IX. ANALOG COMPUTER RESEARCH

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A. THE OPERATION OF PRESENT COMPUTERS

1. Integral Equation Solver

During the past quarter the machine has been used to take a number of Fourier transforms and convolution integrals for various groups in the laboratory.

An initial investigation of the machine's facility in handling nonlinear problems is being conducted with convolution integrals.

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2. The Macnee Differential Analyzer

Some comparative tests have been run on the accuracy of the crossed fields multiplier, and the Angelo multiplier tube. Indications are that the Angelo tube possesses greater accuracy and stability, and it is being modified to operate in conjunction with the Macnee machine. Accuracies of 2 percent of full scale are obtained consistently from the Angelo tube.

Fundamental investigations on gain-bandwidth products in electronic differentiators and integrators are being pursued.

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B. APPLIED NETWORK THEORY

This group has always been active in applied network theory and the construction of special computing machines for solving problems in network synthesis. Work is currently being pursued on potential analogs and RC network synthesis.

1. Potential Analogs

Potential analogy has been extensively used in network synthesis because of the formal similarity of the logarithmic potential of a set of charges and the transfer function of a network with lumped elements.

Kuh (1) developed a potential analogy method to determine the poles of a network which would have a prescribed loss function in a bandpass interval. Using an elliptic transformation, he transformed the p-plane into a rectangular cell, where the potential problem was solved. This method involves a considerable amount of calculation. It is thought that by using some transformation, other than the elliptic one, the required calculations could be reduced.

Seeking a Tchebycheff approximation to the equivalent problem, Darlington (2) solves it by transforming the useful interval (in this case, lowpass) onto a circle. Darlington's work has been extended to the bandpass case, by using the transformation (bandpass interval onto a circle):

$$p^2 = \frac{a + b\left(z - \frac{1}{z}\right)^2}{1 + c\left(z - \frac{1}{z}\right)^2}.$$

One rather simple case, namely that of approximating a prescribed gain $(\pi/2)\omega^2$ over the interval, has been successfully solved by Kuh's method. Other cases will also be investigated, although it is to be expected that the calculations will involve numerical integration procedures. It is also hoped that some relationship between Darlington's method and Kuh's will be found, since the approximations considered are both in the equal-ripple sense.

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References

1. E. S. Kuh: A Study of the Network-Synthesis Approximation Problem for Arbitrary Loss Functions, Stanford University Electronics Laboratory Technical Report No. 44, Feb. 14, 1952

2. S. Darlington: Network Synthesis Using Tchebycheff Polynomial Series, Bell System Tech. J. 31, 613-665, July 1952

2. RC Network Synthesis

It has been established by Guillemin (1) that except for a constant loss and some phase limitations, any transfer characteristic may be approximated arbitrarily closely by means of a linear passive network containing only R's and C's. Although there exist several synthesis procedures for realizing such networks (1, 2, 3), comparatively little is known about the aspects of the connected approximation problem. The present work is being undertaken in an effort to investigate some of these aspects and ultimately their direct engineering application.

The problem of approximation under the restrictions imposed for an assumed passive RC network has been approached by Guillemin in the following manner. For a given transfer function

$$|Z_{12}(j\omega)|^2 = F(\omega^2) \approx \frac{P(\omega^2)}{Q(\omega^2)}$$

$Q(\omega^2)$ is assumed to be of the form

$$Q(\omega^2) = \frac{\prod_{i=1}^{n}\left(\omega^2 + \omega_i^2\right)}{(1+\omega^2)^n}.$$

The product

$$f(\omega^2) = F(\omega^2) \cdot Q(\omega^2)$$

is then transformed into a periodic function $f(\phi)$ in the ϕ-plane by the transformation

$$\omega = \tan\frac{\phi}{2}$$

where it is approximated by a trigonometric series and then transformed back to the ω-plane in the form

$$P(\omega^2) = \frac{A_o \pm A_1\omega^2 + A_2\omega^4 \pm \ldots, + A_n\omega^{2n}}{(1+\omega^2)^n}$$

which gives the desired expression

$$|Z_{12}(j\omega)|^2 = \frac{A_o + A_1\omega^2 + A_2\omega^4 + \ldots, + A_n\omega^{2n}}{\prod_{i=1}^{n}\left(\omega^2 + \omega_i^2\right)}.$$

Since for RC networks lowpass and highpass structures must be studied separately (there is no frequency transformation), the problem of bandpass filters has been taken up first as a more general case. It is hoped that questions like the following may be answered. (a) What relation, if any, does the exponent n bear to the equivalent Q of the network? (b) Given $F(\omega^2)$ and n, what choice of zeros of $Q(\omega^2)$ results in a function $f(\phi)$ that is best approximated by the n terms of the trigonometric series?

The answers to these questions will permit us to establish criteria for designing practical RC filters.

Extensive numerical computation work is being carried on in connection with the ideas expressed above.

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References

1. E. A. Guillemin: Synthesis of RC networks, J. Math. Phys. 28, 22, 1949

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3. A. Fialkow, I. Gerst: Transfer Function of RC Ladder, J. Math. Phys. 30, 49, 1951