V. MISCELLANEOUS PROBLEMS

A. ELECTRONIC DIFFERENTIAL ANALYZER

Staff: Professor H. Wallman
A. B. Macnee

Electron-Velocity Multiplier. A complete working model of this multiplier has been built and tested. The scheme used has been modified and substantially improved as follows.

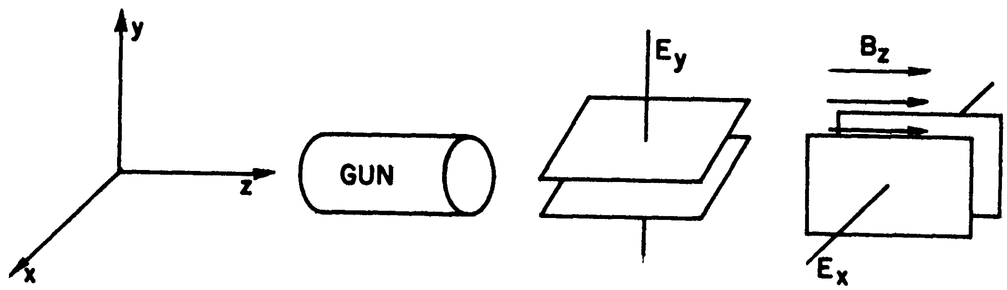


Figure 1. Electron-velocity multiplier

Referring to Fig. 1, an electron beam leaves the electron gun with an axial velocity v_z and no velocity components in the x or y directions. A voltage E_y is applied to the first pair of electrostatic deflecting plates. The electron beam after passing through these plates has a velocity component in the y direction

$$v_y \sim \frac{E_y}{v_z} . \qquad (1)$$

The electron beam next passes through a uniform axial magnetic field B_z which produces a force acting in the x direction

$$F_{xm} = e(v_y \times B_z). \qquad (2)$$

A second pair of electrostatic deflecting plates oriented in the x direction are located in this uniform field; a voltage E_x on these plates exerts a force on the electron beam

$$F_{xe} = e\, E_x. \qquad (3)$$

If through some feedback scheme the two forces F_{xe} and F_{xm} are made equal and opposite, then from Eqs. (2) and (3) one has

$$E_x = -(v_y \times B_z), \qquad (4)$$

and from Eq. (1) therefore

$$E_x \sim E_y \times B_z. \qquad (5)$$

Since B_z can be made proportional to some voltage E_z, this gives a means of obtaining a voltage E_x proportional to the product of two other voltages E_y and E_z.

In the experimental multiplier the required feedback is obtained by placing two photocells in front of the fluorescent screen of the cathode-ray tube in such a manner as to detect any deflection of the electron beam in the x direction, and feedback a voltage E_x in the proper phase and magnitude to keep this deflection zero. The electrostatic plates and electron gun are obtained by using a standard Dumont 5LP5 cathode-ray tube, and the axial field is produced by an air-core coil surrounding the second pair of electrostatic plates.

The model multiplier has the following measured characteristics:

Output - - - - - - - - - - - - $\pm$ 50 volts d-c.
Output rate of change - - - $\pm$ 0.5 volts/μsec. maximum
Normalized output - - - - - $E_y x E_z + .0047E_y + .013E_z + .0057$

where

E_y = electrostatic input voltage,
E_z = voltage driving magnetic coil = kB_z,
Output = feedback voltage = E_x.

This unit has been used to solve a simple non-linear differential equation as indicated below.

<u>Differential Equations</u>. A model differential analyzer utilizing two integrators, an inverter, and a gated adder has been set up to solve the following differential equations:

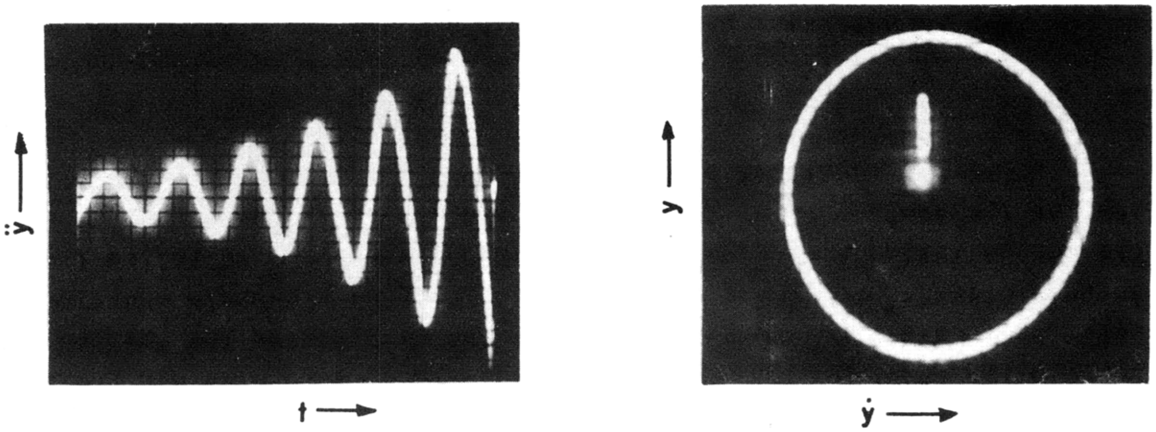
$$\frac{dy}{dt} \pm ky = 0, \qquad (6)$$

and

$$\frac{d^2y}{dt^2} \pm k_1 \frac{dy}{dt} + k_2 y = 0, \qquad (7)$$

where the k's are positive fixed numbers. It will be noted that taking the negative signs in these equations leads to solutions which increase exponentially with time. This type of behavior presents special problems in an electronic analyzer which is to solve equations periodically at a reasonably high repetition rate, and is the reason for the <u>gated</u> adder indicated above. As the result of tests made on these equations, it is concluded that in a general electronic differential analyzer of this type it is necessary to preserve the d-c level of all voltages. This can be accomplished by using d-c amplifiers throughout or by using a-c coupled amplifiers with d-c restoration.

The experimental analyzer has a basic repetition frequency of 60 cps; that is, the solution is repeated every 1/60 second. The solutions $y(t)$, $\dot{y}(t)$, or $\ddot{y}(t)$ are displayed on a cathode-ray tube face. Photographs of two typical solutions are given in Fig. 2.



(a) an exponentially growing sine wave, solution of the equation $\ddot{y} = +k_1\dot{y} - k_2y$.

(b) plot of y versus $\dot{y}$ in the solution of equation $\ddot{y}=-k_2y$

Figure 2. Typical solutions.

A single integrator together with a gated adder and multiplier have been connected to solve a non-linear equation of the form

$$\frac{dy}{dt} = -2yt, \qquad (8)$$

whose solutions are of the form of a Gaussian error curve $y = e^{-t^2}$. Figure 3 is a double exposure photograph of the solutions for $-y$ and $\dot{y}$ in this case.

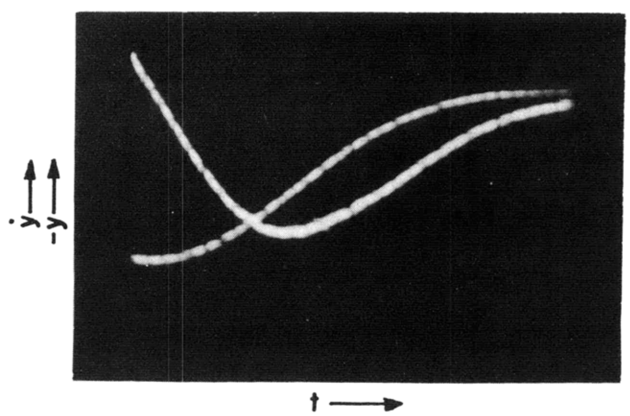


Figure 3. Solution of equation for the Gaussian error curve, $y=e^{-t^2}$.

V. B. MATHEMATICAL INSTRUMENTATION

Staff: Professor H. Wallman
R. E. Scott

Some preliminary investigations have been made in the following three fields of mathematical instrumentation:

1. The solution of algebraic polynomials by machine methods.
2. The solution of simultaneous algebraic equations by electronic feedback circuits.
3. The solution of network problems by a two-dimensional field analog.

<u>The Solution of Algebraic Polynomials</u>. A mechanical device to obtain the roots of high-order algebraic polynomials has been constructed at the Bell Laboratories[1]. In this machine the variable is a cyclic motion and the various powers of the unknown are represented as harmonics of the fundamental frequency. Several workers have pointed out that this problem can be solved equally well by the use of harmonic voltages.[2,3] The requirements are identical with those for a harmonic synthesizer. The harmonic voltages must be of constant amplitude and phase. The complexity of electronic circuits to meet these requirements has led us to investigate the tone wheels from a Hammond Organ.

<u>The Solution of Simultaneous Algebraic Equations</u>. Some time ago Professor Taylor and Professor Thomas of MIT suggested mechanizing the "method of steepest descent" for the solution of simultaneous algebraic equations. In this method feedback is used to constrain the variables to follow the gradient of the surface which they form in n-dimensional space.

Let the equations be

$$\sum_{j=1}^{n} a_{ij}x_j = b_i$$

For an arbitrary point in space the equations will not be solved but will yield remainders of the form

$$\sum_{j=1}^{n} a_{ij} x_j - b_i = R_i.$$

If S is made equal to the sum of the squares of these remainders, then S-equal-to-a-constant represents a family of curves through the point

1. R.L.Dietzold, "The Isograph, A Mechanical Root-Finder", Bell Laboratory Record, December, 1937.
2. O. Vierling's device to find the complex roots of a polynomial, as described in F. J. Murray, "The Theory of Mathematical Machines", King's Crown Press, N.Y., 1947, p.84.
3. S. Leroy Brown, "A Graphical Solution of Network Impedances with a Mechanical Harmonic Synthesizer", Bulletin of the American Physical Society, Vol.22, No. 5, November 28, 1947.

in space. At this point (-grad S) represents the direction of steepest descent from the family of curves S-equal-to-a-constant. Accordingly, if the rates of change of the x_j's are made equal to the components of (- grad S), they will automatically arrive at values which make S = 0, and these values are the solution of the equations.

Mechanization of the method requires an integrator for each variable. The principal difficulty encountered is that the convergence time depends on the value of the determinant. When the convergence time is long, the integrators must operate over a long period. With electronic integrators this necessitates very high gain circuits. It might be thought that long time constants could be obtained by using large resistors and condensers. This is not possible, however, without reducing the magnitude of the output voltage, and this requires the addition of further amplification before the loop can be closed.

An experimental circuit has been built for a two by two matrix which verified the considerations which have just been given. A high-gain d-c amplifier of the sub-miniature package type has been designed for the purpose of checking more accurately the limitations of the method.

The Solution of Network Equations by a Two-Dimensional Field Analog. Linvill[1] and Hansen and Lundstrom[2] have made use of an electrolytic tank to solve network problems. Poles and zeros in the complex plane are represented by current flowing into and out of the electrolyte. The potential along the imaginary axis is the gain function in logarithmic form.

In order to obtain some familiarity with the method, an electrolytic tank was set up and the experiments of Linvill were repeated.

It is hoped eventually that the electrolytic tank may be replaced by some dry, uniform conducting medium such as the "Teledeltos" paper produced by the Western Union Company.

1. J. Linvill, "An Experimental Approach to the Approximation Problem for Driving Point and Transfer Functions", E. E. Thesis, MIT, 1945.
2. W. W. Hansen and O. C. Lundstrom, "Experimental Determination of the Impedance Function by the Use of an Electrolytic Tank", Proc. I.R.E. 33, 528, August, 1945.

V. C. LOCKING PHENOMENA IN MICROWAVE OSCILLATORS

Staff: Professor J. B. Wiesner
E. E. David, Jr.

Investigation of a c-w microwave oscillator synchronized by an invariant external signal has been completed. Methods of analysis have been developed which accurately describe the operation of the locked oscillator and a report on this work is now being written.

A pulsed magnetron, synchronized by a signal from a klystron amplifier, is being used to study further the frequency and phase characteristics of the locking action.

D. ELECTRONIC POTENTIAL MAPPING

Staff: Dr. S. Goldman
C. K. Chien
W. E. Vivian

Since the Quarterly Progress Report of October 15, 1947, preamplifiers for the sixteen pickup elements have been built. The new scanning pickup tube was tested, but because of the development of internal shorts and gas, it had to be returned to the tube shop for repair. It is now again just about ready for use.

The first area potential display, with three pickup elements and the old pickup tube, was tried on a man's chest. The heart potential could clearly be seen moving across the chest. We are awaiting the completion of the sixteen-element system before starting physiological investigations.

E. INVESTIGATION OF ELECTROENCEPHALIC ACTIVITY OVER A WIDE FREQUENCY BAND

Staff: D. F. Winter

The purpose of this project is to investigate electroencephalic activity (brain waves) over a frequency range higher than normally used in clinical work. Commercial encephalographs are available to record brain waves with frequency components up to 100 cps; definite signals are present and have biological significance in that range. Several people have suspected that higher frequencies may also be present, and at the suggestion of Dr. Eric Levin of Johannesburg, South Africa, a quick check was made in June 1947. Amplitude vs. time records were made with the Fast Sweep Synchroscope (R.L.E. Report No. 27). The results were encouraging, but further work was postponed until September 1947.

A new setup has been made to improve the shielding of subject and amplifiers. The subject and the battery-operated amplifiers (gain of

30,000) are placed inside a doubly shielded room. The amplifiers have a pass band from about 300 cps to 11,000 cps. Subsequent ac-operated amplifiers raise the brain potentials to a suitable level for application to the deflecting plates of a high writing-speed cathode-ray tube. Photographic records are obtained on a continuously moving film.

Results so far are very encouraging and indicate that frequencies of the order of 200 to 900 cps are present. A regular laboratory report covering this work is planned.

F. PHYSICAL LIMITATIONS OF R-F RADIATING SYSTEMS

Staff: Professor L. J. Chu
Dr. M. V. Cerrillo
Dr. M. Loewenthal

A radio-frequency radiating system usually consists of a pair of terminals connected to an assembly of conducting or dielectric material or both, which is defined as the antenna. As a transmitter the r-f power is fed through the pair of terminals to the antenna which guides the power into space with a specific distribution at any one frequency. The radiation pattern of the antenna describes the angular distribution of the radiated power at a sufficiently large distance from the antenna. This power together with the power dissipated in the radiation system has a time-average value and is supplied through the pair of terminals. On the other hand, the antenna is also an energy storage device. With a sinusoidal time variation, the time rate of the change of the stored energy beyond the pair of terminals is mathematically related to the reactive power which must be supplied through the terminals. The time-average power and the reactive power which the pair of terminals has to supply, set a definite limitation to the impedance characteristic at the pair of terminals. The present program is an attempt to relate the impedance characteristics of an antenna to its radiation characteristics over a band of frequencies.

Some years ago Professor Stratton expanded the field in the neighborhood of the antenna, as well as the distance field, into a set of spherical vector wave functions, and arrived at the conclusion that it would be impossible to have an antenna of a finite size with an infinite radiation gain in any direction. The current interest in supergain antennas makes it advisable to extend Professor Stratton's original theory on a more quantitative basis. A supergain antenna can be defined as one which has a gain higher than what can be achieved in an antenna of conventional design with the same size limitation. By using the spherical wave functions, it is possible to determine the minimum amount of the reactive power to maintain a given amount of radiated power and certain radiation characteristics.

Some simple examples have been worked out which indicate quantitatively that a supergain antenna requires a high ratio of the reactive power to the real power. It means a narrow impedance bandwidth as well as a high conduction loss in the radiating system.

In low-frequency radiating systems, the radiation gain is of secondary importance. Because of the size limitation, the conduction loss is usually excessive and the radiation efficiency is correspondingly low. A study of the optimum performance of low-frequency radiation systems under various conditions might lead to a definite conclusion about the physical limitations of low-frequency radiating systems.

G. TRANSMISSION PROBLEMS FOR MILLIMETER WAVES

Staff: Professor L. J. Chu
T. Moreno

Introduction. An investigation has been initiated into the properties of transmission lines and other transmission techniques which may be suitable for millimeter waves. By theoretical and experimental investigations, it is hoped to determine which techniques may be most suitable for use with sources of millimeter energy when these sources become more generally available. It is also anticipated that more efficient transmission systems for centimeter waves may be forthcoming.

Results to Date. A theoretical analysis has been undertaken of the properties of waveguides large enough to support more than one transmission mode. The radiation resistance of dipole antennas inside these oversize waveguides has been calculated for rectangular guides, and the efficiency of dipoles when used for reception has been determined for a number of different operating conditions.

The existence of f-m distortion similar to that resulting from multipath propagation has been predicted, and calculations have been made of its severity. The possibility of inter-mode resonances has also been predicted.

Future Plans. It is planned to extend the analysis of oversize waveguides, and to consider in particular antenna systems other than simple dipoles. The inter-mode resonances, which may prove very troublesome, will also be more fully investigated. A comparison of these oversize waveguides with other possible transmission systems will be further extended.

Experimental work will also be undertaken in an attempt to verify some of the theoretical predictions, and to determine the practicality of these oversize waveguides for different applications. The experimental work will probably take the form of scale model investigations at centimeter wavelengths.

V. H. THE SCATTERING OF ELECTROMAGNETIC WAVES; MATRIX METHODS

Staff: Professor H. Mueller
N. G. Parke

The scattering of radiation by matter has been extensively studied both experimentally and theoretically. These studies have contributed to our knowledge of molecular structure and to the structure of matter in bulk. Mueller has suggested a general method for the solution of scattering problems. The principal mathematical tool is a matrix algebra, the matrices characterizing the scatterers, and the corresponding vectors characterizing the incoming and outgoing radiation. It has become increasingly apparant to us that to handle such problems as scattering from electrolytes, the calculus must be strengthened sufficiently to allow

(1) The treatment of polychromatic radiation.

(2) Handling cases where the superposition of scattered radiation from separate particles could not be treated as purely coherent or purely incoherent.

Research to date indicates that Wiener's generalized harmonic analysis is the proper mathematical tool for this problem. We have established the connection between Wiener's coherency matrices and Mueller's matrix algebra. At present we are applying this technique to obtain an explanation of experiments on the scattering of radiation from electrolytes which elude treatment by the earlier calculus.

I. ACTIVE NETWORKS

Staff: Professor E. A. Guillemin
Dr. M. V. Cerrillo
J. G. Linvill

In the last quarter's work on the theory of active networks, particular attention has been paid to the effect of linear constraints in passive linear systems, for which the corresponding F function to be minimized is a real quadratic form. Not many constructive results have been obtained in this connection, mainly because it was recognized that currents or voltages are not the suitable coordinates to use. The future lines of attack rest upon the introduction of frequencies as coordinates. Unit frequency vectors, along the natural frequencies of the system, will be taken as a basis, so that the applied frequency can be expressed as a linear combination of these basic vectors. The general work will be continued by searching for some circuit properties which remain a minimum under the required trans - formations.

V. J. TRANSIENT PHENOMENA IN WAVEGUIDES

Staff: M. V. Cerrillo

The transient phenomena in cylindrical waveguides can be expressed in terms of Lommel's functions and transient functions. Results in this connection have already been reported.[1] New integral expressions for the Lommel's and transient functions have been obtained, as well as the separation of the real and imaginary parts in the case of complex z_k poles. Simple asymptotic solutions, (for the precursoy, main formation, and coda regions) can be derived from the new expression. Complete details on this subject will be given in a separate report.

K. MATHEMATICAL PROBLEMS

Staff: R. M. Redheffer

A report "Microwave Antennas and Dielectric Surfaces" has been prepared for publication in the Journal of Applied Physics. A report "Design of a Circuit to Approximate a Prescribed Amplitude and Phase" has been prepared for, and accepted by, the M.I.T. Journal of Mathematics and Physics.

A few results, based on work of N. Levinson, have been obtained on the problem of separation of variables in Laplace's equation.

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1. R.L.E. Quarterly Progress Report, April 15, 1947, p. 84.