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W. G. TULLER

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by

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Abstract

An analysis is made of the distortion inherent in the Pound microwave discriminators. Balanced and unbalanced types are considered, both with linear and square law detectors. The expression for the output of the balanced microwave discriminator as a function of input signal frequency is shown to be the same as that for the Foster Seeley discriminator, so that all calculations made for the former are directly applicable to the latter.

The effect on harmonic distortion of the law of the detector used is shown to be negligible for small distortion. The magnitudes of harmonics produced by the unbalanced discriminator are shown to differ from those produced by the balanced discriminator only in the addition of even order terms.

Curves are given for the balanced discriminator showing the magnitude of harmonic components up to the seventh as a function of normalized peak deviation.

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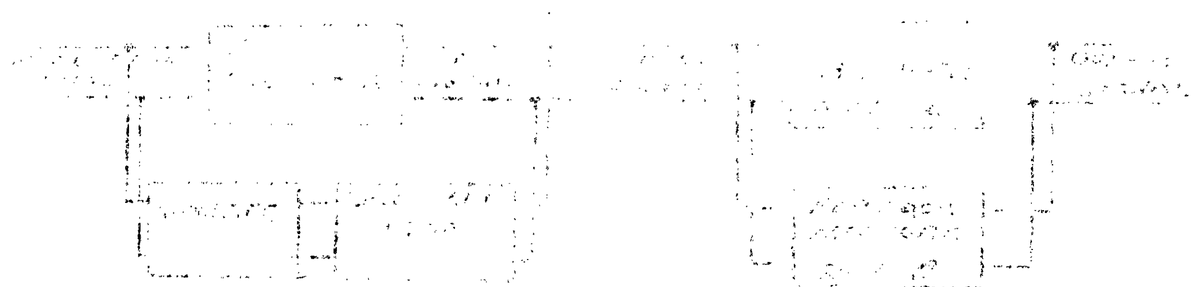
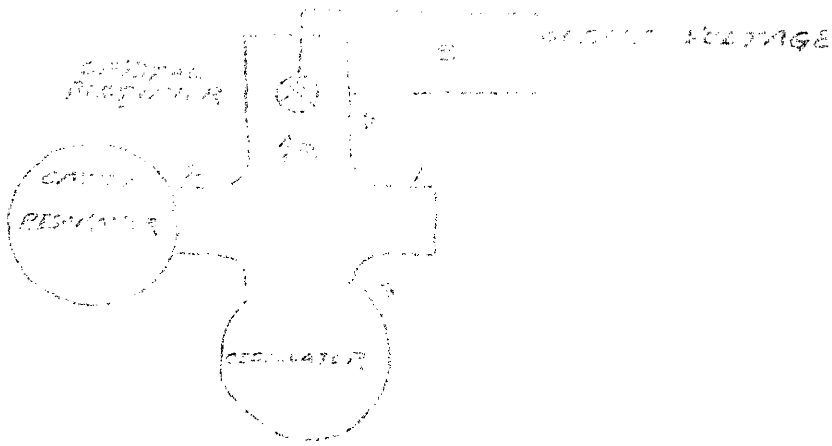


Fig. [illegible]

The linearity of modulation is particularly important when the modulating voltage consists of [illegible] of information stacked one above the other in the frequency spectrum [illegible] telephone systems. Unless linearity [illegible] conditions of cross-talk among the various channels [illegible] at the [illegible] that wide deviations are desirable [illegible]

The following [illegible] carried out with a view to providing data useful [illegible] FM systems. Particular attention [illegible] developed by [illegible] [illegible] of [illegible]

[illegible]



We shall compute p_1, the power delivered to the crystal, since $v = kp_1$ for a square law detector.

According to ...'s calculations for this case

$$p_1 = \frac{1}{2}\,\frac{1 + (ax)^2}{x^2 + (a+1)^2} \qquad (1)$$

where $x = \frac{2D}{\delta_o}$ (2)

$$D = \frac{\nu_1 - \nu_o}{\nu_o} = \frac{\Delta\nu}{\nu_o} \qquad (3)$$

ν_1 = frequency of signal applied to discriminator

ν_o = resonant frequency of cavity

$$\delta_o = \frac{1}{Q_o} \qquad (4)$$

Q_o = unloaded Q of cavity

$$a = \frac{\delta_1}{\delta_o} \qquad (5)$$

$$\delta_1 = \frac{1}{Q_L} - \frac{1}{Q_o} \qquad (6)$$

Q_L = Q of cavity when loaded by a matched generator.

If we expand this expression for p_1 into a power series in a, we obtain

$$p_1 = \frac{\text{[illegible]}}{2(1+a)^2}\left\{ \begin{array}{l} \text{[illegible]} \; a^2\left[-\frac{2a}{(1+a)}\right] + a^3\left[-\frac{2a}{(1+a)^2}\right] \\ \text{[illegible]}\left[-\frac{2a}{(1+a)^4}\right] \; \text{[illegible]}\left[-\frac{2a}{(1+a)^4}\right] - a^6\left[-\frac{2a}{(1+a)^6}\right] \\ - a^7\left[-\frac{2a}{(1+a)^6}\right] - \ldots \end{array} \right\} \quad (7)$$

This expression may now be converted by simple substitution into one involving the various circuit parameters and the deviation from mid-frequency, $\Delta\nu$.

This results in:

$$p_1 = \frac{2(Q_o - Q_L)Q_L}{\nu_o Q_o}\left\{ \begin{array}{l} \frac{\nu_o(Q_o^2 - 2Q_oQ_L + 2Q_L^2)}{4Q_oQ_L(Q_o - Q_L)} + \Delta\nu + \frac{\nu_o}{2Q_o}\left(\frac{2Q_L}{\nu_o}\right)^2(\Delta\nu)^2 \\ - \left(\frac{2Q_L}{\nu_o}\right)^2(\Delta\nu)^3 - \frac{\nu_o}{2Q_o}\left(\frac{2Q_L}{\nu_o}\right)^4(\Delta\nu)^4 \\ + \frac{2}{\nu_o}\left(\frac{2Q_L}{\nu_o}\right)^4(\Delta\nu)^5 + \frac{\nu_o}{2Q_o}\left(\frac{2Q_L}{\nu_o}\right)^6(\Delta\nu)^6 \\ - \frac{2}{\nu_o}\left(\frac{2Q_L}{\nu_o}\right)^6(\Delta\nu)^7 \ldots \end{array} \right\} \quad (8)$$

If we let $\Delta\nu_p$ be the difference between the frequencies at which the output of the discriminator is a positive maximum and that at which the output is a negative maximum, then:

$$\frac{Q_L}{\nu_o} = \frac{1}{\Delta\nu_p} \quad (9)$$

and

$$p_1 = \frac{2Q_L}{\nu_o Q_o}(Q_o - Q_L)\left\{ \begin{array}{l} \frac{\nu_o(Q_o^2 - 2Q_oQ_L + 2Q_L^2)}{4Q_oQ_L(Q_o - Q_L)} + \Delta\nu + \frac{\nu_o}{2Q_o}\left(\frac{2\Delta\nu}{\Delta\nu_p}\right)^2 \\ - \left(\frac{2\Delta\nu}{\Delta\nu_p}\right)^2\Delta\nu - \frac{\nu_o}{2Q_o}\left(\frac{2\Delta\nu}{\Delta\nu_p}\right)^4 + \left(\frac{2\Delta\nu}{\Delta\nu_p}\right)^4\Delta\nu \\ + \frac{\nu_o}{2Q_o}\left(\frac{2\Delta\nu}{\Delta\nu_p}\right)^6 - \left(\frac{2\Delta\nu}{\Delta\nu_p}\right)^6\Delta\nu - \ldots \end{array} \right\} \quad (10)$$

If all terms of this expression are small compared to the direct current term, the effect of a linear instead of a square-law detector may be approximated by using only the first term of the binomial series. This will be feasible if $4\Delta\nu \ll \Delta\nu_p$ which will be generally true if total harmonic distortion [illegible] equation (10) [illegible] to replaced [illegible] percentage of higher order terms [illegible], for very small deviations, the [illegible] of the detector [illegible] in this case.

The percentage of each of the higher order terms as compared to the fundamental are tabulated below for reference:

$$F_2 = \frac{2\Delta\nu}{\Delta\nu_p} \times \frac{\nu_0}{Q_0 \Delta\nu_p} \qquad (11)$$

$$F_3 = \left(\frac{2\Delta\nu}{\Delta\nu_p}\right)^2 \qquad (12)$$

$$F_4 = \left(\frac{2\Delta\nu}{\Delta\nu_p}\right)^3 \times \frac{\nu_0}{Q_0 \Delta\nu_p} \qquad (13)$$

$$F_5 = \left(\frac{2\Delta\nu}{\Delta\nu_p}\right)^4 \qquad (14)$$

$$F_6 = \left(\frac{2\Delta\nu}{\Delta\nu_p}\right)^5 \times \frac{\nu_0}{Q_0 \Delta\nu_p} \qquad (15)$$

$$F_7 = \left(\frac{2\Delta\nu}{\Delta\nu_p}\right)^6 \qquad (16)$$

Balanced Discriminator

The expression giving the output voltage of a balanced microwave discriminator diagrammed in Fig. III as a function of circuit parameters and frequency of input signal is, in our previous terminology, for square law detectors:

$$e = \frac{2\omega K}{a^2 + (\omega l)^2} \qquad (17)$$

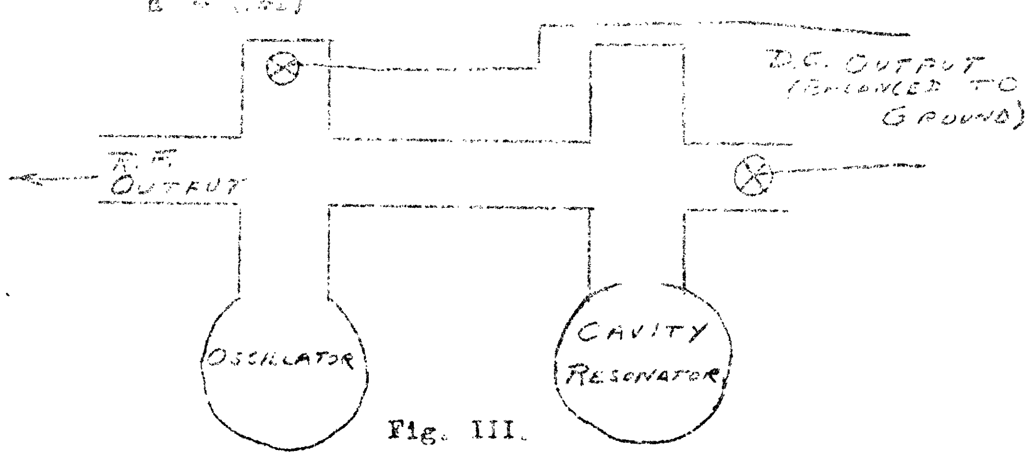


Fig. III.

Balanced Microwave Discriminator

If this expression [illegible]

$$e = \frac{2aK}{(a+1)^2}[a - \frac{a^3}{(a+1)^2} + \frac{a^5}{(a+1)^4} - \frac{a^7}{(a+1)^6} + \dots] \quad (18)$$

where all even harmonics, as well as the steady state term, are cancelled out by the inherent balance of the circuit, assuming the law of each detector to be identical.

Expressing (18) in terms of circuit parameters and deviation gives:

$$e = \frac{(2Q_o Q_L - 2Q_L^2)K}{Q_o \nu_o}[\Delta\nu - \Delta\nu(\frac{2\Delta\nu}{\Delta\nu_p})^2 + \Delta\nu(\frac{2\Delta\nu}{\Delta\nu_p})^4 - \Delta\nu(\frac{2\Delta\nu}{\Delta\nu_p})^6 + \dots] \quad (19)$$

from which it may be noted that the percentage higher order terms as compared to the fundamental is the same as for the unbalanced case, except for the omission of the even order terms.

To get the expression for the output of the balanced discriminator with linear detectors, we use the equation

$$e = K\sqrt{\frac{1}{2}[\frac{1+(\alpha+a)^2}{a^2+(\alpha+1)^2}]} - K\sqrt{\frac{1}{2}[\frac{1+(\alpha-a)^2}{a^2+(\alpha+1)^2}]} \quad (20)$$

We already have the expansion for the first term, since it is the same as the expression used in computing the output of the unbalanced discriminator with a linear detector. If the expression for the second term be expanded in a power series and treated as before, it is found that its terms are equal in magnitude to those of the first expression but that the odd order terms are reversed in sign. Therefore the equation for e is:

$$e = \frac{Q_o K}{\nu_o}\sqrt{\frac{(Q_o - Q_L)^2(Q_o - 2Q_o Q_L + 2Q_L^2)}{Q_o^2 Q_L^2}}\left[\Delta\nu - \Delta\nu(\frac{2\Delta\nu}{\Delta\nu_p})^2 + \Delta\nu(\frac{2\Delta\nu}{\Delta\nu_p})^4 - \Delta\nu(\frac{2\Delta\nu}{\Delta\nu_p})^6 + \dots\right] \quad (21)$$

and the percentage distortion is the same as for a balanced discriminator using square law detectors. From this we conclude that the distortion produced by a balanced discriminator is independent of the laws of the two detectors, as long as the distortion is small and the laws are equal. Further, if the detector laws are equal, only odd harmonic distortion will be present. If the detector laws are unequal, the odd harmonic distortion will be unaffected, but even order terms will be added to the output signal. The magnitude of these even order terms will always be less, percentage-wise, than the corresponding terms in the output of the unbalanced discriminator.

[illegible] discriminator [illegible] the output voltage will be:

$$e = \frac{2bQ_2 F}{1+Q_2^2 F^2} \tag{22}$$

$$\text{where } F = \frac{2\Delta f}{f_m} \tag{23}$$

Δf = deviation

f_m = mid-frequency

Q_2 = loaded Q of coil and condenser combination forming the discriminator secondary

$$b = Q_2 K \sqrt{\frac{L_2}{L_1}} \tag{24}$$

K = coefficient of coupling

L_2 = secondary inductance

L_1 = primary inductance

Converting this into something more nearly resembling our previous notation results in:

$$e = \frac{2ba}{1 + a^2} \tag{25}$$

which may be expanded in a power series in a as before and gives as a final result:

$$e = b\frac{4Q_o}{\nu_o}\left[\Delta\nu - \Delta\nu\left(\frac{2Q_o\Delta\nu}{\nu_o}\right)^2 + \Delta\nu\left(\frac{2Q_o\Delta\nu}{\nu_o}\right)^4 - \Delta\nu\left(\frac{2Q_o\Delta\nu}{\nu_o}\right)^6 + \ldots\right] \tag{26}$$

If external loading, such as from diode detectors, is small, as will generally be the case, we have:

$$\Delta\nu_p = \frac{\nu_o}{Q_o} \tag{27}$$

[illegible] Sturley, K. R. - The Phase Discriminator. Wireless Engr., 21, No. 245, pp. 72-78, Feb., 1944.

[illegible] balanced [illegible] of using linear [illegible] small [illegible]

[illegible]

[illegible] generally used as a stabilizer [illegible] with a modulating crystal, [illegible] the required system gain, [illegible] identical to that in the [illegible]

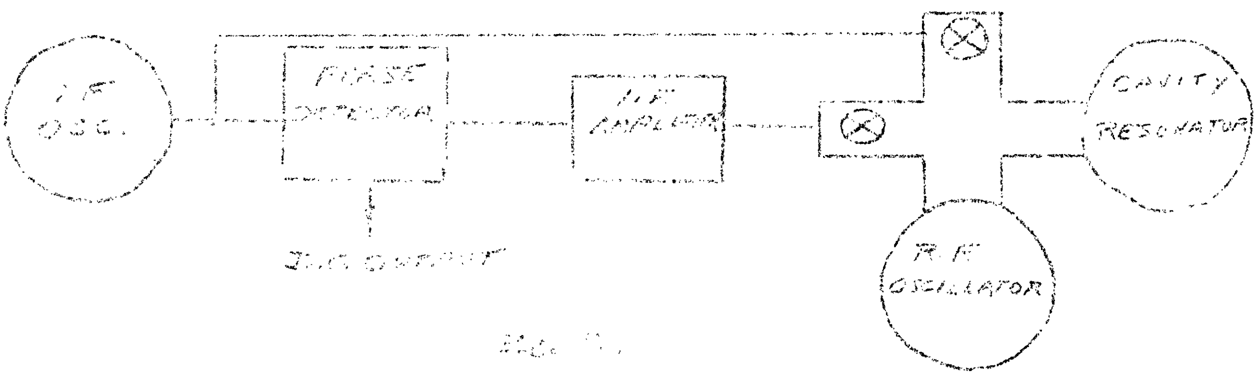


Fig. [illegible]

Harmonic distortion

The preceding analysis has resulted in terms containing various powers of the frequency deviation. While this sort of result is adequate for much work, it is also useful than a harmonic series when the frequency deviation is a sinusoidal function of time. Accordingly the harmonic series has been calculated for the balanced microwave discriminator. The results obtained, as mentioned above, are valid for the Foster-Seeley discriminator and the intermediate frequency amplifier type of microwave discriminator as well. The series given converges adequately if the third harmonic distortion is 5% or less, i.e., peak-to-peak deviations up to about 80% of $\Delta\omega_p$. Under this condition, the third harmonic distortion will be correct to 5% of its computed value, the fifth to 30%, and the seventh to 50%. If the series be expressed as:

$$\epsilon = c\,(a_1 \sin x + a_3 \sin^3 x + a_5 \sin^5 x + a_7 \sin^7 x) \qquad (29)$$

The various harmonic terms are:

$$\left(a_1 + \frac{3}{4}a_3 + \frac{5}{8}a_5 + \frac{35}{64}a_7\right)\sin x \qquad (30)$$

$$\left(-\frac{1}{4}a_3 - \frac{5}{16}a_5 - \frac{21}{64}a_7\right)\sin 3x \qquad (31)$$

$$\left(\frac{1}{16}a_5 + \frac{7}{64}a_7\right)\sin 5x \qquad (32)$$

$$\left(-\frac{a_7}{64}\right)\sin 7x \qquad (33)$$

where as before:

$$a_n = \frac{(2B)^n}{(\Delta\nu_p)^{n-1}} \sin\frac{n\pi}{2} \qquad (34)$$

where B is the peak deviation.

Curves showing the ratio of each harmonic component to the fundamental as a function of the ratio of peak deviation to $\Delta\nu_p$ are given in Fig. V. For example, if we consider a 10,000 mc. discriminator having 100 mc. between peaks a 10 mc. sinusoidal deviation will produce an output voltage containing approximately 1 % third harmonic, 0.01% fifth harmonic, and 0.0001% seventh harmonic. If $\Delta\nu_p$ be lowered to 20 mc., this same 10 mc. deviation will produce a 15% third harmonic, 0.38% fifth harmonic, and 0.03% seventh harmonic.

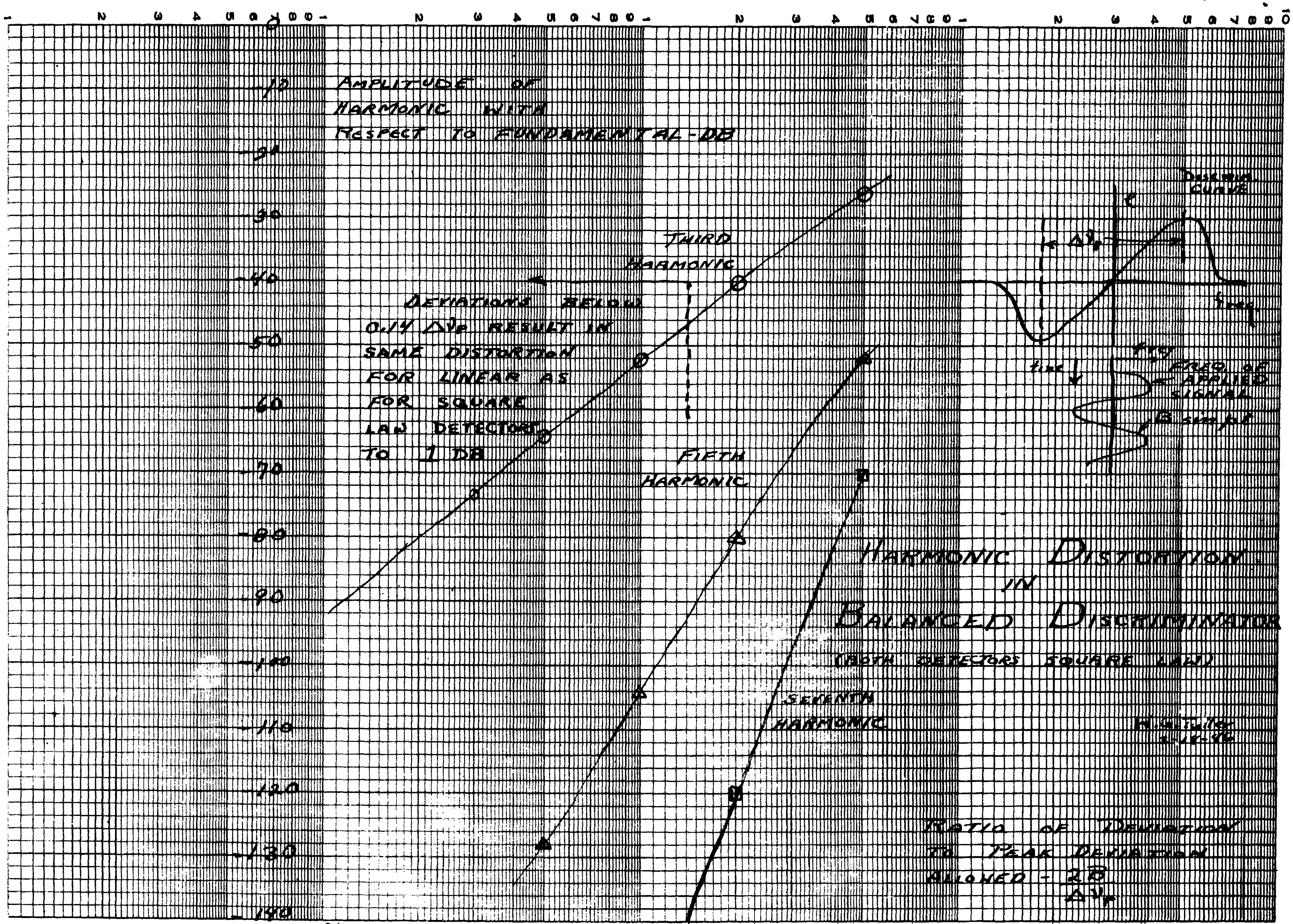

AMPLITUDE OF HARMONIC WITH RESPECT TO FUNDAMENTAL - DB
THIRD HARMONIC
FIFTH HARMONIC
SEVENTH HARMONIC
DEVIATIONS BELOW 0.14 $\Delta\nu_p$ RESULT IN SAME DISTORTION FOR LINEAR AS FOR SQUARE LAW DETECTORS TO 1 DB
DISCRIM. CURVE
HARMONIC DISTORTION IN BALANCED DISCRIMINATOR
(BOTH DETECTORS SQUARE LAW)
RATIO OF DEVIATION TO PEAK DEVIATION ALLOWED - $\frac{2D}{\Delta\nu_p}$
-10
-20
-30
-40
-50
-60
-70
-80
-90
-100
-110
-120
-130
-140
.01
.1
1

FIG. V