IX. ANALOG COMPUTER RESEARCH

Prof. E.A. Guillemin
Prof. R. E. Scott
J. Petrishen
N. DeClaris
S. Fine
W. A. Koelsch, Jr.
F. F. Lee
M. Macrakis
T. E. Stern

A. THE OPERATION OF PRESENT COMPUTERS

1. Integral Equation Machine

A number of convolution integrals have been solved during the previous quarter.

S. Fine

2. The Macnee Differential Analyzer

A problem in nonlinear servomechanism theory is being solved on the Macnee machine.

T. E. Stern

B. THE DESIGN OF COMPUTING ELEMENTS

1. The Angelo Multiplier

The Angelo multiplier has been tested for ac operation as a component of the Macnee computer. Because of its constant current characteristics, the multiplier will operate from a low-current bleeder. The equipment required to adapt this push-pull multiplier to a repetitive computer is not much greater than that required for a single-ended multiplier.

S. Fine

2. Scale-of-10 Counters

Scale-of-10 counters can be constructed from binary units, either by the insertion of six extra pulses at any time during the cycle in a scale-of-16 counter or by blocking two pulses entering the unit in a scale-of-8 counter.

Several systems were examined experimentally at low frequencies (approximately 10 kc/sec), one of which is shown in Fig. IX-1.

The method of operation is as follows: All tubes are in a normally reset position. The eighth pulse resets the 2^0, 2^1, and 2^2 stage, as in a normal four-stage binary counter, and sets the 2^3 stage. The 6AS6 pentode-suppressor grid is consequently cut off. Two pulses now occur in the 2^0 stage, the second pulse resetting the 2^3 stage. Since the 2^1 stage is required to operate at only one half the rate of the 2^0 stage, it is possible to utilize a slight time delay circuit in the feedback path to the suppressor of the 6AS6 pentode. Consequently, it is not necessary to depend on relative pulse amplitudes to a single point for conversion to a scale of 10.

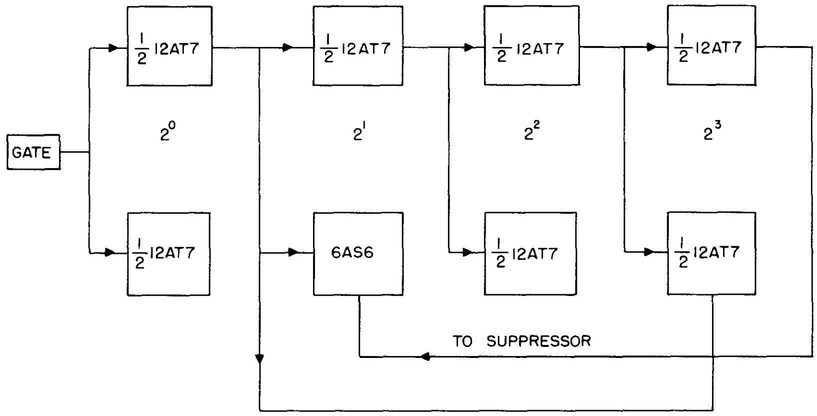

Fig. IX-1
Scale-of-10 counter.

If pentode circuits are used for all stages, it may be possible to build a high-speed scale-of-10 counter on this principle.

S. Fine

3. Two-Dimensional Visual Display

Methods are being investigated for the visual display of digits and other potential patterns obtained from electronic computing equipment. One such device uses a two-dimensional matrix of possible light spots in the form of either separate neon-glow tubes or pairs of small electrodes in a single neon-filled glass envelope. The advantage of the proposed unit over other matrices of a similar type lies in the fact that the coordinates of the spot at which light is desired are to be obtained in an analog fashion from two positioning voltages as in the deflection of a cathode ray tube spot. A third voltage will then control the intensity of the light at the selected location. This operation will eliminate the necessity for the complex switching and gating circuits formerly employed in light matrices.

Extensive calculation has resulted in a set of design criteria for the driving networks to obtain optimum accuracy of spot location and selection for a given number of possible spots in a matrix. The maximum number of spots that can be used in the analog-driven matrix is limited by the variation in firing and operating voltages for the light sources used. Neon-glow tubes of type NE-2 are sufficiently similar for matrices up to approximately 10 by 10, but for more than 100 light spots it may be necessary to use some other light source with a sharper and more consistent light-vs-voltage threshold.

W. A. Koelsch, Jr.

4. A Fourier Transformer

Multiple-order sinusoids can be considered as analog operators to perform such functions as Fourier synthesis, Fourier analysis, and polynomial root extraction. A unit for Fourier analysis, utilizing integral order sinusoids with a base frequency of 100 kc/sec, has been previously constructed by C. L. Searle.

The production of multiple-order sinusoids with RC oscillators for the purpose of performing Fourier analysis is being investigated. Using a highly unsymmetrical multivibrator which provides pulses at a 200-cps rate, it has been possible to synchronize RC oscillators up to the twentieth harmonic of the base frequency. By the use of a dual pulse injection scheme, it has been possible to minimize the distortion of the sine wave caused by the synchronization. A stable constant phase relationship between the harmonically related frequencies is also provided by this synchronization method.

It is planned to complete a unit consisting of 20 sinusoidal generators.

J. Petrishen

C. COMPUTERS FOR NON-COMPUTATIONAL PURPOSES

1. A Chinese Typesetting Machine

Chinese is a difficult language. It is so, even for the Chinese themselves. It is difficult to read, to write, and to print. Such difficulties arise primarily from one source, the fact that written Chinese consists of a large number of symbols called characters.

A character is identified by its topological feature rather than by its sound. In fact, two people from regions which differ in dialect may pronounce the same character in completely different ways. Although they cannot understand one another's speech, two Chinese may communicate freely in writing. It has generally been agreed that approximately 5000 characters are in current usage; the mastery of them requires an extended period of training and practice. For this reason, the symbolic language is difficult to learn. Despite many years of effort on the part of the Chinese Government, the percentage of illiteracy in China is still among the highest in the world.

Since the invention of movable type, the process of printing Chinese has remained essentially unchanged, except for technical improvements to the machinery. There is no Chinese Linotype nor a practical Chinese typewriter. The most time-consuming part of the printing process is typesetting since the setter must search among several thousand characters for each one he needs in the text. To print a page consisting of 400 characters, for example, would require more than an hour of typesetting time. The typesetting staff of a daily newspaper is, therefore, quite large. Many different attempts have been made to obtain a solution to these problems. Some started from the broad aspect of language reform, others studied how the types should be grouped together by their joint probabilities for faster selection. It is not the purpose of this research to revolutionize the Chinese language. Our major objective is to examine the Chinese characters logically, in order to determine differentiating properties which will allow recent technological achievements to be employed to increase the rate at which Chinese can be printed.

A character consists of a number of strokes arranged in a definite spatial relationship. There are about 10 to 20 basic strokes; the exact number is dependent on the desired degree of differentiation. A basic stroke has only a fixed shape. Its relative size and position within the space allotted may change from character to character. Therefore, it is not possible to superimpose a number of standard strokes to form all the characters in the language.

It is extremely fortunate that there is a generally accepted stroke order in which each character is written. Yuen Ren Chao of the University of California, an authority on the Chinese language, has estimated that the usage of stroke order differs only by about 5 to 10 percent, in spite of the fact that there is no fixed standard. If we ignore this small deviation from common practice, we can utilize the stroke order to great advantage.

We are now working on a possible scheme to mechanize the typesetting process so that up to 10 times the present-day speed may be achieved. The method is based on the accepted stroke order of writing Chinese. For example, the stroke order of the character 元 is

(a) (b) (c) (d)

A type of code may be designed based on the fundamental strokes. For instance, the character 元 may have the binary code 0001, 0001, 0101, 1100. Without consideration of exceptions, a properly designed code would have a unique code number for every Chinese character. A modified typewriter keyboard with the basic Chinese strokes in place of the alphabet may be used in conjunction with a relatively simple encoder to generate the code number as rapidly as the typist can feed the machine.

All the Chinese characters which are in use at the present time are to be arranged as a matrix on a photographic negative plate. These characters, which would be about 5000 in number, may be arranged in a rectangular array; thus each character would have an (x, y) coordinate. Each time a character is desired, it is only necessary to position this photographic negative and expose a small portion of film properly placed in an optical system. The film may be used as the master for the photo-offset process.

An electromechanical computer can be designed to bridge these two parts of the machine; that is, it will translate the code number into the (x, y) coordinate automatically.

Work has already begun on the statistical study that is pertinent to the design of a suitable and mistake-proof code. The design of the encoder will follow shortly.

F. F. Lee

D. APPLIED NETWORK THEORY

1. RC Network Synthesis

It was pointed out in the previous Quarterly Progress Report (1) that the approximation problem connected with the synthesis procedures of RC networks presents a great deal of difficulty and very little has been done in that direction.

Guillemin's (2) approach to this problem (the only one of its kind to the knowledge of the writer), though of a general nature, has the main disadvantage that it requires a great deal of numerical computation. Following the notation of the previous discussion (1), once the function $Q(\omega^2)$ of degree n has been assumed, the degree of the function $P(\omega^2)$ has also been decided and the approximation of the function $P(\phi)$ $[P(\omega^2) \to P(\phi)]$ is limited to the first n terms of the trigonometric series. It is possible, however, that the function does not converge fast enough to assure the tolerance requirements with these terms, in which case $Q(\omega^2)$ must be reconstructed and the whole process of the approximation must be repeated for higher n.

As an attempt to solve this difficulty for filter networks, a method of approximation has been devised. Because of its length, the method will appear as a separate technical report. A brief description of the method as applied to the lowpass case, omitting the necessary proofs, is presented here.

Let the desired filter characteristics be of the step type, with a tolerance of ϵ percent allowable for the approximation and with certain cutoff sharpness requirements. In finding a transfer function to fulfill these requirements while preserving the RC character of the final network, the method proceeds as follows:

a. At first an approximation is found of the well known (4) type

$$|f(\omega)|^2 = \frac{K^2 f}{1 + C_f^2 V_n^2(\omega)}$$

where $s = \sigma + j\omega$, which satisfies the cutoff requirements by the use of a proper $V_n(\omega)$ Tschebyscheff polynomial while allowing a tolerance of $(\epsilon/3)$ percent in the bandpass. For reasons of simplicity, which will become apparent later, it is preferred that n be an odd integer. The resultant transfer function is of the form

$$f(s) = \frac{K_f}{(s+t) \prod_{i=1}^{n} (s^2 + a_i s + g_i)} .$$

The distribution of the poles is a semiellipse symmetrical about the real axis.

Let r(s) be the RC transfer function (poles along the negative real axis) which interpolates f(s) at the points $\beta_1, \beta_2, \ldots, \beta_m$ along the positive and negative j-axis. Then

$$e(s) = f(s) - r(s)$$

where r(s) is of the form

$$r(s) = K_r \frac{P + P_1 s + P_2 s^2 + \ldots}{\prod_{i=1}^{2m-1} (s + a_i)}.$$

Another way of expressing e(s) is

$$e(s) = K_e \frac{\prod_{i=1}^{m} (s^2 + \beta_i)}{\prod_{i=1}^{2m-1} (s + a_i) \times (s+t) \prod_{j=1}^{n} (s^2 + a_j s + g_j)}$$

where

$$K_e = K_f \frac{\prod_{i=1}^{2m-1} (a_i - t)}{\prod_{i=1}^{m} (t^2 + \beta_i^2)} = \text{real number}.$$

It follows that

$$|e_s(j\omega)|^2 = K_e^2 \frac{\prod_{i=1}^{m} (\beta_i^2 - \omega^2)^2}{\prod_{i=1}^{2m-1} (\omega^2 + a_i^2)} \times \frac{1}{(\omega^2 + t^2) \prod_{j=1}^{n} \left[(g_j - \omega^2)^2 + \omega^2 a_j^2\right]}$$

where $s = \sigma + j\omega$. It can be easily shown that in this particular case, for $\beta_i \neq 0$ and $a_i \neq t$, $|e(\omega)|^2$ has its maximum value when $\omega = 0$.

b. The second step in this method is to find a sequence of β_i's and a_i's such as

$$|e(0)|^2 = \prod_{i=1}^{2m-1} \left(\frac{a_i - t}{a_i}\right)^2 \prod_{i=1}^{m} \left(\frac{\beta_i^2}{\beta_i^2 + t^2}\right)^2 \leqslant \left(\frac{\epsilon}{300}\right)^2.$$

It is assumed that K_f is chosen to result in $|f(0)|^2 = 1$. It must be observed here that m can be greater than n; thus a sufficient number of poles can be taken to satisfy the above inequality. A convenient value for one of the β' s is 1, which preserves the same cutoff frequency for both functions.

Once the values of β_i's and a_i's have been determined, the transfer function r(s) is uniquely specified by the generalized Lagrange formula of interpolation (3).

c. In order to construct the function r(s), let

$$\mu_k = f(j\beta_k) + j\gamma_k$$

$$z_k = z(s)\big|_{s=j\beta_k} = \left. \frac{\prod_{i=1}^{m} (s^2 + \beta_i^2)}{(s - j\beta_k) \prod_{i=1}^{2m-1} (s + \alpha_i)} \right|_{s=j\beta_k} = m_k + j\nu_k$$

and

$$\frac{\mu_k}{Z_k} = \frac{b_k m_k + \gamma_k \nu_k}{m_k^2 + \nu_k^2} + j\,\frac{m_k \gamma_k - b_k \nu_k}{m_k^2 + \nu_k^2} = A_k + jC_k.$$

The final expression of the r(s) function takes the form

$$r(s) = 2 \sum_{k=1}^{m} \frac{(A_k s + C_k) \prod_{i=1}^{m} (s^2 + \beta_i^2)}{(s^2 + \beta_k^2) \prod_{i=1}^{2m-1} (s + \alpha_i)}$$

Although the method presented here has been applied, for clarity, to the case of lowpass structures, it is easily modified to cover both the highpass and bandpass cases. Actual numerical examples of this method will be included in the technical report to appear soon.

N. DeClaris

References

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3. J. L. Walsh: Interpolation and Approximation, Am. Math. Soc. Colloquium Publications, Vol. 20, 1935
4. R. M. Fano: A note on the solution of certain approximation problems in network synthesis, Technical Report No. 62, Research Laboratory of Electronics, M.I.T. 1948

2. Potential Analogs

The work discussed in the Quarterly Progress Report, January 15, 1953, has been completed and a thesis is being prepared.

The transformation from the s-plane to the z-plane used is

$$s^2 = -\omega_1^2 + \frac{\omega_2^2 - \omega_1^2}{4} \left(z - \frac{1}{z}\right)^2 \qquad (1)$$

The problem of determining the natural modes for a prescribed gain $\bar{a}(\omega)$ over an interval $\omega_1 \leq \omega \leq \omega_2$, is transformed to a filter problem. This is achieved by construction of a function which, solely over the interval of interest, has the negative values of $\bar{a}(\omega)$. A criterion for the location of the natural modes is then developed on the z-plane. This criterion is similar to that given by S. Darlington (Network Synthesis Using Tscheby-scheff Polynomial Series, Bell System Tech. J. 31, 613-55, 1952). It states that the n natural modes should lie on a contour defined by the equation

$$2n \log z + \sum_{k=0}^{\infty} \frac{A_{2k}}{2}\left(\frac{1}{z^{2k}} + z^{2k}\right) = \log K \qquad (2)$$

Different values of the constant $\mathrm{Re}[\log K]$ define equipotentials in the z-plane. The extent to which K influences the type of approximation is also discussed.

The simple case of the gain $\bar{a}(\omega) = p\omega^2$ has been thoroughly investigated. For the case of a general $\bar{a}(\omega)$ the expansion

$$\bar{a}(\omega) = \sum_{n=1}^{\infty} P_{2n}\omega^{2n} \qquad (3)$$

is used. By proper truncation of the series, together with economization of its terms and superposition, we have taken advantage of the results of the simple case $\bar{a}(\omega) = p\omega^2$ to solve the general problem.

M. Macrakis