_i$, $l_j$, and angles $\theta_i$, $\theta_j$ as shown in Figure 6.1. Figure 6.1 also shows that the vertical distance between features $i$ and $j$ can be found by

$$y_{ij} = l_i \sin(\theta_i) + l_j \sin(\theta_j). \tag{6.3}$$

Thus the distance $y_{ij}$ can be described as a function of 4 variables,

$$y_{ij} = f(l_i, l_j, \theta_i, \theta_j). \tag{6.4}$$

Using quality dimensional data and the relationship given in Equation 6.3, values for the coefficients $\beta_0$ and $\beta_1$ in Equations 6.1 and 6.2 can be statistically determined if the variations on $l$ and $\theta$ are known. Hence these variations will be discussed next. In this case,

$$\sigma_l = \sigma_{\parallel}. \tag{6.5}$$

However, the perpendicular variation component must be transformed into a variation in $\theta$, so this transformation will be discussed next. The perpendicular variation can be thought of as a variation in the arc length $ds$. The arc length can be calculated by

$$ds = l d\theta, \tag{6.6}$$

where $ds$ is the arc length, $l$ is the radius of the arc, and $d\theta$ is the angle of the included arc. If $d\theta$ is small enough, then the chord shown in Figure 6.3 will be approximately equal to the arc length. Then,

$$\sigma_{\perp}^2 = \sigma_s^2 = l^2 \sigma_\theta^2. \tag{6.7}$$

but $\sigma_{\perp} = \beta_0$. Therefore,

$$\sigma_\theta = \frac{\beta_0}{l} \quad . \tag{6.8}$$

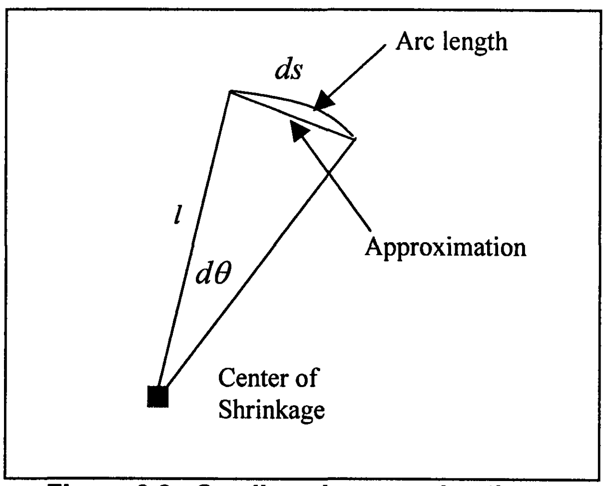



**Figure 6.3: Small angle approximation.**

Using Equation 6.3, we can now derive the variation on inputs $l_i$, $l_j$, $\theta_i$, $\theta_j$ from the quality assurance data on the characteristic $y_{ij}$. The important issue is to take the measurement data and decompose it into implied input variable variation with respect to the center of shrinkage. If this statistical fitting produces consistent variation constants across many different sets of dimensions from different parts, then the model is validated.

## 6.4 Variation from the Center Of Shrinkage

There are two possible analytical methods that can be employed to determine the relationship between quality measurements taken between features and a two-dimensional positional model that describes how a particular feature will vary. The first analytical method uses a Taylor series expansion of Equation 6.3. The second analytical method approaches the problem at a geometric level by stating that the actual position of a feature can be decomposed into two pieces: the nominal position, and a stochastic error. The variation for the stochastic error is composed of two terms with one being parallel and the other being perpendicular to $l$, shown in Figure 6.2. Both the Taylor series approximation and the geometric modeling techniques yield the same fundamental variational equations, but use different approaches.

### 6.4.1 Taylor Series Expansion

Expanding Equation 6.3 using a first order Taylor series approximation about $(x-a)$ [Salas and Hille, 1990], one obtains

$$\begin{aligned} y_{ij} &= f(l_{io}, l_{jo}, \theta_{io}, \theta_{jo}) + \\ &\frac{\partial f(l_{io}, l_{jo}, \theta_{io}, \theta_{jo})}{\partial l_i}(l_i - l_{io}) + \frac{\partial f(l_{io}, l_{jo}, \theta_{io}, \theta_{jo})}{\partial \theta_i}(\theta_i - \theta_{io}) \\ &\frac{\partial f(l_{io}, l_{jo}, \theta_{io}, \theta_{jo})}{\partial l_j}(l_j - l_{jo}) + \frac{\partial f(l_{io}, l_{jo}, \theta_{io}, \theta_{jo})}{\partial \theta_j}(\theta_j - \theta_{jo}) \end{aligned} \tag{6.9}$$

where $f$ is given by Equation 6.4 and $l_{io}, l_{jo}, \theta_{io}$, and $\theta_{jo}$ represent the nominal design target values with respect to the center of shrinkage.

The application of the Taylor series assumes two things brought to light by [Park and Himmemblam, 1980]. First, $l$ and $\theta$ must be linearly independent where linear independence is defined as $E(xy) = E(x)E(y)$ when $E(xy)$ is the expected value of $xy$. This is true here, as the shrinkage variation ($\sigma_{\parallel}$) is not related to the other physical contributors to variation ($\sigma_{\perp}$). Second, the original function $f$ must be approximately linear in the region of interest so that there is no difficulty when using the Taylor series approximation. The original equation describing $y_{ij}$ (Equation 6.3) is non-linear because there are trigonometric terms. However the small magnitude of $\sigma_l$ and $\sigma_\theta$ allow this approximation to hold.

Applied probability theory will then manipulate Equation 6.9 to derive the variance of $y_{ij}$ using the knowledge that if two random variables are linearly independent, their variances will add. The following equation will illustrate this point along with the method by which variances of random variables also treat constants and multipliers:

$\mathrm{var}(2x+3y+5)=2^2\,\mathrm{var}(x)+3^2\,\mathrm{var}(y)$, where x and y are linearly independent. If a random variable is multiplied by a constant, the variance is multiplied by the square of that constant. Also the variance of a constant is zero because that constant has only one single value.

Applying these probability rules to Equation 6.9 yields:

$$\begin{aligned}\mathrm{var}(y_{ij})&=\left(\frac{\partial f(l_{io},l_{jo},\theta_{io},\theta_{jo})}{\partial l_i}\right)^2\mathrm{var}(l_i-l_{io})+\left(\frac{\partial f(l_{io},l_{jo},\theta_{io},\theta_{jo})}{\partial \theta_i}\right)^2\mathrm{var}(\theta_i-\theta_{io})\\ &\left(\frac{\partial f(l_{io},l_{jo},\theta_{io},\theta_{jo})}{\partial l_j}\right)^2\mathrm{var}(l_j-l_{jo})+\left(\frac{\partial f(l_{io},l_{jo},\theta_{io},\theta_{jo})}{\partial \theta_j}\right)^2\mathrm{var}(\theta_j-\theta_{jo})\end{aligned}\quad (6.10)$$

The quantity $\mathrm{var}[f(l_{io},l_{jo},\theta_{io},\theta_{jo})]$ is zero because $f(l_{io},l_{jo},\theta_{io},\theta_{jo})$ is a constant, and all the partial derivative terms are squared because they are also constants.

Equation 6.10 is often used to model a manufacturing system and can be generalized using the same linearity and linear independence assumptions stated before. This generalization is shown in Equation 6.11.

$$Var\{Y\}=\sum_{i=1}^{n}\left(\left[\frac{\partial f(x_o)}{\partial X_i}\right]^2 Var\{X_i\}\right)\quad (6.11)$$

where $Y$ is the dependent variable and is a function of several independent random variables $X_1,X_2,X_3,\ldots,X_n$.

Equation 6.10 can be simplified into the following form:

$$\begin{aligned}\sigma^2_{y_{ij}}&=\left(\frac{\partial f(l_{io},l_{jo},\theta_{io},\theta_{jo})}{\partial l_i}\right)^2\sigma^2_{l_i}+\left(\frac{\partial f(l_{io},l_{jo},\theta_{io},\theta_{jo})}{\partial \theta_i}\right)^2\sigma^2_{\theta_i}\\ &\left(\frac{\partial f(l_{io},l_{jo},\theta_{io},\theta_{jo})}{\partial l_j}\right)^2\sigma^2_{l_j}+\left(\frac{\partial f(l_{io},l_{jo},\theta_{io},\theta_{jo})}{\partial \theta_j}\right)^2\sigma^2_{\theta_j}\end{aligned}\quad (6.12)$$

Taking the partial derivatives of Equation 6.3 and substituting them into Equation 6.12 yields:

$$\sigma^2_{y_{ij}}=(\sin(\theta_i))^2\sigma^2_{l_i}+(l_i\cos(\theta_i))^2\sigma^2_{\theta_i}+(\sin(\theta_j))^2\sigma^2_{l_j}+(l_j\cos(\theta_j))^2\sigma^2_{\theta_j}\,.\quad (6.13)$$

Substituting Equations 6.1 and 6.5, and Equation 6.8 into Equation 6.12 yields:

$$\sigma^2_{y_{ij}}=(\sin(\theta_i))^2\beta_1^2 l_i^2+(\cos(\theta_i))^2\beta_0^2+(\sin(\theta_j))^2\beta_1^2 l_j^2+(\cos(\theta_j))^2\beta_0^2\,.\quad (6.14)$$

Simplifying Equation (6.14) yields:

$$\sigma^2_{y_{ij}} = \left(l_i^2 \sin^2(\theta_i) + l_j^2 \sin^2(\theta_j)\right)\beta_1^2 + \left(\cos^2(\theta_i) + \cos(\theta_j)\right)\beta_0^2 . \tag{6.15}$$

A similar equation can be derived if the known dimension is in the X direction, as shown in Figure 6.1:

$$\sigma^2_{x_{ij}} = \left(l_i^2 \cos^2(\theta_i) + l_j^2 \cos^2(\theta_j)\right)\beta_1^2 + \left(\sin^2(\theta_i) + \sin(\theta_j)\right)\beta_0^2 . \tag{6.16}$$

## 6.4.2 Random Variable Derivation

It is possible to rewrite Equation 6.3 to reflect the constant and stochastic components of the position of both features. Figures 6.1 and 6.2 have been redrawn to reflect this relationship and are shown in Figures 6.4a and 6.4b. Figure 6.4a shows the nominal and actual positions of both features i and j.

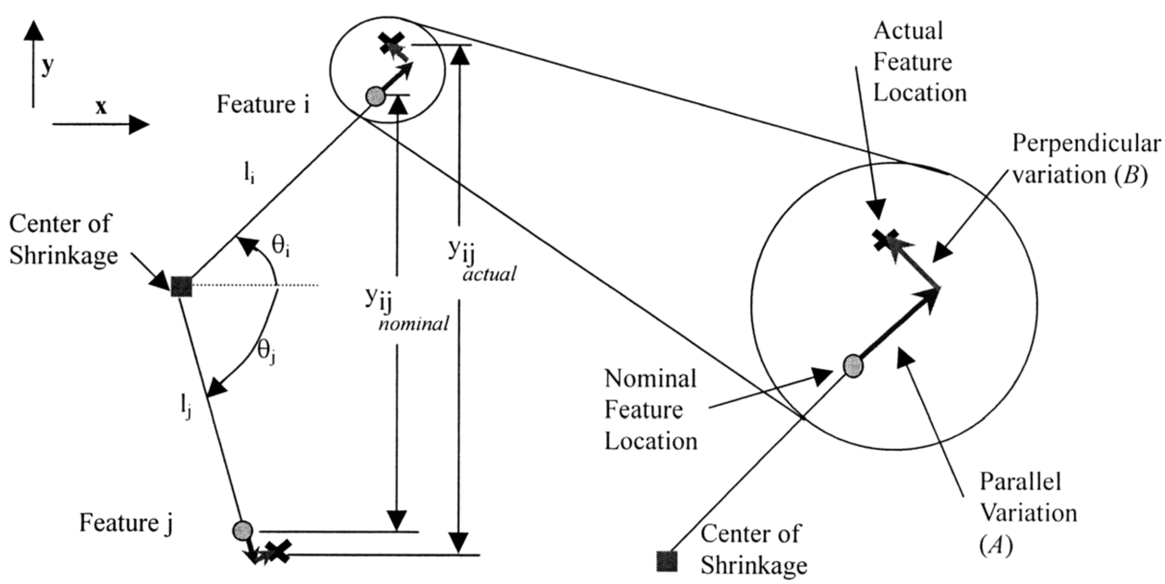



**Figure 6.4a: Center of Shrinkage Geometry.** **Figure 6.4b: Nominal and Actual Feature Position**

Due to variations in material, the injection molding process, and the manufacturing of the mold, the actual position of a feature will be displaced from its nominal position. This variation will be decomposed into two stochastic variables that are parallel and perpendicular to the shortest distance from the center of shrinkage. The stochastic component parallel to $l$ has been termed $A$ and the stochastic component perpendicular to $l$ has been termed $B$. From the geometry shown in Figures 6.4a and 6.4b, it can be determined that the actual distance, $y_{ij}$ is given by Equation 6.17.

$$y_{ij}\Big|_{Actual} = y_{ij}\Big|_{Nominal} + y_{ij}\Big|_{Error}. \tag{6.17}$$

where $y_{ij}\Big|_{Error}$ is given by Equation 6.18.

$$y_{ij}\Big|_{Error} = A_i \sin(\theta_i) + B_i \cos(\theta_i) + A_j \sin(\theta_j) + B_j \cos(\theta_j) \tag{6.18}$$

Random variable $A$ in Equation 6.18 has a normal distribution with standard deviation,

$$\sigma_A = \beta_1 l, \tag{6.19}$$

and represents the variability parallel to $l$. Random variable $B$ in Equation 6.18 has a normal distribution with a standard deviation,

$$\sigma_B = \beta_0, \tag{6.20}$$

and represents the variability perpendicular to $l$. A comparison can be made between the standard deviations for A and B to those shown in Equations 6.1 and 6.2 respectively.

Taking the variance of Equations 6.17 and 6.18, assuming linear independence between features, yields:

$$\begin{aligned} \operatorname{var}\left(y_{ij}\Big|_{Actual}\right) - \operatorname{var}\left(y_{ij}\Big|_{Nominal}\right) &= \operatorname{var}(A_i \sin(\theta_i)) + \operatorname{var}(B_i \cos(\theta_i)) \\ &+ \operatorname{var}(A_j \sin(\theta_j)) + \operatorname{var}(B_j \cos(\theta_j)) \end{aligned}. \tag{6.21}$$

Equation 6.21 can be simplified by realizing that the variance of a constant, $y_{ij}\Big|_{Nominal}$, is zero, and also noticing that the *sine* and *cosine* terms refer to angles at their nominal positions, and are also fixed constants. The position of a feature has been modeled to vary in two perpendicular directions, thus both a radial and angular component have been used to determine the feature's actual position. Equation 6.22 incorporates these simplifications,

$$\sigma^2_{y_{ij}}\Big|_{Actual} = \sin^2(\theta_i)\sigma^2_{Ai} + \cos^2(\theta_i)\sigma^2_{Bi} + \sin^2(\theta_j)\sigma^2_{Aj} + \cos^2(\theta_j)\sigma^2_{Bi}. \tag{6.22}$$

Substituting Equations 6.19 and 6.20 for $\sigma^2_A$ and $\sigma^2_B$ yields:

$$\sigma^2_{y_{ij}} = \sin^2(\theta_i)\beta_1^2 l_i^2 + \cos^2(\theta_i)\beta_0^2 + \sin^2(\theta_j)\beta_1^2 l_j^2 + \cos^2(\theta_j)\beta_0^2. \tag{6.23}$$

Equation 6.23 can be manipulated to obtain the same result found by using a first order Taylor series approximation (Equation 6.15) and is shown in Equation 6.24.

$$\sigma^2_{y_{ij}} = \left(l_i^2 \sin^2(\theta_i) + l_j^2 \sin^2(\theta_j)\right)\beta_1^2 + \left(\cos^2(\theta_i) + \cos^2(\theta_j)\right)\beta_0^2 \tag{6.24}$$

Again a similar result can be obtained if the direction of interest is in the X direction shown in Figures 6.4a and 6.4b:

$$\sigma_{x_{ij}}^2 = \left(l_i^2 \cos^2(\theta_i) + l_j^2 \cos^2(\theta_j)\right)\beta_1^2 + \left(\sin^2(\theta_i) + \sin(\theta_j)\right)\beta_0^2. \tag{6.25}$$

Both Equations 6.15 and 6.16 and Equations 6.24 and 6.25 relate measured dimensional data variation ($\sigma_{y_{ij}}$) to feature variation parameters ($\beta_0, \beta_1$) that can be used to determine feature variation given only preliminary design information ($l, \theta$) through Equations 6.1 and 6.2. The only stipulation is that Equations 6.15 and 6.16, and Equations 6.24 and 6.25 do indeed fit production data from many different feature dimensions from many different parts. This will be discussed next.

## 6.5 Statistical Validation of Dimensional Data

Now that a relationship has been derived that relates the output of a typical quality measurement ($\sigma_{y_{ij}}^2$) and the preliminary design variables, a least squares approach can be used to statistically determine the equation constants $\beta_0$, and $\beta_1$. To do this, some additional practices in the injection molded part industry, first introduced in section 4.4.1.2 are applied. Generally, dimensions of parts fall into two categories, those that are tightly controlled and those that are not. [SPI, 1993] This "Fine" and "Commercial" distinction is common in the plastic injection mold making industry. The concept of a

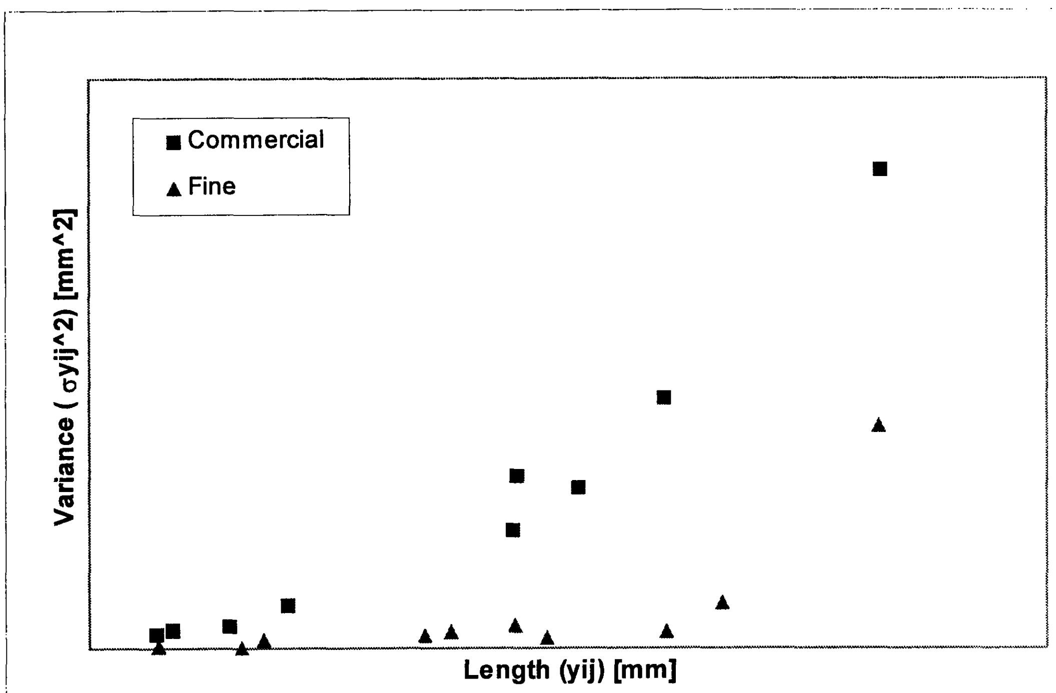



**Figure 6.5: Dimension Variation *vs.* Dimension Length**

Commercial and Fine line was applied to data in Figure 6.5 by visually grouping each data point into either category depending on where each point was located relative to all other points. Thereby two sets of manufacturing capability constants were needed in Equations 6.15 and 6.16: ($\beta_{0\,\text{fine}}$ and $\beta_{1\,\text{fine}}$), and ($\beta_{0\,\text{commercial}}$ and $\beta_{1\,\text{commercial}}$).

Before a least squares linear regression model was applied to determine the constants $\beta_0$, and $\beta_1$, a graph of actual variance versus the dimension length was plotted to observe any obvious trends in the data. Figure 6.5 shows this graph. Note the two distinct clusters of data; the triangular and rectangular data sets represent the SPE/SPI Fine and Commercial lines respectively.

Given Equation 6.15, a least squares statistical analysis will take the form

$$Y = C_1 X_1 + C_2 X_2, \tag{6.26}$$

where $Y = \sigma^2_{y_{ij}}$, $C_1 = \beta_0^2$, $C_2 = \beta_1^2$, $X_1 = \cos^2(\theta_i) + \cos^2(\theta_j)$, and finally $X_2 = l_i^2 \sin^2(\theta_i) + l_j^2 \sin^2(\theta_j)$.

Here Y is the standard deviation of measurements from quality control, $X_1$ and $X_2$ are derived using nominal values of the parts measured, and $C_1$ and $C_2$ are statistically fit.

For each set of features, the center of shrinkage was determined from the approximate center of mass and insights on mold shrinkage. Inputs to the regression model were $l_i$, $l_j$, $\theta_i$, and $\theta_j$. Data was collected from parts made with multiple cavities in multiple tools, and the total variance over all these tooling parameters was used in the regression model.

Using the part dimensional data, the coefficients $\beta_0$, and $\beta_1$ were stochastically determined for the fine and commercial data clusters shown in Figure 6.5. The commercial line fit to a regression statistic of $r^2 = 0.98$, and the fine line fit to a regression statistic of $r^2 = 0.70$. This is graphically shown in Figure 6.6. Next a comparison was made between the actual dimensional data, the center of shrinkage model predicting feature variation, and also the SPI charts. This is also shown in Figure 6.6.

By calculating the coefficient of determination, it can be numerically shown that the center of shrinkage model represents the current data set to a higher degree than the SPI charts do. The coefficient of determination between the actual data and the center of shrinkage line (shown in Figure 6.6) is 0.974 for the commercial line and is 0.562 for the fine line. The corresponding coefficient of determination between the actual data points and the SPI lines (shown in Figure 6.6) is 0.759 for the commercial line and 0.080 for the fine line. Thus, it is clear that the variation model based on the center of shrinkage can be more accurately used as a preliminary design feature variation tool, as it does predict variations well. Note that the regression line derived from the model is much different from the SPI line, especially at longer lengths.

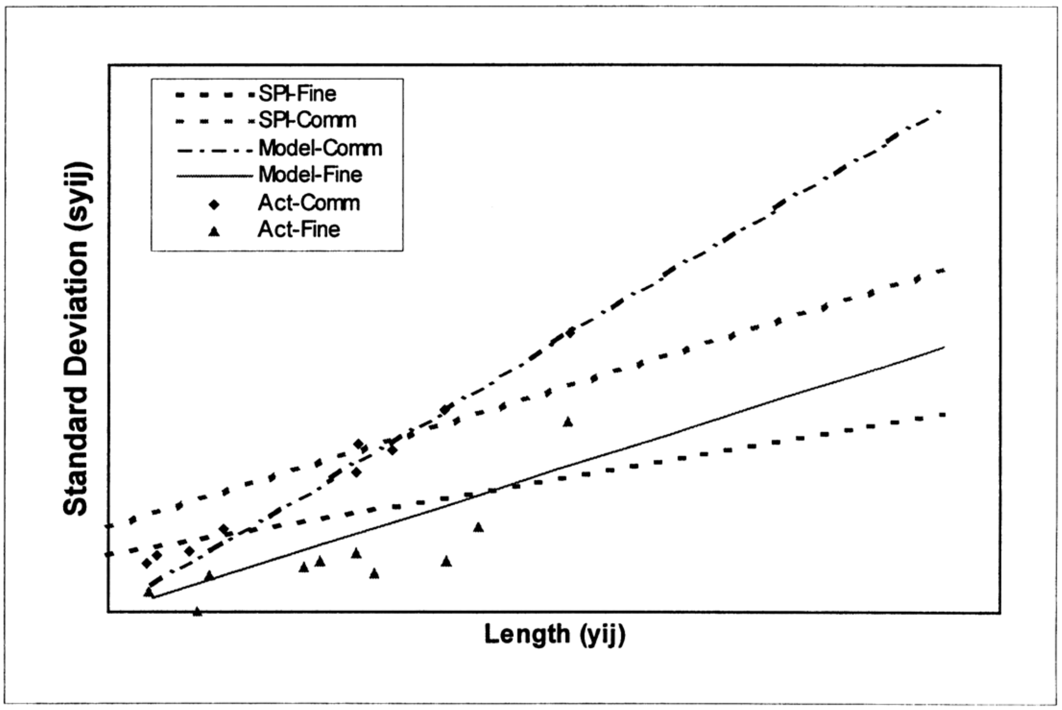



**Figure 6.6: Comparison to SPI Guide.**

# Chapter 7

## Plastic Hole Diameter Variation

The center of shrinkage concept developed in Chapter 6 was revisited and adapted to develop a variational model that will predict the diametrical variation of hole.

Section 7.1 discusses several different methods that are commonly used to measure the diameter of a hole including a concept first introduced in Chapter 3. This method discussed how a coordinate measuring machine (CMM) measures a hole by determining the location of the hole's wall at several different points around the hole. Furthermore, an algorithm was discussed that converted these X and Y measurement into a single measure of diameter. This method was called the method of least squares.

Sections 7.2 and 7.3 discuss the overall geometry of how a hole can be modeled using the theory of a center of shrinkage. Although a generalized approach is derived first, a rather simple variational equation is derived after several geometric observations are made.

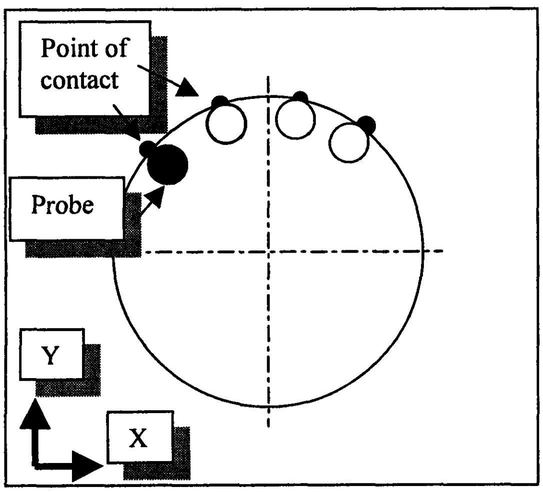
Section 7.4 uses production data to determine the accuracy of the model. After the original model is considered, a secondary term is added to the model to more accurately reflect the center of shrinkage concept developed in the previous Chapter.

Section 7.5 describes a method that can be used to incorporate the statistical nature of a cavity's mean in the variational model derived from the theory of a center of shrinkage. This method decomposes the variation of a part into the variation expected within a cavity and the variation expected between cavities.

## 7.1 Hole Measurement Techniques

There are several different techniques that are commonly used to measure and report the diameter of different sized holes including gaging blocks, coordinate measuring machines (CMM), and vision systems. These types of measurement techniques were discussed in more detail in Chapter three. If an automated technique, such as a CMM or vision system, is used to determine the diameter of the hole, then the method used to report this diameter is important to understand.

Regardless of the automated inspection tool used to measure the part of interest, the fact remains that measuring the diameter of a hole requires measuring several different points on the hole. A CMM accomplishes this task by moving and recording the location of the probe tip at several different locations around the inside wall of the hole. This is described below in Figure 7.1. Also the depth at which a hole's diameter is to be




1. Probe is originally located in the center of where the hole is expected to be.
2. Probe is moved toward the wall of the hole diameter, at an angle perpendicular to the estimated hole diameter wall
3. When the probe touches the wall with a predefined amount of force, or displacement, the X and Y location of the probe is recorded.
4. Steps 2-3 are repeated for a predefined number of points around the hole.

**Figure 7.1: Simple Algorithm for Measuring a Hole Diameter**

measured must be specified, because if a hole is tapered, the measurements will yield different values. This depth is usually defined in the measurement plan before the measurements are taken, so an algorithm can be specified from which the measurement machine will be programmed.

A vision system measures the diameter of a hole in a significantly different way, because a vision system does not use a mechanical method to measure the part. Instead of using a mechanical probe to determine location of the inside wall of the hole, a vision system measures differences in the amount of light defining the location of the hole's diametrical wall. This is accomplished by shining a light from underneath the part so that the light illuminates the inner portion of the hole, whereas the surrounding material is dark. The use of light limits the use of this technology because it often becomes difficult to measure blind holes, and also transparent parts. However, the addition of a light source placed over the part, in conjunction with the traditional light source placed beneath the part, such measurements are possible. After the proper level of light is achieved, the vision system can determine where the hole-diameter wall is located. These measurements are accomplished by detecting distinct differences in light, and then recording the X and Y locations of these locations.

Regardless of whether a CMM or vision system is used, the results obtained from measuring the diameter of a hole is a list of X and Y coordinates where either the probe encountered the wall, or the vision system recorded a distinct difference in light intensity. This list of points corresponding to the X and Y locations around the outside of the hole diameter must now be converted into a single number that is reported in a final inspection report. Several different options are available when converting this array of points into a single number. Two simple options, although not the most widely used, are to report the smallest and largest possible circles that will fit within and outside the given data points respectively. The smallest possible circle is called an inscribed circle, and the largest possible circle is called a circumscribed circle. This distinction is shown in Figure 7.2.

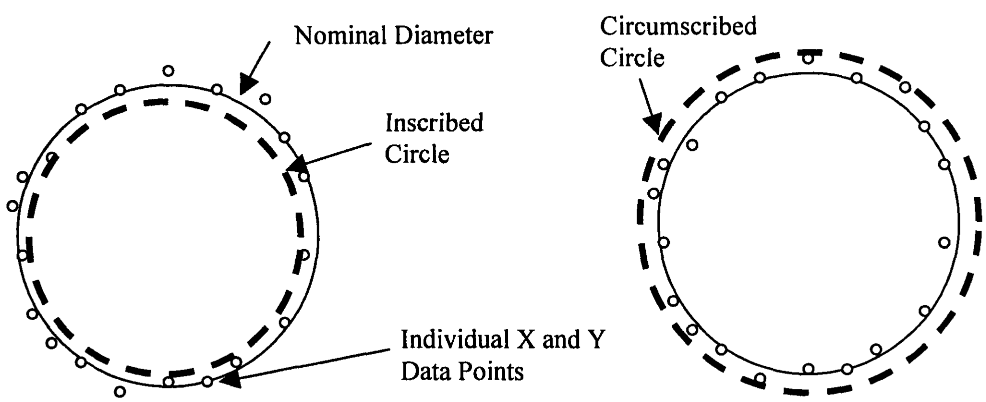



**Figure 7.2: Inscribed and Circumscribed Circles Approximating Hole Diameter Data**

The most common conversion transforming these multiple data points into a single number, is accomplished using a least squares algorithm, where this algorithm is described next.

## 7.1.1 Method of Least Squares Used to Determine a Hole Diameter

There are several options available when reporting the diameter of a hole, with the most popular being the method of least squares. The derivation used in the following section has been derived from [ANSI, 1972].

> "From the center of the chart (see Figure 7.3) draw a sufficient number of equally spaced radii. In the illustration they are shown, numbered 1-12. Two of

these at right angles are selected to provide a system of rectangular coordinates -XX and -YY. The distances to the points of intersection of the polar trace with these radii, P1 to P12, are measured from the axes -XX and -YY, taking positive and negative signs into account. The distances $a$ and $b$ of the least squares center from the center of the paper are calculated from the following approximate formulae: $a = \frac{2\sum x_i}{n}$, and $b = \frac{2\sum y_i}{n}$ ....The radius of the least squares circle, if wanted, is calculated as the average radial distance of the points P from the center,

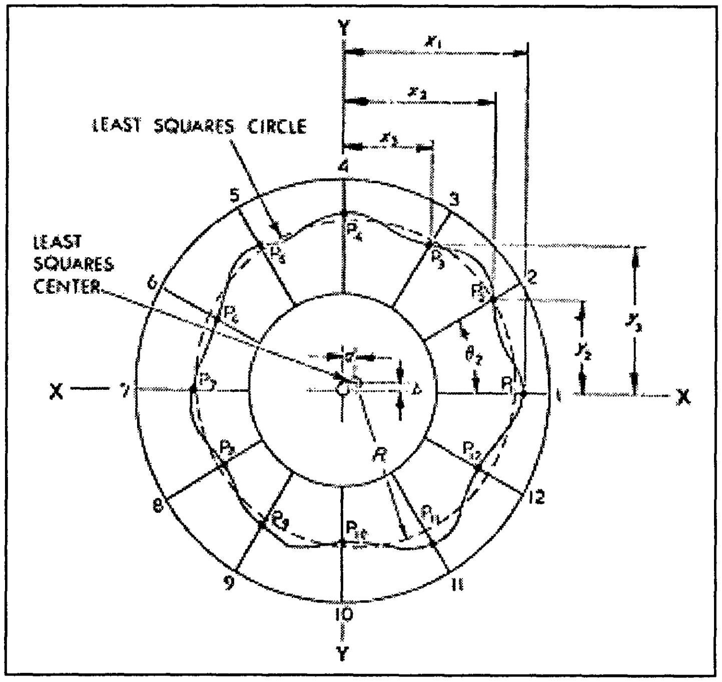



**Figure 7.3: Determination of Center for Least Squares Circle**

that is: $R = \frac{\sum R_i}{n}$ ....The accuracy of determination, both of the center and of the width of the radial zone, depends on the number of ordinates taken."

This ANSI specification can be used to determine the least squares center of a hole given $n$ equally spaced X and Y coordinates around the outside of a hole.

## 7.2 Center of Shrinkage

The concept called center of shrinkage developed in Chapter 6 will now be adapted to model the diametrical variation of a hole. The center of shrinkage is defined as the point to which all features will approach, as they are able, as the part cools. It is possible that the center of shrinkage will not lie within the material of the part. For a part with shapes whose surfaces all shrink away from the mold in cooling, the center of shrinkage is approximated by the center of mass. For parts that are comprised of features that shrink on to the mold wall, such as with holes, undercuts and the like, the center of shrinkage is more toward that feature than from the center of mass. While the center of shrinkage is not completely defined objectively without a detailed analysis, it nonetheless can be used effectively in the preliminary design phase through rough estimation. Estimating the center of shrinkage in the following derivations has been one of approximating the center of mass using two-dimensional techniques, and not through the assistance of a CAD solid modeler. Because this approach has been designed to be used in the preliminary design stage, when limited geometric information is known, and is subject to change, this approach may still be used.

In Chapter 6, the position of a feature was modeled with respect to the center of shrinkage. A similar approach was taken for the diameter of a hole. Figure 7.4 illustrates this point. The square in the figure is the center of shrinkage, and the large circle is the hole diameter being modeled. The definition of the diameter of a circle is a chord connecting two points that lie on the circle, where this chord also passes through the center of the circle. Therefore two points were arbitrarily chosen to satisfy this criteria, and are labeled Point i and Point j in Figure 7.4. Point's i and j can be completely described relative to the center of shrinkage when using $l_i$, $l_j$, $\theta_i$, and $\theta_j$.

The models derived for features moving on a plane in Chapter 6 stated that the position of each feature can be modeled as being both parallel and perpendicular to the shortest lines connecting the features to the center of shrinkage. This model stated that as features i and j cooled, the positional variation of each feature would grow as the distance from the center of shrinkage increased. This model assumed that other sources of variation would not grow with this distance. Similarly, Points i and j in Figure 7.4, will be modeled using similar methods. Therefore the technique of decomposing the variation into two perpendicular components is again used. The geometric derivation of the variational equation for the diametrical variation of a hole is discussed next.

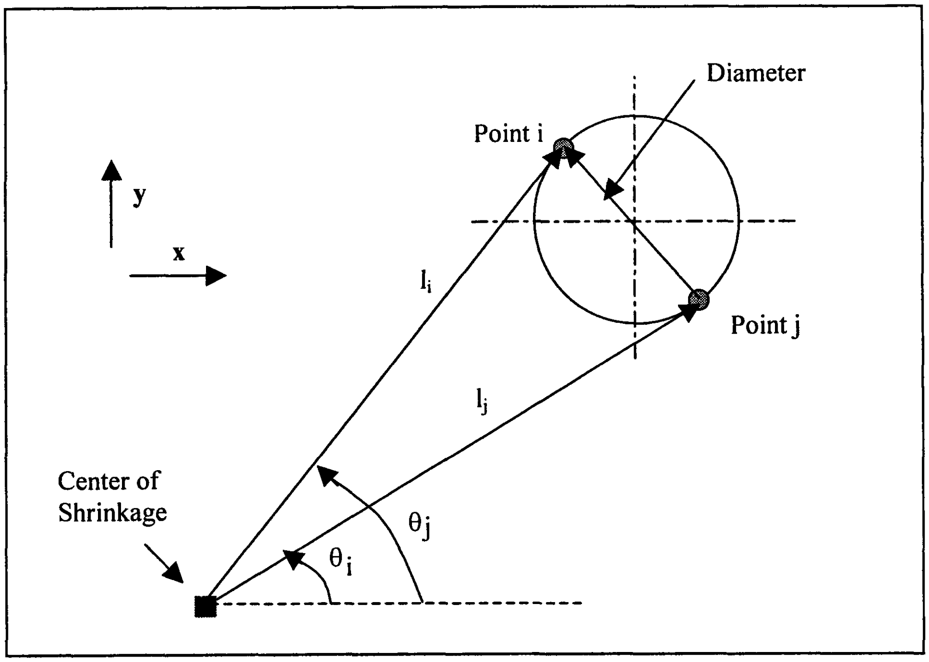



**Figure 7.4: Center of Shrinkage Geometry**

## 7.3 Variation from the Center of Shrinkage

To assure that multiple combinations of location of holes relative to the center of shrinkage can be modeled, a generalized model will be derived, and later simplified to reflect the specific case being considered. From the geometry shown n Figure 7.4 it can be shown through vector algebra that the diameter of a hole can be expressed by:

$$\overrightarrow{diameter} = \vec{l_i} - \vec{l_j} \tag{7.1}$$

where $\vec{l_i}$ and $\vec{l_j}$ are vectors that are described by Equations 2 and 3:

$$\vec{l_i} = (l_i \cos(\theta_i))\vec{i} + (l_i \sin(\theta_i))\vec{j} \tag{7.2}$$

$$\vec{l_j} = (l_j \cos(\theta_j))\vec{i} + (l_j \sin(\theta_j))\vec{j}\ . \tag{7.3}$$

Equations 7.2 and 7.3 can now be substituted into Equation 7.1 yielding:

$$\overrightarrow{Diameter} = (l_i \cos(\theta_i) - l_j \cos(\theta_j))\vec{i} + (l_i \sin(\theta_i) - l_j \sin(\theta_j))\vec{j}\ . \tag{7.4}$$

Equation 7.4 represents the diameter of the hole as a vector, including magnitude and direction, whereas the diameter reported on quality measurements does not possess direction. To enable the variational model for a hole diameter to be experimentally validated, Equation 7.4 must be transformed so that it only possesses magnitude. Therefore the magnitude of Equation 7.4 will be taken and yield:

$$Diameter = \left|\overrightarrow{Diameter}\right| = \left[\left(l_i \cos(\theta_i) - l_j \cos(\theta_j)\right)^2 + \left(l_i \sin(\theta_i) - l_j \sin(\theta_j)\right)^2\right]^{1/2} \quad (7.5)$$

which can be symbolically simplified by making the following substitutions:

$U = l_i \cos(\theta_i) - l_j \cos(\theta_j)$, $V = l_i \sin(\theta_i) - l_j \sin(\theta_j)$, and $W = U^2 + V^2$. Equation 7.5 can now be simplified to:

$$Diameter = [W]^{1/2} \quad (7.6)$$

while the diameter can be represented as a function of four variables:

$$Diameter = f(l_i, l_j, \theta_i, \theta_j). \quad (7.7)$$

Equation 7.7 is a function of the same four variables that described the positional variation of a feature on a plane. Just as the features on a plane used a Taylor series approximation to estimate the variance of a position's variation, the variational model for the diameter of a hole will use the same Taylor series approximation. Again the assumptions of statistical independence of each of the four variables expressed in Equation 7.7, and also of linearity of the function in the region of interest, are still valid when applying Equation 6.11 to Equations 7.5, 7.6 and 7.7. First, it is clear that points i and j are not independent because of the nature by which the geometry correlating the hole to the steel are related, but this dependence is expressed through Equation 7.14. This equation relates $l_i$ and $l_j$ to the diameter of the hole and the distance from the center of the nominal hole to the center of shrinkage. Second, Equation 7.6 is not linear, but due to the magnitude of the variations expected for the diameter of a hole, it can still be used.

Applying Equation 6.11 to Equation 7.7 yields:

$$\sigma^2_{Diameter} = \sigma^2_D = \left(\frac{\partial f}{\partial l_i}\right)^2 \sigma^2_{li} + \left(\frac{\partial f}{\partial \theta_i}\right)^2 \sigma^2_{\theta i} + \left(\frac{\partial f}{\partial l_j}\right)^2 \sigma^2_{lj} + \left(\frac{\partial f}{\partial \theta_j}\right)^2 \sigma^2_{\theta j}. \quad (7.8)$$

Evaluating the partial derivatives of Equation 7.6 and substituting them into Equation 7.8 yields:

$$
\begin{aligned}
\sigma_D^2 &= \left[\frac{1}{2}W^{-\frac{1}{2}}\left(2U\cos(\theta_i)+2V\sin(\theta_i)\right)\right]^2\sigma_{li}^2 \\
&+\left[\frac{1}{2}W^{-\frac{1}{2}}\left(-2U\sin(\theta_i)l_i+2V\cos(\theta_i)l_i\right)\right]^2\sigma_{\theta i}^2 \\
&+\left[\frac{1}{2}W^{-\frac{1}{2}}\left(2U\cos(\theta_j)-2V\sin(\theta_j)\right)\right]^2\sigma_{lj}^2 \\
&+\left[\frac{1}{2}W^{-\frac{1}{2}}\left(-2U\sin(\theta_j)l_j-2V\cos(\theta_j)l_j\right)\right]^2\sigma_{\theta j}^2
\end{aligned}
\tag{7.9}
$$

Next, expressions for $\sigma_{li}^2, \sigma_{\theta i}^2, \sigma_{lj}^2$ and, $\sigma_{\theta j}^2$ should be substituted into Equation 7.9. The variation parallel to the line connecting the center of shrinkage to the point of interest will be modeled as being $\sigma_{\parallel} = \beta_1 l$ (Equation 6.1) and the variation perpendicular to this same line is modeled as $\sigma_{\perp} = \beta_0$ (Equation 6.2). The perpendicular component of variation can be derived to yield the variation in the angular direction by using Equations 6.6, 6.7 and yielding Equation 6.8: $\sigma_\theta = \frac{\beta_0}{l}$. Equations 6.1, and 6.8 are then substituted into Equation 7.9. Substitution of these quantities, followed by algebraic

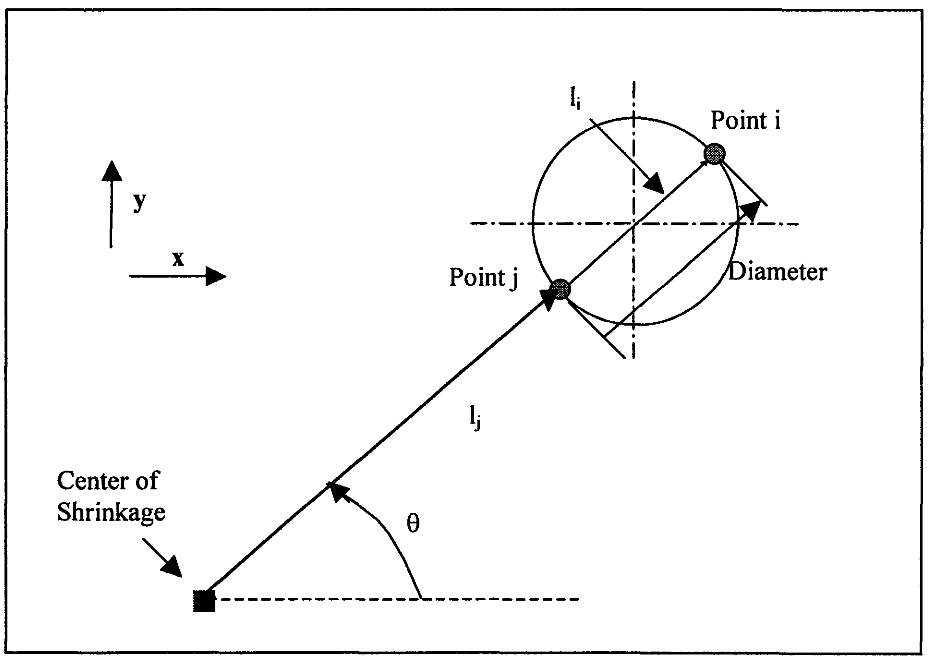



**Figure 7.5: Points I and j are Collinear**

simplification yields:

$$\begin{aligned}\sigma_D^2 &= \beta_0^2\left\{W^{-1}\left[\left(U\cos(\theta_i)+V\sin(\theta_i)\right)^2 l_i^2+\left(U\cos(\theta_j)-V\sin(\theta_j)\right)^2 l_j^2\right]\right\}+\\ &\beta_1^2\left\{W^{-1}\left[\left(-U\sin(\theta_i)+V\cos(\theta_i)\right)^2+\left(-U\sin(\theta_j)-V\cos(\theta_j)\right)^2\right]\right\}\end{aligned} \quad (7.10)$$

Several observations considering the geometry of Figure 7.3 can be made to simplify Equation 7.10. First if $\theta_i = \theta_j = \theta$, Points i and j will be collinear with the diameter of the nominal hole diameter, as shown in Figure 7.5. This colinearity will result in the simplification of Equation 7.10 to yield:

$$\begin{aligned}\sigma_D^2 &= \beta_0^2\left\{\frac{1}{(l_i-l_j)^2}\left[4(l_i-l_j)^2\sin^2(\theta)\cos^2(\theta)\right]\right\}+\\ &\beta_1^2\left\{\frac{1}{(l_i-l_j)^2}\left[(l_i-l_j)^2 l_i^2+(l_i-l_j)^2\left(2\cos^2(\theta)-1\right)^2 l_j^2\right]\right\}\end{aligned} \quad (7.11)$$

The next geometric observations that, once made, will simplify Equation 7.10 are that Points i and j, the nominal circle of the hole, and the center of shrinkage are all collinear. The knowledge that the angle θ, as defined in Figure 7.5, is arbitrarily defined will also greatly simplify Equations 7.10 and 7.11. Therefore the coordinate system shown in Figure 7.5 can be rotated until the X-axis is directed along the same direction as the line connecting Points i, j, the center of the nominal circle, and the center of shrinkage. Rotating about the center of shrinkage equates θ to zero. Combining these observations simplifies Equation 7.11 to Equation 7.12, where the results are graphically illustrated in Figure 7.6.

$$\sigma_D^2 = \beta_1^2\left(l_i^2 + l_j^2\right) \quad (7.12)$$

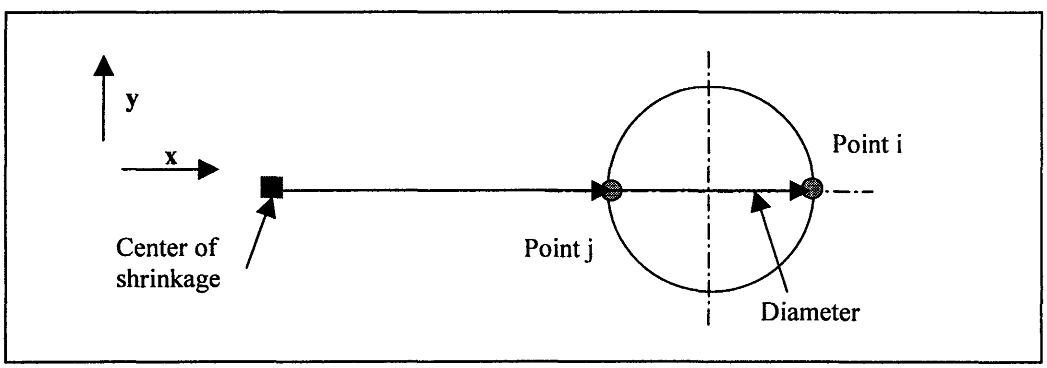



**Figure 7.6: Points I and j are Collinear with the Center of Shrinkage**

Equation 7.12 relates measured dimensional information $(\sigma_D^2)$ to feature variation parameters $(\beta_1)$. This relationship has been developed using only information that will be

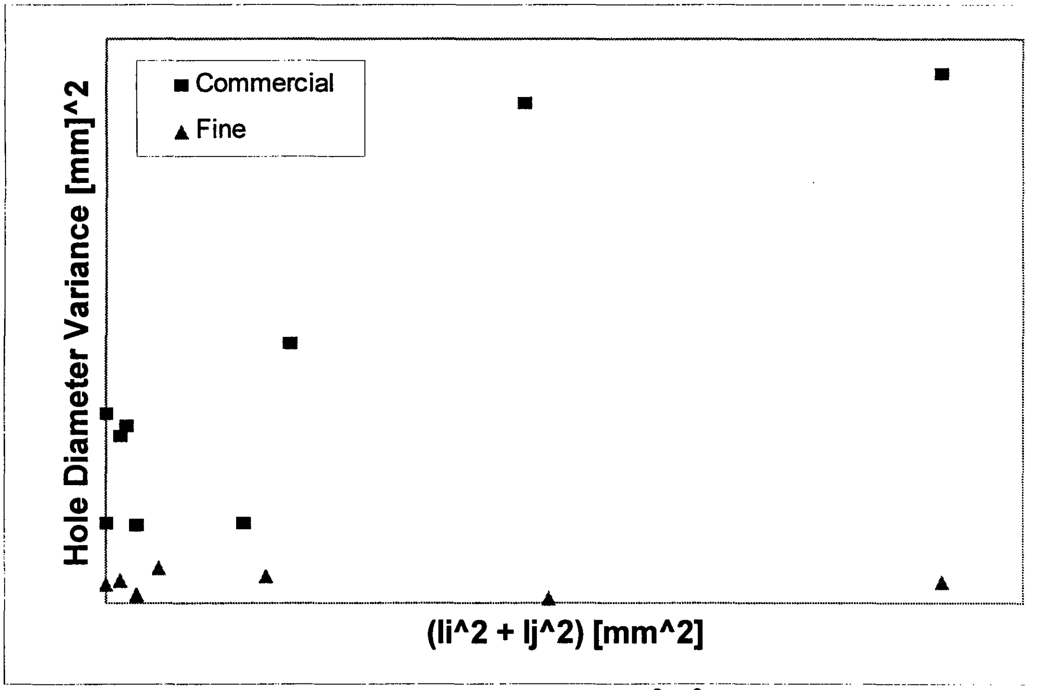



**Figure 7.7: Hole Variation *vs.* ($li^2$+$lj^2$) Graph**

known during the preliminary design of the part. This will enable the above model to be used as a preliminary design tool that aids the design team when making critical design decisions with only very limited information. The task of validating this model will be discussed in the next section.

## 7.4 Statistical Validation of Hole Diameter Variational Model

Now that a relationship has been developed that relates manufacturing quality measurements to preliminary design variables, a least squares approach is used to statistically determine the constant $(\beta_1)$. First, a graph of diametrical variance *vs.* the quantity $(l_i^2 + l_j^2)$ was plotted to determine any obvious trends in the data. This graph is shown in Figure 7.7. Two different series were plotted in Figure 7.7 because it was postulated that the data could be classified into either a "Commercial" or "Fine" category as was discussed in Chapter 6. The concept of a Commercial and Fine line was applied to the data in Figure 7.7 by visually grouping each data point into either category depending on where each point was located relative to all other points. This classification is analogous to the SPE/SPI guidelines that also classify a part into two distinct categories when estimating the tolerances to be placed on a dimension. [SPI, 1993]

Given Equation 7.12, a least squares analysis will take the form:

$$Y = C_1 X_1 \tag{7.13}$$

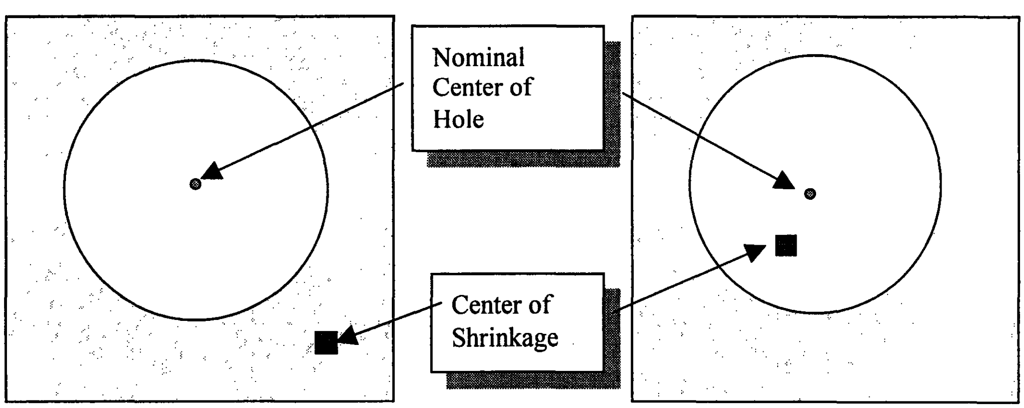
where,

$Y = \sigma_D^2$, $C_1 = \beta_1^2$, and $X_1 = \left(l_i^2 + l_j^2\right)$. In Equation 7.13, Y represents the diametrical hole variance provided from quality control, $X_1$ is derived using nominal values for $l_i$ and $l_j$, and $C_1$ is statistically determined. For each hole diameter, the center of shrinkage was determined from the approximated center of mass, and insights on mold shrinkage. If a center of shrinkage was determined in Chapter 6 for the part, then this same center of shrinkage was used in the hole diameter model. Data from 16 hole diameters from parts manufactured using only one material was collected, where the parts were being molded in multiple tools and multiple cavities. In practice, $l_i$ and $l_j$ were calculated using Equations 7.14

$$l_i = \left| l_C + \frac{Diameter}{2} \right|, \text{ and } l_j = \left| l_C - \frac{Diameter}{2} \right|, \tag{7.14}$$

and the hole sizes used in this model varied from about 1.5mm to about 13mm.

There are two possible geometrical combinations for the center of shrinkage and the nominal center of the hole. The center of shrinkage could be located either inside or outside the diameter of the hole. These two possibilities are shown in Figure 7.8. Regardless of where the center of shrinkage is located with respect to the nominal center




a) Center of shrinkage is located outside the hole diameter

b) Center of shrinkage is located within the hole diameter

**Figure 7.8: Location of the Center of Shrinkage Relative to Hole Diameter**

of the hole, Equations 7.14 hold true.

A regression analysis was performed using Equation 7.13, and was aimed at experimentally determining the unknown coefficient $\beta_1$. $\beta_1$ was determined and used to plot the solid line in Figure 7.9. As is expected, the model predicts a linear relationship between the diametrical variance of a hole and the quantity $\left(l_i^2 + l_j^2\right)$ with the constant of proportionality being $\beta_1^2$. As Figure 7.9 shows, this linear relationship does not accurately predict the experimental data to a high degree.

Two possible solutions were implemented that improved the degree of fit between the model and the experimental data. First the observation that there were two trends in the data led to the theory that two different models were required. These two different models represented the SPE/SPI Commercial and Fine lines as described earlier in this

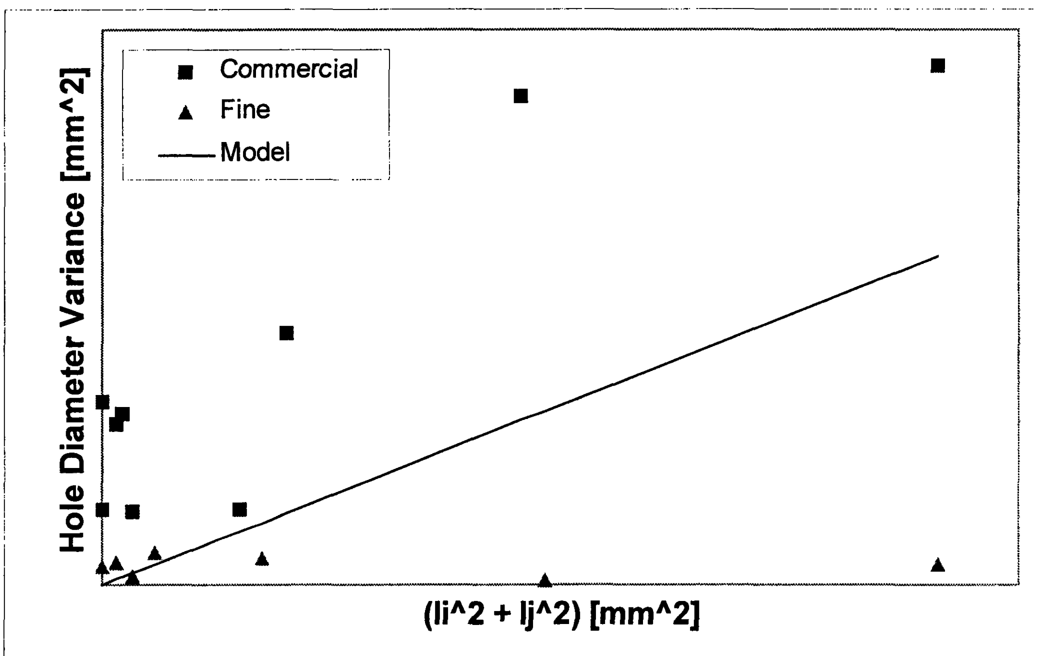



**Figure 7.9: Regression Analysis Modeling the Diametrical Variation of a Hole**

section. Second, it was also observed that the models did not contain a Y-intercept, because that quantity was eliminated from the model because of the co-linearity of the center of shrinkage, with the center of the circle, and Points i and j, shown in Figure 7.6. Therefore the model developed for the diametrical variation of a hole, Equation 7.12, was modified and is described next.

## 7.4.1 Modification of Original Model

The variational model describing the diametrical variation of a hole, resulting in Equation 7.12 was modified to include a Y-intercept. The Y-intercept in question is similar to the one proposed in Chapter 6 and was modeled as $\beta_0$. This term is intended to include variational contributors that do not grow with the Point's distance from the center of shrinkage. For example, the variability introduced when machining the mold steel, or the arbitrary placement of a cavity into a multi-cavity mold, or the temperature gradients that exist in a multi-cavity mold.

The Equations that derived the variational equation using the first order Taylor series approximation (Equations 7.8 and 7.9) were revisited to add the $\beta_0$ term. $\beta_0$ was added to the model of variation parallel to the center of shrinkage and the nominal center of the hole resulting in Equation 7.15.

$$\sigma_{\|} = \sigma_l = (\beta_0 + \beta_1 l). \tag{7.15}$$

Substituting Equation 7.15 into Equation 7.9 and also the geometric observation from Figure 7.5 that $\theta_i = \theta_j = \theta = 0$, Equation 7.8 can be simplified to obtain:

$$\sigma_D^2 = 2\beta_0^2 + 2\beta_0\beta_1(l_i + l_j) + \beta_1^2(l_i^2 + l_j^2), \tag{7.16}$$

where $\sigma_D$ is diametrical variation determined from experiments, and, $l_i$, and $l_j$ have the same meanings as described in Equation 7.14.

Now that a relationship has been developed which takes into account other non-shrinkage-related factors, a regression analysis was initiated. The regression analysis in this case was non-linear, as opposed to the linear model used before. Therefore a nonlinear statistical fitting approach was used to determine the statistical constants $\beta_0$, and $\beta_1$. Again, the data was classified into two groups motivated by the industry standard SPE/SPI charts. [SPI, 1993]

Figure 7.10a shows the results when using the model shown in Equation 7.16 to approximate the experimental results. Figure 7.10a is plotted on the same X-axis as Figure 7.9 so a comparison can be made. However, because Equation 7.16 is a non-linear equation, a three-dimensional plot would be required to fully describe the relationship between $\sigma_D^2$, $l_i$ and $l_j$. This is why Figure 7.10b shows a graph of diametrical variance *vs.* part number, where both the actual and modeled variances are shown for comparison. A useful method that can be used to compare the original model to the current model is to use the sum of squared errors, or SSE. This quantity is defined as being the sum of the squared differences between the actual data point and the corresponding point on the least squares fit line, with the same X coordinate, for all experimental data points. The difference between the experimental data point and the corresponding point on the model with the same X-coordinate, is also called the residual, so the SSE term is also called the residual sum of squares. This residual sum of squared term is also the function that's minimized when determining the statistical constants $\beta_0$, and $\beta_1$. The SSE terms are summarized in Table 7.1 below.

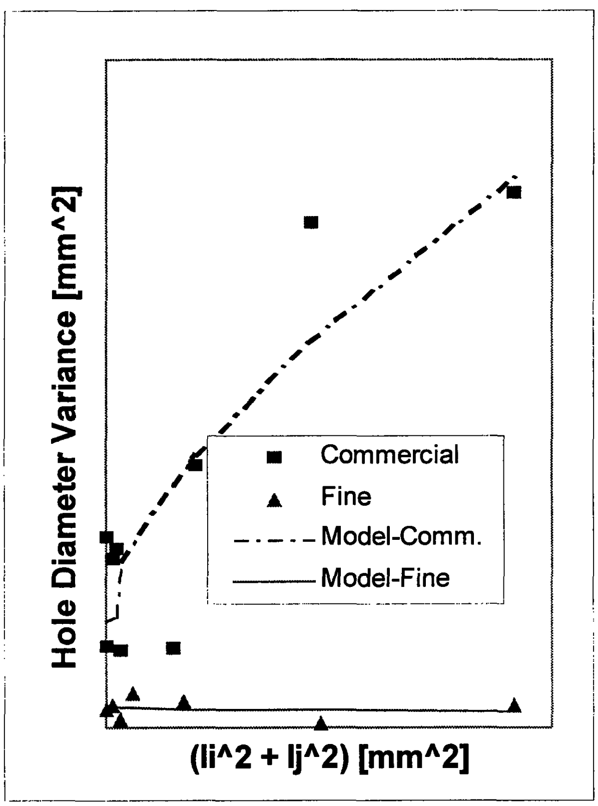


Figure 7.10a: Non-linear Model Verification

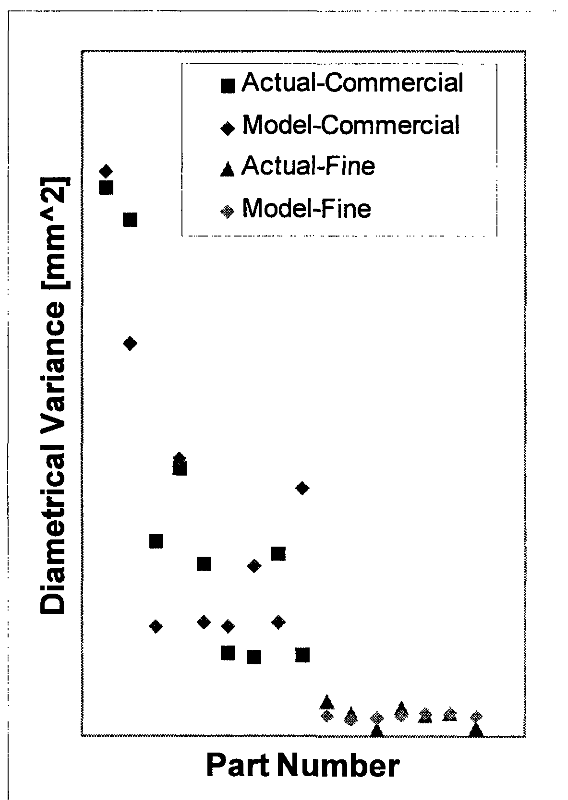


Figure 7.10b: Non-linear Model Verification

|  | SSE | SSE |
|---|---|---|
| Model Type | Commercial | Fine |
| Linear | 4.27E-08 | 7E-10 |
| Non-linear | 1.75E-08 | 2E-10 |

**Table 7.1: Summary of Model Error**

The SSE decreased in both the Commercial and Fine cases when the non-linear model was used, as opposed to the linear model. The reason for these decreases is because there was a source of variation that was not being modeled. After this other source of variation was modeled accordingly, the relationship between the experimental data points and the model improved, and the SSE decreased.

## 7.5 Incorporating Cavity Mean Offset into Variational Model

When investigating the individual points in Figure 7.10, it has been discovered that the variance over all cavities is directly related to the probability that the mean of each cavity will be significantly offset from the other cavity means, and the nominal specification for the same part. This has led the author to investigate the effect of cavity mean offset when using the center of shrinkage derivation.

The variational model that has been derived from the theory of center of shrinkage is now expanded and modified to incorporate the concept of predicting cavity mean offset. The variational model presented in Equation 7.16 is decomposed so that these two different sources of variation could be modeled separately.

This new variational model is defined in Equation 7.17:

$$\sigma_D^2 = \left[2\beta_0^2 + 2\beta_0\beta_1\left(l_i + l_j\right) + \beta_1^2\left(l_i^2 + l_j^2\right)\right] + \sigma^2_{\substack{cavity\\mean\\offset}}, \tag{7.17}$$

where the term $\sigma^2_{\substack{cavity\\mean\\offset}}$ represents the effect that cavity mean offset will have on the diametrical variation of a hole.

Cavity mean offset can also be decomposed into two different types: displacement of an individual cavity with respect to part's average, and also the displacement of the part's average relative to the nominal specification. Here the part's average is defined as the average of all data points over all cavities. These two different types of cavity mean offset are illustrated in Figure 7.11. The first type of cavity mean offset is when the mean

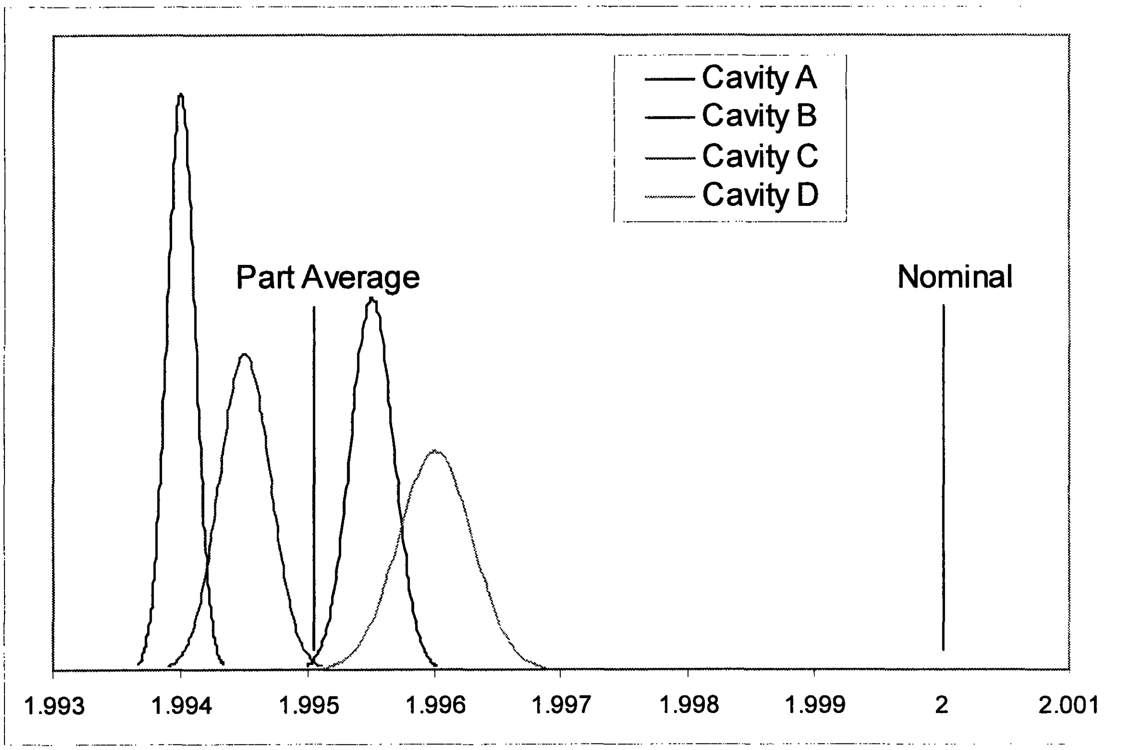



**Figure 7.11: Illustration of Two Different Types of Cavity Mean Offset**

of each cavity is displaced relative to the part average. The second type of cavity mean offset is illustrated by the part average, being displaced relative to the nominal specification.

The distinction between these two different types of cavity mean offset is made, because the first type will effect the diametrical variation of a hole, where the second will not. The second type of cavity mean offset will not effect diametrical variation because, the variational model given in Equation 7.17 has been derived to predict the diametrical variation of a any one hole for any one part with respect to that part's average, and not the diametrical variation for all possible holes for all possible geometries. Given the nominal geometry of a single hole, and the part's material, one can use Equation 7.17 to predict the diametrical variation of a hole with respect to the part's average. Equation 7.17 should not be used to predict how much the part's average will be offset from the intended nominal of the part.

Equation 7.17 uses the center of shrinkage concept to model the diametrical variation of a hole, given that the mean of each cavity is producing parts at the part's average. In order to apply this model, the statistical parameters $\beta_0$ and $\beta_1$ are determined using a regression analysis, similar to the one described in section 7.4.1, except that the mean of each cavity is adjusted to be equal to the part's average. This procedure will be discussed next through the use of an example.

### 7.5.1 Equating the Means of All Part Cavities to the Part's Average

In order to shift the mean of each cavity so that it equals the part's average, a relationship between the mean of a single cavity and the overall variance of all the cavities must be developed. This was the inspiration for developing Equation 4.8 that described the affect cavity mean offset has on the overall variance across several cavities. This idea will be illustrated through an example where the means of four cavities will be offset so that the mean of each cavity will be equal to the mean of all cavities, or the part's average. For the purposes of this example, it has been previously determined that cavities one through four consistently produce parts with dimensions that can be described using normal distributions with the statistical parameters shown in Table 7.2. Figure 7.12 shows the distribution of these four cavities.

| Cavity | Average | StdDev |
|---|---|---|
| 1 | 2.1736 | 0.000696 |
| 2 | 2.172405 | 0.002126 |
| 3 | 2.1681075 | 0.004095 |
| 4 | 2.17366 | 0.001555 |
| All | 2.171943 | 0.003334678 |

**Table 7.2: Statistical Parameters for Cavities One through Four**

Because the distribution for each cavity will be moved from its current position, represented by $a$, to the part's average, represented by $x$, the quantity $(x-a)$ is equal to (Part Average - Cavity Average). Tabulated values for the quantities (Part Average - Cavity Average) for cavities one through four are shown in Table 7.3

| Cavity | (Part Average - Cavity Average) |
|---|---|
| 1 | -0.00165688 |
| 2 | -0.00046188 |
| 3 | 0.00383562 |
| 4 | -0.00171687 |

**Table 7.3: Cavity Mean Offset Values**

Therefore cavity one will be required to decrease its mean by 0.00165688 so that the cavity will be centered on the part average. It is not expected that cavity one will experience a change in its standard deviation as a result of this move, because the entire distribution will be offset.

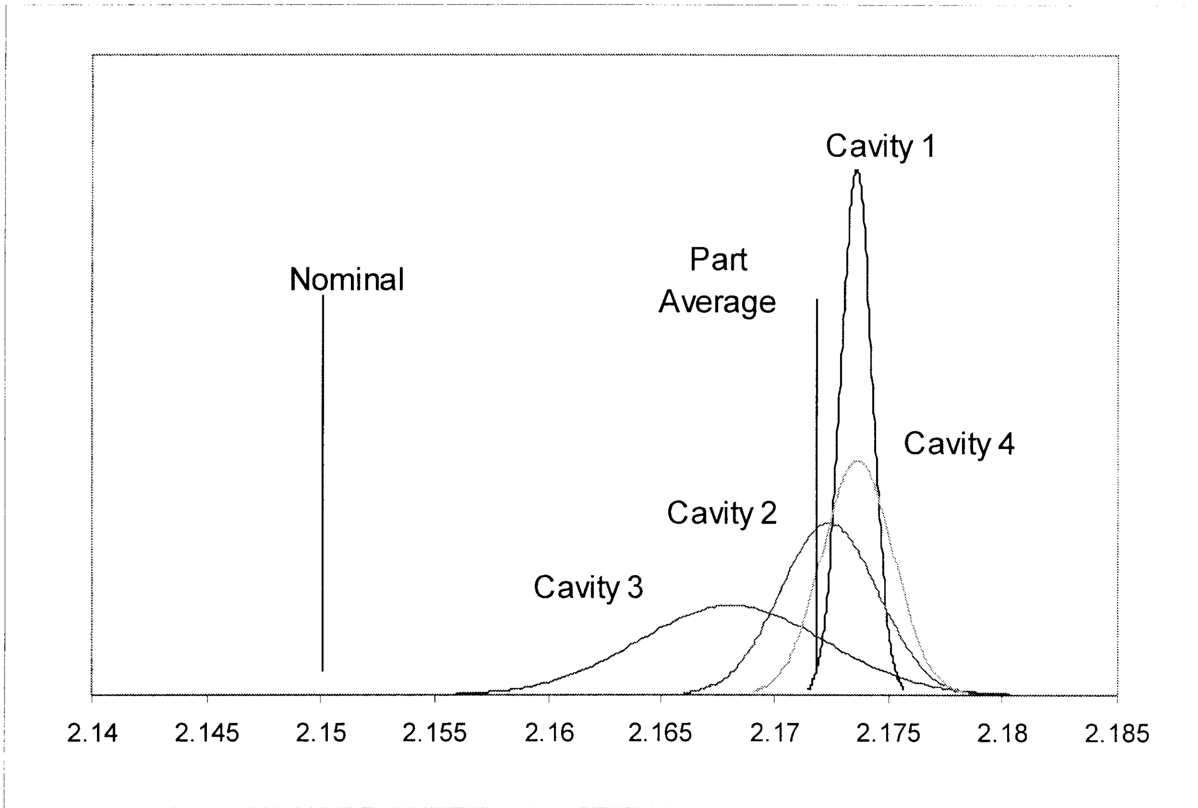



**Figure 7.12: Four Cavities being Nominalized**

Applying the appropriate mean shift to cavity one, substituting the appropriate values into Equation 4.8, and also knowing that there were forty samples taken for each cavity yields:

$$\Delta\sigma_{total}^2 = \frac{2*40}{[(40+40+40+40)-1]}*$$
$$\left[2.1736 - \frac{(2.1736*40+2.172405*40+2.1681075*40+2.17366*40)}{40+40+40+40}\right](-0.00165688)$$
$$+\frac{2*40}{[(40+40+40+40)-1]}\left[1-\frac{40}{40+40+40+40}\right]\frac{(-0.00165688)^2}{2!}$$

$$\Delta\sigma_{total}^2 = -8.6328E-07 .$$

Therefore, after cavity one has been shifted, its average decreased by 0.00165688, and the resulting change in the overall variance of all four cavities is $\sigma_{New}^2 = \sigma_{Old}^2 + \Delta\sigma_{total}^2 = (3.334678\text{E-}03)^2 - 8.6328E-07 = 1.02568\text{E-}05$. This result can be experimentally verified, by adding (-0.00165688) to each data point in cavity one, and then calculating the cavity average, cavity standard deviation, and the resulting overall standard deviation. This will verify that the mean of cavity one is now equal to the part's average (2.171943), its standard deviation will not have changed, and the overall variance will have decreased by the appropriate amount.

Next the mean of cavity two will be equated to the part's average, and the corresponding change in the overall variance of all four cavities will be calculated as before. However, the new average for cavity one (2.171943) must be used in Equation 4.8. This calculation is shown below.

$$\Delta\sigma^2_{total} = \frac{2*40}{[(40+40+40+40)-1]}*$$
$$\left[2.172405 - \frac{(2.171943*40+2.172405*40+2.1681075*40+2.17366*40)}{40+40+40+40}\right](-0.00046187)$$
$$+\frac{2*40}{[(40+40+40+40)-1]}\left[1-\frac{40}{40+40+40+40}\right]\frac{(-0.00046187)^2}{2!}$$
$$\Delta\sigma^2_{total} = -1.6335E-07$$

Therefore, the updated overall variance of all four cavities, resulting from the means of cavities one and two being equated to the part's average, is: $\sigma^2_{New} = \sigma^2_{Old} + \Delta\sigma^2_{total} = 1.02568E\text{-}05 - 1.6335\text{E}-07 = 1.00934\text{E}-05$. This procedure is repeated for cavities three and four, yielding the results in Table 7.4, where the final result from shifting cavities one through four yields an overall variance of 5.9331E-6. This is considerably less than the original diametrical variance before the means of all four cavities were adjusted to be equal to the part's average: $0.003334678^2$=1.11201E-05. The results obtained from adjusting all four cavities can be observed in Figure 7.13. Again this process has not effected the spread of any of the cavities. Cavity one still has the tightest spread, or smallest variance, whereas cavity three has the widest spread, or largest variance, and now all four cavities are centered about the part's average.

| | Delta $\sigma^2$ | Current $\sigma^2$ |
|---|---|---|
| Cavity 1 nominalized | -8.6328E-07 | 1.02568E-05 |
| Cavity 1, and 2 nominalized | -1.6335E-07 | 1.00934E-05 |
| Cavity 1, 2, and 3 nominalized | -3.6041E-06 | 6.48937E-06 |
| Cavity 1, 2, 3, and 4 nominalized | -5.5624E-07 | 5.93313E-06 |

**Table 7.4: Resulting Change in Overall Variance**

The procedure described above was repeated for all four cavities, and for each of the sixteen parts used in the variational model for hole diameter.

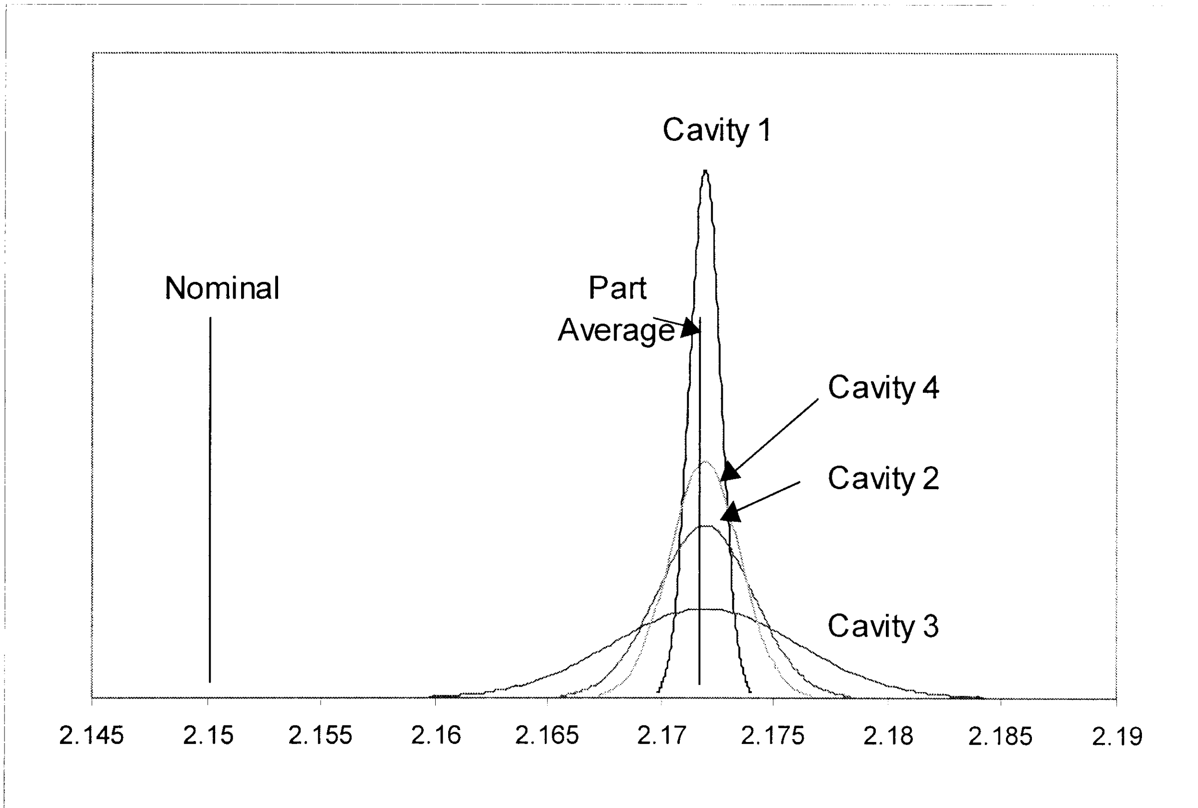



**Figure 7.13: Nominalizing All Four Cavities**

## 7.5.2 Determining the Statistical Parameters $\beta_0$ and $\beta_1$

As was stated in section 7.5, the statistical parameters $\beta_0$ and $\beta_1$ are determined using variances that are derived from cavities having means equal to their specific part averages. This procedure, described in section 7.5.1, was completed for all parts, resulting in the mean of all individual cavities, for a specific part, being equal to the mean over all cavities in that part. A regression analysis similar to the one described in section 7.4.1, was performed where the following model was used:

$$Y = C_1\beta_0^2 + C_2\beta_0\beta_1 + C_3\beta_1^2 \tag{7.18}$$

where

$Y = \sigma^2_{\substack{Part\\Average}}$, $C_1 = 2$, $C_2 = 2(l_i + l_j)$, and $C_3 = (l_i^2 + l_j^2)$. Again, the data was classified into two groups motivated by the industry standard SPE/SPI charts [SPI, 1993]. These two sets of constants were used to derive the graph shown in Figure 7.14 that plots diametrical variance *vs.* part number for both the model and the actual data points. This graph is very similar to the graph shown in Figure 7.10b, except for the scale on the vertical axis, because this graph compares the diametrical variances (given that all cavity means equal to the part averages) to the variances predicted by the model. Table 7.5 shows the Sum of Squared Error (SSE) for the diametrical model for a hole when the effect of cavity mean offset is removed from the model

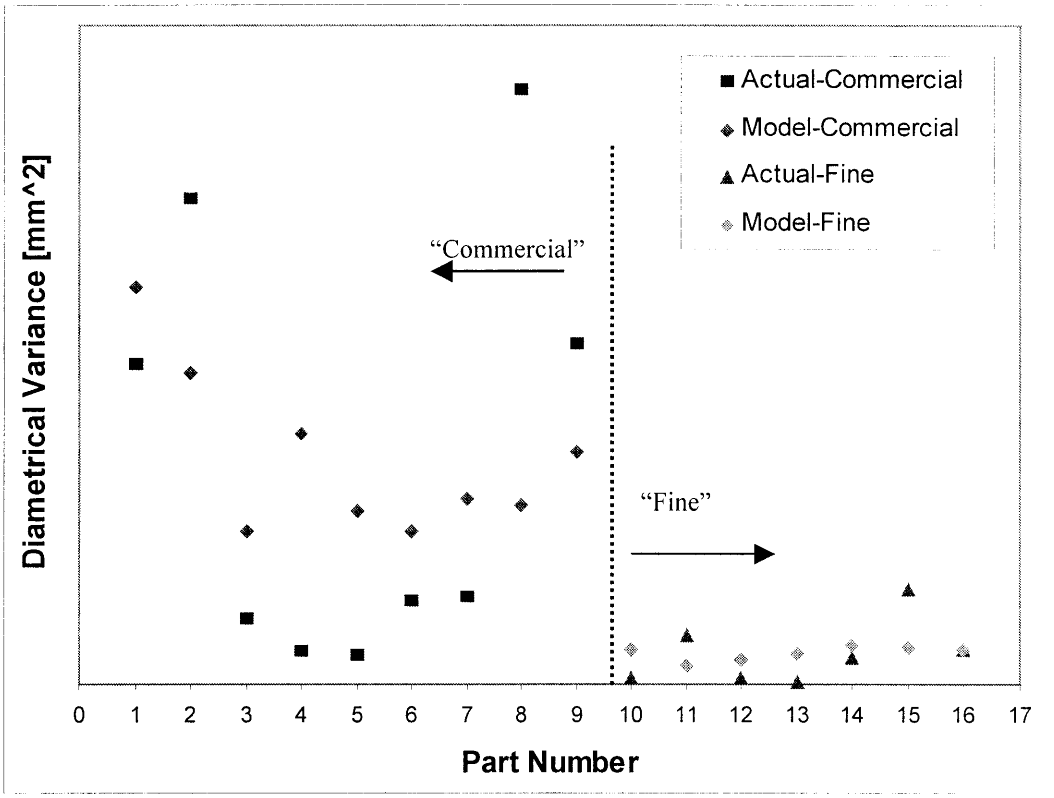



**Figure 7.14: Graph of Diametrical Variance *vs.* Part Number when Cavity Mean = Part Mean**

Next, a relationship will be developed to determine the probability that a particular cavity will be offset from its nominal position. This will be discussed in the following section.

| | Comm | Fine |
|---|---|---|
| No cavity mean offset effects | 4.6E-09 | 9.53E-11 |

**Table 7.5: Summary of SSE Model Error**

## 7.5.3 Determining Cavity Mean Offset With Respect to Part Average

Now that the first part of Equation 7.17 has been completed by using the center of shrinkage derived model to predict the diametrical variance of a hole with respect to the part's average, a relationship must be determined to express the $\sigma^2_{\substack{cavity\\mean\\offset}}$ term in Equation 7.17. This term describes how the mean of each cavity will be expected to vary with respect to the part's average, or the average over all cavities, and is expected to be a single numerical value. It is expected that the probability that each cavity will be equal to the part's average is not dependant on the particular part or the size of the hole under consideration.

To determine the appropriate value of $\sigma^2_{\substack{cavity\\mean\\offset}}$, a histogram of the quantity (Cavity Average - Part Average) was plotted verify the independence assumption stated above. This histogram is shown in Figure 7.15. The form of the histogram appeared to originate

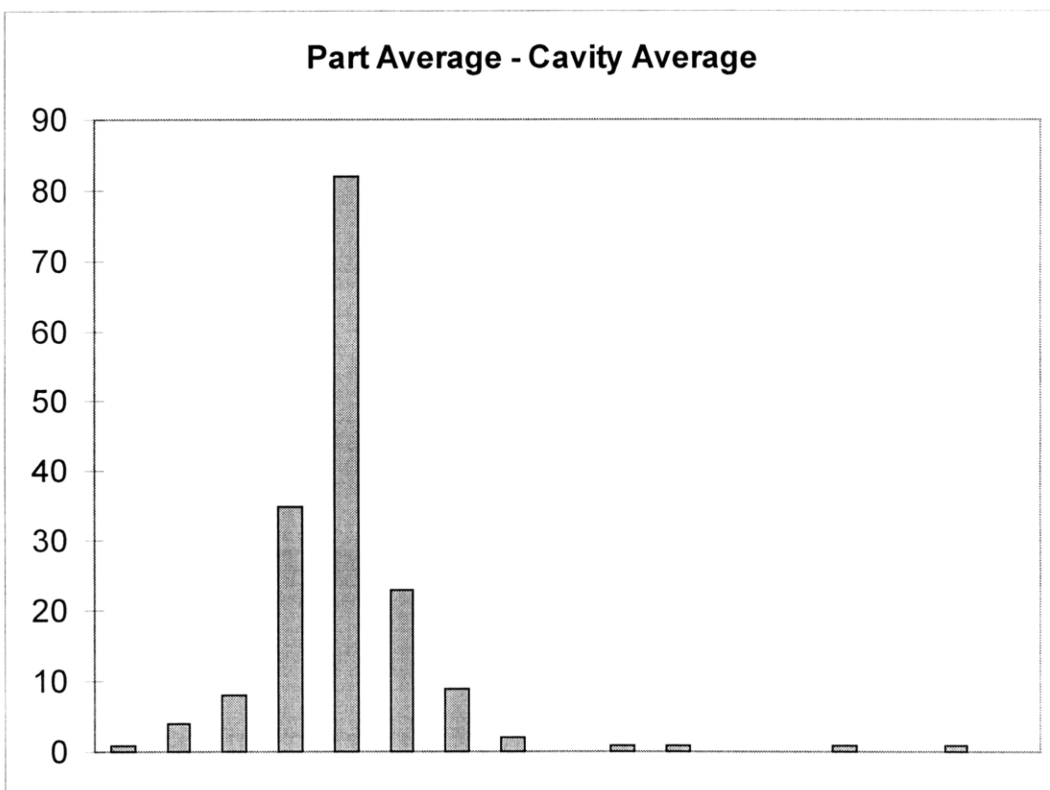



**Figure 7.15: Histogram of (Cavity Average - Part Average)**

from a normal distribution, as expected, because the amount a particular cavity will be offset relative to the part's average is independent of the type of part and regardless of the cavity. The statistical properties of the quantity (Cavity Average - Part Average) were next determined. To obtain better estimates of the statistical parameters of the data shown in Figure 7.15, the four right most points were not included in this analysis because these four points would significantly drive up the estimates when these points may in fact be outliers.

The value of $\sigma^2_{\substack{cavity \\ mean \\ offset}}$ is exactly equal to the variance of the data used to generate the histogram shown in Figure 7.15. The term $\sigma^2_{\substack{cavity \\ mean \\ offset}}$ has been defined as being the variance of how far a cavity will be offset with respect to the part's average, which is exactly the quantity (Cavity Average - Part Average).

### 7.5.4 Summarizing Cavity Mean Offset Variational Model

Two models have been developed that combine to form a variational model that predicts the diametrical variance of a hole. The first part of this model predicts the variation within a cavity with respect to the part's average and is based on the theory of a center of shrinkage. The part's average has been defined as the average over all cavities for a particular hole dimension on a particular part. Next, a model describing how the mean of each cavity is expected to be different from the part's average was developed directly from plotting a histogram of the quantity (Cavity Average - Part Average).

These two models can now be combined to estimate the total diametrical variance of a hole. This new estimate of $\sigma_D^2$ is shown in Figure 7.16. The sum of squared error (SSE) has been calculated for this new model and compared to the other models developed in this thesis for the diametrical variation of a hole. This comparison is shown in Table 7.6. The SSE has been found to be greater than the error for the linear and the non-linear model for the commercial data set. However the error for the new data set is greater than the error of the non-linear, but less than that of the Fine data set.

Comparing the error obtained for the cavity mean offset model in Table 7.6 to the error in Table 7.5, leads to the conclusion that the major contributor to the SSE error for the cavity mean offset model of variation is in the process used to model the term, $\sigma^2_{\substack{cavity\\mean\\offset}}$.

This is not an unexpected result, given the amount of data that was used to derive the histogram shown in Figure 7.15

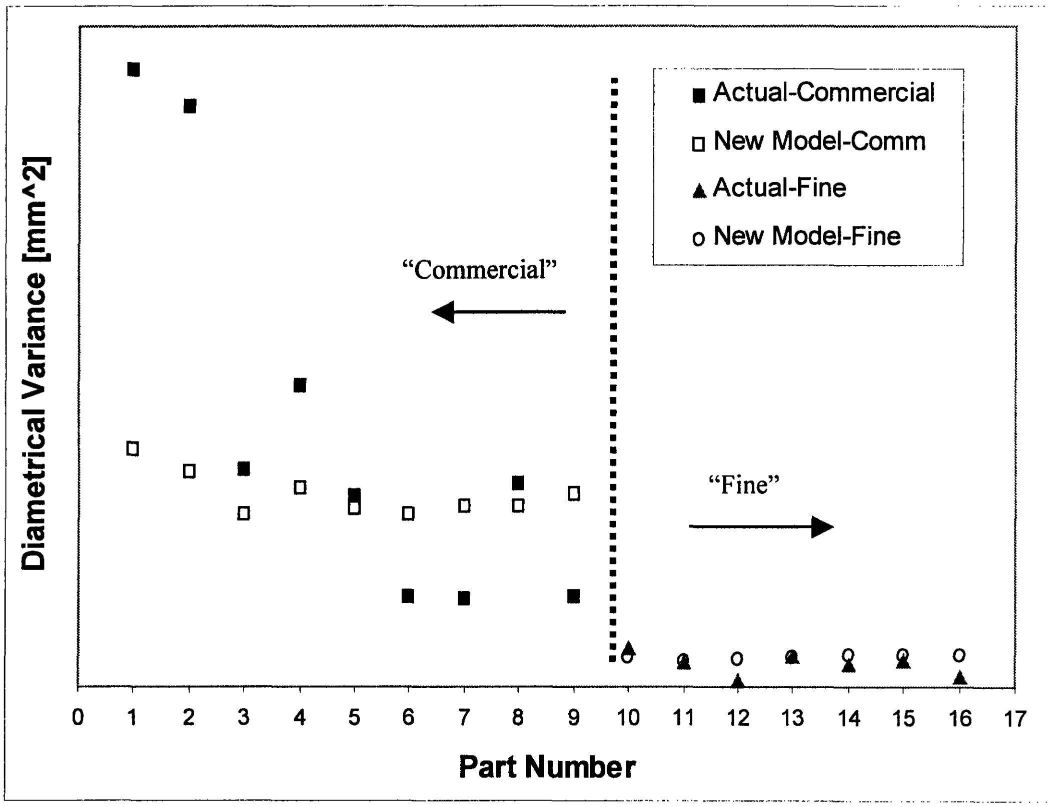



**Figure 7.17: Comparison of New Cavity Mean Offset Model to Production Data**

A model that incorporates the effects of cavity mean offset has been developed. This variational model is decomposed into two sections aimed at predicting the variation within a cavity with respect to the part's average, and between cavities with respect to the part's average.

| | Comm | Fine |
|---|---|---|
| linear | 4.27E-08 | 7E-10 |
| non-linear | 1.51E-08 | 2.0E-10 |
| modified for cavity mean offset | 6.53E-08 | 2.6E-10 |

**Table 7.6: Comparison of SSE for diametrical models**

# Chapter 8

## Summary

Although the goal of this thesis is to determine a physics based method for predicting the variation of injection molded part features, for use during the preliminary design phase, many other issues relating to several different areas have been addressed in this thesis. These areas include tolerance analysis, (Chapter2), metrology (Chapter 3), and some general techniques used to design injection molded parts (Chapter 4).

Chapter two defined a tolerance analysis, what it can accomplish, and how it can be realistically used as a tool to help create robust designs. Several different methods were described using illustrative examples that are commonly used to determine a part's quality before it is actually manufactured. These methods range in complexity and accuracy of results, and depend on the amount of information used as inputs to these models.

Chapter three discussed several different issues that are commonly raised when specifying and carrying out a measurement plan. This included two different types of automated measurement equipment where specific attention was paid to their advantages and disadvantages of use. Also the question of what to record on a measurement summary report was discussed including different measures that are commonly used to report the capability of a process. Chapter three also introduced the definitions of mean and variance, where the overall variance and mean of several cavities was derived as a function of only the cavity means, variances, and sample sizes.

Chapter four introduced several different general topics involving the design and modeling of injection molded parts. First, several of the difficulties associated with designing injection molded parts were listed. Next, several different methods that are commonly used to design products that use injection molded parts were discussed. These methods varied in the level of information required to initiate them ranging from only preliminary geometric and material information, to detailed geometric, material, and process information. Finally, Chapter five laid the framework from which the variational models developed in this thesis were built.

Chapters six and seven developed two variational models for the position of features moving on a plane, and hole diameters respectively. These models were based on a concept called the center of shrinkage that stated that the incurred variation grows with the feature's general distance from the center of shrinkage. Although an analytical approach for determining the center of shrinkage was not developed, the center of shrinkage for each part was determined through a two-dimensional approximation of the center of mass. These variational rules were statistically verified by using current production data. Also the model describing the position of features moving on a plane was favorably compared to the SPI design guidelines.

Furthermore, Chapter seven details the influence that cavity mean offset will have on the overall mean and variance of all cavities combined. This exact relationship between cavity mean offset and the overall change in variance across all cavities has been determined as a function of only the individual cavity averages, variances, and sample sizes, because sometimes this is the only readily available measurement information. Also, the variational model for hole diameter has been decomposed into variation within a cavity, and variation between cavities. These models have been favorably compared to production data in accuracy.

## 8.1 Application of Variational Rules

There are several different techniques that can be used to implement the variational rules developed in this thesis. The first application would be to integrate these rules into a tolerance analysis package, thus making it semi-automated. The user can follow the steps for running a tolerance analysis, described in section 2.2.2, except that the variations applied to a particular feature can be derived from the variational rules. The user could possibly select whether they wanted to use either the commercial or fine models developed in Chapters 6 and 7, and then the tolerance analysis tool would prompt the user on whether they wanted to accept, reject or modify these predictions of variation automatically placed on the selected features.

Automating this step in the tolerance analysis process would significantly speed up the time require to run a tolerance analysis because often a large number of features are active in a tolerance analysis. Automating the variational inputs in a tolerance analysis will also help to standardize the feature variations used in the analysis from user to user. Often different inputs are used because the current sources of variational information are often inconsistent from source to source. If a central location is used to supply feature variational data, then all users can have the most updated predictors of variation as the variational rules are improved with more data sources.

Having a central location which all users can access these variational models indicates that the source of the rules must be platform independent and accessible at all times, because the overall design of a product is rarely isolated to one geographic location. It is possible that the marketing for a product could be designed in the United States, the design developed in Europe, and the manufacturing be done in Asia. The global economy that today's engineered products are brought to market in, poses several problems, but can also yield several advantages including cost savings, and fresh perspectives on difficult problems.

One type of technology that currently seems to address most of the issues presented above is the Internet. The web uses a network of machines located all over the world, but the most updated information can be instantly obtained by just the click of a button. The web is also platform independent, and does not restrict the flow of information to only business hours.

It can be postulated that the variational rules developed in this thesis could be ultimately integrated with the web and a CAD tool so that users could have the most current variational rules regardless of the time. This level of integration is currently difficult to achieve without a considerable effort, but the technology exists today to transform this vision into a reality.

## 8.2 Conclusion

This thesis has developed a systematic method that can be used to predict feature variations using preliminary design information. Two examples were developed showing how the planar position of a plastic injection molded part feature, and how the diameter of a hole can be expected to vary in manufacturing production. This information can be used as an input to a tolerance analysis, thus aiding the design team when making critical preliminary design decisions. The variational model derived from the concept of shrinkage has compared well to production data. The center of shrinkage model makes use of available dimensional production data, therefore implementing this theory is not costly because the data set already exists. Feature dimensions are commonly measured to assure that current manufacturing quality standards are being met. Furthermore the center of shrinkage model, for features on a plane better predicts manufacturing variation than the industry standard SPI charts, which were developed without aid of production data. The completion of such models for different features can possibly lead to new design guidelines for injection molded parts where these models are aimed at aiding in the understanding of feature tolerances and variation during preliminary design.

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# Appendix A

## Possible Sources of Variation for Injection Molded Parts

An extensive list of possible sources of variation for injection molded parts was generated to aid during the development of the different variational models throughout this thesis. They are group into several main categories for clarity. This list represents the efforts of a large team of industry and University engineers and scientists, and not one particular's individual's effort.

**A.1 Assembly**
**A.2 Feature**
**A.3 Material**
**A.4 Measurement**
**A.5 Injection molding machine**
**A.6 Mold**
**A.7 Part**
**A.8 Process**
**A.9 Other**

## A.1 Assembly

1.0 Level of automation for assembly - importance of assembly speed. Higher speed assembly lines tend to drive tighter piece part tolerances

2.0 Assembly variations (slop, press fits, etc.) often tend to drive individual piece part dimensional requirements

## A.2 Feature

1.0 Relative size of features. Different sized features tend to have different variation characteristics.

2.0 Where features meet, possible defects can occur. It is possible that the machining processes used to create the area between two closely spaced features is not large enough to permit the ideal machining environment.

3.0 Relatively long expanses. It is a common industry standard to reinforce large flat areas aimed at preventing warpage

4.0 Relative distance from other important features. The concept of linear shrinkage developed throughout this thesis is based on the relative distance between features. Features that are very far apart from one another may lead to large predicted variations, while it may be difficult to use conventional modeling techniques on relatively closely packed features.

5.0 Aspect ratio of feature (2D or 3D)

6.0 Whether it crosses parting line from other important features

7.0 Does feature straddle the parting line

8.0 Flexibility/rigidity of feature itself or with respect to other important features on the part. For example the tip of a long thin feature would be expected to have more variation then the tip of a short thick feature.

9.0 Is the feature an internal or external feature. This is important to know because the feature can shrink onto the mold or shrink away from the mold.

## A.3 Material

1.0 Material preparation before molding,. *e.g.* material drying. It is possible for the plastic to absorb moisture during storage and retrieval, causing inconsistent material properties

2.0 Inconsistent blending of material in screw, hopper, or delivery system This might be important contributor to material property variation if the injected material is being mixed from 2 different sources.

3.0 In most handbook design guides, variation prediction rules are classified according to material. Therefore it is important to classify most of the major plastics into two different categories:

- Thermoplastics become soft when exposed to sufficient heat and harden when cooled, irrelevant of how often the process is repeated.

- Thermosetting materials are set into a permanent shape when heat and pressure are applied to them during molding. Re-heating the parts will not soften these materials.

4.0 Percent of regrind, and fillers. It is well documented how the percentage of fillers, like glass, can significantly change the shrink rates of the base materials.

5.0 Material properties (E, $\nu$, $\alpha$, shrink rate, etc.). The published values found in handbooks for material properties are industry averages, and should only be used as first approximations, and not final detailed design parameters.

6.0 Accuracy of initial shrink rate prediction, or unknown or estimated shrink rate. An injection mold is designed with a specific shrink rate in mind, whereas the actual shrink rate, just like the other material properties, will vary in production.

7.0 Effects of high coefficient of thermal expansion

- Plastic can shrink and grow as a function of temp, moisture, time during storage, and transportation.
- If the plastic is a thermoset, curing effects can continue after molding has been completed.
- Part storage in heated area for thermoplastics. This type of environment may cause warpage and exhibit creep over time

8.0 Foreign particle contamination introduced into the plastic melt. Residues from previous batches, and residual cleansing agents can all contaminate the plastic melt and thus increase the variability of the part.

## A.4 Measurement

The measurement of features on injection molded parts is not a trivial task and is often complicated by the complex geometry achievable through injection molding. The points on the part where it is to be held into a measurement fixture will often distort the part yielding incorrect data. Other causes of measurement variability are listed below.

1. Temperature effects. Due to the high coefficient of thermal expansion of plastics, the temperature at which the measurement is taken should be specified.

Also the issue at whether to measure the part at room temperature or its operating temperature will often effect the outcome of the measurement study.

2. Measurement probe deflection. For traditional Coordinate Measuring Machines (CMM), a probe is used to determine exactly where different points on the part are with respect to a reference coordinate frame. Thus it is possible for a small thin feature on the part to deflect before the probe takes a measurement.
3. Unreliable staging/fixturing during inspection. Incorrect fixturing can often lead to distorting the part, thus recording false dimensional data.
4. Measurement tool repeatability. A measurement tool has a certain accuracy and repeatability associated with it and should not be used to try and measure a feature that requires more capability than the measurement tool can provide.
5. It is common in industry for easy to measure features, both in the tool steel and in the part, to have a lower variation than those features that are difficult to reach. For example a feature that is difficult to reach may require the measurement machine to extend beyond its intended limits, thus producing a higher variation.
6. Programming variation in inspection. It is common for the dimensions on a part to be measured in more than one sequence, although different sequences will yield different dimensional variations.

## A.5 Injection molding machine

It is often difficult for the manufacturer of an injection molding machine to hold all process settings constant throughout a production run. Also manufactures are often faced with the fact that they posess several different types of injection molding machines, of quite possibly different ages. These, and the below list, all contribute to dimensional variation.

1. Inconsistency of measuring machine parameters or inconsistent machine parameters. Such parameters might include:
   - Screw speed
   - Barrel wall temp
   - Die design (for extrusion)

- Accuracy to keep a consistent cooling time
- Accuracy to keep a consistent cycle time
- Accuracy to keep a consistent pack time and pressure
- Accuracy to keep a consistent hold time
- Accuracy to keep a consistent peak cavity pressure
- Accuracy to keep a consistent injection pressure
- Accuracy to keep a consistent pack pressure
- Accuracy to keep a consistent hold pressure
- Accuracy to keep a consistent back pressure
- Accuracy to keep a consistent oil temp
- Room humidity in which the press is operating
- Room Temperature (ambient), in which the press is operating.
- Accuracy to measure clamping force supplied by hydraulic or electric means
- Measured distance which screw moves back and measured angle of screw position (angular) might result in the wrong volume of polymer to be injected into the mold. If too much polymer is injected then there is too much pressure, pushing the plastic out of the cavity resulting in flash. However if too little polymer is injected then a short shot will result
- Measured distance which screw moves forward

2. If several machines, of different ages, are simultaneously used, added variability might result because older and newer machines will not operate identically.
3. Inability to measure the part's cycle time might result in the part being ejected too soon. Then the part will not be fully cooled and will still be flexible, warpable and ductile, thus resulting in a slightly deformed part
4. Which mold machine is planned (brand/model). Presses, made by different manufacturer, specified with the same clamping force may posses different variability.
5. Offset of mold halves. Mold half mismatch can cause all the features on one half of the mold to be offset relative to the other mold half.

## A.6 Mold

Machining the mold used in the injection molding process is probably the most crucial step of the entire process. Special care must be taken when machining the mold so as to ensure the most accurate mold. The following are possible ways in which the mold in the injection molding process can contribute to the final variability of the part.

1. Mold Creation
   - Vibration during the machining of mold
   - Variation caused by method of mold manufacture: Prismatic Machining, Grinding, EDM, *etc.* There are different process capabilities associated with each different manufacturing process
   - Different tolerances characteristics can be achieved repeatably with the same process. For example:
     1. Grinding: surface finishes can be maintained in grinding by using different grit wheels
     2. Prismatic machining - different tooling, feeds, speeds, and cooling conditions effect tolerances.
   - These types of process variations can be tied to cost (a coarse grinding wheel may be cheaper than a fine one).
2. Mold Assembly
   - Techniques and technologies used to assemble the mold can effect the tolerance of features relative to one another in different portions of the mold. *e.g.* Welding, bolting together, press fit, etc.
3. Mold material. It is a well established fact that the mold will tend to wear onver time, thus introducing a dimensional shift in the part's quality data
4. Placement of gates and runners can also effect dimensional stability.
5. The need for cores and inserts placed into an injection mold can significantly change the way in which a mold is designed, thus changing its variability.
6. The number of cavities required for a part is directly related to its production requirements and planned schedule. These production estimates are often used to predict when a tool or cavity should be replaced because of excessive wear. The

number of cavities also introduces a significant amount of variation, because of the methods used to create the cavities themselves have process capabilities. It often the case where the cavity is not the same material as the tool, thus introducing more variation due to different coefficients of thermal expansion.

7. The placement of cooling channels is one of considerable study today, because molds are often thermally unbalanced, thus introducing variation.
8. Knockout, or ejector, pins are used to remove the part from the mold, but they can also deform the part if it is not entirely solidified.
9. The three basic types of molds can all introduce different variations for a part.
   - cold runner 2 - plate system
   - cold runner 3 - plate system - the runner system is separated from the parts when the mold opens
   - hot runner "Runnerless" - molten plastic is kept hot in a heated runner all the way up until the cavity and gate. Cycle times are shorter because only the part needs to be cooled
10. The geographic location where the cavities and molds will be made is important to specify during design because different locations often posses different machining process capability.
11. Can all features be made using a straight draw between 2 mold halves, or are more complex requirements necessary (like side draws, etc.). As the number of moving parts required in a mold increases, so does the variability.
12. Mold maintenance for dimensional stability. Mold maintenance can often prevent catastrophic failures, such as fracture, but can also increase the variability accrued during re-assembly of the mold.

## A.7 Part

There are several different things that can effect the variation of an entire part, including the environment in which the part is stored after molding.

1. Part storage in heated area for thermoplastics. Parts stored in this type of environment may warp and exhibit creep over time

2. It is ideal to design a part with a constant wall thickness. Thin walls tend to freeze quickly before thicker sections causing defects. If a change in wall thickness is unavoidable, then a gradual changes in thickness is best, as opposed to a sudden change in thickness
3. Where parts will be made (what vendor, what country)
4. Flexibility of part as a whole
5. Surface finish
6. The overall size of part will determine the capacity and size of the molding machine on which it can be made.

## A.8 Process

1. See Injection Molding Machine above for repeatability of process settings
2. Certain conditions may not be optimized on tolerances, but on cycle time, or possibly just getting the part out of the mold
    - Mold Temperature
    - Melt Temperature
    - Shot time
    - Packing time
    - Cooling time
    - Material composition
    - Method of removal from mold (where are ejector pins/parting line located in cavity)
3. Feedback vs. non-feedback control systems
4. SPC vs. part control
5. Part handling after molding. Different variations would be expected on the same part if they were dropped into a pile, and if they were taken out of the mold by a robot and placed on a flat storage tray.
6. Continuous molding (24 hrs a day, 7 days a week) vs. molding runs of a day or less will also cause the variability of a part to differ.

7. Variations introduced by the operator, including start-up set-points can significantly effect the history of a part.

## A.9 Other

1. Inserting a metal filter before the gate can filter out impurities expected in the melt. This technique is often used in plastic extrusion.
2. The pressures exerted on manufacturing for scheduling reasons may cause the cycle time of a part be reduced down below what is recommended, thus increasing variation.

# Appendix B

## Features vs. Variation Document

### Project DART
### Features vs. Variation
### Types

Before rules can be developed which predict the amount of variation on part features represented in the TARGET tolerance analysis package, we must establish:

- What features will be automatically varied in TARGET
- What TYPES of variation will be applied to each type of feature.

"Variation", as used here, is the set of transformations applied to a feature in a tolerance model to represent all the possible positions, orientations, sizes, and forms the feature is apt to take, due to the manufacturing process. The values used in the transformations (such as distances and angles) are statistical distributions, not scalars.

Variation differs from tolerance in subtle but important ways, addressed in the appendix (see "Variation vs. Tolerance"). A suggested list of possible variation types, analogous to GD&T tolerance types, is also listed (see "Types of Variations").

# Premises

Variation types:

Are for plastic injection molding (although they may apply to other processes). Are chosen to reflect process capability first, then to support product function and ease of TA modeling. Err on the conservative side when the decision is ambiguous. May be refined as measurement data is analyzed.

# Variations Applied to Sets of Features

Depending on the causes of a variation, it may apply to:

- the entire part
- the half of the part in one mold half
- a group of features
- a single feature
- a local area within a feature

Examples of these might be:

- shrink rate
- mis-matching of the mold halves
- mis-placement of an electrode burning in several features
- size of a post or hole
- waviness of a surface due to cyclic error in the electrode machining

The DART project will investigate variations both in single features and in sets of features -- multi-feature variations, if true, might simplify the rules greatly.

Two variation possibilities which should be investigated are:

- variation from shrink rate, as a function of distance between features
- variation across a parting line

Both of these are represented as simple "rules" on the Kodak general tolerances shown on every plastic injection molding drawing. Others may be added as possibilities suggest themselves.

# Choices of Variation Types for Features

Typically, a feature used in tolerance analysis varies in:

- mis-location or offset of the entire perfect feature
- mis-orientation of the entire perfect feature
- irregularity of the feature

and may vary in:

- size
- amount of draft

How did we decide which ones to measure and turn into rules?

# Factors which cause variation:

One, two, or three of the first group of variations may happen in the actual process.

For example, if the main cause of variation in a particular case were electrode burn imprecision, perhaps irregularity alone would describe the variation well.

If a cavity were set up to be machined or burned and was located on a very short footprint, mis-orientation alone might describe most of the variation.

If mold halves were not lined up well, a combination of mis-location and mis-orientation might describe the bulk of the variation, without assuming any irregularity.

# Factors which matter to tolerance analysis:

We may choose to disregard some of the actual variations if they are seldom important to a tolerance analysis, to simplify the DART investigation and rule set.

For example, mis-orientation of a planar face may only matter during calculation of reflection angle, pressure angle, and a few other angular issues.

Rounded surfaces such as spheres and toroids may typically be intended to contact at a single point. Then, if we used only an irregularity variation to show how much that point could vary, it might be enough.

# Features that may be used by TARGET:

*All features contain their edges and vertices as sub-features.*
*Sub-features will not be varied by TARGET directly--only through their main features.*
*Features with a * will be addressed by the DART project.*

Faces (Features)

- Planar *
- Cylindrical *
  Subfeature--Centerline
- Conical *
  Subfeature--Centerline
- Spherical *
  Subfeature--Center Point
- Toroidal
  Subfeatures--Center Point, Center Arc
- Other face
  Special-Use Features (Features of Size)
  - Post *
    Subfeature--Centerline
  - Hole *
    Subfeature--Centerline
  - Slot *
    Subfeature--Centerplane
  - Rib *
    Subfeature--Centerplane

  Artifacts of Molding (Not Required by Design)
  - Fill Radius *
- General Solid
- Wireframe
  - Point
  - Line
  - Arc
    Subfeature--Center Point

# Feature: Planar Face

How TARGET and analyst will recognize it:

- Planar face in UG
- Not part of a slot or rib in actual part
- Not used anywhere in the TARGET model in a slot or rib, at the time autovarying is done

TARGET should generate these variations:

- Taper variation from draft (if applicable)
- Offset variation of whole surface (analogous to profile)
- Additional offset variation of each instance of use of the surface (i.e., how much each part of the surface can vary with respect to every other part, analogous to parallelism or perpendicularity)
- Variations of surfaces bounding this surface *

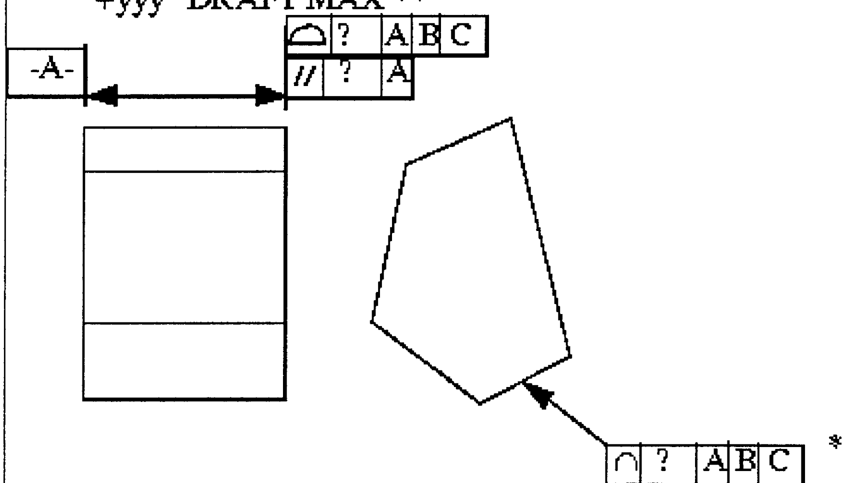
Not to be addressed by TARGET at this time:

- Slope of the face in the region(s) used for mating or measurement

Related GD&T tolerances, noted by "?":

xxx BSC

+yyy° DRAFT MAX **




* Generated indirectly through varying features which adjoin this feature

** Max draft is called out, but TARGET must model the expected range in draft.

# Feature: Cylindrical Face

How TARGET and analyst will recognize it:

- Cylindrical face in UG, interior or exterior
- Not part of a post or hole or fill radius in actual part
- Not used anywhere in the TARGET model as a pin, hole, or fill radius, at the time autovarying is done

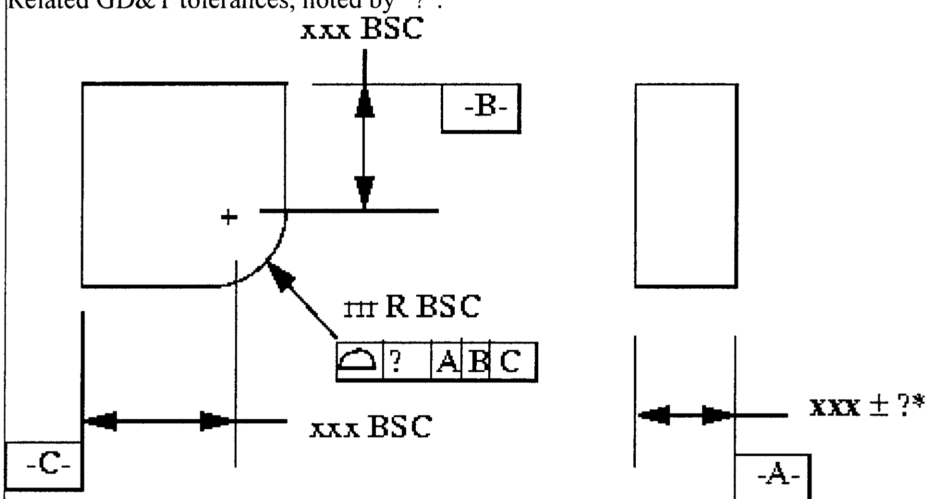
TARGET should generate these variations:

- Offset variation, ONE applied EACH time the cylinder is used for mating or measurement in TARGET (analogous to profile and form variation)
- Variations of surfaces bounding this surface at the ends. (If surfaces bound the cylinder at a tangent, TARGET will assume that those surfaces get their offset variations from the cylinder's variation. Otherwise the bounding surfaces may vary independently.)*

Not to be addressed by TARGET at this time:

- Draft (cylinders with draft must be modeled in UG as conical faces)
- Slope of the face in the region(s) used for mating or measurement

Related GD&T tolerances, noted by "?":




* Generated indirectly through varying features which adjoin this feature

# Feature: Conical Face

How TARGET and analyst will recognize it:

- Conical face in UG, interior or exterior
- Not part of a post or hole or fill radius in actual part
- Not used anywhere in the TARGET model as a pin, hole, or fill radius, at the time autovarying is done

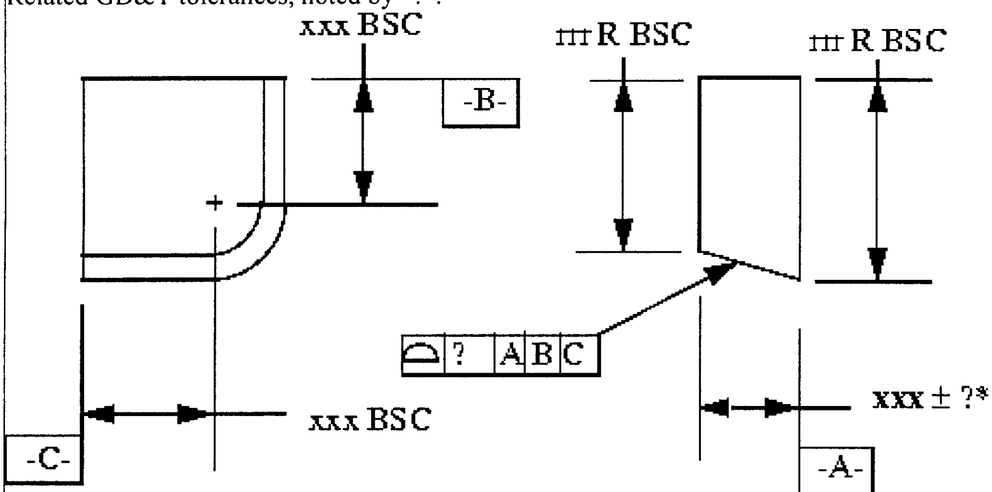
TARGET should generate these variations:

- Offset variation, ONE applied EACH time the cone is used for mating or measurement in TARGET (analogous to profile and form variation)
- Variations of surfaces bounding this surface at the ends. (If surfaces bound the cone at a tangent, TARGET will assume that those surfaces get their offset variations from the cone's variation. Otherwise the bounding surfaces may vary independently.)*

Not to be addressed by TARGET at this time:

- Slope of the face in the region(s) used for mating or measurement
- Draft (cone angle is assumed sufficient for ejection)

Related GD&T tolerances, noted by "?":




* Generated indirectly through varying features which adjoin this feature

# Feature: Spherical Face

How TARGET and analyst will recognize it:

- Spherical face in UG, interior or exterior
- Not part of a fill radius in the actual part
- Not used anywhere in the TARGET model as a fill radius, at the time autovarying is done

TARGET should generate these variations:

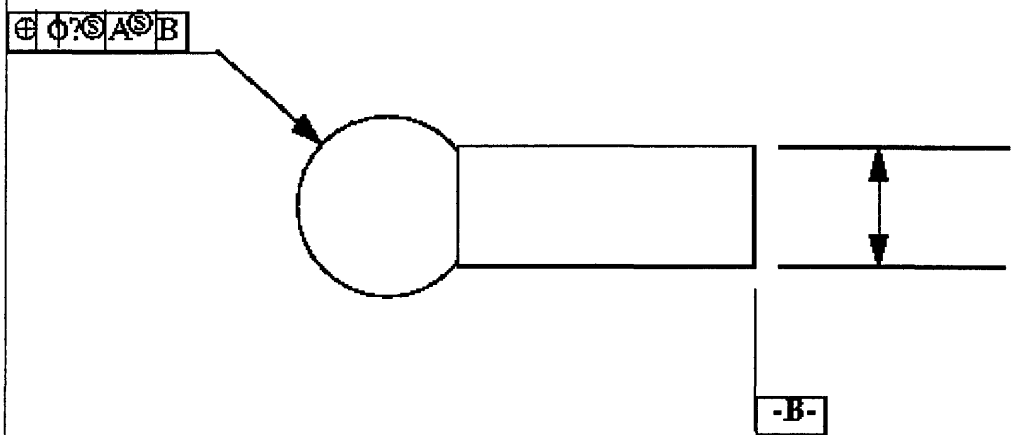
- Offset variation, ONE applied EACH time the sphere is used for mating or measurement in TARGET (analogous to profile and form variation)
- Variations of surfaces bounding this surface. (If surfaces bound the sphere at a tangent, TARGET will assume that those surfaces get their offset variations from the sphere's variation. Otherwise the bounding surfaces may vary independently.)*

Not to be addressed by TARGET at this time:

- 

Related GD&T tolerances, noted by "?":

SRxxx ± ?




* Generated indirectly through varying features which adjoin this feature

# Feature: Toroidal Face

How TARGET and analyst will recognize it:

- Toroidal face in UG, interior or exterior
- Not part of a fill radius in the actual part
- Not used anywhere in the TARGET model as a fill radius, at the time autovarying is done

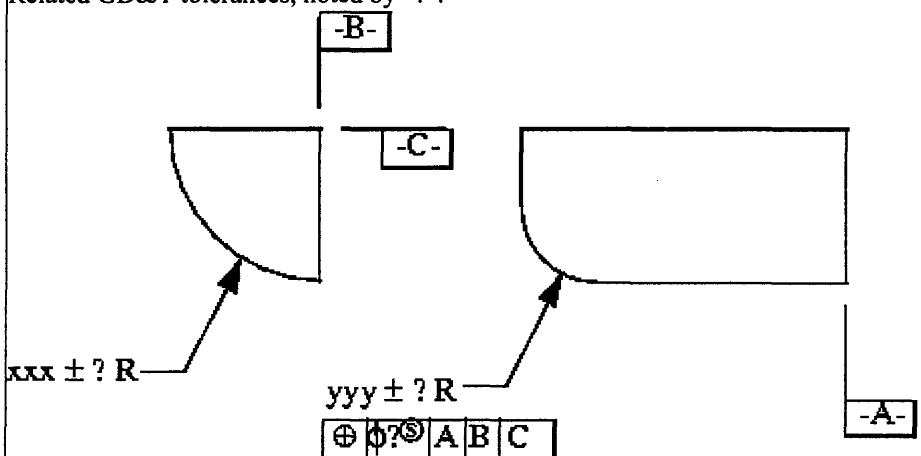
TARGET should generate these variations:

- Offset variation, ONE applied EACH time the toroid is used for mating or measurement in TARGET (analogous to profile and form variation)
- Variations of surfaces bounding this surface at the ends. (If surfaces bound the toroid at a tangent, TARGET will assume that those surfaces get their offset variations from the toroid's variation. Otherwise the bounding surfaces may vary independently.)*

Not to be addressed by TARGET at this time:

- Slope of the face in the region(s) used for mating or measurement

Related GD&T tolerances, noted by "?":




* Generated indirectly through varying features which adjoin this feature

# Feature: Other Face

How TARGET and analyst will recognize it:

- Not part of any other face or feature used in TARGET

TARGET should generate these variations:

- Offset variation, ONE applied EACH time the surface is used for mating or measurement in TARGET (analogous to profile and form variation)
- Variations of surfaces bounding this surface *

Not to be addressed by TARGET at this time:

- Slope of the face in the region(s) used for mating or measurement
- Draft

Related GD&T tolerances, noted by "?": **tbd**

* Generated indirectly through varying features which adjoin this feature

# Feature: Post

How TARGET and analyst will recognize it:

- Cylindrical (or conical with draft) with material on the inside
- 360 degrees, not split lengthwise by a parting line
- Might be used as a "pin" in a TARGET constraint
- Might be created as "boss" in UG
- Probably used as a pin, boss, shaft, etc. which mates with a hole or slot in the actual assembly

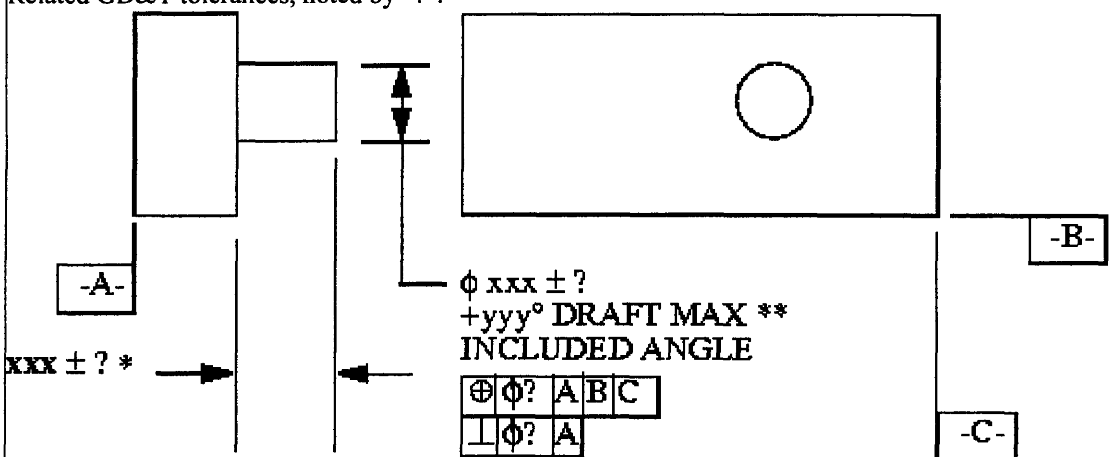
TARGET should generate these variations:

- Height variation *
- Diameter size variation
- Taper variation from draft **
- Circular-translate variation about the center axis
- Float-in-a cone variation about the base of the post's axis (analogous to perpendicularity)

Not to be addressed by TARGET at this time:

- Straightness (since plastic posts/shafts are seldom used with 3 or more points of contact/measurement)

Related GD&T tolerances, noted by "?":




* Generated indirectly through varying features which adjoin this feature
** Max draft is called out, but TARGET must model the expected range in draft.

# Feature: Hole

How TARGET and analyst will recognize it:

- Cylindrical (or conical with draft) with material on outside
- 360 degrees
- Might be used as a "hole" in a TARGET constraint
- Might be created as "hole" in UG
- Probably used as a hole which mates with a pin, boss, shaft, etc. in the actual assembly
- May be a through-hole or blind hole

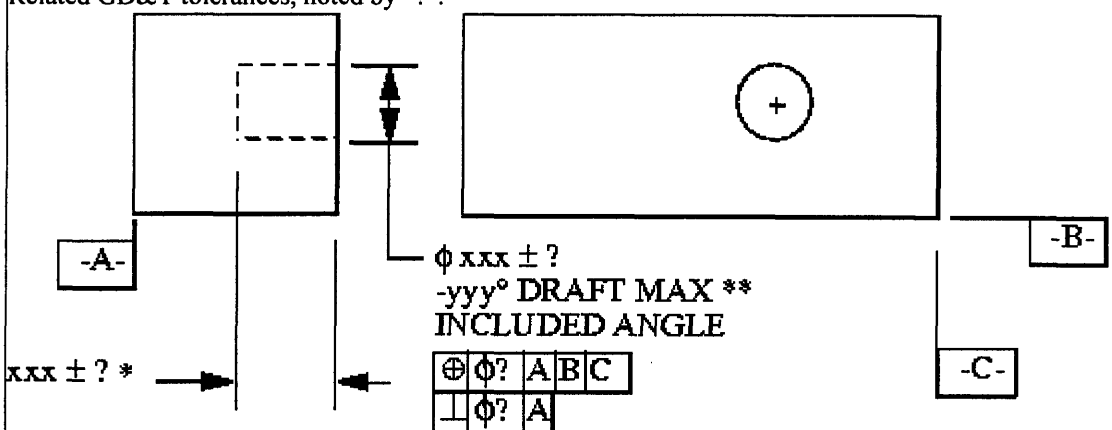
TARGET should generate these variations:

- Depth variation
- * Diameter size variation
- Taper variation from draft **
- Circular-translate variation about the center axis
- Float-in-a cone variation about the center of the hole's axis (analogous to perpendicularity)

Not to be addressed by TARGET at this time:

- Straightness (since plastic holes are seldom used with 3 or more points of contact/measurement)

Related GD&T tolerances, noted by "?":




* Generated indirectly through varying features which adjoin this feature
** Max draft is called out, but TARGET must model the expected range in draft.

# Feature: Slot

How TARGET and analyst will recognize it:

- Used as a "slot" in a TARGET constraint, at the time autovarying is done
- Used to mate with a pin, boss, shaft, pad, etc. in the actual assembly
- Probably "slot", "pocket", or two interior parallel planes in UG
- May be a through-slot or blind slot
- Includes only the two side-planes, not the end-faces

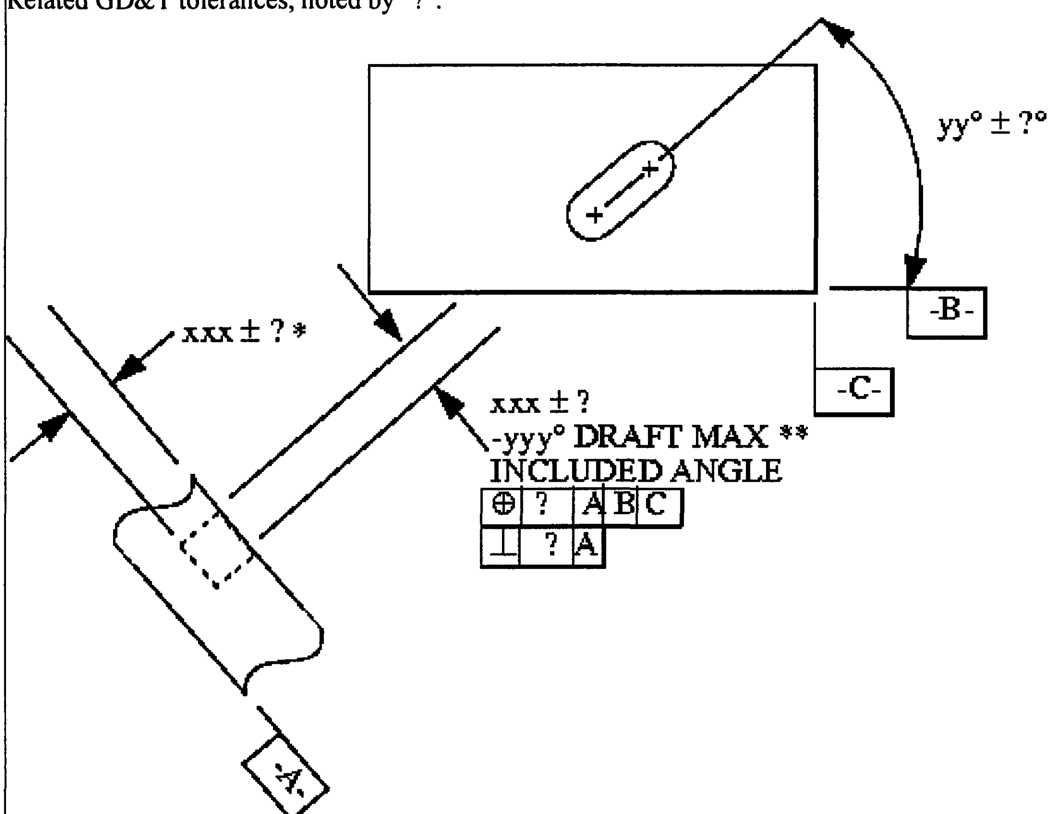
TARGET should generate these variations:

- Depth variation *
- Width size variation
- Taper variation from draft **
- A linear-translate variation perpendicular to the planar faces of the slot (analogous to positional tolerance)
- Additional linear-translate variations perpendicular to the planar faces of the slot, ONE applied EACH time the slot is used for mating or measurement in TARGET (analogous to the angular tolerance on the slot, or straightness ALONG the slot, and analogous to the perpendicularity shown below)

Not to be addressed by TARGET at this time:

- Straightness in and out of the slot (since plastic slots are seldom used with 3 or more points of contact/measurement in that direction)
- Length of slot

Related GD&T tolerances, noted by "?":




* Generated indirectly through varying features which adjoin this feature
** Max draft is called out, but TARGET must model the expected range in draft.

# Feature: Rib

How TARGET and analyst will recognize it:

- Used as a "rib" in a TARGET constraint, at the time autovarying is done
- Used to mate with a slot in the actual assembly
- Probably a "pad" or two exterior parallel planes in UG
- Includes only the two side-planes, not the end-faces

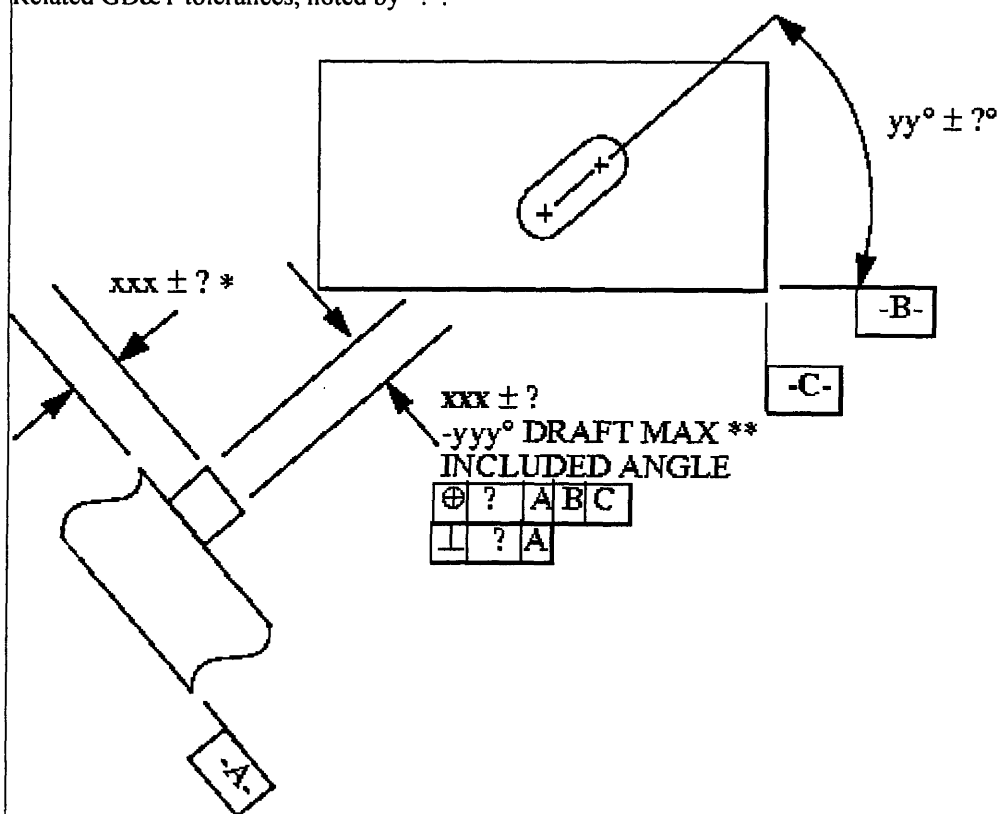
TARGET should generate these variations:

- Height variation *
- Width size variation
- Taper variation from draft **
- A linear-translate variation perpendicular to the planar faces of the rib (analogous to positional tolerance)
- Additional linear-translate variations perpendicular to the planar faces of the rib, ONE applied EACH time the rib is used for mating or measurement in TARGET (analogous to the angular tolerance on the rib, or flatness ALONG the rib, and analogous to the perpendicularity shown below)

Not to be addressed by TARGET at this time:

- Straightness up and down the rib (since plastic ribs are seldom used with 3 or more points of contact/measurement in that direction)
- Length of rib

Related GD&T tolerances, noted by "?":




* Generated indirectly through varying features which adjoin this feature

** Max draft is called out, but TARGET must model the expected range in draft.

# Feature: Fill Radius

How TARGET and analyst will recognize it:

- Cylindrical, conical, spherical, or toroidal face in UG, interior or exterior (TARGET will not handle other kinds of blends)
- Identified as a "fill radius" in TARGET, at the time autovarying is done
- Representing fillets or corners "shown sharp", not "shown round", on the part drawing
- May be an inside or outside edge or corner

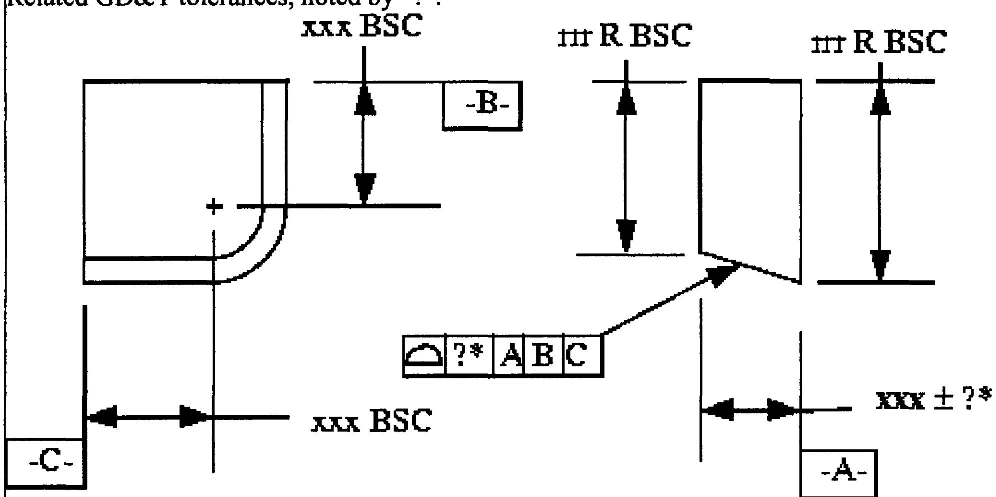
TARGET should generate these variations:

- Radial size variation, ONE applied EACH time the blend is used for mating or measurement in TARGET
- Variations of surfaces bounding this surface. (TARGET will assume that the blend gets its locational tolerance from the surfaces to which it is tangent.) *

Not to be addressed by TARGET at this time:

- tbd

Related GD&T tolerances, noted by "?":




General Notes:
Outside corners shown sharp:
R ? min to R ? max

* Generated indirectly through varying features which adjoin this feature

# Feature: General Solid

If an entire solid is chosen for mating or measurement in TARGET, TARGET will vary each of the faces of the solid independently, following the rules above for specific face or feature type.

# Feature: Wireframe--Point, Line, or Arc

Although TARGET will support these features, they will not be auto-varied by TARGET. The TARGET user can apply appropriate variations by hand.

# Appendix C

## C.1 List of Characteristics Which May Make the Injection Molding of a Part Difficult

1. Undercuts
2. No draft
3. Side pulls
4. Cores
5. Screw threads
6. Fine Cosmetic Features
7. Sharp corners
8. Crooked or irregular parting line

## C.2 List of Defects Which Can Occur in an Injection Molded Part

1. Warpage
2. Sink marks can occur on the opposite side of a large volume section, caused by shrinkage
3. Closing of a U shaped section
4. Curving of flat surfaces in the direction of a boss, protrusion, or added material
5. Bad surface finish
   - made from bad position of injector pins
   - part was ejected while not fully cooled
6. Porosity
7. Flash
8. Weld lines
9. Glossy surface finished tend to accentuate surface inconsistencies