VIII. COMMUNICATION RESEARCH

A. MULTIPATH TRANSMISSION

Prof. L. B. Arguimbau   W. L. Hatton   R. A. Paananen
Dr. J. Granlund   E. E. Manna   G. M. Rodgers

1. Speech and Music

a. Field-test receivers

A new receiver has been built similar to the one discussed in earlier reports. It differs from the previous model primarily in the selectivity; the new unit has a choice of two bandwidths, 200 kc and 40 kc. The wider band filter uses seven tuned circuits, staggered and adjusted for optimal flatness, the narrow band filter uses three additional tuned circuits.

J. Granlund, E. E. Manna

b. Noise-reducing circuits

As mentioned in the October 1950 report, three-path reception is troubled by impulsive noise. Each noise impulse is accompanied by an ultrasonic burst. This suggests that the ultrasonic signal be rectified and used to operate gate circuits for repairing the audio output of the receiver.

A block diagram of the receiving system emphasizing the repair circuits is shown in Fig. VIII-1. The receiver output is passed through a high-pass filter to separate the ultrasonic bursts. These are rectified and shaped into rectangular pulses which are used to operate a gate circuit in the "first-order repair circuit".

The gate circuit follows work done by Robert Wilson in 1948 while an M.I.T. cooperative student at Philco Corporation. The general plan is indicated in Fig. VIII-2a. The input signal is connected to a condenser through two series tubes that act as rheostats. When the tubes are biased by the gating pulse the resistors become infinite and the condenser voltage is held constant for the duration of the trouble. The circuit is indicated in Fig. VIII-2b. Wilson experienced difficulty in isolating the gating pulse circuit from the series triodes. The trouble has been avoided by using the gating pulse to modulate a radio-frequency oscillator as shown in Fig. VIII-3. The modulated radio signals are rectified and fed to the grids to supply two thoroughly isolated gating voltages. In effect the oscillator-rectifier combination serves as a d-c transformer, and has many advantages over conventional pulse transformers. It operates on direct voltages and can be made to have a very short rise time.

Figure VIII-4 shows some of the waveforms involved. It will be noticed that the first-order repair circuit just discussed in effect cuts off the noise impulses, avoiding the discontinuity in the signal.

An attempt has been made to go further than this by removing the discontinuity in the derivation. The scheme is shown in Fig. VIII-5. The original wave (a) is differentiated (b) and then the derivative is repaired (c). At times of trouble the derivative is gated (d).

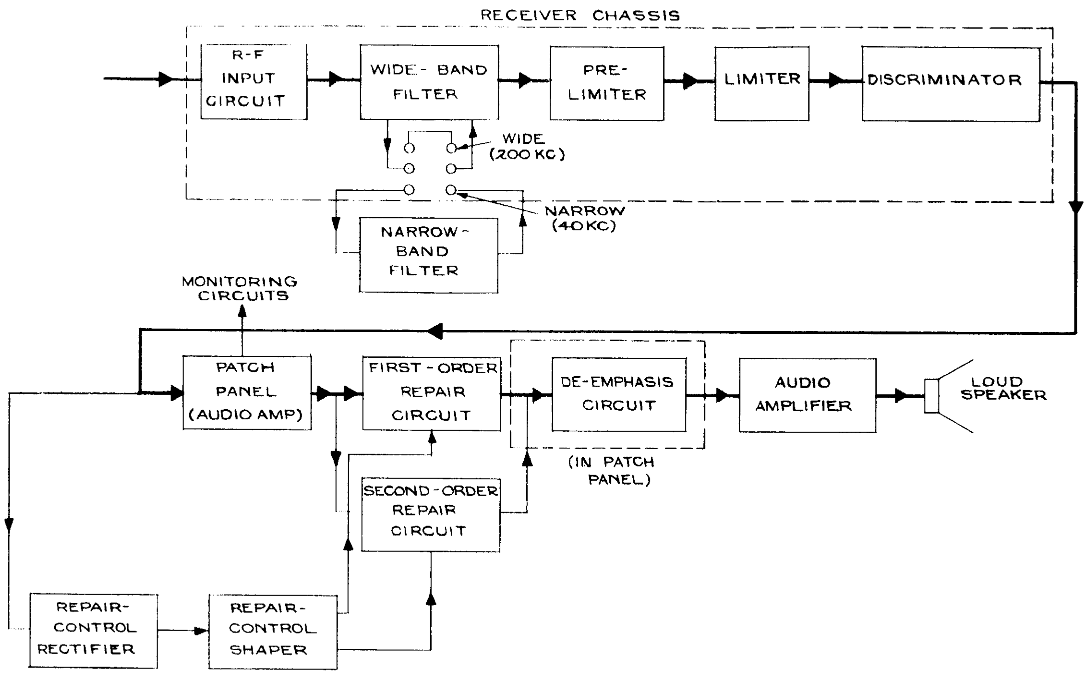


Fig. VIII-1 Block diagram of FM receiving system.

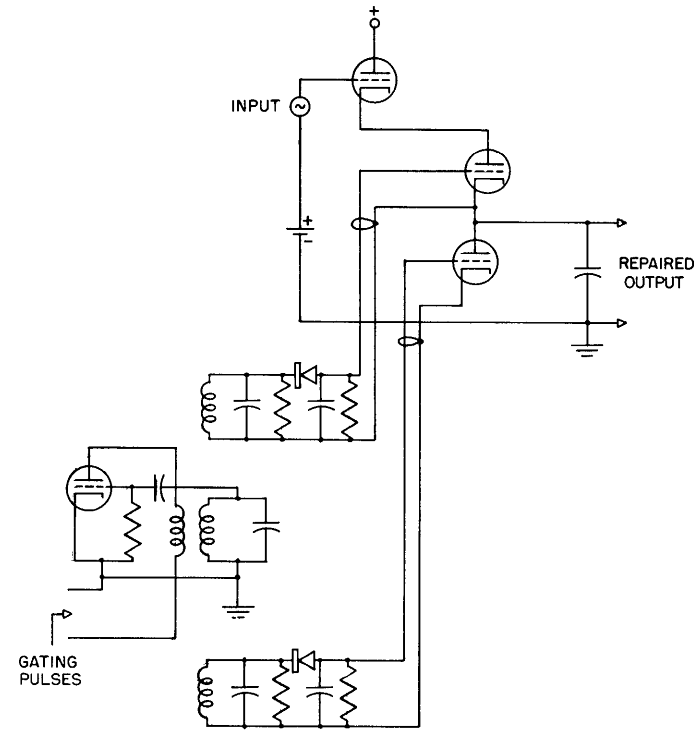


Fig. VIII-3 First-order repair circuit.

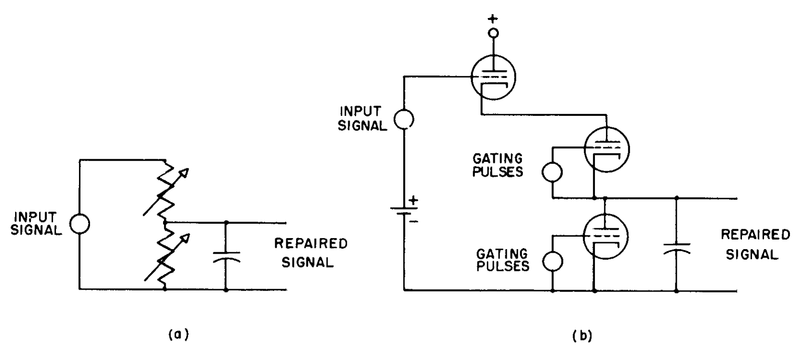
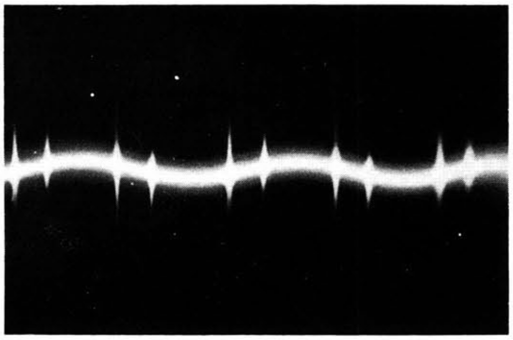
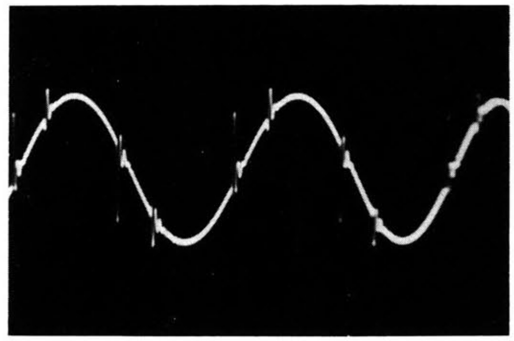


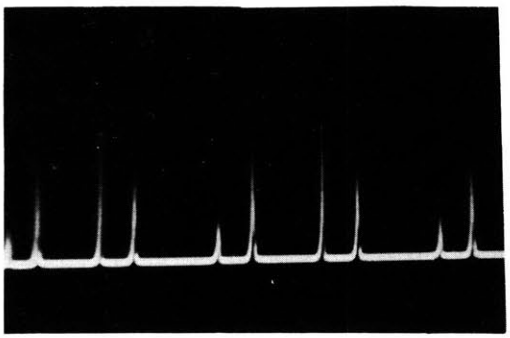
Fig. VIII-2 a) Functional diagram of repair circuit.
b) The rheostats of (a) are replaced by tubes.



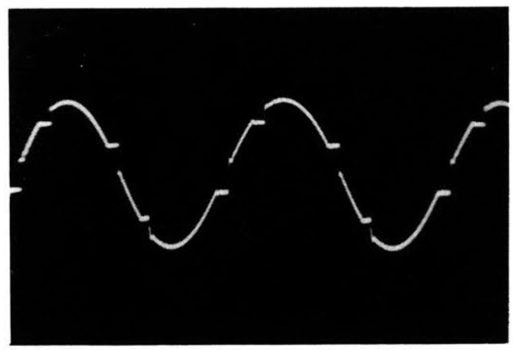
a) Output of discriminator without filtering. Ultrasonic bursts were actually much higher than shown photographically.



b) Output of discriminator after amplification and low-pass filtering. Audio frequency content of bursts in (a) are in evidence.



c) Repair-control rectifier output before shaping.



d) Output after first-order repair. Usually the repair was limited to shorter time intervals.

Fig. VIII-4 Waveforms involved in repair of signals.

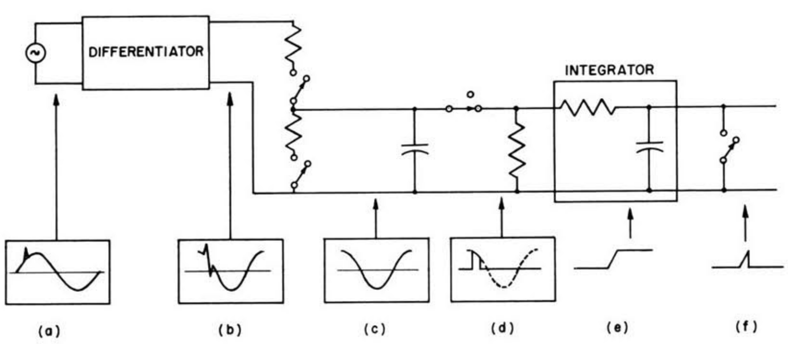


Fig. VIII-5

Functional diagram of second-order repair circuit.

The gated derivative is then integrated (e) and again gated (f). The triangle thus found is the portion missing after the first-order repair. The triangle is added to the earlier repaired wave.

The repair circuits have been tried out on musical signals with simulated 3-path fading. It has been found that they make a marked reduction in impulsive noise.

L. B. Arguimbau, R. A. Paananen, G. M. Rodgers

c. Television

In conventional FM broadcast work the audio signal is pre-emphasized before transmission. This corresponds approximately to differentiating the signal. When this procedure is applied to television, the sharp transitions in the video waves give rise to spikes which increase the peak-to-peak amplitude of the signal greatly.

The possibility of reducing the amplitude of such spikes by means of phase distortion has been investigated at some length. The first investigation consisted in determining the effect of single-section, all-pass, phase-shifting networks. A number were tried out by numerical convolution, which can be done fairly rapidly on a computing machine. Some other phase characteristics were investigated by the method of paired echoes (H. A. Wheeler, Proc. I.R.E. 27, 359, 1939) which is useful when the amplitude or phase distortion is small.

The results of these calculations were rather discouraging: It seemed that the positive peak could be reduced, but an accompanying negative peak was produced, and the peak-to-peak amplitude remained constant or even increased slightly.

The pre-emphasis network which is used to increase the amount of highs in the signal is, by itself, a minimum phase network. The phase characteristic associated with it can be considered the natural phase of the system, with the phase corrector networks creating additional phase shift. A calculation was made to estimate the effect of the amplitude characteristic and the natural phase characteristic of the pre-emphasis network separately. This was done by assuming a linear phase instead of the natural phase and comparing this result with that in which the natural phase was used. The linear phase resulted in a wave having symmetrical positive and negative peaks whose total height was considerably larger than the single positive peak which resulted from the natural phase. It could be suspected from this that the minimum peak-to-peak amplitude might be connected with the natural phase. However, a single-section network which had not been tried up to that point gave a 16 percent reduction in peak-to-peak amplitude, which was felt to be more than calculated error would allow. Therefore, the minimum phase is not the optimum on the basis of peak-to-peak amplitude.

A new method of attack is now considered necessary and is being carried out.

W. L. Hatton

d. Future plans

J. Granlund and C. A. Stutt are starting a second set of field tests on transatlantic speech and music transmission. The remainder of the multipath group is switching to the television portion of the work. There is an indication the multiple image effects can be studied with ease on a facsimile machine, and plans are being made to do this.

B. STATISTICAL THEORY OF COMMUNICATION

Prof. J. B. Wiesner
Prof. W. B. Davenport, Jr.
Prof. R. M. Fano
Prof. Y. W. Lee
Prof. J. F. Reintjes
Dr. O. H. Straus
F. K. Bennett
B. A. Basore
J. J. Bussgang
L. Dolansky
P. E. Green, Jr.
B. Howland
L. G. Kraft
A. J. Lephakis
F. L. Petree
C. A. Stutt
D. E. Ullery
I. Uygur
L. Weinberg

1. Single-Channel Electronic Analog Correlator

Design and construction of all circuits of the analog correlator have been completed, and tests of the over-all unit are about to be undertaken.

J. F. Reintjes

2. Multichannel Electronic Analog Correlator

A multichannel electronic analog correlator project has been initiated. Fifty channels, corresponding to fifty different values of $\Delta\tau$, are being planned. During a single sampling period, the input signal is sampled at all values of $\Delta\tau$, and the correlation results are to be displayed on a cathode ray tube screen as a complete correlation curve. The time required to obtain a complete curve will be of the order of 5 seconds to 30 seconds, depending upon the magnitude of the $\Delta\tau$ increments.

J. F. Reintjes, Y. W. Lee, D. E. Ullery, F. L. Petree

3. Noise in Nonlinear Devices

A report is being completed on this work and will be published as Technical Report No. 178.

Y. W. Lee. L. Weinberg

4. Techniques of Optimum Filter Design

A detailed report on the results of this study is being prepared and will be published as Technical Report No. 182.

Y. W. Lee, C. A. Stutt

5. Interference Filtering

Approximation methods for the design of interference filters (Quarterly Progress Report, October 15, 1950) have been developed and tested with good results. The following advantages are claimed for the method of approximation used:

a. The resulting networks are composed only of resistances and capacitances.

b. The solutions of simultaneous linear equations only are required.

c. For a given network complexity the filtering error may be readily computed and compared to the irremovable error.

d. For a given network complexity the method minimizes the filtering error rather than the approximation error between the actual and ideal response.

e. The network configuration and parameter values are available without complex mathematical work.

f. Although the one and two channel cases were considered in detail, the method may be extended to cover the filtering of any finite number of channels.

A detailed report on the results of this study is being prepared.

Y. W. Lee, J. P. Costas

6. Digital Electronic Correlator

The digital correlator has been used in obtaining data as reported in the sections on Noise in Nonlinear Devices, Correlation Techniques in Electro-Acoustic Measurements, and Brain Wave Investigations.

Y. W. Lee, L. G. Kraft, I. Uygur

7. Correlation Techniques in Electro-Acoustic Measurements

Several crosscorrelations of the white noise input to a loudspeaker with the output of a microphone have been run.

Fig. VIII-6 shows the schematic diagram of the system investigated. Fig. VIII-7 shows an oscilloscope picture of the time response to an impulse excitation. Fig. VIII-8 shows the crosscorrelation curve obtained with the digital electronic correlator for the same system.

The latter curve is not affected by the presence of any internal noise generated independently of the input. At the present time more experiments are being carried out to determine under what conditions best results are obtainable.

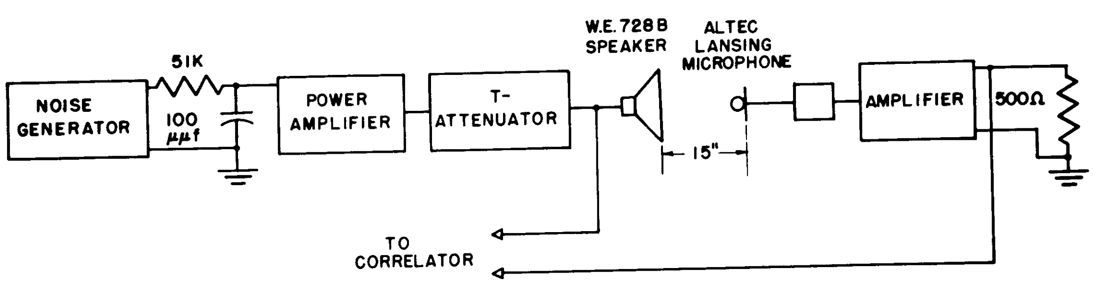


Fig. VIII-6 Block diagram of circuit for obtaining crosscorrelation functions.

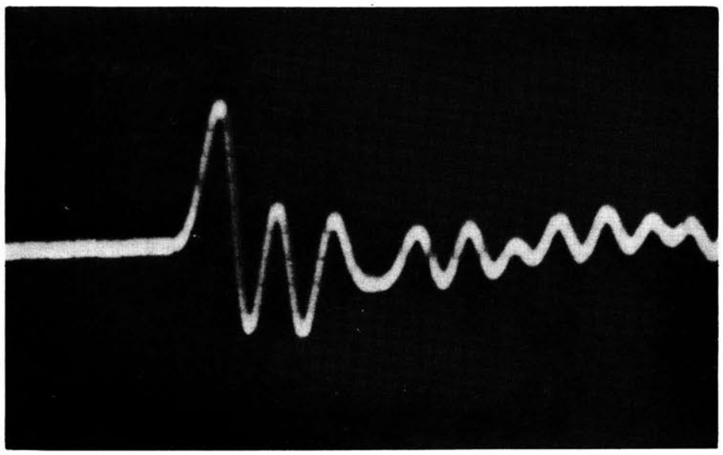


Fig. VIII-7 Time response to an impulse excitation.

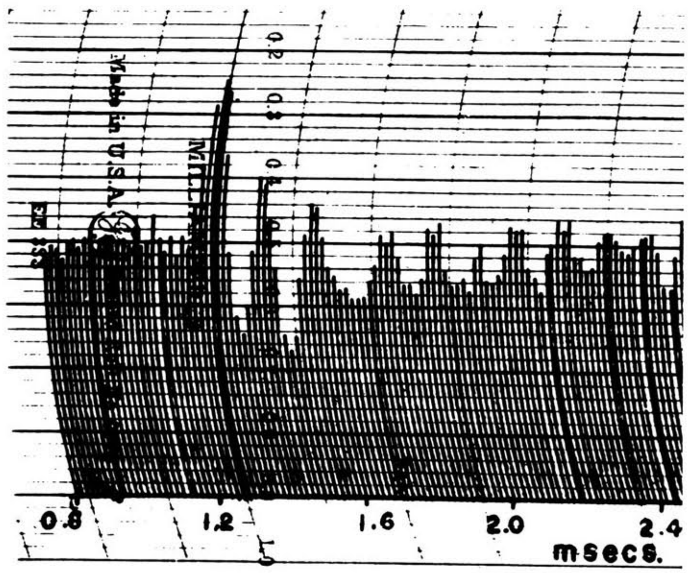


Fig. VIII-8 Input-output crosscorrelation function.

J. B. Wiesner, J. J. Bussgang

8. Speech Studies

a. A short-time correlator for speech waves

The technical report covering this project has been held up pending completion of certain circuit revisions for improving stability.

As reported in the Progress Report, October 15, 1950, the multiplying strips were required to amplify and detect a product component at 455 kc that had undergone phase shifting in the delay line. As a result, a carrier frequency shift of 0.1 percent corresponded to a phase shift of about 270° at the synchronous detector of channel 12, for example, and proportionately smaller phase shifts for earlier channels.

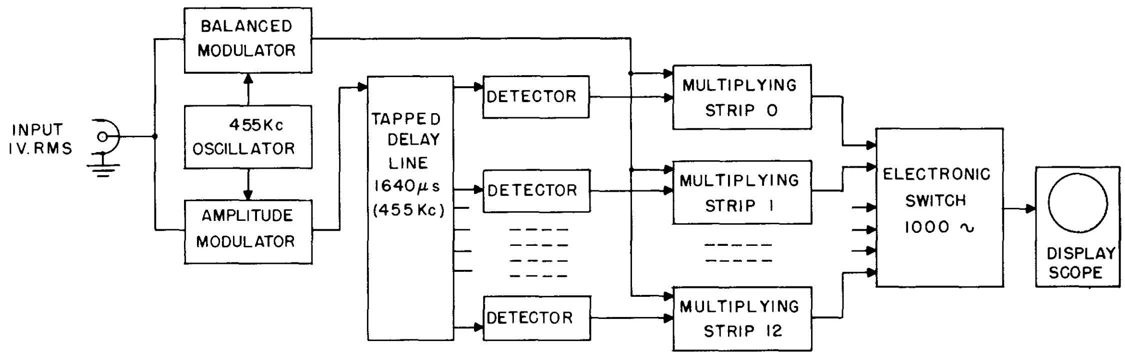


Fig. VIII-9 Block diagram of modified short-time correlator.

Rather than attempt frequency stabilization of the degree indicated as necessary, the block diagram has been modified to the present form shown in Fig. VIII-9.

The changes indicated have been completed and testing of the apparatus has reached the final stages.

R. M. Fano, B. L. Basore

b. An electrical analog of the cochlea

The experimental design procedure previously described (1) led to a 35-section line, the nth section of which is shown in Fig. IX-11 of Reference 1, having the following parameters:

$$\omega_o(n) = 2\pi \times 20 \times 10^3 \ e^{-0.1278n} \text{ rad/sec}, \ 1 \leqslant n \leqslant 24$$

$$\omega_o(n) = 2\pi \times 1.098 \times 10^3 \ e^{-0.334(n-24)} \text{ rad/sec}, \ 25 \leqslant n \leqslant 35$$

$$m(n) = 0.1 \ e^{0.092n}, \ 1 \leqslant n \leqslant 24$$

$$m(n) = 0.5, \ 25 \leqslant n \leqslant 35$$

$$\left.\begin{aligned} R(n) &= 5000 \text{ ohms} \\ R_1(n) &= \omega_o(n)\, L_1(n)/6 \text{ ohms} \\ R_2(n) &= \omega_o(n)\, L_2(n)/3 \text{ ohms} \end{aligned}\right\} \ 1 \leqslant n \leqslant 35$$

The behavior of a model of this line, in which the frequency was scaled up by a factor of 10, is illustrated in Figs. VIII-10 to VIII-13. The end of the line was short-circuited. Figure VIII-10 shows the amplitude of the response as a function of position for several frequencies. In Fig. VIII-11 the response is plotted as a function of frequency for several values of n. In both cases, the response is the charge Q_c on $C_2(n)$ for $1 \leqslant n \leqslant 24$, and $1.93Q_c$ for $25 \leqslant n \leqslant 35$. The input charge amplitude, I/ω, was maintained at a constant value during all measurements. The corresponding responses of the cochlea are

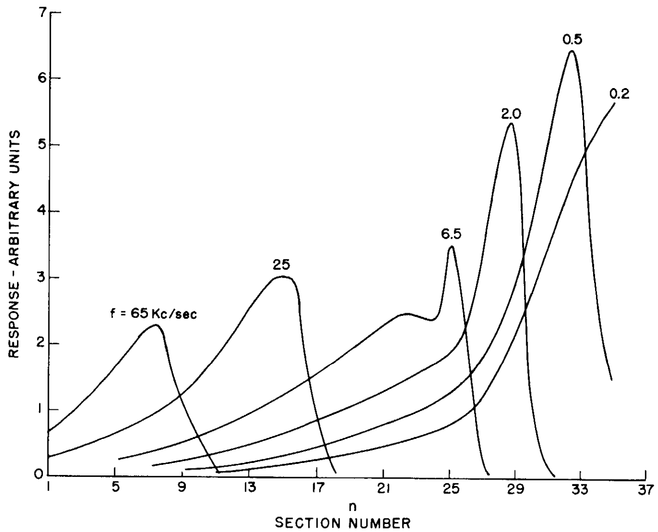


Fig. VIII-10 Response of the line as a function of section number, for several frequencies.

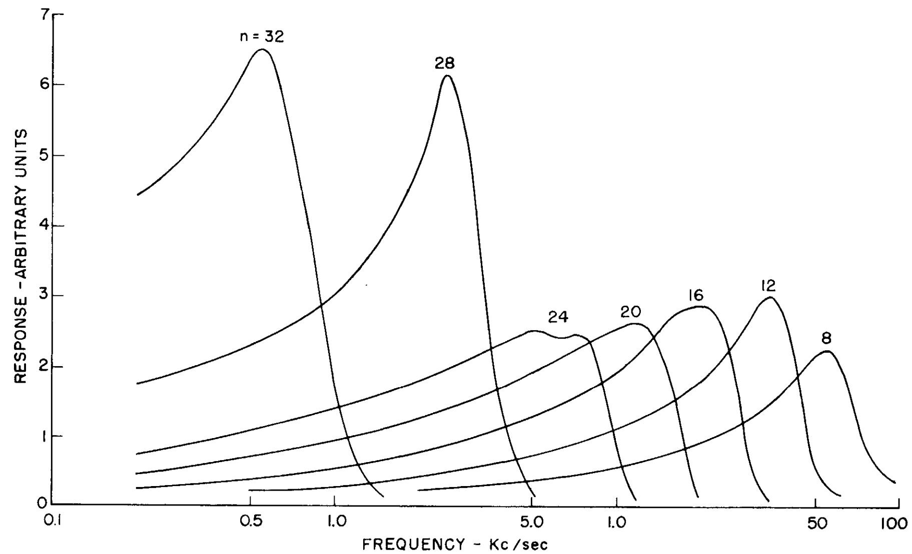


Fig. VIII-11 Response of the line as a function of frequency, for several sections.

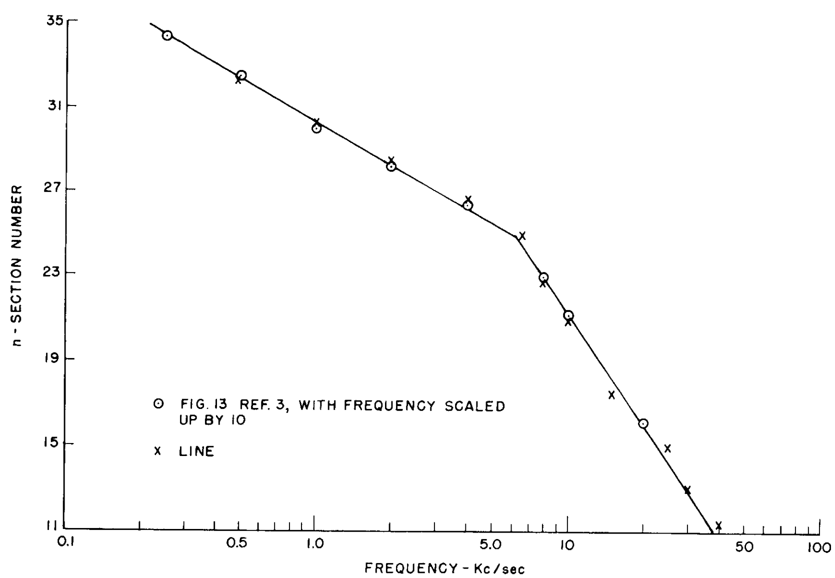


Fig. VIII-12 Positions of the maxima of vibration in the line and in the cochlea.

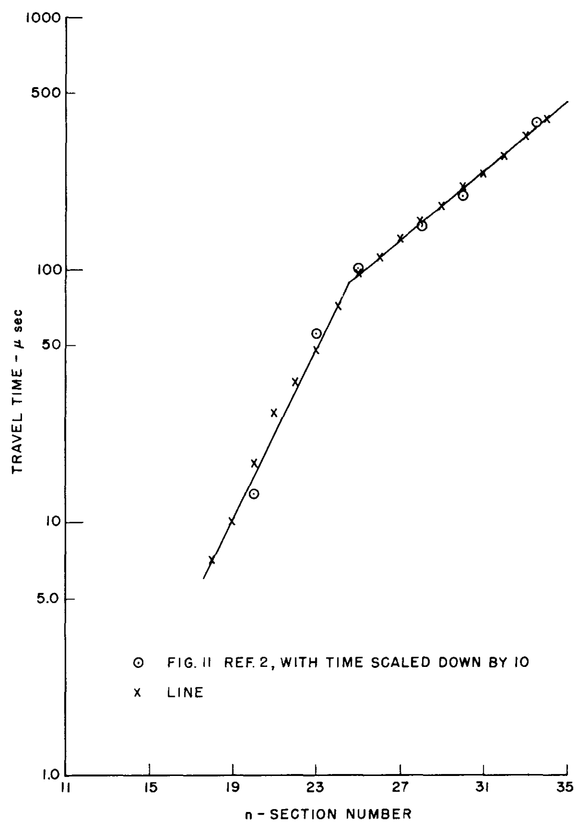


Fig. VIII-13 Travel time of a pulse wave in the line and in the cochlea.

given in Figs. 1 and 7 of Reference 2. The main discrepancy between the responses of the line and of the cochlea lies in the variation of the peak amplitudes of the curves. In the former, the peak amplitude increases as frequency decreases, while in the latter the opposite is true (Fig. 8 of Reference 2). This discrepancy may be minimized by placing an inductance across the input of the line. Another effect noticed in the line was that the curves which peaked in the vicinity of sections 24-25, such as the curve for f = 6.5 kc/sec in Fig. VIII-10, have secondary peaks. This condition is probably due to the discontinuity in the variation of line parameters between sections 24 and 25. The positions of the maxima of the response curves of Fig. VIII-10 and the corresponding maxima in the cochlea (Fig. 13 of Reference 3) are compared in Fig. VIII-12.

When a 0.5-μsec pulse was applied at the input of the line, a damped sinusoid was observed at each section. The logarithmic decrement of this sinusoid was approximately 1.7; this value compares favorably with the value 1.4 to 1.8 which occurs in the cochlea (p. 254 of Reference 2). The delay between the input pulse and the start of the sinusoid is shown in Fig. VIII-13, together with the corresponding delay in the cochlea (Fig. 11 of Reference 2).

It was attempted to improve the performance of the line by experimentally adjusting the parameters of each section. This procedure was found to be impractical, however, because the adjustment of one section had a considerable influence on preceding and following neighboring sections.

Work at present is directed toward synthesizing a line in which vacuum tubes are used between adjacent sections for purposes of isolation. Experimental adjustment of such a line will be simplified, since adjustment of one section will not affect preceding sections. The synthesis will be based on a set of response curves which can be sketched from data given by v. Békésy. To facilitate the determination of these curves, a three-dimensional model of the response surface showing the relation between frequency, amplitude, and position has been constructed. The model, which is based on the curves in Figs. 1, 7 and 8 of Reference 2, is shown in Fig. VIII-14.

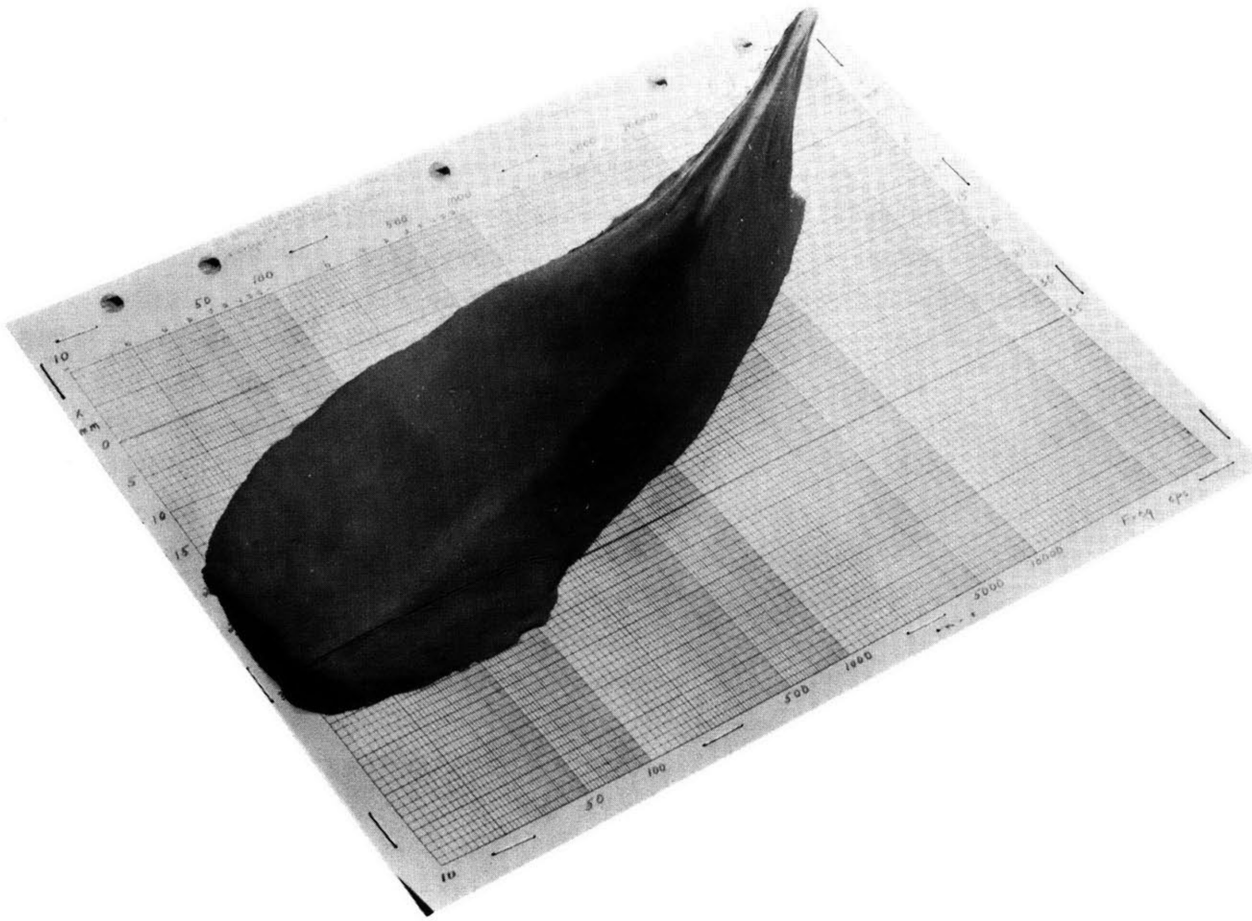


Fig. VIII-14 Model of the surface generated by the response curves of the cochlea.

References

1. Quarterly Progress Report, Research Laboratory of Electronics, M. I. T., October 15, 1950, p. 48-49.
2. G. v. Békésy: J. Acous. Soc. Amer. 21, 245-254 (1949).
3. G. v. Békésy: J. Acous. Soc. Amer. 21, 233-245 (1949).

R. M. Fano, A. J. Lephakis

9. Information Theory

Digital coding

An investigation was made of some properties of a digital code suggested by Shannon (C. E. Shannon, W. Weaver: The Mathematical Theory of Communication (University of Illinois Press, Urbana, 1949). Shannon says, "----- consider a source which produces a sequence of A's and B's with probability p for A and q for B. If $p << q$ we have

$$H \approx p \log \frac{e}{p} \qquad .$$

In such a case one can construct a fairly good coding of the message on a 0, 1 channel by sending a special sequence, say 0000, for the infrequent symbol A and then a sequence indicating the number of B's following it. This could be indicated by the binary representation with all numbers containing the special sequence deleted. All numbers up to 16 are represented as usual; 16 is represented by the next binary number after 16 which does not contain four zeros, namely 17 = 10001, etc.

"It can be shown that as $p \to o$ the coding approaches ideal provided the length of the special sequence is properly adjusted."

The optimum length, S, of the above-mentioned special sequence, and the resulting code efficiency for a range of values of p were determined. It was found that the optimum value of S does not exceed 4 for values of p as small as 10^{-10}.

Any binary sequence not containing the special sequence will be called admissible. Let the number of admissible binary sequences of length exactly n symbols, be Y_n, and the number of admissible binary sequences of length less than or equal to n, be Z_n. Recurrence formulas, or difference equations, were developed for Y_n and Z_n. These difference equations are of the S^{th} or $(S + 1)^{st}$ order. An approximate solution (with upper and lower bounds) was found for the S^{th} order characteristic equation which results from the S^{th} order difference equation. Tables of Y_n and Z_n were constructed for various values of S.

It was found that this code has properties which might enable an electronic or mechanical device to perform the required encoding and decoding operations in a simple manner. By this is meant that the device would not have to have built into it a complete table of code elements to which it would be necessary to make constant reference. For instance, to each admissible binary sequence there corresponds in this code some integer. Given an admissible binary sequence to be decoded, ... gfedcba (where a, b, c, etc. are each either 1 or 0), the integer to which the sequence corresponds is $aY_1 + bY_2 + cY_3 + dY_4 + \dots$, where Y_n is defined above.

The source producing the A's and B's might operate by some sort of scanning process. In this event, it is desirable to be able to transmit synchronizing data concerning the scanning process. A slight modification of the suggested code can achieve this desired result with only a slight loss in efficiency.

Finally, a proof was exhibited of Shannon's statement that the suggested coding approached ideal as $p \to o$. This proof is based on the existence of a less efficient code which can be shown to approach ideal as $p \to o$.

The foregoing results will be incorporated in a technical report.

R. M. Fano, F. K. Bennett

10. Pulse Code Magnetic Recorder

To eliminate the undesired vertical motion of the tape in the region of the head assembly, several changes have been tried. Since the reliability of obtaining the playback pulses is still not high enough, more mechanical changes are being considered.

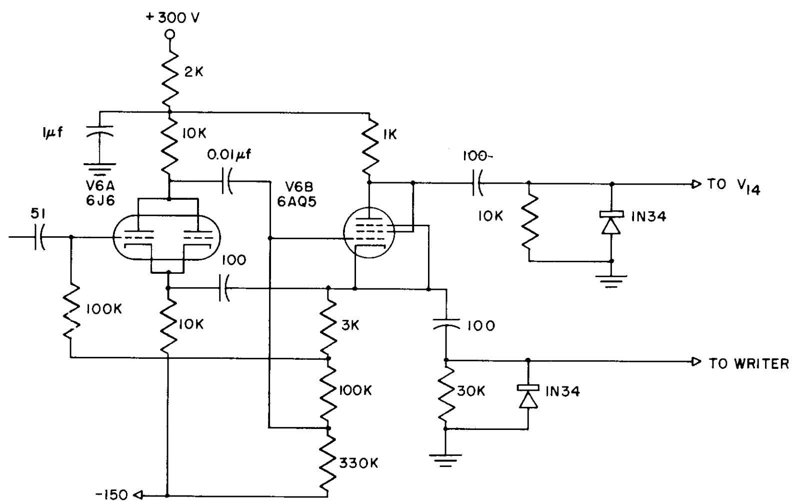


Fig. VIII-15 Final stage of the fundamental timing circuit.

The decoder part has been revised to ensure that all the pulses corresponding to the individual digits of the same sample will be of equal width (during the decoding procedure). With the new decoder the width of all the pulses is now derived from the same source. A first model was built in temporary form and tested. On the basis of these results the original design has been corrected and a new model is being built at the present time.

To remove the undesired higher frequencies (above 10 kc) in the input signal, an input filter has been designed and constructed. The filter includes a cathode follower, so that the impedance seen by the filter will not change according to impedance of the particular signal source. The filter is being tested at the present time.

The timing, sampling, coding and writing circuits have been constructed in permanent form. It was found necessary to change the cathode resistance of V_9 to 510 ohms and to replace V_6 by a two-tube circuit shown in Fig. VIII-15. Refer also to Fig. VIII-24, Quarterly Progress Report, April 15, 1950.

During the testing the greatest difficulties were encountered in the high-speed binary counters V_{15}, V_{17}, V_{19}, V_{21}, V_{23}, and V_{25}. For this reason the equipment was tested for reliability on a long-time basis. At present the equipment needs about 20 minutes to warm up, but otherwise it has worked satisfactorily in all-day runs for several weeks. The reliability test is still in process.

In Figs. VIII-16 to VIII-28 the waveforms at various points of the timing, sampling,

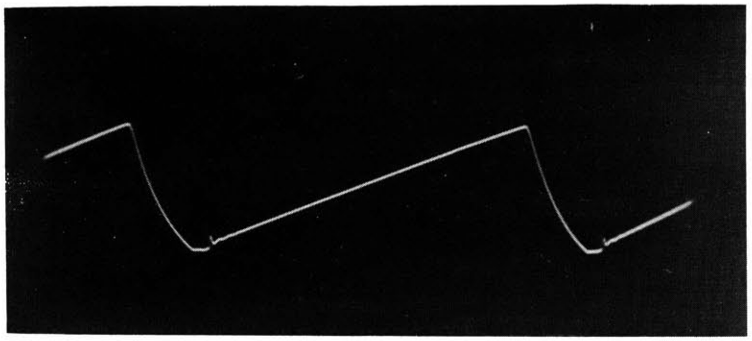


Fig. VIII-16 Phantastron plate voltage (V_{11}) for maximum audio input voltage corresponding to 127 counter pulses.

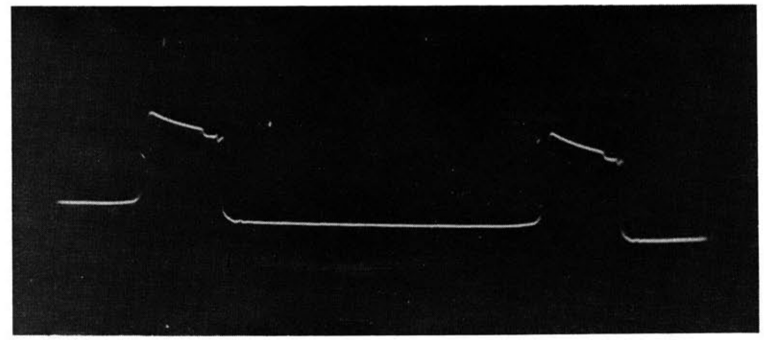


Fig. VIII-17 Screen voltage of the same tube (V_{11}).

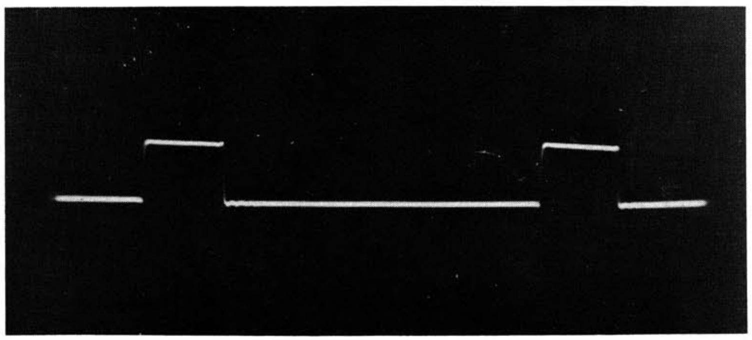


Fig. VIII-18 Corresponding gate voltage (suppressor grid of V_8).

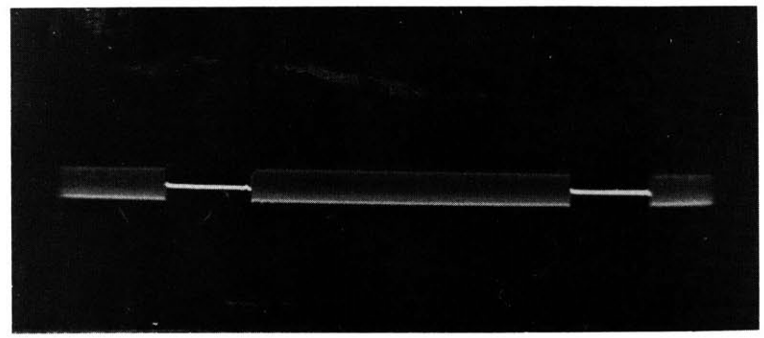


Fig. VIII-19 Gated pulse train for maximum audio signal voltage (output of V_7).

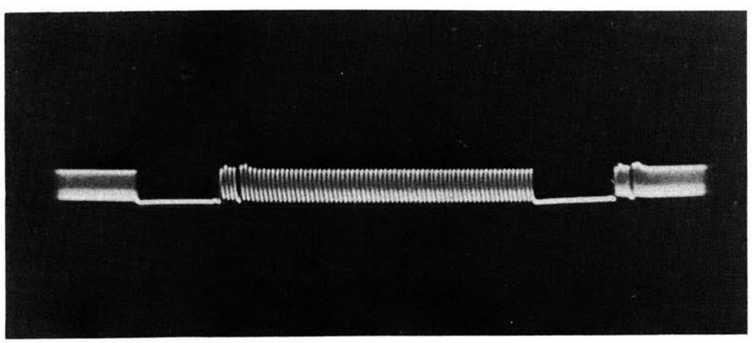


Fig. VIII-20 Output of the first counter stage (V_{15}).

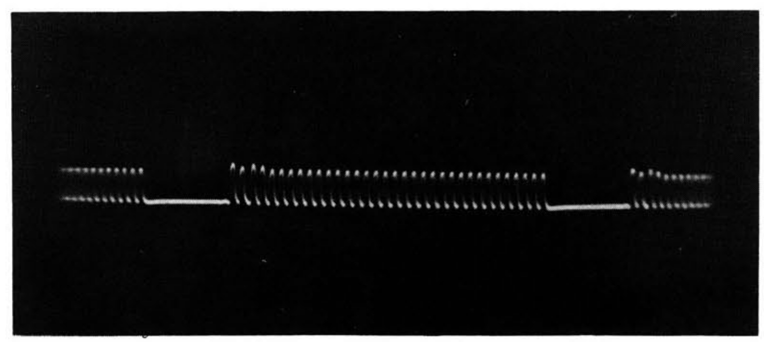


Fig. VIII-21 Output of the second counter stage (V_{17}).

In Figs. VIII-16 to VIII-27 a downward deflection means a positive deviation of voltage. The basic repetition rate of the waveforms is 25 kc.

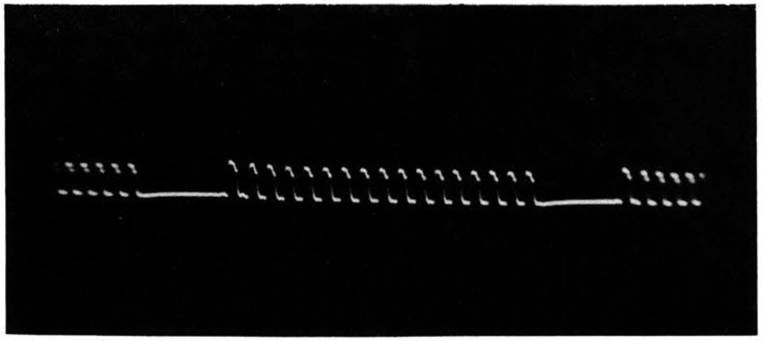


Fig. VIII-22 Output of the third counter stage (V_{19}).

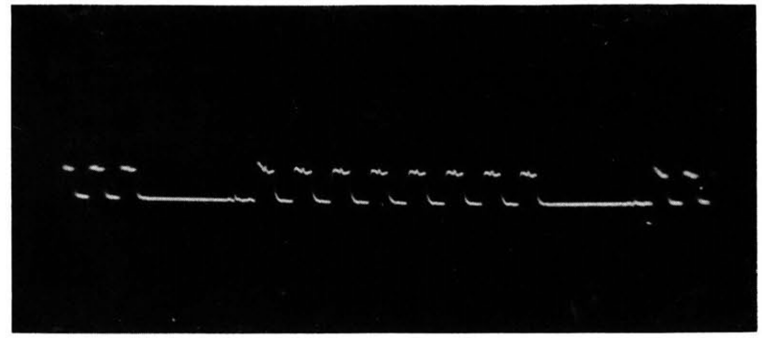


Fig. VIII-23 Output of the fourth counter stage (V_{21}).

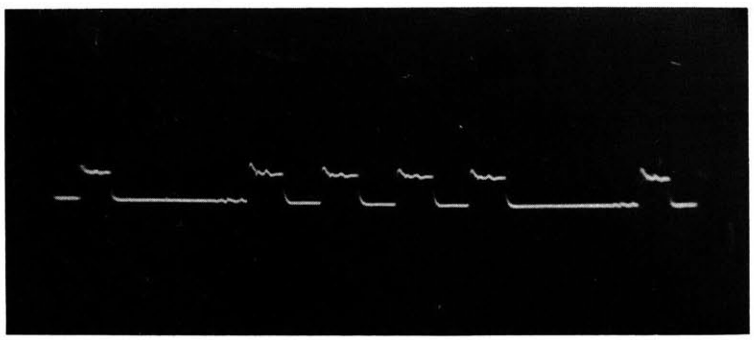


Fig. VIII-24 Output of the fifth counter stage (V_{23}).

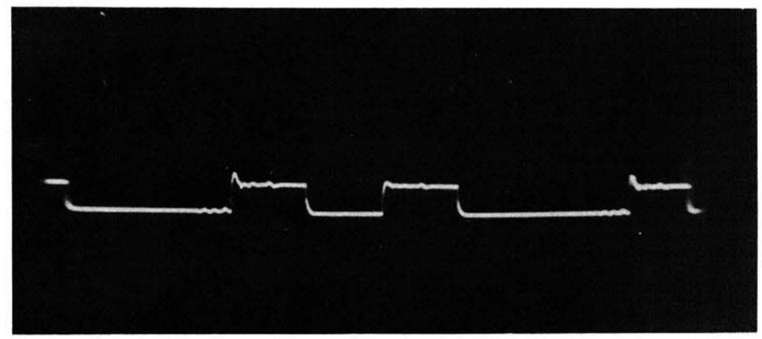


Fig. VIII-25 Output of the sixth counter stage (V_{25}).

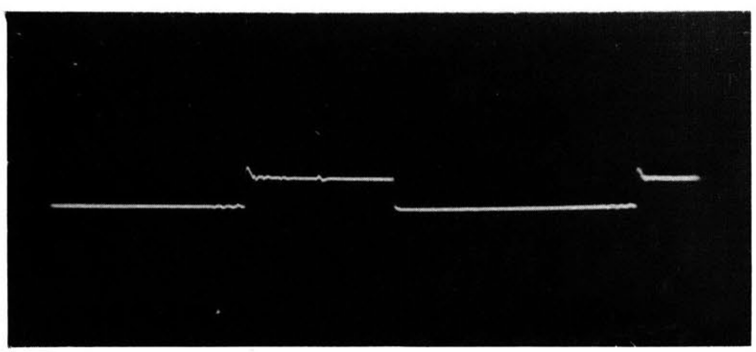


Fig. VIII-26 Output of the seventh counter stage (V_{27}).

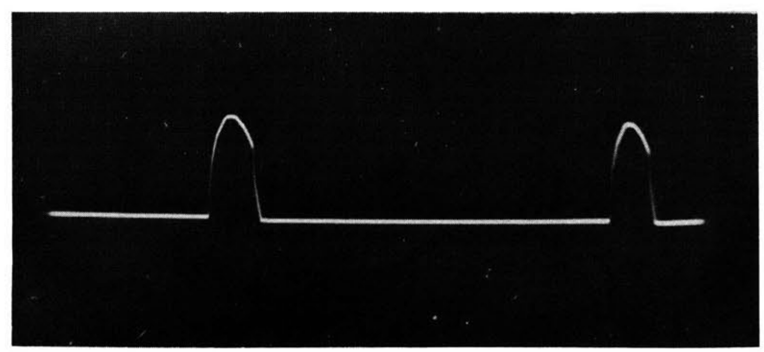


Fig. VIII-27 Plate voltage of a writer output stage.

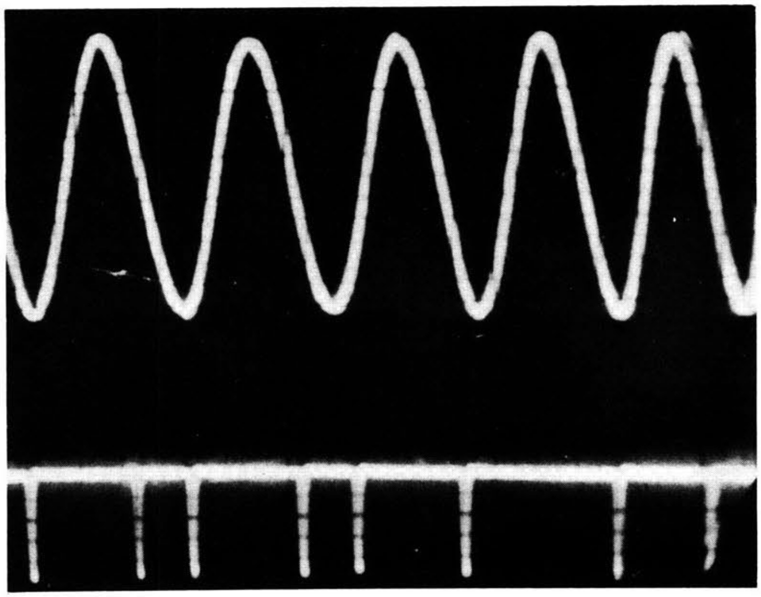


Fig. VIII-28 Audio input and corresponding writing pulses of the most significant stage (i.e. 2^6 stage) under dynamic conditions. The input signal is a 10 kc sine wave. Writing pulses occur only when sampling takes place during the positive cycle of the audio waveform. (In the upper part of the picture a downward deflection means a positive deviation of voltage).

coding and writing circuits are given. For a description of the operations and diagrams refer to the Quarterly Progress Reports of January (page 73-75), April (page 59-60) and July 1950 (page 79-80).

J. B. Wiesner, L. Dolansky

11. Felix (Sensory Replacement)

The task of transforming speech into signals intelligible through the skin for the use of the totally deaf entails the solution of two problems: (a) a large reduction of the quantity of information contained in the speech signal; and (b) matching of the output signals to the capability of the tactile rather than the auditory sense. Both these problems are imposed by the small channel capacity of an area of skin; i.e. by the skin's low rate of signal discrimination in time, and by the small number of just noticeable differences in sensation level between the thresholds of pain and perception.

These two requirements are fairly well met by the analyzer unit of the Bell Telephone Laboratory's "Vocoder" which breaks speech up into signals corresponding to the amount of energy in each of a number of frequency bands. The total information content of these signals is a small fraction of that of the original speech signal, and the signals vary at the phonemic rate, which fairly well matches the rate of perception of tactile stimuli.

This decomposition of speech in terms of energy in different frequency bands has been used for Felix, and work in the last few months has been largely concerned with amplitude quantization of the several output signals of the frequency filters. This has several advantages for our purposes: (a) The amplitude levels at which discrimination takes place will be determined by settings of the apparatus rather than by the performance of the tactile stimulators, and thus will not be subject to variations from one test to the next. (b) Objective recordings of the signals to be perceived can be made with a multiple pen impulse recorder, enabling a check on the consistent performance of the apparatus. (c) The amplitude quantization circuits may be used to stimulate the information loss due to the limited dynamic range of the tactile barrier; similarly, time quantization of these signals will correspond to the loss of information due to imperfect time resolution of the skin. The outputs of time and amplitude quantization circuits thus represent information which should be available to the subject through tactile stimulation. Graphic records of such on-off signals give little clue as to the information contained therein; but recently, in a preliminary experiment, it was found possible to produce barely intelligible speech by passing these on-off signals through the synthesizer unit of an eight-channel Vocoder. This technique thus enables one to monitor by ear the information available to the subject through tactile stimulation.

We have constructed a twelve-channel sampling circuit which will permit amplitude quantization with more than one level for several of the eight channels of our Vocoder, and will also permit time quantization by sampling at an arbitrary rate. It is proposed to run word and sentence articulation tests with this apparatus and the Vocoder for a number of different settings of amplitude quantization levels, and also with different types of input volume compression circuits. While it is not certain that adjustment for maximum intelligibility of the synthesized speech wave will correspond to the best arrangement for tactile communication, insofar as these are related we will be able to optimize the system before teaching an individual the art of "skin hearing".

B. Howland, O. H. Straus, J. B. Wiesner

12. Brain Wave Investigations

A program of investigation of some properties of brain-wave signals has been undertaken in cooperation with Massachusetts General Hospital. As a starting point, preliminary investigations of the particular type of brain wave known as "kappa rhythm" have been started. This type of wave manifests itself as a simultaneous appearance of signals of frequency of approximately 8 cps at opposite sides of the head, and with a phase difference of roughly 180°. It has not been definitely established whether these waves are caused by two sources near opposite sides of the head firing 180° out of phase, or whether they are the same signal arriving at the two places with delays differing by approximately 62 milliseconds (180° at 8 cycles).

In an attempt to resolve this question, the digital correlator, in conjunction with the low frequency tape recorder, has been used to obtain the crosscorrelation function of these two signals. (The displacement τ between signals was obtained by displacing the pickup heads along the magnetic tape.) On the basis of examination of one crosscorrelation curve of these two functions, we now think that the two rhythms are occuring at the same time (within 10 msec), but are of opposite polarity. This experiment is to be repeated several times.

Details concerning the frequency-modulated tape recorder used in this work have appeared in the Review of Scientific Instruments, 21, 893 (1950).

P. E. Green, Jr. with J. U. Casby of Massachusetts General Hospital

C. HUMAN COMMUNICATION SYSTEMS*

Prof. A. Bavelas | G. Bromfield | J. Macy, Jr.
Prof. J. B. Wiesner | J. B. Flannery | S. L. Smith
F. D. Barrett | R. D. Luce | P. F. Thorlackson

1. Communication Pattern and the Adaptability of Task-Oriented Groups

The following question has been asked: if a group which has learned to perform a given task must re-learn parts of it due to a change in the task or in the environment, will the group's ability to adapt to the new situation be related to the new communication network which they are constrained to use? Experiments were performed with groups operating in each of three patterns: circle, chain and wheel (see Fig. VIII-29).

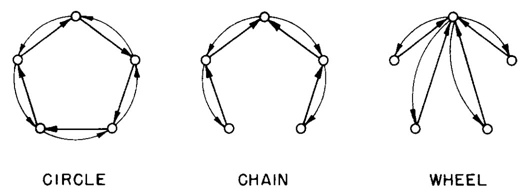


Fig. VIII-29

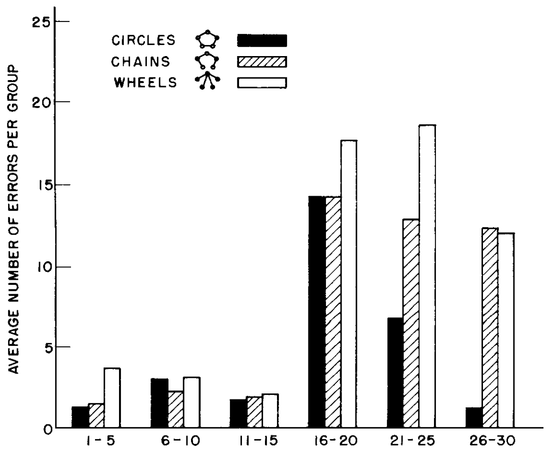


Fig. VIII-30 Average number of errors per group over 30 trials.

Each group performed thirty tasks, all of the same general nature and requiring the same process for completion. The first fifteen of these tasks required the transmission of information concerning the color of a number of objects which in every case were solid and easily discriminated colors. The second fifteen required the transmission of information regarding objects which were variegated in color – that is, each object was streaked with several colors. These objects were distinguishable, one from another, but not easily describable. Figure VIII-30 gives the average number of errors committed by the groups working in each of these three patterns. Figure VIII-31 gives the number of tasks performed correctly plotted against the time of completion.

Notice that in the first 15 trials there is no significant difference of performance among the three patterns (see Fig. VIII-30). After the change from solid to variegated color (the introduction of noise), the difference between the performance of the circle pattern and the other two patterns is striking. Also, in Fig. VIII-31, it is shown that after the introduction of noise the circle is not only making fewer errors, but is solving the problems more rapidly.

S. L. Smith

* This particular section has been supported in part by the Rand Corporation.

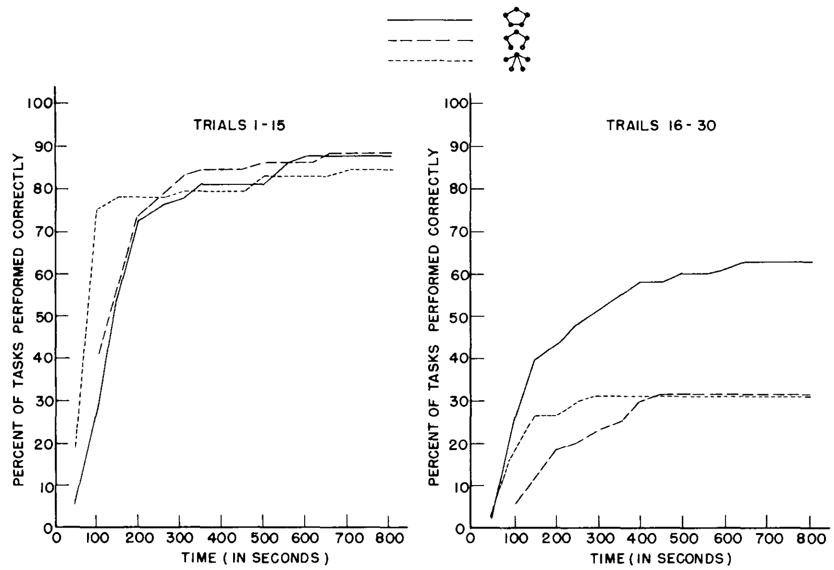


Fig. VIII-31 Tasks performed correctly vs time of completion.

2. Experiments on Network Patterns and Group Learning

A series of experiments is under way to investigate some particular questions concerning the behavior of five-man, task-oriented groups operating under the condition of specific imposed network patterns. The experimental apparatus to be used is a five-man partitioned table with slots through which the subjects may communicate by written messages. The position of these slots may be varied by the experimenter. This apparatus has now been modified by a simple arrangement of push-buttons, relays and a bell to make the flow of messages discrete in time rather than continuous. The condition imposed is that when all persons have pushed a button, and not until then, a bell rings and all five messages are sent at this instant. This arrangement permits precise tracking of the whole sequence of events occurring during the experimental period. Furthermore, by using printed message cards the content of message flow is restricted to those precise categories of theoretical interest to us. At the beginning of each trial, each member will be presented with one piece of information. The group will have solved its problem when each man knows what information the other four were given.

The following questions are to be considered in the experiment:

a. Are the modes of solution, the times required for solution and the amount of error significantly dependent on the imposed patterns? If so, in what way? Is there a strong correlation between any of these factors and any mathematical properties or parameters of the network?

b. Are the actions of the individual in some sense rational on the basis of the information he receives? For example, can it be shown that for any individual the a priori equi-partition of probabilities of transmission over the allowable links is progressively altered so as to weight more heavily those links to people whom the sender feels, on the basis of the information he has accumulated, are most able to spread information throughout the group? Are the direction and the degree of this probability change predictable?

c. If the messages received specify the origin of each piece of the task information or, in addition, the persons through whom it has been channeled, will this additional second-order information be utilized by the individuals, and hence by the group collectively? Will it be used in such a way as to increase the total efficiency of first-order information dissemination? If so, is this utilization statistically predictable?

On the completion of Octopus (discussed in Sect. IX-C4) it is hoped that many of these experiments can be carried out on that apparatus more effectively and rapidly than on the table we plan to use for the interim period.

F. D. Barrett, R. D. Luce

3. Mathematical Approaches to Networks

Following earlier work (1, 2, 3) the networks of groups of $m \geqslant 2$ nodes are considered to be represented by matrices of order m with entries 0 or 1; 0 when there is no link from i to j, and 1 when there is. This, of course, allows nonsymmetrical situations. A link present can be considered as a probability 1 of the transfer of information, and not present as 0 probability. This suggests that it may be more general to consider probability matrices A with entries $0 < A_{ij} < 1$; however, from the mathematical point of view of classifications, any such probability matrix may be considered as a well-defined set of 0, 1 matrices with probabilities assigned to each matrix in the set. For other purposes, it is fruitful to consider the probability matrices themselves. For example, W. H. Huggins (4) has developed a theory of information flow based on a probability matrix (corresponding in electrical theory to the impedance matrix) which arises with the assumption of linear characteristic node responses. This leads to the study of the characteristic transients of the nodes and permits operational definitions of concepts such as the "leader" and "stability" of the group and provides some criterion as to when these conditions are satisfied. Much of our proposed study is ultimately an attempt to account for the development of such an impedance matrix and possibly to predict the stabilized form in terms of the prescribed network and certain other assumptions about the characteristic model responses.

We are restricting our study to networks which are connected, i.e. for any pair of nodes i, j we assume there exists an ordered set of intermediate nodes such that there is a link from i to the first, a link from the first to the second, etc., and one from the last to j. Thus, if A is the matrix representative of a connected network,

$$\sum_{i=1}^{n} A > 0 \quad \text{for some n.}$$

The least n such that this is true is called the degree of connectivity. If p is the number of links, then the possible pairs (p, n) define classes of connected networks. Attempts are being made to determine what are the possible pairs, and to characterize these classes of matrices. Some, but by no means complete, success has been achieved on this problem.

A second tool being used in the study is the class of Boolean matrices of order m (i.e. matrices whose entries are taken from a Boolean algebra rather than from a field, as in the case of ordinary matrices). The primary difficulty in this approach is that this algebra has not been extensively studied in the literature; this difficulty is partially offset by the following assets:

If o is the null set and e the universal of the given Boolean algebra, then matrices with entries o and e form a closed subset under operations union, intersection, and multiplication. Furthermore, the determination of whether of two matrices with entries o, e (or equally 0, 1), one is simply a permutation of the labels of the other, can be simplified to the general similarity problem for the matrices formed from a two-element Boolean algebra. This is markedly more simple than the equivalent formulation for ordinary matrices. It is for these reasons that we propose to continue studying the structure of this algebraic system (5).

A third tool for the study of symmetrical networks lies in the theory of algebraic topology. Any symmetric network is an abstract topological complex in which the simplices are the cliques (1), their subsets, and all symmetric links. It may also be possible to study the nonsymmetrical cases by the appropriate use of the concept of oriented simplices. This study has barely begun and so it is impossible to predict what may come of it.

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R. D. Luce

4. "Octopus"

An experimental apparatus designed by Straus is nearing completion. It will permit a quantitative analysis of the rate of communication over the various links of a network connecting human beings whose task is to exchange information. The ultimate purpose of this work is to answer the question: Given a task involving a number of individuals working together to do some assigned job, how does the organization of the group, and particularly the structure of the communication channels in the group, affect the speed of learning of the group, and the final degree of proficiency that can be learned?

It has been shown (in unpublished work by Straus) that such a communication net can be mapped as a four-dimensional matrix in which the elements are the number of communications per unit time or per unit task. The purpose of such matrix representation is to permit partitioning of the messages into meaningful categories. In some cases these may be analysed by linear network methods, such as node pair impedance functions.

The apparatus, called Octopus, is a binary switching network operated by five people, permitting free choice of message, destination, and time of transmission. Messages successfully received are acknowledged to the sender. The subjects are given no information about the presence or absence of links for a particular experiment, and so must deduce this information from the presence or absence of acknowledgments to their transmissions.

Messages which may be sent may be written in the general case as "JKL(0, 1)(t, r)", read as "node J sends to node K the information that node L has a 0(or a 1). This message is a transmission (or a reception)." This last symbol is necessary since all transmissions do not necessarily result in receptions. For the particular case of a five-man group with these restrictions, the matrix referred to above may be simplified to a three-dimensional $5 \times 5 \times 5$ matrix with entries identifying the message as 0 or 1, t or r, or "no message". This specialized matrix is referred to as the "master" matrix, and the apparatus is laid out with this matrix in mind.

Each subject has in front of him a panel, called his station, carrying two matrices of switches. On his right is a matrix representing a section of the master matrix in the KL plane at the value of J for that station. The main diagonal and the row of entries KK are omitted, since a man will not send to himself, nor will he send to K that K has a certain piece of information. On the subject's left is a similar matrix representing a section of the master matrix in the JL plane at the value of K assigned that station and with similar omissions. The right-hand matrix consists then of 16 momentary-contact, spring-return, neutral-center switches, used for the acknowledgment of the subject's receipts. Each switch has two possible positions, corresponding to the sending or receiving of the information 0 or 1, and each switch has two signal lights which signal the transmission or reception of information. The method used in sending is as follows:

J sends to man K that man L has a 0 or a 1 by pushing the corresponding switch. This action, if such a link exists, lights the corresponding light on man K's receive panel, and man K acknowledges receipt by throwing his receive switch. This permanently lights the signal light on J's transmit matrix, allowing a check on both the receipt of the message and the correctness of the reception. In addition to the send and receive matrices, each man has a pair of lights giving him his input, 0 or 1, for the particular trial; a switch and signal lights with which to transmit his answer, or output; and two lettered panels which light up to show starting and stopping instructions for each trial. A trial is run by giving to each subject an input of 0 or 1. The task of the group is to find out whether the sum of the inputs is even or odd. This answer may be required to be known to any or all of the subjects, and is signaled with the station's output switch. The trial ends automatically when the required number of stations have submitted the answer.

The central station of the apparatus is in two parts. One is the control station, at which the net structure, the inputs, and the answers required are selected. All necessary control equipment is mounted here, and the apparatus may be operated by only one operator. The second part is the recording, or "concise" station, at which matrices of lights show projections of the master matrix on the JK and KL planes. A matrix of counters tallies the messages and indicates whether they are successful transmissions or not. The answers required and delivered, the inputs, and the net structure are also shown, along with the experiment number, time for completion, and similar pertinent data. This panel is continuously photographed by 16- or 35-mm motion picture equipment. In this manner a short strip of film will give complete data already set up in matrix form for analysis. Provision has been made at appropriate points for addition of circuits for special purposes, such as "liar" circuits, which make some or all of a station's outputs randomly or certainly false; the introduction of various forms of "noise"; and the operation of the net under a pulsed system, in which transmissions are permitted only at specified intervals; etc. The wiring includes interlocks which make guessing and premature transmission of the answer impossible.

The apparatus described above has been designed to enable us to obtain answers to a number of basic questions in regard to group communication. We feel that the apparatus outlined meets the following conditions, which we consider desirable for such a purpose:

a. The elapsed time from the start of an experimental task to the completion of the task must be as brief as possible, preferably less than one minute.

b. For purposes of more adequate sampling, the task used should require as large a number of transmissions during the task period as possible.

c. The possible categories in which messages might fall should be kept as low as possible, and made as rigid as possible.

d. The experiment should require a maximum of learning on the part of the group, and a minimum of learning on the part of any individual in the group.

e. The structure of the net should be variable at any time at the will of the experimenter, if possible without the direct knowledge of the experimental subjects.

f. Successive tasks by the same group under the same net structure should not be of such a nature that they become trivially simple after a few trials.

g. It should be possible to allow inputs to, and outputs from, the group to enter at any or all of the nodes.

h. The activity within the net should be recorded automatically, and in such form as to make interpretation of it easy. It should be possible to record completely the processes – i. e. transmissions, inputs, answers, outputs, and their times, the structure used, conditions of the experiment, etc. – which occur during an experimental trial.

i. The experiment should not be so restrictive in nature that the subjects do not exhibit human behavior, but merely act as machinery.

These conditions are felt to be necessary for the efficient collection of usable experimental data. It is hoped that Octopus, by meeting these conditions, will make possible substantiation of various elements of the mathematical theory which has been developing, and possibly will suggest new and fruitful hypotheses. O. H. Straus, J. Macy, Jr.

D. TRANSIENT PROBLEMS

Prof. E. A. Guillemin — W. H. Kautz
Dr. M. V. Cerrillo — H. M. Lucal

1. Linear Frequency Variation Transients

a. Statement of the problem

The transient behavior of passive networks when excited by driving forces possessing a linear variation of frequency can be obtained with the aid of the methods of approximate integration. We will illustrate this statement with the following problem:

Let a passive network having a system function G(s), $s = \sigma + i\omega$, be acted on by the stimulus

$$e(t) = K \sin (ht - at^2)$$

$$\text{where } K,\ h,\ a = \text{constants},\ t = \text{time}. \tag{1}$$

Let f(t) be the corresponding transient response. Then, the problem is to determine f(t), as accurately as we wish, by using the methods of approximate integration. (In this particular case, the "pocket method").

The integral expression of f(t) is given by

$$f(t) = \frac{1}{2\pi i} \int_{\gamma} E(s)\, G(s)\, e^{st}\, ds \tag{2}$$

where

$$E(s) = \mathcal{L}_t\, e(t) \quad .$$

b. The Laplace transform E(s)

Equation 1 can be written as

$$\left.\begin{aligned} e(t) &= e_1(t) + e_2(t) \\ e_1(t) &= \frac{K}{2i}\, e^{i(ht-at^2)} \\ e_2(t) &= \frac{K}{2i}\, e^{-(ht-at^2)} \end{aligned}\right\} \quad . \tag{3}$$

If

$$E_1(s) = \mathcal{L}\, e_1(t)$$

$$E_2(s) = \mathcal{L}\, e_2(t) \quad .$$

Then

$$\left.\begin{aligned} E_1(s) &= \frac{K}{2i} \int_0^{\infty} e^{-t(s-ih)}\, e^{-iat^2}\, dt \\ E_2(s) &= -\frac{K}{2i} \int_0^{\infty} e^{-t(s-ih)}\, e^{-iat^2}\, dt \end{aligned}\right\} . \tag{4}$$

The transforms of (4) can be simply obtained by using the property

$$\int_0^{+\infty} e^{-p\tau}\, e^{-\lambda^2\tau}\, d\tau = \frac{\sqrt{\pi}}{2\lambda}\, e^{(p/2\lambda)^2}\, \operatorname{erfc}\left(\frac{p}{2\lambda}\right) \quad . \tag{5}$$

(See tables of Laplace transforms, e.g., W. Magnus, Speziellen Funktionen, etc. page 174).

After some algebraic manipulations one gets

$$\left.\begin{aligned} E_1(s) &= -\frac{\sqrt{i}K}{4}\sqrt{\frac{\pi}{a}}\, e^{-\frac{h^2}{4ia} - \frac{h}{2a}s + \frac{1}{4ia}s^2} \times \operatorname{erfc}\left\{\frac{h}{2\sqrt{ia}}\left(\frac{s}{h} - i\right)\right\} \\ E_2(s) &= \frac{K}{4\sqrt{i}}\sqrt{\frac{\pi}{a}}\, e^{+\frac{h^2}{4ia} - \frac{h}{2a}s + \frac{is^2}{4a}} \times \operatorname{erfc}\left\{\frac{-\sqrt{i}h}{2a}\left(\frac{s}{h} + i\right)\right\} \quad . \end{aligned}\right\} \tag{6}$$

c. Appropriate expansions of $E_1(s)$ and $E_2(s)$. (Small values of t)

The exact integration of (2), when Eq. (6) is therein substituted, is a difficult problem even for a very simple expressions of G(s). Here enter the methods of approximate integration to produce approximate solutions of f(t), say $f^*_{(n)}(t)$, such that:

$$\left.\begin{aligned} &1.\; f^*_{(n)}(t) \underset{n\to\infty}{\to} f(t) \quad \text{(uniformly)} \\ &2.\; \text{The convergence of } f^*_{(n)}(t) \text{ towards } f(t) \text{ is very rapid in a} \\ &\quad\; \text{given interval of } t. \end{aligned}\right\} \tag{7}$$

By applying the "pocket method" we will produce a sequence $f^*_{(n)}(t)$, which converges very rapidly toward f(t) for t relatively small. (First part of the transient)†. Simple considerations of the "pocket method" indicate immediately the following expression: Take the first equation of (6) and let $z = (s/h) - i$; $\delta = h/2\sqrt{ai}$.

By definition of the complementary error function we have

† The solutions for large values of t are not given in this condensed report.

$$\operatorname{erfc}(\delta z) = \frac{2}{\sqrt{\pi}} \int_{(\delta z)}^{\infty} e^{-u^2} du \quad . \tag{8}$$

The integral (8) admits the following continued fraction expansion.

$$\frac{2}{\sqrt{\pi}} \int_{(\delta z)}^{\infty} e^{-u^2} du = \cfrac{e^{-(\delta z)^2}}{2(\delta z) + \cfrac{1}{(\delta z) + \cfrac{2}{2(\delta z) + \cfrac{3}{(\delta z) + \ddots}}}} \quad . \tag{9}$$

Immediately one gets

$$\left.\begin{aligned} E_1(s) &= -\frac{K}{2}\sqrt{\frac{i}{a}}\, F(W)\ ;\ W = X_1 \\ E_2(s) &= +\frac{K}{2\sqrt{ia}}\, F(W)\ ;\ W = X_2 \end{aligned}\right\} \tag{10}$$

where

$$X_1 = \frac{h}{2\sqrt{ai}} \left(\frac{s}{h} - i\right);\quad X_2 = -\frac{h\sqrt{i}}{2\sqrt{a}} \left(\frac{s}{h} + i\right) \tag{11}$$

and

$$\left. F(W) = \cfrac{1}{2W + \cfrac{1}{W + \cfrac{2}{2W + \cfrac{3}{W + \ddots}}}} \quad . \right\} \tag{12}$$

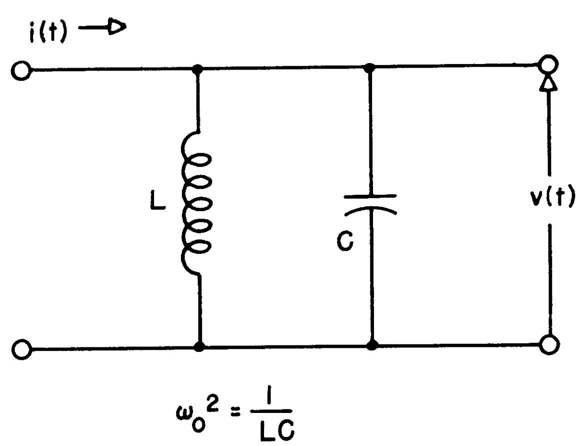

Fig. VIII-32 Tank circuit illustrating transient response.

The first 5 approximants of F(W), say $F^*_{(n)}(W)$, are †

$$\left.\begin{aligned}
F^*_{(1)}(W) &= \frac{1}{2W}\\
F^*_{(2)}(W) &= \frac{W}{2W^2+1}\\
F^*_{(3)}(W) &= \frac{W^2+1}{W(2W^2+3)}\\
F^*_{(4)}(W) &= \frac{W(2W^2+5)}{4(W^4+3W^2+\frac{3}{4})}\\
F^*_{(5)}(W) &= \frac{2W^4+9W^2+4}{W(4W^4+20W^2+15)}\\
&\text{-------------------}
\end{aligned}\right\}
\quad
F^*_{(n)} = \cfrac{1}{2W+\cfrac{1}{W+\cfrac{2}{2W+\cfrac{3}{W+\ddots+\cfrac{n-1}{cW}}}}}
\qquad c=\begin{cases}1;\ n\ \text{even}\\ 2;\ n\ \text{odd}\end{cases} \tag{13}$$

All the star functions of F are rational functions of W and consequently of s (see transformations (11)).

d. The approximate response $f^*_{(n)}(t)$.

Let G(s) be the Laplace transform of the system function. Then, the network response will be

$$\left.\begin{aligned}
f(t) &= f_1(t) + f_2(t)\\
\text{where}\qquad f_1(t) &= -\frac{K}{2\sqrt{a}}\sqrt{i}\,\frac{1}{2\pi i}\int_\gamma F(X_i(s))\,G(s)\,e^{+st}\,ds\\
f_2(t) &= \frac{K}{2\sqrt{ai}}\,\frac{1}{2\pi i}\int_\gamma F(X_2(s))\,G(s)\,e^{+st}\,ds
\end{aligned}\right\}. \tag{14}$$

---

† Still better and more rapid convergent approximation can be obtained by a "constant" termination of (12) as

$$F^*_{(n)}(W) = 1\rfloor 2W + 1\rfloor W + 2\rfloor 2W + 3\rfloor W + \ldots + n-1\rfloor cW + \Delta_{(n)};$$

$$c = \begin{cases}1 \text{ for } n \text{ even}\\ 2 \text{ for } n \text{ odd}\end{cases}$$

$$\Delta_{(n)} = \text{constant}$$

The constant $\Delta_{(n)}$ is selected so that the aperture of the corresponding window function, in a given time interval, is almost a minimum.

The approximate solutions can be immediately written as follows: Let

$$\left.\begin{aligned} F(X_1) &\approx F^*_{(n)}(X_1) \\ F(X_2) &\approx F^*_{(n)}(X_2) \end{aligned}\right\} \quad . \tag{15}$$

Since G(s) and $F^*_{(n)}(X_1(s))$ and $F^*_{(n)}$ are rational functions of s or respectively of X_1 and X_2, then $F^*_{(n)}(X_1)G(X_1)$ and $F^*_{(n)}(X_2)\,G(X_2)$ can be expressed in partial fractions of the form. (Case of simple poles is here shown).

$$\left.\begin{aligned} F^*_{(n)}(X_1)\,G(X_1) &= \sum_{k=1}^{m} \frac{A_{1,k}}{X_1 - X_{1,k}} \\ F^*_{(n)}(X_2)\,G(X_2) &= \sum_{k=1}^{m} \frac{A_{2,k}}{X_2 - X_{2,k}} \end{aligned}\right\} \tag{16}$$

where m is the total number of poles of the product F(x) G(x), A_k their residues. Let us introduce (15) in (14) and using the notation $f^*_{1,(n)}(t)$ and $f^*_{2,(n)}(t)$ for the approximate solutions of (14), one gets finally

$$f(t) \approx f^*_{(n)}(t) = f^*_{1,(n)}(t) + f^*_{2,(n)}(t) \tag{17}$$

where

$$\left.\begin{aligned} f^*_{1,(n)}(t) &= -\,iKe^{-iht} \sum_{k=1}^{m} A_{1,k}\, e^{\mu_1 X_{1,k}}; \quad \mu_1 = 2\sqrt{ait} \\ f^*_{2,n}(t) &= +\,iKe^{-iht} \sum_{k=1}^{m} A_{2,k}\, e^{\mu_2 X_{2,k}}; \quad \mu_2 = -2\sqrt{\frac{a}{i}}\,t \end{aligned}\right\} \quad . \tag{18}$$

e. Application to a tank circuit

We will illustrate the above procedure with a simple example. For simplicity in the presentation we will use the first approximant $F^*_1(W)$. The approximation with n = 1 is, of course, a rough one. For more accurate solutions n must be larger, but the general procedure is systematically the same.

Figure VIII-32 shows a tank circuit LC. The entering current is, by hypothesis, of the form

$$i(t) = K \sin\left\{ht - at^2\right\} \quad .$$

We want to find the voltage v(t) across the tank terminals. The corresponding system function is

$$G(s) = \frac{1}{c} \frac{s}{s^2 + \omega_o^2} \quad . \tag{19}$$

After simple algebraic operations one gets

$$v^*_{(1)_{n=1}}(t) = -\frac{Kh}{c(\omega_o^2 - h^2)} \; 2 \sin \frac{1}{2} (\omega_o + h) \, t \sin \frac{1}{2} (h - \omega_o) \, t \quad . \tag{20}$$

This result is valid only for $o \lesssim t < \pi/(\omega_o + h)$.

Note that the first approximation is independent of "a". The second, etc., is not.

Equation 20 shows that the transient wave begins, in the first approximation, with an instantaneous angular velocity of $1/2\,(h - \omega_o)$. The envelope increases sinusoidally with angular velocity of $1/2\,(\omega_o + h)$. If it happens that $\omega_o = h$, then

$$v^*_{(1)}(t) = -\frac{Kh}{c(\omega_o + h)} \; t \sin \frac{1}{2} (\omega_o + h) \, t \quad . \tag{21}$$

The approximation will be materially increased by choosing larger values of n. For n = s, for example, a very accurate solution will be obtained for values of t much larger than $2\pi/(\omega_o + h)$.

f. Other solutions to the problem stated in section a, and which are obtained by other methods of approximate integration will be reported. M. V. Cerillo

2. Synthesis in the Time Domain

An attack on the problem of synthesizing a finite, lumped-constant, linear network for prescribed transient behavior is being made by controlled, frequency-domain approximation techniques. The process consists of (1) a transformation of the desired transient response to the frequency domain, (2) an expansion of the resultant function of frequency about an appropriate point in the frequency plane, (3) termination of this expansion after a finite number of terms to achieve a realizable system function of some sort, and (4) network realization by known synthesis techniques. The key to the entire process lies in step 2; not only must the proper point for expansion be chosen, but the proper type of expansion must be made to achieve rapid convergence and to insure the physical realizability of its partial sums of "approximants".

The present approach is directed toward the utilization of a class of basic expansions representing fairly simple time-functions which may be used as constituent approximating elements of the given, desired response. In the frequency domain these "constituent transients" become simple transcendentals, the character of each of which is fixed principally by the nature of its one or two singularities. The application of these fundamental responses (and their corresponding expansions) as "building blocks" for more general and complex transients offers the following possible merits:

(a) Control of error for prescribed intervals of time, without plotting the final approximation.

(b) Improved economy in the number of network elements required for a specified response and set of tolerances.

(c) Application of general theory of expansions to the representation of the desired response by "constituent transients".

(d) Possible introduction of frequency-domain constraints (e.g., on bandwidth) simultaneous with the approximation.

(e) Simplicity of meeting realizability criteria, permitting a choice of the network-type at the start.

(f) Generation of an entire sequence of approximating networks (not just one), each corresponding to less time-domain error.

Recent work has been concerned with the investigation of the so-called "constituents transients", and the time-domain convergence of various types of frequency-domain expansions of such functions. The most useful type of development is the continued-fraction expansion, primarily because of its rapid convergence, but also for the reason that its approximants (partial sums) can be made to satisfy any one of several realizability criteria.

Although the eventual solution to the transient synthesis problem will probably be one in which a single, rapidly-convergent expansion is found for a given desired response, it is felt that the present approach will lend valuable insight into the more difficult general solution, and provide as well a synthesis technique applicable to a large number of cases, including many practical problems.

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E. SLIGHTLY LOSSY NETWORKS

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Compensation for Incidental Dissipation in Network Synthesis

A number of synthesis techniques for networks employing lossy elements (coils) with effectively lossless elements (crystals and capacitors) have been studied this quarter. The aim has been to develop a suitable method whereby one can use a few high-quality elements in an otherwise lossy structure to obtain results which would be unattainable in a uniformly lossy structure. This work is a further study in connection with the methods presented for evaluating the effects of incidental loss reported in the October 15, 1950 Progress Report. The particular problem being considered is the design of filters with very sharp discrimination characteristics; and the form of network sought is a practical ladder structure. To the present none of the methods tried has been satisfactory; however, some progress has been made, and it seems that further work will yield a good answer.

Initially, an extension of a well known technique for synthesis of resistance-terminated lossless ladders (Fig. VIII-33a) with prescribed transfer impedance was

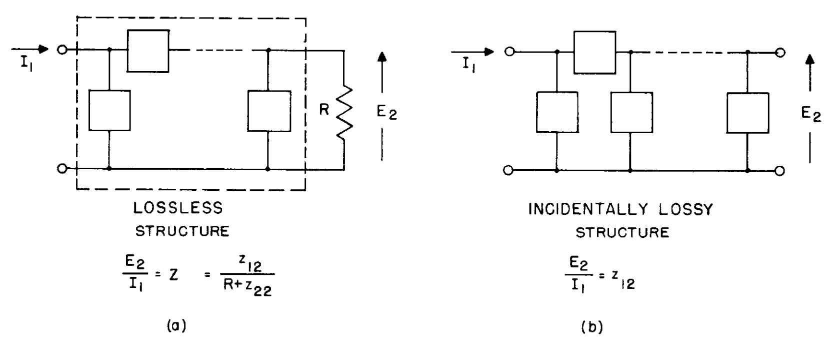

Fig. VIII-33 Resistance-terminated lossless ladder network and ladder network with incidental loss.

attempted. This technique consists of prescribing Z_{12} from which z_{12} and z_{22} (the open-circuit transfer and driving-point impedances of the lossless structure) are determined. The synthesis procedure is to develop a lossless ladder with the prescribed z_{22} in such a way to provide zeros of z_{12} at the required frequencies. If the ladder is to be constructed of incidentally lossy elements, the poles and zeros of Z_{12} are displaced small amounts from their location with a similar structure of strictly lossless elements. In the particular case of uniform loss, the poles and zeros are displaced to the left in the complex-frequency plane by the loss factor ($R/L = G/C$) and the practical network using lossy elements may be synthesized by developing the related lossless structure in which the critical frequencies have been pre-distorted to the right by $R/L = G/C$. In the case of nonuniform loss, the displacement of critical frequencies from their locations in the related lossless structure can be evaluated by the methods reported last quarter. However, one cannot say before the synthesis is completed how the critical frequencies are displaced and the technique of pre-distortion is no longer applicable. On the basis of the study made during the previous quarter, it was learned that the loss in any element caused a shift in any natural frequency inversely proportional to the rate of change of admittance seen by that element at the natural frequency. This suggested that the distortion of critical frequencies caused by loss might possibly be made negligible provided that one managed the synthesis of z_{22} to place the lossy elements in an environment of rapidly changing admittance. For the examples considered, no means was found whereby one could make the displacements of critical frequencies negligible by appropriately placing the lossy elements. Moreover, since the use of crystals (which are represented by three elements in the network) imposes a severe constraint on the synthesis procedure, the addition of further constraints intolerably complicates matters.

Attention was shifted to unterminated ladder networks of nonuniformly lossy elements (Fig. VIII-33b). Reflection on the nature of the networks of Figs. VIII-33(a and b) reveals certain similarities between the networks shown and leads one to think that resistance termination of lossy structures is ordinarily both unnecessary and undesirable. In Fig. VIII-33a the load resistance plays a role interestingly analogous to the loss in the ladder elements of Fig. VIII-33b. The load resistance may be considered to distort the poles of z_{22} (or z_{12}) into the positions held by the poles of Z_{12}. In the same way, nonuniform loss in the elements of Fig. VIII-33b moves the poles of its z_{12} to the left from the imaginary axis where they would be if the elements were lossless. The synthesis technique for networks of the form of Fig. VIII-33a may be said to consist of the prescription of a Z_{12} and the subsequent determination of a z_{22}, the poles of which will be appropriately displaced by the load resistance. If one uses lossy elements in the ladder structure of a filter which is to provide sharp discrimination, the difficulty ordinarily encountered is that loss shifts the critical frequencies too far into the left half-plane and the use of a resistance termination only accents this problem. Hence the appropriate network form seems to be the unterminated structure and the appropriate functions on which to focus attention seem to be the z_{22} and z_{12} of the related lossless structure.

Reactance functions developable into ladder structures of suitable form for filters are being studied. The particular aim is to obtain some experience with the effects of loss in different network configurations. The obtaining of such information should suggest a practical method of design.

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