

VIII. AIR NAVIGATION*

Prof. L. J. Chu

Prof. R. B. Adler

Prof. S. J. Mason

S. J. Fricker

The major work of setting up the punched cards for the case of aircraft flying without enroute control has been accomplished. The formulation of this problem was described in the Quarterly Progress Report, April 15, 1950 as Case (1). A few of the results which are currently available will be presented here to illustrate the general form of the computed data, and to indicate some of the preliminary conclusions.

The histograms shown in Figs. 1, 2 and 3 represent sample enroute deviation statistics for the three basic types of distribution

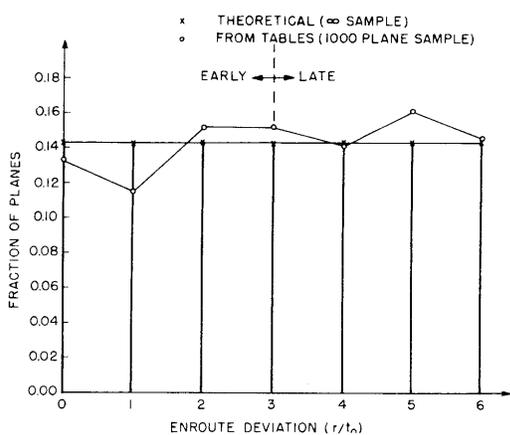


Fig. VIII-1 Frequency distribution of enroute deviations (box car, $S = 6$).

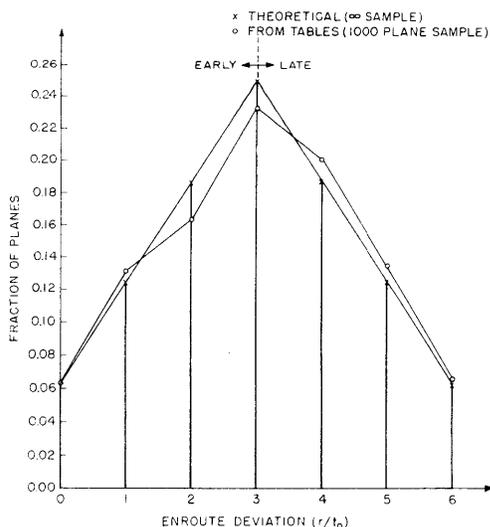


Fig. VIII-2 Frequency distribution of enroute deviations (triangular, $S = 6$).

chosen. The theoretical histograms give the probability of any particular enroute deviation. Those labelled "From Tables" summarize the frequency distributions actually obtained in a sample of 1000 numbers from tables constructed for each case. The deviations are quantized and normalized in units of the minimum time separation (t_0) between consecutive landings. All other results are therefore also quantized and normalized with t_0 as unity.

To render all numbers positive, the "on time" position of a plane has been set at the left end of the distribution during the computation. In all places where it affects the results, this origin has been shifted back to the center subsequently, as shown by the designations "early" and "late" on each figure. The parameter S represents the normalized spread of these enroute deviation statistics. The maximum amount by which a plane can be early or late is therefore $S/2$ units.

There appears to be a slight tendency toward lateness in the samples for $S = 6$. This is shown on the figures by the fact that all the curves are skewed to the right of center. The average value, or first moment, is about 12 percent above 3.00 in Fig. 1, 6 percent above in Fig. 2, and 2 percent above in Fig. 3.

*The work reported in this section is supported in part by the Air Navigation Development Board of the Department of Commerce.

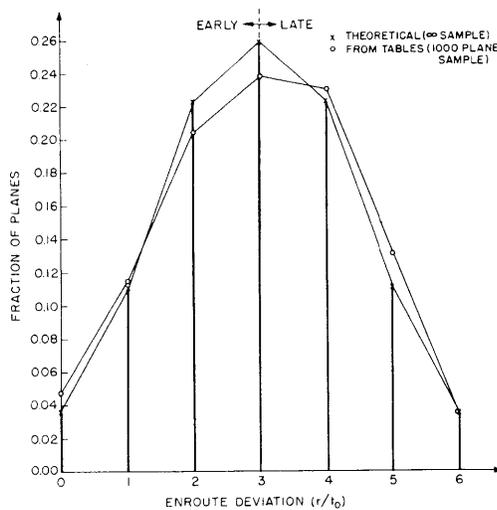


Fig. VIII-3 Frequency distribution of enroute deviation ("gauss," $S = 6$)

The normalized stack delay (τ/t_0) is computed in sequence for 1000 planes scheduled to arrive at integer multiples of t_0 but delayed enroute according to the deviation statistics. For values of traffic parameter ϵ less than unity, blanks are left in the schedule at random to obtain the proper ratio of average arrival rate to acceptance rate. Figures 4, 5 and 6 show the average stack delay as a function of ϵ for all the runs which have been completed to date. The figures refer respectively to the box car, triangular and "gauss" enroute deviation statistics, with $S = 6, 12, 18, 24$ and $\epsilon = 1.0, 0.95, 0.9, 0.8, 0.5$. The values of ϵ given above, and quoted later, are nominal. Because the method

used to write the time-tables involves the use of a random number table to drop out a certain fraction of planes, the actual values of ϵ obtained in each case were 1.000, 0.958, 0.907, 0.810, 0.505. These correspond to the calculated points shown on Figs. 4, 5 and 6. For comparison purposes the curve of average delay vs. ϵ for a Poisson distribution of arriving planes is also given (1). It is clear that even for quite large values of S the average delay lies considerably below that given by the Poisson assumption. The fundamental reason for this may be understood from the fact that no stack greater than $S + 1$ planes can ever occur under the assumption of a finite spread S . The probability that any particular number of planes will arrive in a unit time interval therefore depends heavily upon the height of the stack at the beginning of the interval. Whenever the arrival probabilities are assumed to be independent of stack conditions, it can be shown (1, 2) that in statistical equilibrium infinite average delays and average stack heights will result when $\epsilon = 1.0$. The Poisson arrival distribution is merely illustrative in this respect.

When planes have an enroute deviation statistic with finite spread S , the results of any calculation of stack delays for a sample of 1000 planes will depend to some extent upon the condition of the stack when the first plane arrives. It has been shown that for $\epsilon \leq 0.9$ this dependence is, at worst, not very important. The calculations for $\epsilon = 0.5, 0.8$ and 0.9 have therefore been carried out starting from a zero stack when the first plane arrives. The cases $\epsilon = 0.95$ and 1.00 have, on the other hand, been calculated under the most pessimistic assumption, namely that the stack already has S planes in it when the first scheduled plane arrives. In the particular case $\epsilon = 1.00$ it can be shown theoretically that under these starting conditions any symmetric deviation statistics must lead to an average stack delay of $S/2$ (for an infinite sample). The curves of Figs. 4, 5 and 6 corroborate this conclusion. In addition they show that for a given value of the

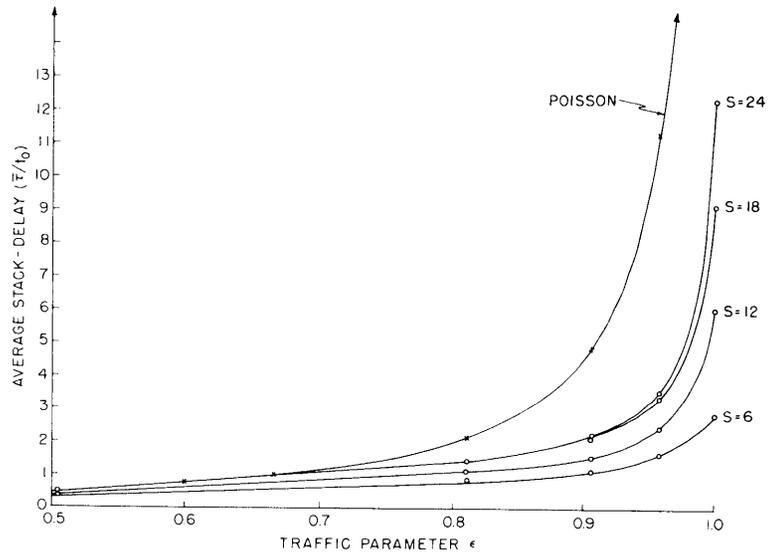


Fig. VIII-4 Average stack-delay vs. traffic parameter for box car enroute errors.

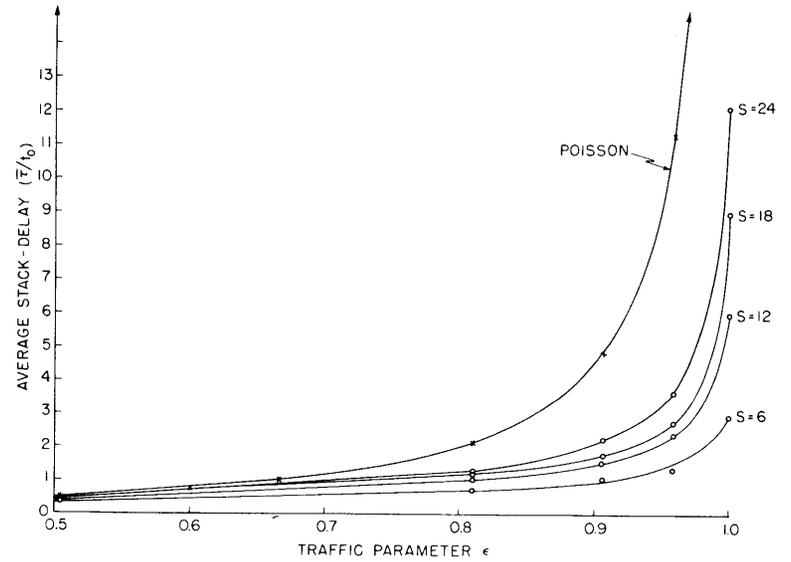


Fig. VIII-5 Average stack-delay vs. traffic parameter for triangular enroute errors.

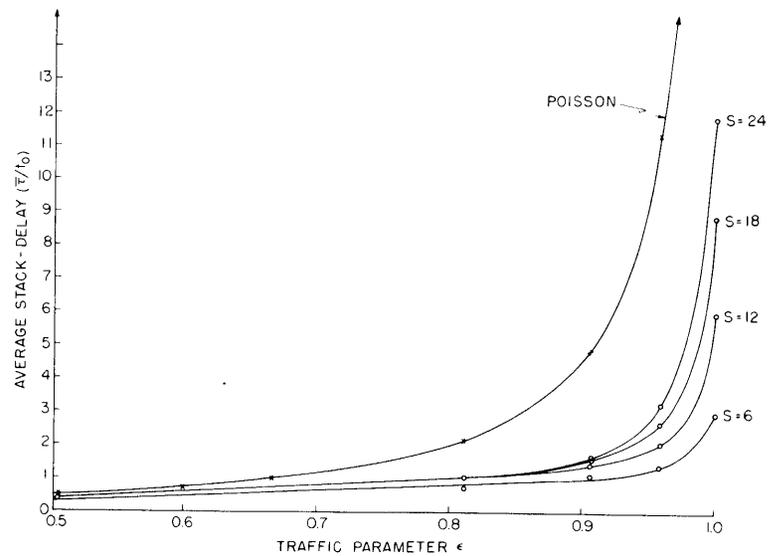


Fig. VIII-6 Average stack-delay vs. traffic parameter for "gauss" enroute errors.

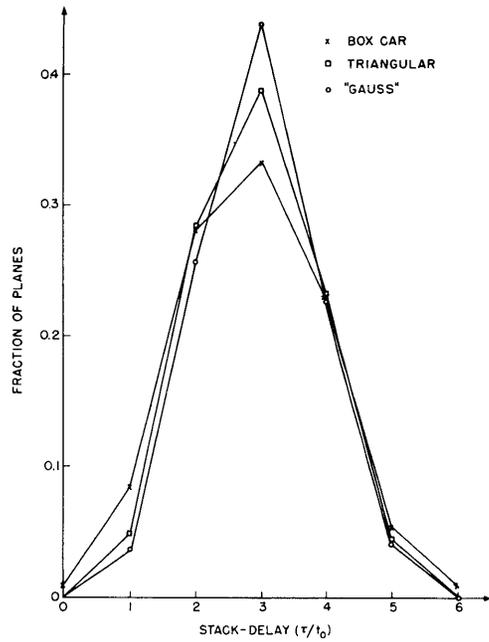


Fig. VIII-7 Frequency distribution of stack-delay ($S = 6, \epsilon = 1.00$).

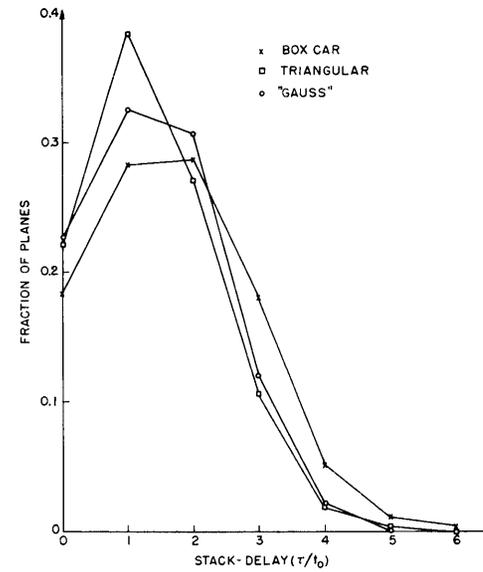


Fig. VIII-8 Frequency distribution of stack-delay ($S = 6, \epsilon = 0.95$).

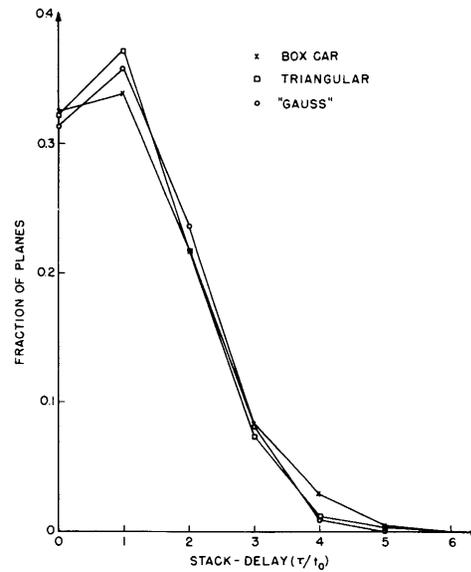


Fig. VIII-9 Frequency distribution of stack-delay ($S = 6, \epsilon = 0.90$).

(VIII. AIR NAVIGATION)

spread S , the average delay is not very much affected by the detailed shape of the deviation distribution. There is, nevertheless, a slight tendency toward reduction of the average delay as the deviation statistics become more peaked. When $\epsilon \leq 0.8$, the average delay also becomes quite independent of S , provided S exceeds about 12.

The average stack delay is not the only property of the traffic which deserves consideration. In Figs. 7, 8 and 9 are shown examples of the relative frequencies with which the values of (τ/t_0) occur in sample runs of 1000 planes each. The minimum stack delay is of course zero, and the maximum is always S (corresponding to a maximum stack of $S + 1$ planes). Only the case $S = 6$ is shown.

Figure 7 corresponds to $\epsilon = 1.00$. It can be shown that the distributions should all be symmetric about $S/2$ (the average value), and hence that the average and the most probable delays should be identical. The slight skewing toward delays less than $S/2$ in these particular cases corresponds to the tendency for late arrivals illustrated by Figs. 1, 2 and 3. Figure 8 applies to $\epsilon = 0.95$. Already the most probable delay is reduced from 3 to 1.

In Fig. 9, the value of ϵ has been reduced to 0.9, but the most probable delay is still unity.

From the three figures referred to above, it appears that:

a. The shape of the enroute deviation statistics is of relatively minor importance with respect to the delay distribution, although there is a general tendency to reduce the frequency of high delays when the deviation statistic becomes more peaked.

b. Reduction of ϵ rapidly reduces the most probable delay and decreases the frequency of larger ones. The frequency of small delays (zero, for example) increases quite rapidly as ϵ is reduced. Because the total delay spectrum must have unit area, the frequency of the most probable delay remains fairly constant until that delay becomes zero. Eventually, of course, the probability of zero delay must approach unity as $\epsilon \rightarrow 0$.

Additional delay distributions for other values of S and ϵ are available on the punched cards, but have not yet been analyzed. Detailed machine programs have been written for a further extension of all of these results. These extensions will be needed in connection with the enroute control problems outlined under Case (3) in the previous report. General programs have also been formulated for the Case (3) enroute control procedure itself, using one and two intermediate control points. They will shortly be reduced to detailed machine programs. Along with these computations, some theoretical work has been carried on continuously in order to check the results wherever possible. The formulation of Case (4) – discrete control with rescheduling – will begin in the near future.

References

1. G. Temple: Ministry of Civil Aviation, ORS/MCA Report No. 2 (June 1948).
2. G. E. Bell: op. cit., Report No. 3.