

VII. COMMUNICATION RESEARCH

A. MULTIPATH TRANSMISSION

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1. Speech and Music

a. Field tests on frequency-modulation reception

Field tests have been completed. They indicate that frequency modulation can give excellent quality under favorable conditions, virtually eliminating noise and selective fading. Under most conditions met in the field frequency modulation appeared to be quite favorable in comparison with typical amplitude-modulation reception at the same wavelength.

L. B. Arguimbau, J. Granlund

b. Simplified receiver

The simplified FM receiver has been constructed and tested. A schematic drawing of the unit appears in Fig. VII-1.

Functionally the unit consists of a flat stagger-tuned i-f amplifier, a two-stage limiter using 1N56 crystals with 2 volts of bias, a single-tube Granlund-type discriminator of 2-Mc bandwidth, and de-emphasis and audio circuits. A side amplifier and rectifier to provide automatic volume control voltages and tuning indication is also included.

Fig. VII-2 shows the signal-to-interference ratios needed for effective suppression of interference. The optimum operating range as seen from the graph is quite small.

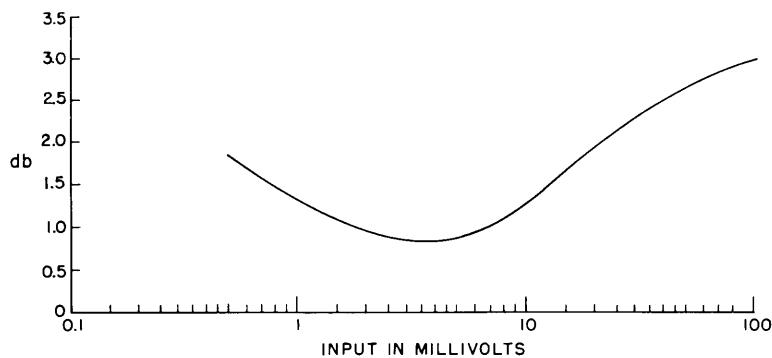


Fig. VII-2 Suppression threshold as a function of input.

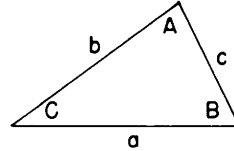
Some improvement can be made by rearranging the stagger-tuned amplifier. The presence of automatic volume control in the r-f unit will tend to keep the operating point in the desired range.

R. A. Paananen

c. Three-path propagation interference

The expression for the average frequency of the sum of three carriers, which appeared in the Quarterly Progress Report, April 15, 1950, has been considered in more detail and found to be misleading. The average frequency can be expressed in a very simple form:

$$\bar{\omega}_i = \frac{A\omega_1 + B\omega_2 + C\omega_3}{\pi}$$



The three carriers have strengths a , b , and c , and frequencies ω_1 , ω_2 , and ω_3 respectively. The angles A , B , and C are determined from the signal strength triangle as indicated. If the strength of one carrier is greater than the sum of the other two strengths, the triangle cannot be formed, and the average frequency is exactly the frequency of the strongest carrier. If the differences of the carrier frequencies are commensurate, the expression above represents the average, over all possible initial phases of the carriers.

The reasoning which led to the result above is rather interesting. The addition of the three carriers is indicated in the vector diagram of Fig. VII-3. If $(\omega_3 - \omega_1)$ and

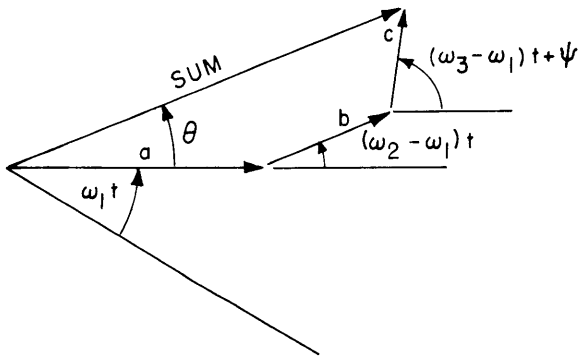


Fig. VII-3 Vector diagram showing addition of three carriers.

$(\omega_2 - \omega_1)$ are incommensurate, the diagram never repeats itself exactly. In other words, the sum is aperiodic, and any averaging must be done over all time. The instantaneous frequency of the sum is

$$\omega_i = \omega_1 + \frac{d\theta}{dt}$$

and the average frequency is

$$\bar{\omega}_i = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \omega_i dt.$$

Now the instant from which time is to be measured was chosen as the time when the

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a and b signals are in phase, and the c signal leads them by ψ . If another time reference had been chosen, another value of ψ would have resulted. Considering all possible time references, all values of ψ are equally likely. Thus it is correct to average, over all possible values of ψ :

$$\overline{\omega_i} = \frac{1}{2\pi} \int_0^{2\pi} \overline{\omega_i} \, d\psi = \frac{1}{2\pi} \int_0^{2\pi} d\psi \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \omega_i \, dt \quad .$$

Changing the order of integration, we have

$$\overline{\omega_i} = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T dt \frac{1}{2\pi} \int_0^{2\pi} \omega_i \, d\psi \quad .$$

Functionally, the instantaneous frequency is $\omega_i [(\omega_2 - \omega_1)t, (\omega_3 - \omega_1)t + \psi]$. After performing the definite integral on ψ , or on $[(\omega_3 - \omega_1)t + \psi]$ since t is constant during the integration, the result is some function of the angle $(\omega_2 - \omega_1)t$. This function is clearly periodic in $2\pi/(\omega_2 - \omega_1)$, so that the average over all time amounts to an average over a period, similar to the integration on ψ .

The advantage of this devious method is that the two integrations can be performed easily, while the limiting process initially proposed presents serious difficulties. In addition, the method suggests a way of finding the average frequency of the sum of more than three carriers: all but two are given initial phases, ψ_1, ψ_2 etc., over which averages may be performed.

J. Granlund

2. Television

Preliminary work has been completed by F. Cole on the subject of the use of pre-emphasis and clipping for FM video. The results are difficult to evaluate because of the inferior quality of video signal available for test purposes. The signal was obtained from a commercial television receiver tuned to a local station.

The exciter for the FM transmitter has been completed. It obtains a deviation of 5 Mc/sec at a center frequency of 50 Mc/sec with a conventional reactance tube. The speed of response needs improving, as it takes about 0.15 μ sec to go from 10 percent to 90 percent of the total frequency change. This figure should be around 0.08 μ sec or better. A modulation monitor working over a band about 12 Mc/sec wide was built for checking the frequency dynamically. From the construction of this unit it would seem possible that a discriminator 60 Mc/sec can be built for the receiver. However, the current requirement for fast action is such that 829B tubes would have to be used.

A few preliminary calculations have been made on the possibility of limiting the overshoot due to pre-emphasis in video signals by the use of phase shifting networks. The

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phase of the video signal is distorted in the transmitter and restored in the receiver.
The system shows promise. W. Hatton

B. STATISTICAL THEORY OF COMMUNICATION

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Prof. R. M. Fano	E. H. Gibbons	O. H. Straus
Prof. Y. W. Lee	P. E. Green, Jr.	C. A. Stutt
Prof. J. F. Reintjes	L. G. Kraft	I. Uygur
Dr. E. E. David, Jr.		L. Weinberg

1. Analogue Electronic Correlator

A design for the analogue electronic correlator has been worked out, and most of the circuits constructed and tested.

In the present design, master timing pulses are formed in a blocking-oscillator type of pulse generator, and sampling pulses are obtained from the trailing edges of two phantastron delay circuits. The sampling pulses are separated by a time interval which is controllable between 0.5 μ sec and 2000 μ sec. Each sampling pulse is used to store, on a capacitor, a charge proportional to the input-signal function at the instant of sampling. During each sampling interval, the storage capacitor is charged or discharged to the proper voltage level through a triode which is held cut-off except during the time of sampling. The circuit has been designed for a charge and discharge time constant of approximately 0.2 μ sec. The sampling pulses have a duration of 1 μ sec.

One of the variable-amplitude capacitor voltages produced is converted to a pulse-duration-modulated signal by means of an overdriven cathode-coupled amplifier, a sawtooth generator, and a bistable multivibrator. The sawtooth-voltage wave is initiated at the instant the input signal is sampled, and the sawtooth wave, together with the variable-amplitude voltage produced across the storage capacitor by the input signal, is applied to the cathode-coupled amplifier. An output pulse delayed (with respect to the instant of sampling) by an amount proportional to the input-signal amplitude is obtained from the cathode-coupled amplifier. This pulse and the sampling pulse trigger the bistable multivibrator, which provides a rectangular-voltage wave having a duration proportional to the amplitude of the input-signal function at the instant of sampling.

The pulse-duration-modulation circuit has been designed for a maximum modulation of ± 50 μ sec about a quiescent value of 100 μ sec. Maximum modulation is obtained for an input-signal voltage of 60 peak-to-peak volts.

The duration-modulated and variable-amplitude waves are combined in a coincidence circuit, and the combined pulses are integrated to produce the correlation function. Circuits for performing these two operations are now being constructed.

J. F. Reintjes

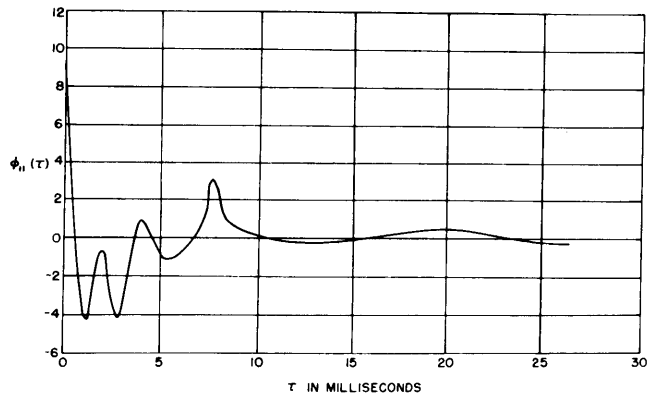


Fig. VII-4 Autocorrelation of speech, male voice.

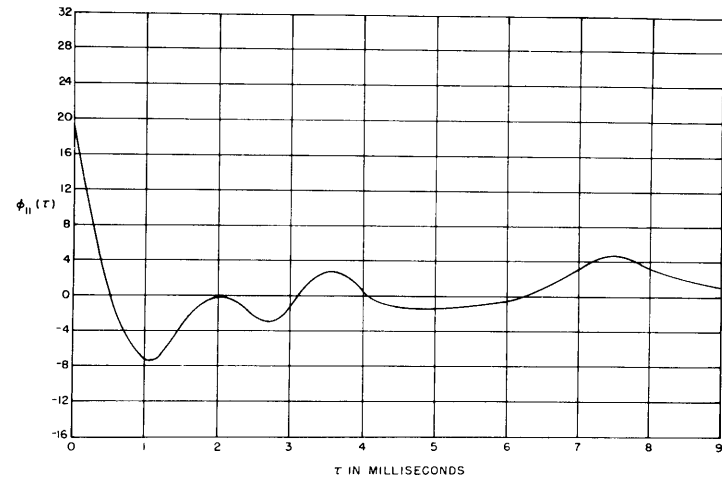


Fig. VII-5 Autocorrelation of speech, male voice, for a smaller range of τ .

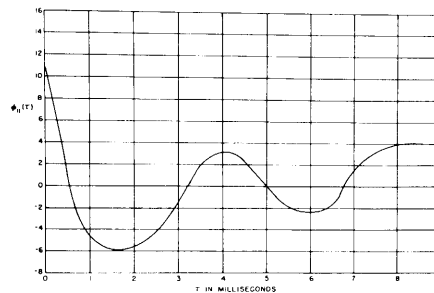


Fig. VII-6 Autocorrelation of speech, female voice.

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2. Autocorrelation of Speech

Three determinations of long time speech correlation have been made. Magnetic tape recordings 15 minutes in length were used for speech samples. Figure VII-4 is the autocorrelation of a male voice over a τ -range of 25 milliseconds. Approximately 14,000 product pairs were averaged for each point obtained on the curve. Figure VII-5 is an autocorrelation obtained from the same speech recording over the smaller range of τ indicated. Approximately 28,000 product pairs were averaged in obtaining this curve. Figure VII-6 is a similar curve for a female voice.

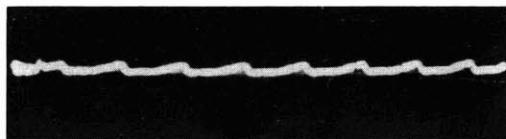
Y. W. Lee, L. G. Kraft, I. Uygur

3. Detection of Small Signals in Noise

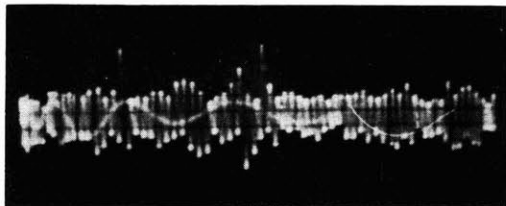
The digital electronic correlator has been used to detect the waveshape of several small periodic signals mixed with noise. In addition to the detection of a square wave, as reported in the Quarterly Progress Report, April 15, 1950, sawtooth, exponential, and staircase waveforms have been detected.

Figure VII-7 shows the results in the case of the sawtooth waveform.

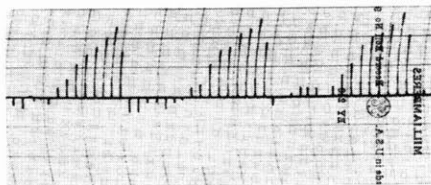
Y. W. Lee, L. G. Kraft, I. Uygur



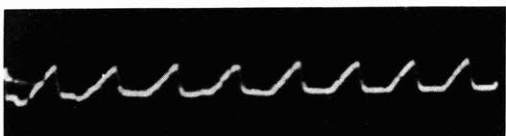
a



b



c



d

Fig. VII-7 Detection of periodic waveform in noise.

- a. Sawtooth (actual size)
- b. Sawtooth plus noise (input to correlator)
- c. Output of correlator
- d. Sawtooth (enlarged)

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4. Experimental Determination of System Functions by the Method of Correlation

The theoretical result that internal noise should not affect the determination of system functions by the method of correlation has been experimentally verified. An

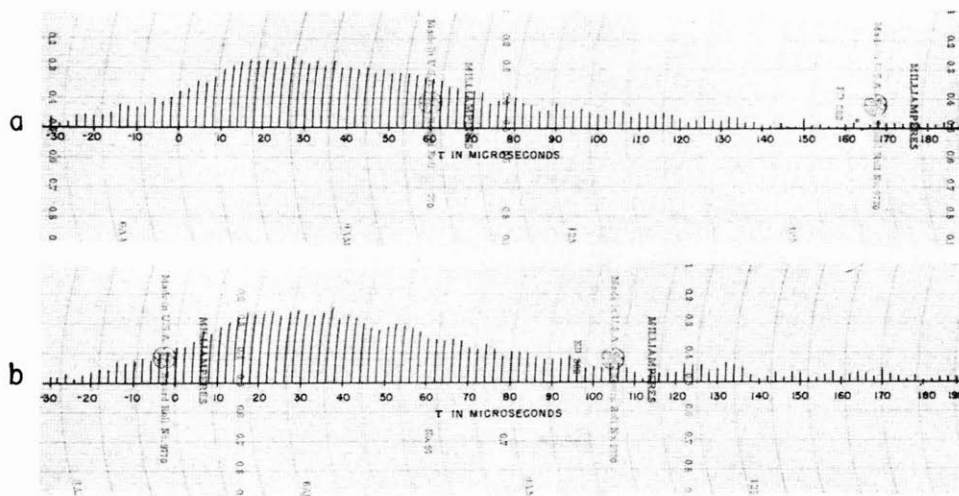


Fig. VII-8 Crosscorrelation curves obtained
(a) without internal noise (b) with internal noise.

amplifier incorporating an internal source of noise was constructed and its system response found on the correlator as described in the Quarterly Progress Report, April 15, 1950. Figure VII-8 shows the results obtained when the internal noise source is set at zero and at the maximum value consistent with linearity considerations of the theory.

J. B. Wiesner, Y. W. Lee, L. G. Kraft

5. Noise in Nonlinear Devices

The statistical characteristics of the output of a nonlinear device for a noise input or an input of noise plus a sinusoidal signal (modulated or unmodulated) have been known for a number of years. An attempt is being made to verify the theoretical results experimentally. A band-pass filter with variable Q and a noise amplifier have been built. The output of the amplifier is fed to a linear rectifier. It is intended that the correlation functions of the input and the output of the rectifier will be obtained by means of the electronic correlator; it will then be possible to obtain the power-spectrum curves through use of the electronic differential analyzer.

Y. W. Lee, L. Weinberg

6. Techniques of Optimum Filter Design

The flexible transmission system for approximating arbitrary transient responses is nearing completion. The basic feature of the device is a tapped delay line having a total delay of 500 μ sec with tap points at 10- μ sec intervals. This line is made up of

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100 line sections similar to those pictured in Fig. VII-9. Each section provides a 5- μ sec delay, and is made up of sixteen prototype and m-derived sections. The nominal characteristic impedance of the line is 1000 ohms, and the cut-off frequency is 1 Mc/sec. The

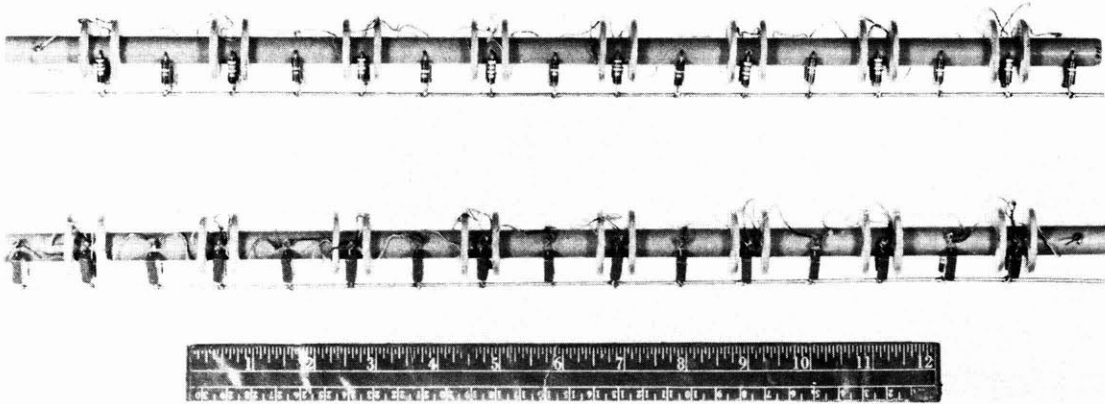


Fig. VII-9 Typical line sections in delay-line filter.

relatively high cut-off frequency is used so that the shape of the pulses used in setting up a desired impulse response will be preserved as they progress along the line. To insure further that the pulse shape is preserved, and to insure proper functioning of the line as a filter in the audio frequency range, sufficient shunt loading is inserted to produce distortionless transmission.

A section of line 60 μ sec in length has been completed. Its performance in affording a seven-point approximation of arbitrary time responses of that duration has been quite satisfactory.

In addition to its primary function as a flexible linear system, the delay-line filter has several other applications of interest in communication and network theory problems. For example, it may be used as

a. Computer of Fourier transforms. By observing the frequency response of the line when set up for an arbitrary time function, one automatically computes or measures the Fourier transform of the time function. Power spectrum functions may be obtained from correlation functions in this manner.

b. Computer of convolution integrals. On account of attenuation the line is split into two sections with interposed amplifier. Each section may be adjusted for a particular impulse response, then, when connected in cascade, the over-all time response is the convolution of the two functions.

c. Function generator.

d. Storage device.

e. Delay device for computing correlation functions.

Y. W. Lee, C. A. Stutt

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7. Theory of Nonlinear Transducers

Scope of the Theory. A useful theory of nonlinear transducers need not have as great a degree of generality as the present linear theory. Since a given transducer is designed to work only for a well-defined input ensemble governed by a set of probability distribution functions, its behavior need be specified only for the ensemble under consideration. This is in contrast to the linear theory, which applies to any input whatever. The linear theory is also very complete with respect to energy considerations. Insofar as the processing of information is concerned, these energy considerations are irrelevant. What is important from the point of view of information theory is how the transducer operates on the input signal to produce the output signal. By concentrating on this aspect of the transducer's behavior, it seems likely that a useful theory can be developed. Such a theory should be helpful in the study of existing or projected nonlinear networks, in the design of encoding and decoding equipment, and in the design of noise discriminators, predictors, and other operators.

Basis for the Theory. In the linear case the principle of superposition has led to a very general theory based on exponential functions and Fourier techniques. The failure of superposition for nonlinear transducers leads to a search for another base on which to build a theory. A possible answer suggested by Wiener lies in the concept of time-invariance. According to this point of view, the past of the input to a transducer is represented by a series of numbers $u_1(t)$, $u_2(t)$, \dots , $u_n(t)$, and the output of the transducer is a function of the several variables u_1 , u_2 , \dots , u_n :

$$\text{Output} = F(u_1, \dots, u_n) \quad .$$

Since the transducer is time-invariant, its output for any prescribed test input will be the same, independent of when the test input is applied, provided only that transients or other effects due to previous inputs are clear prior to application of the test.

The function F represents the behavior of the transducer under all conditions which are to be encountered, and it can accordingly be considered as the system function of the transducer. For purposes of mathematical manipulation and physical representation F may be expanded in a series (such as a power series, or a series of Hermite functions*); or the multi-dimensional space of the variables u_1, \dots, u_n , which is the domain of F , may be broken up into cells such that F may be taken as constant in any one cell, thus permitting a graphical representation of the system function.

Applications of the Theory. By means of the theory it appears possible to show that any time-invariant transducer can be approximated arbitrarily closely by means of a network composed of resistors, inductors, condensers, rectifiers, and direct-current

*The expansion in Hermite functions and its application to the experimental study of transducers are due to N. Wiener.

sources. The theory also indicates methods for experimentally determining the system function of unknown nonlinear networks. A central problem which needs study is: Given a specified transducer and an input having a specified set of probability distributions, what is the set of probability distributions of the output? This is a problem of analysis. The corresponding synthesis problem is: Given the probability distributions of the input and output of a transducer, what is the system function of the transducer? The question of physical realizability comes in here, as does also the question as to the conditions under which a transducer has an inverse.

The theory is also applicable to the problem of channel matching and to the problem of noise discrimination. The performance of a noise discriminator may be optimized on the basis of the minimum mean square error criterion, or on the basis of minimizing the probability that the error will be greater than a predetermined maximum. A specific example of the use of the theory to determine the optimum noise discriminator on the basis of the minimum mean square error criterion is given below.

Example. We consider the discrete quantized case, i. e. the case in which an ergodic source generates at regular intervals one of a finite set of possible symbols (numbers). The source drives an encoding transducer which has a finite memory. The transducer produces an output symbol u_n for each input symbol x_n ,

$$u_n = F(x_n, x_{n-1}, \dots, x_{n-m+1})$$

so that the encoding transducer operates on a finite set m of the past input symbols. If the transducer is time invariant, F may be expanded in a series:

$$F = \sum a_{i,j,\dots,h} x_n^i x_{n-1}^j \dots x_{n-m+1}^h \quad .$$

The purpose of the encoding transducer is to predistort the input signal in such a way as to combat most effectively the noise on the transmission channel. At a repeater station it will generally be desirable to make use of a nonlinear noise-removing transducer G in order to avoid the cumulative effects of noise. The optimum system function (in the least mean square error sense) of such a transducer may be found as follows: Let v_n be the received time series. Let

$$w_n = G(v_n, v_{n-1}, \dots, v_{n-q+1}) = \sum c_{i,j,\dots,\ell} v_n^i v_{n-1}^j \dots v_{n-q+1}^\ell$$

be the output of the noise-removing transducer G . Then, since u_n is the input to the channel,

$$w_n = u_n + e_n \quad \text{or} \quad e_n = w_n - u_n$$

where e_n is the error remaining after the operation by G . The mean square error is

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$$\mathcal{E} = \overline{e_n^2} = \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{-N}^N (w_n - u_n)^2$$

and by minimizing this error we obtain the constants $[c]$ which define the optimum transducer G . We first rewrite w_n :

$$w_n = \sum_a C_a V_a(n)$$

where each $V_a(n)$ represents one of the products $v_n^i v_{n-1}^j \dots v_{n-q+1}^\ell$, and C_a is the corresponding $c_{i,j,\dots,\ell}$. Then

$$\begin{aligned} \mathcal{E} &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{-N}^N \left[\sum_a C_a V_a(n) - u_n \right]^2 \\ &= \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{-N}^N u_n^2 - 2 \sum_a C_a \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{-N}^N u_n V_a(n) \\ &\quad + \sum_a \sum_\beta C_a C_\beta \lim_{N \rightarrow \infty} \frac{1}{2N+1} \sum_{-N}^N V_a(n) V_\beta(n) \end{aligned}$$

The limits in the above equation are statistical parameters which will be denoted by ϕ_{uu} , ϕ_{uV} , and ϕ_{VV} , so that

$$\mathcal{E} = \phi_{uu}(0) - 2 \sum_a C_a \phi_{uV}(a) + \sum_{a\beta} C_a C_\beta \phi_{VV}(a, \beta) .$$

In order for \mathcal{E} to be minimum, we must have $\partial \mathcal{E} / \partial C_\gamma = 0$ for all γ .

$$\frac{\partial \mathcal{E}}{\partial C_\gamma} = -2 \phi_{uV}(\gamma) + 2 \sum_a C_a \phi_{VV}(a, \gamma)$$

$$\phi_{uV}(\gamma) = \sum_a C_a \phi_{VV}(a, \gamma) \quad \gamma = 1 \dots M$$

where M is the largest value of γ . This is a set of M simultaneous equations in M unknowns, and hence leads to the evaluation of the C_a and of the system function G .

H. E. Singleton

8. Short-Time Correlation Functions and Power Spectra

The use of short-time autocorrelation functions to represent speech sounds, brings up the theoretical question of how such a representation is related to the spectrographic representation which has been used in the past.

By properly defining the short-time autocorrelation function $\phi_t(\tau)$ and power spectrum $G_t(\omega)$ it has been possible to obtain an extension of Wiener's theorem in the form

$$\phi_t(\tau) = \frac{e^{-a|\tau|}}{2\pi} \int_{-\infty}^{\infty} G_t(\omega) \cos \omega\tau \, d\omega$$

$$G_t(\omega) = \int_{-\infty}^{\infty} e^{-a|\tau|} \phi_t(\tau) \cos \omega\tau \, d\tau \quad .$$

A paper on this subject is to be published by the Journal of the Acoustical Society of America.

A study has been made of the effectiveness of correlators in separating a periodic signal from noise when the averaging process in the correlator is performed by means of an RC filter. The results will be presented in a forthcoming paper.

R. M. Fano

9. Speech Studies

a. A short-time correlator for speech waves

Progress has been mainly in mechanical construction and assembly. The delay line has been designed to occupy a 24-inch panel on a standard 19-inch rack. This panel also contains the amplifiers associated with each filter section. The filter coils have been wound and assembly of the components is now in progress.

Other completed work includes redesign and reconstruction of the carrier oscillator and distribution panel, alignment of the product filter amplifiers and adjustment of the switching circuit.

It should be possible shortly to obtain visual short-time correlation patterns corresponding to the speech sounds spoken into a microphone.

R. M. Fano, P. E. A. Cowley



Fig. VII-10



Fig. VII-11

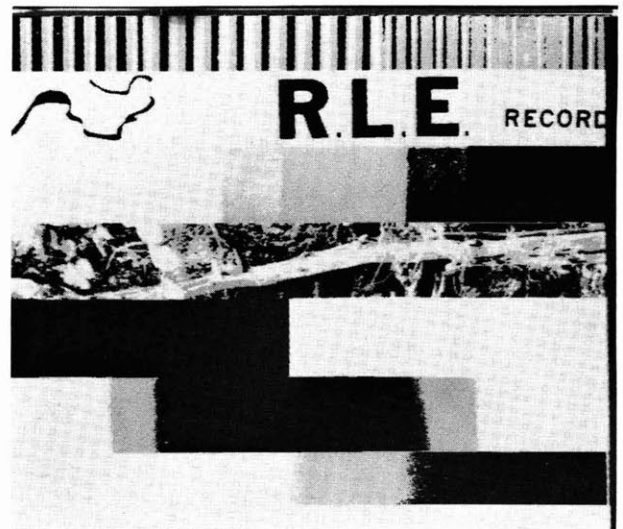


Fig. VII-12



Fig. VII-13



Fig. VII-14

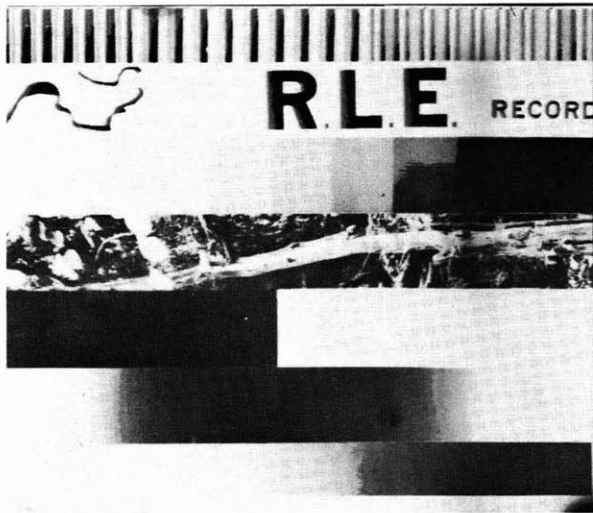


Fig. VII-15



Fig. VII-16

Fig. VII-10 – Fig. VII-16 Original picture (Fig. VII-10) was coded successively with an increasing number of digits in test to determine the number of digits required.

10. Pulse-Coding of Picture Signals

Several tests have been made with the coding equipment. The three which gave the most conclusive results were first, the determination of the number of density levels required to transmit a high-quality picture; second, the amount and type of information in each digit channel; and third, the necessary sampling rate for a given type picture. The running of these tests also brought to light the need for a precise, reproducible photographic process as well as the need for more suitable methods of quality determination.

The first test was made by transmitting the same picture (Fig. VII-10) eight times, each time at a different level in the order 2^1 , 2^2 , 2^3 , ... 2^8 . The first six of these pictures are shown in Figs. VII-11 to VII-16. The seventh and eighth were exactly like the sixth indicating that between 48 and 64 levels were required to transmit a picture containing gradual transitions from one density level to another. When the transitions were less gradual, fewer levels were needed as evidenced by the picture of the tree which appears as good at 16 levels as it does at 64. Since the average lowest-perceptible-difference of the eye corresponds to about 1 out of 50 levels from black to white, any greater refinement than this would be of value only at brightness densities where the actual lowest-perceptible-difference is smaller than the average. Hence unless there is some additional integration in the equipment or in the photographic process it would seem that 64 levels are adequate to transmit any type of picture over this system. The nominal line number of the wire-photo system is 894, based on current television standards.



Fig. VII-17



Fig. VII-18



Fig. VII-19

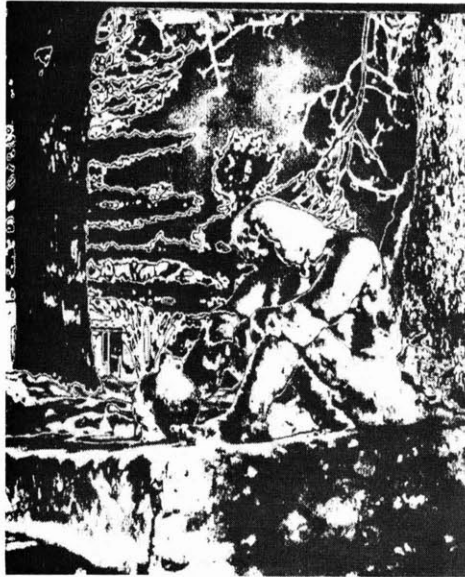


Fig. VII-20



Fig. VII-21

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Fig. VII-22

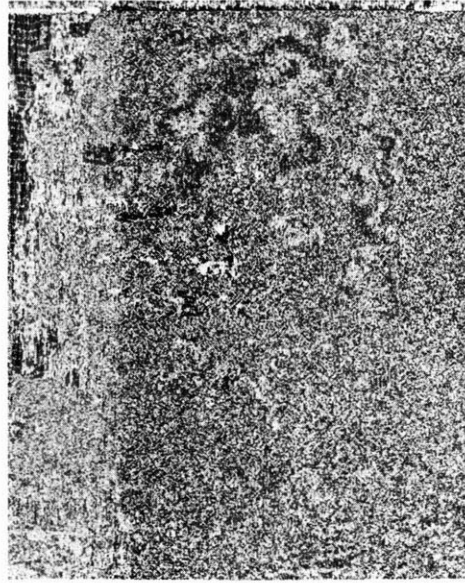


Fig. VII-23

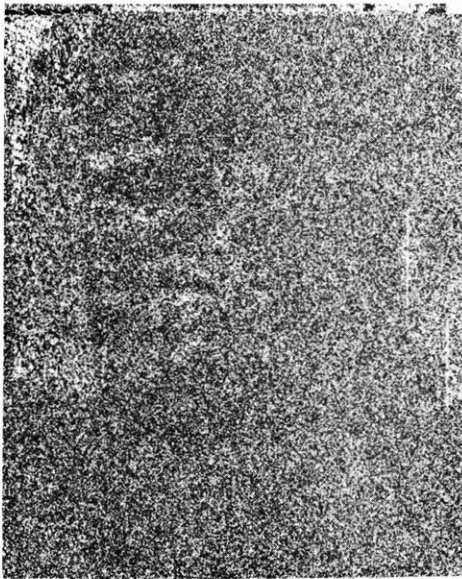


Fig. VII-24

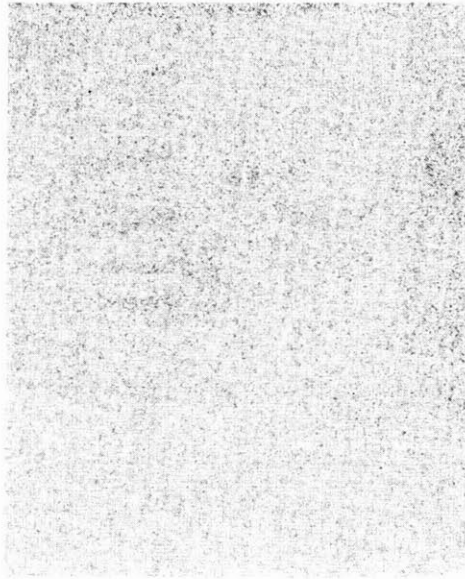


Fig. VII-25

Fig. VII-17 – Fig. VII-25 Original picture (Fig. VII-17) with black and white pictures corresponding to the output of each of the 8-digit channels.

The second test consisted of making a series of black and white pictures corresponding to the output of each of the eight digit channels. Fig. VII-17 is the original picture used in this test. Each of the lines from the reader was successively used as the No. 8 input to the decoder; all the other lines were disconnected. The No. 8 current source,

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being the most significant digit, provided sufficient drive to produce a black and white picture. Figures VII-18 to VII-25 indicate what type of information is carried by each digit, i. e. overall contrast, main detail, fine detail, etc. Conversely they show in what contrast range or ranges various elements of the pictures lie. If all the detail were found in the largest (No. 8) digit, then the picture would be a silhouette, or if it were found in digit No. 7, the picture would be considered "hard" or "contrasty." Since this picture is "soft" the main details appear in digits Nos. 6 and 5. The overall contrast appears in digits 7 and 8, and the fine detail and shadings in 3 and 4. Digits 1 and 2 consist almost entirely of uniform gray dots and so add very little to the picture, thus confirming the results of the first test.

The third test was made by varying the sampling rate from 400 to 1800 cycles in four steps. Fig. VII-10 was used so that bandwidth of 1 1/2 cycles for the strip containing the sinusoidal black-to-white variation to 200 cycles or more for the strip containing the tree would be available. The maximum bandwidth of the equipment was 600 cycles which theoretically required a 1200-cycle sampling rate. It was found that a sampling rate of 400 cycles was adequate to reproduce faithfully the picture of the tree as well as the remaining strips which had lower bandwidths. The only noticeable difference between the pictures at the various rates was that noise effects were accentuated as the sampling frequency was decreased. Therefore it could be concluded that 600 cycles provides sufficient bandwidth for the exceptional picture, but that normally about 300 cycles is all that is needed, so that a sampling rate of 600 cycles would be sufficient. This can be further substantiated by the fact that though theoretically a 140-line/in definition would produce at the most a 900-cycle square wave (for the Times-Photo equipment), the maximum physically realizable square wave repetition rate as found in a chopper pattern is about 471 cycles. Such a pattern is seldom found in a normal picture. Since the picture of the tree contains a more than average amount of fine detail it would seem reasonable to assume that normally a sampling frequency lower than 1200 cycles would be adequate in the interest of conserving bandwidth.

A standard photographic procedure was evolved to insure that errors in the received picture would be caused by the coding process alone. It consisted of calibrating the receiver output current in terms of the gray shades on the photographic paper used for reception. Then by comparing portions of the picture to be transmitted with the gray shades, the receiver and transmitter were adjusted to expose the paper properly. The proper development of the paper was insured by making use of a set of exposed, undeveloped, step-tablet test strips, one of which was developed just prior to the print to make certain that the developer was correctly mixed and at the right temperature.

The quality of the received picture was measured with respect to density levels by means of a recording spectrophotometer and with respect to detail reproduction by means of oscilloscope waveforms. The first measurement permitted the transfer characteristic

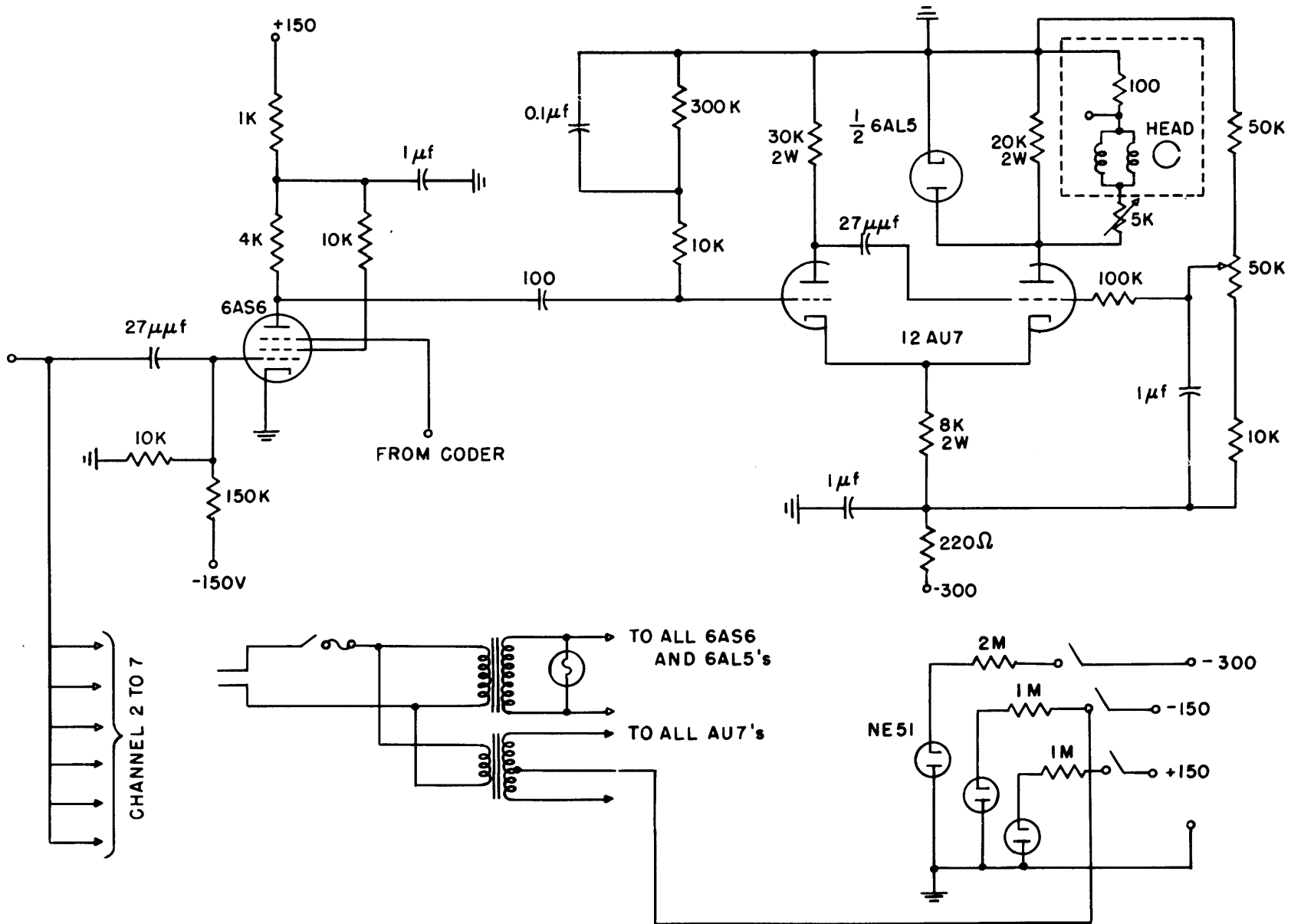


Fig. VII-26 Recording circuit

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of the equipment to be made and discrepancies to be discerned. The second allowed corrections for the transient response of the system. While such measurements help to make certain that the equipment is properly adjusted as far as the link between the transmitter and the receiver is concerned, and can reveal what information can be left out in the coding process, such as the 1st and 2nd digits, they say little about the link between the picture and the observer. It is felt that as the study progresses of what type of information can be left out of a picture, the development of adequate methods of making observer tests will be mandatory.

The actual circuits of the equipment and a description of their operation and limitations may be found in the thesis, "Transmission of Picture Signals by Pulse-Code" by E. H. Gibbons, M.I.T., 1950. J. B. Wiesner, E. H. Gibbons

11. Pulse-Code Magnetic Recorder

Because of the high inductance of the recording heads it was difficult to obtain a recording current waveform which would be independent of the pulse repetition rate and yet strong enough to secure a sufficient signal-to-noise ratio. To meet the requirements, the series connection of the recording coil halves was changed into a parallel one, thereby reducing the inductance by a factor of four; and a special recording circuit (shown in Fig. VII-26) was designed.

These modifications made it possible to obtain a pulse with a steep rise and an exponential decay with a time constant of less than 8 μ sec so that, even at the highest possible repetition rate of the system (i. e. 25 kc), no pulse can be affected appreciably by the preceding pulse. The peak current is about 5 ma which is quite sufficient since reliable playback has been obtained with a peak current of only 2 ma.

As an incidental result it was found that the playback voltage waveform does not depend very much upon the recording current. The playback waveforms do not differ from the ones shown in Fig. IX-18 and IX-19, pp. 66-67, Quarterly Progress Report, July 15, 1949, although those waveforms were produced by square pulses. Reduction in amplitude for frequencies above 10 kc occurred in both types of recording waveforms. The conclusion was that, at high frequencies, no region of zero magnetization exists between two successive pulses.

The timing, coding and recording circuits have been built in permanent form, and most of them have been tested. J. B. Wiesner, L. Dolansky

12. Twin-Track Recorder for Slow Phenomena

Tests of the recording system have been made using brainwave signals. Typical results are reproduced in Fig. VII-27, in which (a) represents the input signal, (b) the pen record made by conventional electroencephalographic equipment, and (c) the speeded-up playback from the tape recording system. The time duration of the traces is 2.25

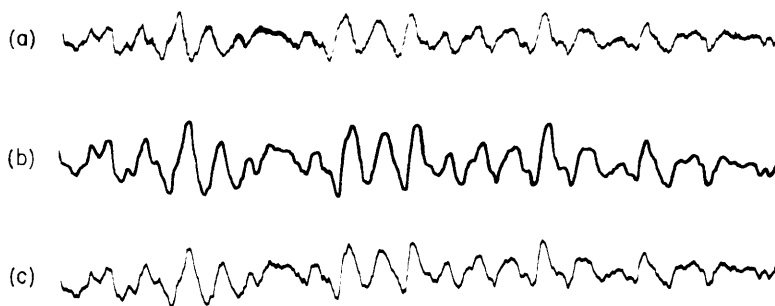


Fig. VII-27 Performance of one channel of the recorder with a brainwave signal. (a) Electrical input to both pen recorder and tape recorder. (b) Record made by conventional pen recorder. (c) Playback from tape recorder.

seconds for (a) and (b), and 0.45 seconds for (c). Details of the equipment will be given in a later progress report.

J. B. Wiesner, P. E. Green

C. HUMAN COMMUNICATION SYSTEMS*

Prof. A. Bavelas	A. Simmel
Prof. J. B. Wiesner	F. L. Skinner
R. P. Abelson	S. L. Smith
W. H. Huggins	N. O. Sokal
L. Levine	O. H. Straus

1. Human Communication Networks

The problem of an efficient group effort is an old one, and in the operation of a group, or team, the effective exchange of information is a critical factor. In many instances, it is quite clear that the quality of performance that may be expected from a team is determined primarily by the limitations of the communication net in which it operates. While many of these limitations are of the technical sort which traditionally concern the electrical engineer, there is a set of limiting factors which may be called psychological and which appear to derive from the pattern properties, or topology, of the existing communication net.

Criteria of "efficiency" of group performance include: (a) The speed and accuracy of performance of a given task. (b) The volume and distribution of messages within the network per unit task completed. (c) The adaptability of the group to changes of task. This includes the notions of "group learning." (d) The emergence and stability of organization, including the development of "leader-follower" relationships. (e) Individual and group morale of network members. Earlier work has shown that the above measures

*The work of this particular section has been supported in part by the Rand Corporation.

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vary among different communication networks, all of which are logically adequate for the successful completion of the given tasks.

Observed values of a set of measures such as these will be some function of (a) the topology of the communication links in the network, (b) the location of the driving points, where information is injected, and of the output points, where the results of the group activity leave the network, (c) the nature and the variability of the assigned group task, and (d) constraints placed upon the content of messages within the network. Other variables, such as the effect of given individual personalities in given positions in the net, are of secondary interest for now, and their effects can be diminished by appropriate statistical methods.

A. Bavelas

a. Structure and function in human networks

The topological structure of a human network may be characterized by the number of individuals in the net (roughly, junction points, or nodes) and the number and arrangement of links connecting the nodes. If each link is considered to be directional, and not more than one link connects a node pair in a given direction, the set of all possible topological structures composed of N individuals can be described by an $N \times N$ matrix. The non-diagonal elements are equal to zero wherever direct communication between node pairs is made impossible by external constraints, such as one-way telephone lines, instructions, or social custom. The remaining non-diagonal elements, representing the rate of message flow, are in general not equal; where nothing is known about the density of message flow, the entries may be unity, and the matrix completely represents the structure imposed by external constraints. This corresponds to the usual notion of organization of a group. The number of such structures equals $2^{N(N-1)} / N!$ (removing the $N!$ duplications due to permutations of labelling the nodes), and this grows rapidly as N increases. An attempt is being made to reduce this set to subsets which may show common behavior.

The subset of networks being studied is that in which messages can get from any node to any other node, including itself. This condition means that the consequences of any message can potentially be felt by the originator of that message. This seems to be a useful and reasonably precise criterion for what is commonly meant by the term, "membership" in a group or organization. It is equivalent to saying that among the "members" of a human organization, the coupling between every node pair is significantly greater than zero. The whole net then forms a single feedback system.

In order to get a description that yields a determinate system, every new piece of information, including ideas from the memory of its members, must be represented explicitly as a driving function at a definite driving point. The information flowing around a task-oriented human network, after it has been operating for a little while, is, however, of two sorts. "First-order information" can be defined as the original information, or

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the same thing after elements of the net have operated upon it. This is like target information in a servo-mechanism. "Second-order information" refers to information about the sequence of messages, or the channel over which they arrive, etc., irrespective of their immediate content. This is information about the functioning of the net itself, which induces the members to change their individual "transfer functions," and hence the overall "transfer function," or performance characteristic, of the total group. This comes close to an operationally meaningful definition of "group learning." Second-order information corresponds to difference measurements and correction factors derived from them in a servo.

In order to describe in an orderly fashion the activity of a network, whose task, for instance, is the distribution or combination of information, the index notation for n-dimensional matrices is useful. Let the general element be $p_{\alpha\beta\gamma\delta}$, where α ranges over the senders of the message, β over the receiver, γ over the original input points of the information, and δ over the contents of the information, which may be thought of as quantized. The matrix, P, represents the probabilities that a given message characterized by the subscripts will occur.

Once formulated in this way, a new notion emerges. Not only what is usually called information, but also structure or organization, or topology, all appear in a single coordinate system. What is called organization is precisely those probability elements, or portions of them, which are predictable in advance of any actual input to drive the system. What is called information, is those elements which are not predictable in advance. It is to be noted that this method provides for variable handling of messages depending on their content, the variable index δ .

A network system of this type using binary inputs, messages and outputs, consisting of electrical switches operated by human subjects, is under construction. The logical design is such as to allow testing of the notions presented here.

O. H. Straus

b. Electrical network analysis and human networks

Although it is recognized that human group behavior is transitory and nonlinear on the microscopic level, there is a possibility that linear methods may be used to study phenomena which involve so many events that the constituent processes may be defined adequately by their Markoff probabilities. When this is possible, the highly developed techniques of linear network theory are applicable and the interaction of the various members within the group may be expressed in terms of admittance and impedance functions. Furthermore, it is possible to express the stress patterns throughout the group in terms of a set of modes which are characteristic of the structure of the group. These suggest measures of the "coupling" between a member and the group as well as the tendency for the group to break up into opposing factions and cliques. Effort at present is directed

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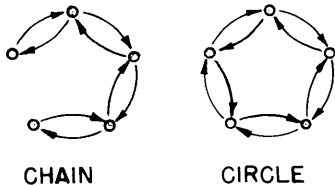
toward a reformulation of the problem on a microscopic, nonlinear basis using the mathematics of game theory. This more concise formulation will then be used to determine the conditions under which the treatment by linear methods is permissible when the number of subevents becomes large. W. H. Huggins

c. Communication in networks required to do collective reasoning

A series of experiments has dealt with the effect of communication networks upon certain aspects of group behavior. A communication network can be best represented by a picture constructed as follows: each individual is represented by a "cell," or "node," and a directed arrow from cell x to cell y indicates a channel from cell x to cell y over which some form of communication may proceed. The existence of a directed link from x to y does not necessarily imply the existence of a link from y to x. The absence of an arrow originating at cell x and terminating at cell y indicates that individual x is unable to contact individual y directly. Any such picture, or "topological complex," of cells and arrows represents some particular communication network.

When research on this subject began it was supposed that the communication network imposed upon a group of people would tangibly affect their performance of experimental tasks. This was found to be the case with group morale, speed, accuracy, and other attributes for a group performing a task which involved the collection of simple information at a central point. The present experiment dealt with an experimental task requiring organizational and evaluative activity as well as information-collecting.

The problem posed to the group was that of hitting a numerical target by having each member contribute some portion to the numerical total. The contributions were made with tokens of two colors; however, the group was not told the numerical worth of each color. After each shot on a target, the group numerical total was announced, and a new target was given. Thus the performance of the task required first that the color code be broken, and second that a leader be chosen who would assign proper numerical quotas to each member so that the target could be hit.



Six five-man groups were tested in each of two different networks, the "chain" and the "circle," shown in the accompanying figure. The members of the group communicated with each other by means of written messages without constraint as to content. A preliminary analysis of the data yields a provocative result: All six of the groups with the circular communication network broke the color code by agreeing that all members would contribute only tokens of one color as a "test." Only three of the six chain groups did this, two of the remaining three relying on arithmetical calculations based upon how many tokens of each color the members had sent, and one of them never breaking the code at all. This result can be very neatly explained on the basis of certain mathematical

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models we have developed. In a chain, information tends to collect faster than in a circle, but the circle allows for more rapid spread of information (and suggestions) than the chain. Broadly speaking, the circle is better adapted to simple planning, the chain better adapted to compiling information in order to decide thereby upon a course of action.

There are other suggestive indications of network differences in our data, but more important, we have recently devised a scheme of message analysis with many promising applications. This scheme divides messages into two classifications: "original" and "reactive." All messages which stem from a thought initiated by someone else are considered reactive, and all messages with no obvious prior referent are considered original. A "reactivity matrix," R , can be constructed, where the element r_{ij} equals the number of messages by j which react to original messages by i , divided by the number of original messages by i . Over and above individual differences, this matrix seems to be a strong function of the network pattern; it is possible to regard it as a network impedance matrix. Given the input at each node, (the number of original messages) the matrix allows us to calculate the resulting reactive flow (current) of messages. Further analysis of data will be made.

R. P. Abelson, A. Simmel

d. Pattern recognition

A study of the logical structure of pattern recognition, and its quantitative formulation in terms of probability functions, is being attempted. Experimental equipment for testing the notions on human subjects has been constructed and is being used to collect data.

F. L. Skinner, O. H. Straus

2. Sensory Replacement

a. Guidance device for the blind

A study has been undertaken to determine how a guidance device for the blind can best present environmental information through one or more of the blind person's remaining sensory channels, at a rate sufficient to allow him to walk confidently through environments of familiar type. Such a device collects and encodes information; its efficiency can be no greater than the ability of the user to decode the signals it presents.

Previous work on guidance devices (1, 2) was with sound or light beams projected from a device carried in the hand. A tone varying in pitch or pulse repetition rate indicated the range of objects as the device was directed, like a flashlight, point by point over the environment. This method of presentation required that the user integrate the point-by-point range signals over the spatial field, and in the time domain. The low rate at which information, thus coded, could be perceived, decoded, and integrated into a picture of the environment, was the bottleneck which limited the usefulness of the

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previous devices.

This project is an attempt to break open the bottleneck by a better method of coding and presenting information. Information is presented in the form of a relief map which can be read with the fingertips, similar to the reading of Braille text. The map will be constructed from an array of seventy pegs which can project through holes in a board; a projecting peg indicates the presence of an obstacle; a retracted peg indicates a clear area. At present, it is planned to map a 45° sector of the environment, to a range of 12 feet; the map scale will be 0.150 inch = 1 foot, and the resolution will be one foot. The peg-board will be carried by the blind subject; for the present, the pegs will be actuated by solenoids controlled through a cable by a human operator who will replace the scanning device of a complete system. As the subject moves in translation or rotation through an obstacle course, the obstacles on the peg map will move correspondingly. Continuous and simultaneous display of range and azimuth in a sector of the environment will allow exploration of the sector by the fingertips, using both touch and finger position sense in a manner similar to the normal activity of the eyes.

Equipment is under construction, and obstacle-course testing by blind subjects will be made available through the co-operation of the Perkins Institution and the Massachusetts School for the Blind, Watertown, Massachusetts.

N. O. Sokal, O. H. Straus

References

1. Research on Guidance Devices for the Blind, a progress report prepared by the Haskins Laboratories, New York, N. Y., for the Committee on Sensory Devices of the National Academy of Sciences (Dec. 1, 1946).
2. Research on Guidance Devices and Reading Machines for the Blind, a final report by the same authors (Dec. 31, 1947).

b. Felix

Felix has had an operational shortcoming. Whenever the subject's ability to receive words varied substantially from one test to another, we could not ascertain to what degree this was the fault of the subject or of the equipment.

We have now installed a monitoring circuit which makes a continuous record of the word patterns received by the subject. The monitor consists of a trigger, relay, and Esterline-Angus recording pen for each of the seven channels. When the voltage input level in a channel exceeds a specified value (adjusted to correspond to the subject's threshold of feeling for that channel), the trigger closes the relay. The relay, which remains closed as long as the input to the trigger exceeds the threshold, controls a voltage fed to the pen. We are thus able to obtain a seven-channel on-off pattern for each word. By comparing this pattern with others of the same word, we will be able to measure the relative degree of variability of the subject and the apparatus.

J. B. Wiesner, L. Levine, O. Straus

D. TRANSIENT PROBLEMS

Prof. E. A. Guillemin	W. H. Kautz
Dr. M. V. Cerrillo	H. M. Lucal
L. Weinberg	

1. Synthesis in the Time Domain Approach

An attempt is being made to find an adequate solution to the transient synthesis problem which is concerned with the design of electrical networks for specified transient behavior. The solution is carried out in two steps; first, an approximation in which the given specifications are converted into a realizable "system-function" of a physical network; and second, the actual synthesis of the network from this system-function. The latter step can usually be carried out without excessive difficulty by means of one or more of a large number of synthesis methods available.

Various attempts to solve the more challenging approximation problem have met with little success except in isolated special cases. The problem is one of passing from a given, desired time response $f(t)$ to a system-function of frequency $F^*(s)$ which is rational and meets certain other restrictions which depend upon the type of system desired. Two methods of attack are available. Either the given function $f(t)$ may be Laplace-transformed immediately into $F(s)$, and the approximation carried out in the s (frequency) domain to obtain $F^*(s)$; or, $f(t)$ may be approximated by a sum of complex exponentials $f^*(t)$ of the proper type, which, when transformed, yield a realizable system-function $F^*(s)$. Each solution has numerous disadvantages; the principal difficulty in the first method is the almost complete lack of correlation between approximation errors in the s -domain and those in the t -domain; with the second method it is difficult to obtain expansions of $f(t)$ which converge sufficiently rapidly to be useful for economical approximations.

The technique of solution to be investigated is based on a single fact which is an outgrowth of a long period of research on approximate theories of integration carried out in the Laboratory. Briefly, a definite correlation between the time and frequency domains has been found for a very large class of general and typical transients, each of which corresponds in the frequency domain to a particularly simple frequency function or "prototype transform." The transition from $f(t)$ to $F^*(s)$ may now be carried out by representing $f(t)$ in terms of the appropriate generating function, $g(t)$, which has a Laplace transform $G(s)$, the prototype transform. $G(s)$ is now approximated by the rational system-function $F^*(s)$. It is anticipated that this division of the approximation problem into two smaller ones will permit a greater control over approximating errors, and will facilitate the introduction of those additional constraints on the time or frequency function which often accompany specific problems. In addition, the use of more powerful

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and more rapidly convergent expansions is appropriate because of the fundamental and simple character of the generating functions and their prototype transforms.

2. Condensed Basic Aspects on the Problem of Integral Approximation

One basic task of this group is the investigation and development of methods of approximate integration of a class of integrals of the type

$$f(\tau_1, \dots, \tau_n) = \frac{1}{2\pi i} \int_{\gamma} F(s) e^{W(s, \tau_1, \dots, \tau_n)} ds \quad (1)$$

particularly the subclass associated with transient phenomena in electrical networks and other linear systems.

Several methods have been investigated and developed. They can be classified into three basic groups: Open methods, Cliff methods and Pocket methods. This classification is in accordance with the way in which the contour of integration, γ , encircles the singularities or passes in their vicinity.

Other subclassification has been made depending on the method in which we approximate the function $F(s)$, or $W(s, \tau_1, \dots, \tau_n)$ or both. These subclassifications are: stationary phase, saddle point, extended saddle point, plain and mixed cliff, plain pocket, essential and substitutional methods. They are listed in the order of increasing generality. The last method is by far the most complete theoretically. It has been found that the substitutional method possesses powerful potentialities because it is a sort of generalization of the preceding ones.

Investigation of this last method began about a year ago in a rather limited extent. Even at that time extensive applications of the method were foreseen in connection with the synthesis problem, and as a consequence considerable attention has been given to the method during the last few months.

The basic ideas contained in this powerful method which will be investigated thoroughly in future work are outlined here in a condensed form.

We are all aware that the singularities of $F(s)$ and $W(s, \tau_1, \dots, \tau_n)$, their branch cuts, etc., have a canonical role in the genesis of the integral formation. With certain conditions on the contour γ , which type (1) may satisfy, it happens that the complete integral contribution comes directly from the neighborhood of the singularities of the integrand and along the banks of the cuts, around which we deform the contour of integration.

The theoretical experience with these integrals indicates that the net quantitative effect of the singularities on the integral solution depends strongly on their nature and more strongly still upon their relative position in the s -plane. Because of the relationship of the parameters, τ_1, \dots, τ_n and the functions in the integrand, it becomes clear

that these parameters influence, in general, the position, or sometimes the nature, of the above-mentioned singularities.

In our integrals, the parameters τ_1, \dots, τ_n possess a variable nature (independent, of course, of s). When they change, the singularities of the integrand may move in the s -plane, changing their relative position, or sometimes their nature, and therefore changing their effect on the process of the integral formation.

We may naturally wonder if this motion of the singularities of the integrand as a function of τ_1, \dots, τ_n can be exploited as a possible basic idea for a method of approximate integration. To illustrate this possibility we must describe the process in more precise terms.

Suppose that we are interested in an approximate solution of an integral contained in class (1), which will be valid in a certain domain of variation of τ_1, \dots, τ_n , say G_ν .* When τ_1, \dots, τ_n vary in G_ν , the singularities of the integrand will, in general, move in the s -plane. They follow certain orbits with a definite law of motion, thus changing their relative position. It may happen that during relative motion a certain group of singularities have, or may attain, an almost dominant quantitative control on the building up of the integral solution when τ_1, \dots, τ_n are in G_ν ; while other singularities have, or may attain, a small or secondary quantitative effect on the integral when τ_1, \dots, τ_n are in G_ν . Let us assume momentarily that this is the case.

The following notation is now convenient. Let τ_1, \dots, τ_n be in G_ν . "Primary" singularities (in G_ν) are those which have a strong quantitative influence and "secondary" singularities (in G_ν) are those which have a minor quantitative effect on the integral solution.

Now, let us follow the motion in the s -plane of the singularities of the integrand, with particular attention given to the primary ones. Let us trace in the s -plane the orbits and find their laws of motion**, in terms of τ_1, \dots, τ_n , for each singularity.

The segments of the orbits, corresponding to the displacements of the primary singularities for τ_1, \dots, τ_n in G_ν , will occupy one or several regions of the s -plane, D_ν . The union of these s -plane regions will be denoted by D_ν . This situation clearly establishes a certain correspondence (not necessarily one-to-one) between G_ν and one, or

*We can think of G_ν as follows: the n -tuple of variable numbers (τ_1, \dots, τ_n) defines an n -dimensional vector space, say R_n , over the field of definition of the numbers $\tau_1, \tau_2, \dots, \tau_n$. Hence G_ν is a subdomain of R_n .

**The reader may have a clearer picture of this process if we particularize the general case by considering that τ_1, \dots, τ_n are, for example, continuous functions of the real variable t and set

$$\tau_k = \tau_k(t); \quad k = 1, \dots, n$$

in the interval $t_a < t < t_b$ (G_ν is $t_b - t_a$). The above equations define a line immersed in R_n .

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several regions of the s-plane, D_ν .

The final wave shape produced by the contributions of the primary singularities strongly depends upon the combined effect of every primary singularity. We may therefore look at them as a group rather than as isolated singularities. This group consideration is important particularly because: (a) For a given set and disposition of primary singularities, the relative displacement of one of them may cause, and this is often the case, a considerable change of the wave shape of the integral solution. (b) In some cases we can replace the group by some other analytical entities* of simpler structure whose effect with regard to the integral solution is equivalent or almost equivalent, and vice versa. These entities sometimes define points in the s-plane which also move as functions of τ_1, \dots, τ_n .

We will agree that when at least one of the primary singularities of a given group drops to a secondary rank, or as soon as at least one new singularity rises to the primary rank, then the given group must be considered as a different one.

Past experience with the approximate integration of a large subclass of integrals of type (1) has revealed and confirmed:

(a) The reality of the primary and secondary singularities associated with a given domain G_ν , of variation of τ_1, \dots, τ_n .

(b) The raising or lowering in rank of certain singularities in different domains, say G_ν and $G_{\nu+n}$.

(c) The decisive influence on the wave form of the primary singularities when they are considered as a group rather than in their individual roles.

(d) A basic structural composition of the primary group. For example, the improper modification of the group elements, say, by cancellation of one or several singularities, alteration of orbits or laws of motion, may produce an incorrect integral approximation.

In the light of all these observations, the following two questions arose. Let $F_\nu^*(s)$ and $W_\nu^*(s, \tau_1, \dots, \tau_n)$ be two functions which satisfy the requisites:

(a) They approximate respectively $F(s)$ and $W(s, \tau_1, \dots, \tau_n)$, i. e.

$$\left. \begin{array}{l} F_\nu^*(s) \sim F(s) \\ W_\nu^*(s, \tau_1, \dots, \tau_n) \sim W(s, \tau_1, \dots, \tau_n) \end{array} \right\} \begin{array}{l} s \in D_\nu \\ \tau_k \in G_\nu; \quad k = 1, \dots, n \end{array}$$

(b) The functions $F_\nu^*(s)$ and $W_\nu^*(s, \tau_1, \dots, \tau_n)$ contain the primary singularities (or corresponding analytical entities) respectively of $F(s)$ and $W(s, \tau_1, \dots, \tau_n)$ for $\tau_k \in G_\nu; k = 1, \dots, n; s \in D_\nu$.

*Among those entities are saddle points, branch cuts, etc. Inversely, branch cuts can be replaced by certain sequences of alternated poles and zeros.

(c) Suppose that the integral defined by

$$f_{\nu}^*(\tau_1, \dots, \tau_n) = \frac{1}{2\pi i} \int_{\gamma} F_{\nu}^*(s) e^{W_{\nu}^*(s, \tau_1, \dots, \tau_n)} ds$$

exists for $\tau_k \in G_{\nu}$; $k = 1, \dots, n$.

Questions:

- I. Are the conditions a to c sufficient to assure that $f^* \approx f$ for $\tau_k \in G_{\nu}$, $k = 1, \dots, n$ (but not necessarily for $\tau_k \notin G_{\nu}$) within a certain set of small tolerances?

Supposing an affirmative answer for I:

- II. If now the orbits of the primary singular points of F and W and, respectively, the orbits of the singular points of F_{ν}^* and W_{ν}^* become closer and closer and the respective laws of motion become more and more nearly equal for $\tau_k \in G_{\nu}$; $k = 1, \dots, n$, then, can we say that the set of tolerances mentioned in I becomes smaller and smaller for $\tau_k \in G_{\nu}$ but not necessarily for $\tau_k \notin G_{\nu}$; $k = 1, \dots, n$?

If one considers as "primary" every singularity of the integrand, then a positive answer may be given to the above questions and the solution is indeed valid for the complete domain of variation of τ_1, \dots, τ_n . But if one considers only the primary group, disregarding or modifying the secondary ones, then, the answer may be positive or negative and the problem is open to a further investigation. Direct mathematical proofs are very hard to conceive and construct.

In view of some indirect consequences and results, which are derived from previous, known theories of approximate integration, we are fully aware, at least in many specific but illuminating cases, that we may affirm the possibility of a positive answer. The illuminating cases above suggest the existence of a positive answer for many other cases.

The above sequence of ideas serves as a possible base for the development of a method of approximate integration. The above ideas, however, are not directly constructive since: (1) No means are provided to find or test the primary singularities corresponding to a given G_{ν} . (2) No methods are given to construct the functions F_{ν}^* and W_{ν}^* . (3) It is presupposed that the integral

$$\frac{1}{2\pi i} \int F_{\nu}^*(s) e^{W_{\nu}^*(s, \tau_1, \dots, \tau_n)} ds$$

exists, but no means are provided to perform the corresponding integration.

An intensive research has been conducted by this group, particularly during the last year, in order to supply the constructive means which are necessary in applying the above ideas. The new method of integration along these lines was called "substitutional."

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It is fairly well advanced, although it is still far from a final goal. To illustrate this we give below a few examples of the approximate integration of a subclass of integrals. The results are striking for their simplicity. Let

$$f(t) = \frac{1}{2\pi i} \int_{\gamma} F(s) e^{st - \phi(s)} ds; \quad n = 1, \tau_1 = t \quad .$$

Let us consider solutions for small values of t in the cases:

Case I $F(s) = C \frac{\prod_1^n (s - s_k^o)}{\prod_1^m (s - s_j^x)} ; \quad \phi(s) \equiv 0; \quad \text{let } p = m - n; \quad q_0 = \sum_1^n s_k^o - \sum_1^m s_j^x$

Case II $F(s) = C \frac{\mu \sqrt{\frac{\alpha}{\prod_1^{\mu} (s - s_{\mu}^o)}} \frac{\delta}{\prod_1^{\delta} (s - s_h^o)}}{\nu \sqrt{\frac{\beta}{\prod_1^{\nu} (s - s_{\nu}^x)}} \frac{\gamma}{\prod_1^{\gamma} (s - s_k^x)}} ; \quad \phi(s) \equiv 0$

let $p = \gamma - \delta + \frac{\beta}{\nu} - \frac{\alpha}{\mu} ; \quad q_0 = \sum_e s_e^o j_e - \sum_m s_m^x j_m ; \quad j_k$ algebraic multiplicity of the pole or zero

Case III $F(s)$ as in cases I and II;

$\phi(s) \neq 0$ but satisfying condition $\phi(s) \rightarrow Ms^N$ with $N \leq 1$.
 $s \rightarrow \infty$

Let $\phi(s)$ be expanded around $s = \infty$ as

$$\phi(s) = s(a_0 + \frac{a_1}{s} + \frac{a_2}{s^2} + \dots)$$

General Solution (three cases)

$$f(t) \sim u_{-1}(\tau) e^{-a_1 \tau} \frac{\tau^{p-1}}{\Gamma(p)} \Lambda_{(p-1)}(X) ; \quad 0 \leq \tau \ll 1$$

where: $u_{-1}(\tau)$ = unit step function

$\Lambda_{(p-1)}$ = Lambda function of order $(p - 1)$

$$\tau = t - a_0 ; \quad X = 2\sqrt{\tau(q_0 + a_0)}$$

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$$\left. \begin{array}{l} \text{Case I} \\ \text{Case II} \end{array} \right\} \begin{array}{l} a_0 = 0 \\ a_1 = 0 \end{array}$$

Note: one single-term solution for this large family of integrals.

E. ACTIVE NETWORKS

Prof. E. A. Guillemin	Dr. M. V. Cerrillo
Prof. J. G. Linvill	J. G. Truxal

The Synthesis of Servomechanisms

This work is being completed and will be described in detail in forthcoming Technical Report No. 162.

E. A. Guillemin, J. G. Truxal

F. LOCKING PHENOMENA IN MICROWAVE OSCILLATORS

Prof. J. B. Wiesner	Dr. E. E. David, Jr.
Prof. R. M. Fano	W. P. Schneider

Work on this project has been completed and technical reports are in preparation. They cover modulation systems, mutual synchronization of oscillators, approximate solutions to the inhomogeneous Van der Pol equation, and preoscillation noise measurements.