VII. TUBE RESEARCH AND DEVELOPMENT

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A. MICROWAVE TUBES

1. Electron Beam Studies

a. Noise in electron beams

The experiments described by H. E. Rowe and C. E. Muehe, Jr. in previous Quarterly Progress Reports had to do with the noise characteristics of magnetically focused electron beams. Although the experiments were conducted with vacua of less than 10^{-6} mm Hg, the long drift tube insured almost complete neutralization of the electron beam by positive ions. Thus, it was never possible, for example, to achieve true Brillouin focusing. We decided to repeat the measurements, using a pulsed electron beam. With a pulse duration of 10^{-6} sec, a pressure of 2×10^{-7} mm Hg, a drift length of 40 cm, and with the assumption that no loss of ions occurs, the ion concentration at the end of the pulse would only amount to about 1 percent of the electron density.

The apparatus used is that built by Muehe for his experiments. It has been modified by adding a motor drive to move the cavity smoothly along the beam, as shown in Fig. VII-1.
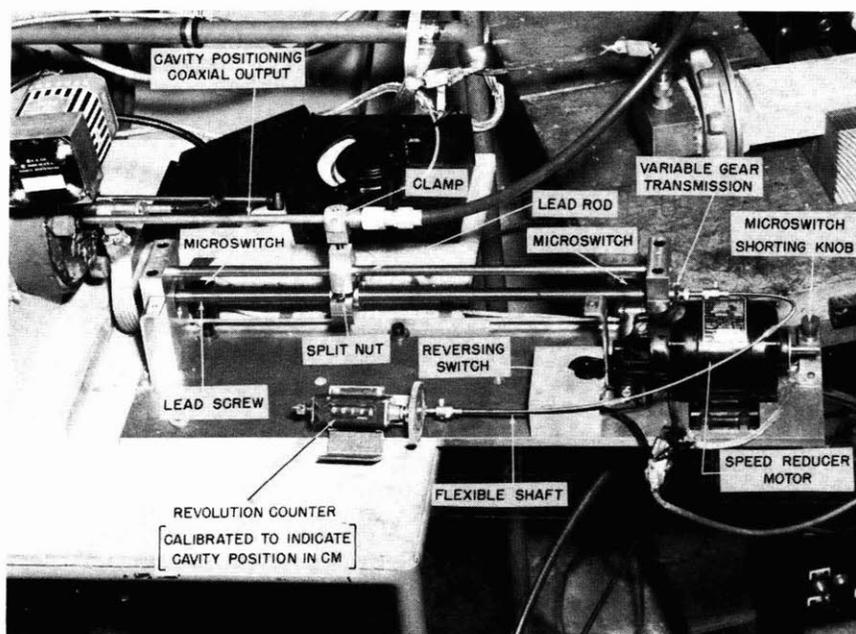
Fig. VII-1
Automatic motor drive for cavity positioning.

Fig. VII-2

Block diagram of apparatus used in measuring the noise output.

A hard tube modulator is used to pulse the beam for 1 μsec at a 4000 cps repetition rate. The pulsed noise power induced in the probing cavity is expected to be the same as the steady noise measured in the earlier dc experiments. The average power will therefore be approximately 24 db less. Since this is less than the noise level of the receiver, it was necessary to gate the output circuit, as indicated in Fig. VII-2. Considerable time and effort were spent in shielding the modulator and receiver in order to keep the modulator video pulse from swamping the receiver.

The initial measurements are on a beam produced by a Pierce gun with the following parameters:

Perveance	0.11×10^{-6} amp/volt$^{3/2}$
Cathode diameter	0.130 inch
r_a	0.236 inch
r_c	0.530 inch
θ_1 (half angle of convergence)	7.2°
r_{min}	0.010 inch

The focusing field for ideal Brillouin flow (V_o = 1500, I_o = 4.5 ma) should be 350 gauss. Tests indicate best focusing at approximately 1.3 times this value. This result must be considered tentative until the solenoid is recalibrated.

Preliminary measurements indicate that a noise standing wave exists, just as in the dc experiments; furthermore, the noise growing wave is still present. As the shielding of the receiver has just been completed, no precise noise data are available at this writing.
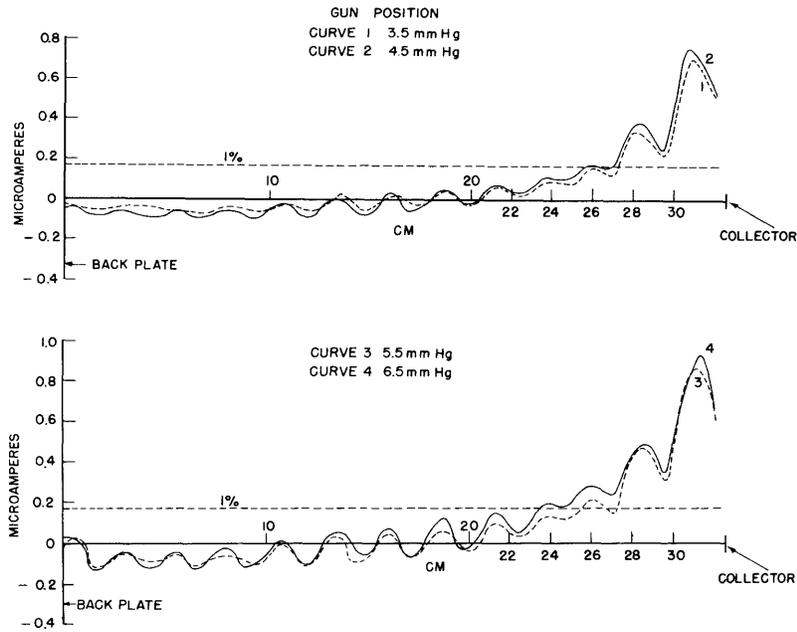
Fig. VII-3

Interception current vs cavity position with gun position as
the parameter. For all four curves V_0 is 1500 volts; I_{coll}
is 17 μa; magnetic field, approximately 355 gauss; pressure,
1.5×10^{-7} mm Hg. The parameter is the gun position
measured from the back plate outside the magnetic field.
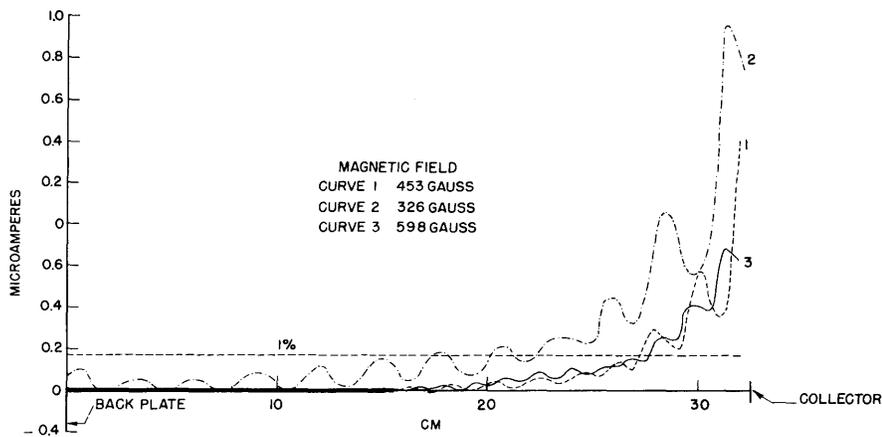
Fig. VII-4

Interception current vs cavity position with magnetic field
as the parameter. For all four curves V_0 is 1500 volts;
I_{coll} is 17 μa; pressure, 1.5×10^{-7} mm Hg; gun position,
3.8 mm from back plate outside the magnetic field.

Figures VII-3 and VII-4 show the results of beam interception current measurements. Figure VII-3 shows the variation of interception current with the gun position at a constant magnetic field. Figure VII-4 shows the dependence of interception current on the magnetic field with the gun fixed at its optimum position.

L. D. Smullin, C. Fried

b. Propagation of signals on electron streams

Measurements are being made of the behavior of a magnetically focused electron beam when the velocity is modulated by a 3000 Mc/sec signal. The apparatus is essentially the one used by H. E. Rowe. Two movable cavities are provided; one is used to modulate the beam and the other is used as a pick-up or catcher cavity. It is hoped that these measurements will shed some light on the details of electron bunching phenomena.

H. A. Haus

2. Backward Wave Oscillator (BWO)

a. Construction

A helix backward wave oscillator is being constructed to work in the range between 3000 and 6000 Mc/sec at beam voltages of 500 to 2500 volts. The essential dimensions of the gun and the construction of the tube are shown in Figs. VII-5 and VII-6. The gun produces a parallel beam and will be immersed in the magnetic field. A first anode is provided to control the current, and the second anode is expected to run at helix potential.

Details of the matching system are described in section XI of this report.

A. G. Barrett

b. Design study of helix backward wave oscillators

Introduction

The observance by Kompfner of wide-range, voltage-tunable oscillations in a traveling-wave tube with a matched collector-end terminal has been explained on the basis of stream interaction with a wave whose phase velocity is directed opposite to the total power flow. Waves of this type, called backward waves, occur rarely in smooth cylindrical structures[*] but are necessarily present in any periodic structure. Periodic interaction structures are therefore of prominent interest for study, and although an analysis by the field theory is in these cases prohibitively complicated with free electron current present, the circuit and stream model of Pierce can lead to useful results. It

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[*]Waves propagating in a dielectric in the frequency region of a resonant absorption may be of this type (cf. J. A. Stratton: Electromagnetic Theory, McGraw-Hill, New York, 1941, p.340); the sheath helix modes having angular variation can also be of this type (See ref. 1).
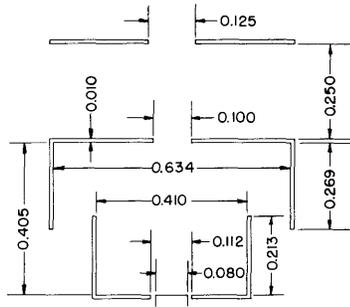
Fig. VII-5

Essential dimensions of the gun.   Transverse parts are
Nichrome V and cylindrical parts are 304 stainless steel.
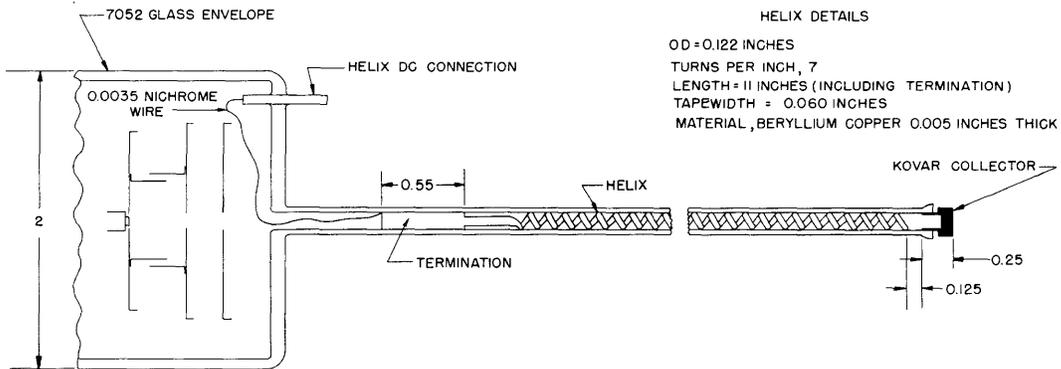
Fig. VII-6

Backward wave oscillator tube.

is the purpose here to apply some of Sensiper's results for the tape-helix problem (1) to Pierce's model to obtain design information for helix backward wave oscillators.

Criterion for Starting of Oscillations: Zero Space Charge Approximation

The foundation for the theory of the backward wave oscillator is given in Chapter XI of Pierce's text (Traveling Wave Tubes, Van Nostrand, 1950). By extending this analysis somewhat, Heffner has found the condition for start-oscillation (2). In the zero space charge approximation (only this case is considered here since, otherwise, there is considerable difficulty in evaluating the space charge parameter) he finds the necessary conditions

$$(h - \beta_e)L = 3.00 \tag{1a}$$

$$CN = 0.312 \tag{1b}$$

where h = propagation constant of the circuit wave with stream absent, $\beta_e = \omega/v_e$, v_e is the velocity of the electron stream, and L = active length of the circuit. The quantities C and N have their usual meaning

$$C = \frac{1}{2}\left(z_c Y_b\right)^{1/3}$$

$$N = \frac{hL}{2\pi}$$

where

$$z_c = \frac{E_z E_z^*}{h^2}\left(\frac{1}{P_z}\right)$$

is a thin-beam definition, with E_z the electric field at the beam and

$$Y_b = \frac{\text{direct beam current}}{\text{direct beam voltage}}.$$

Equations 1a and 1b are the conditions for start-oscillation. Should the CN product be greater than the given critical value, oscillations are nevertheless sustained, but the frequency-determining equation, Eq. 1a, is slightly modified. The circuit properties of interest in these equations are the impedance Z_c and the propagation constant h of the wave which is to interact with the stream.

Properties of the -1 Space Harmonic on the Tape Helix

The simplicity of the helical wire waveguide makes this structure very appealing for practical application. The tape wire is felt to be superior to round wire because it allows the beam to pass in regions of higher field strength. Of the possible backward waves on the tape helix, the -1 space harmonic is felt to be best adapted to BWO operation and is thus investigated here. Sensiper has worked out many of the details of the

tape-helix problem, and from his work one finds that the -1 space harmonic component of axial electric field for the region inside the tape radius is given by

$$E_{z,-1} = \left\{ \left(\frac{\mu}{\epsilon}\right)^{1/2} \frac{I}{p} \frac{1}{ka} \left[(\gamma_o a)^2 \tan\psi - h_o a \right] K_1(\gamma_{-1}a) \frac{\sin x}{x} \right\}$$

$$\times \left\{ I_1(\gamma_{-1}r) \exp j\left[\omega t + \theta - \left(\frac{2\pi}{p} - h_o\right) z \right] \right\} \tag{2}$$

where

I = amplitude of current flowing on tape

$k = \frac{\omega}{c}$, a = helix radius

h_o = fundamental propagation constant

$$\gamma_o = \left(h_o^2 - k^2 \right)^{1/2}$$

$$\gamma_{-1} = \left(h_{-1}^2 - k^2 \right)^{1/2}, \quad h_{-1} = h_o - \frac{2\pi}{p}, \quad p = \text{the helix pitch}$$

$x = \frac{\pi\delta}{p}$, δ = the tapewidth

ψ = helix-pitch angle measured from planes z = constant.

Although Eq. 2 is exact only in the limit of vanishing tapewidth, it is felt that this expression is a very good approximation for tapewidths as great as one-fourth of the pitch. One notes from the $I_1(\gamma_{-1}r)$ dependence that the field would couple very weakly to electrons in the region of the cylinder axis; but because of the difficulty in producing hollow beams, it will be assumed that the beam to be used is solid and completely fills the cross section within the tape radius. Because of the θ-dependence of the interacting field and the fact that the beam has a finite cross section, there is some question as to a proper definition of circuit impedance. If one adopts the simple picture of considering the total beam to be composed of many independent parallel beams, it is appropriate to take the average of the quantity $(E_{z,-1} E_{z,-1}^*)$ over the beam cross section. Using this definition, one finds from Eq. 2, after a little manipulation

$$\frac{\overline{|E_{z,-1}|^2}}{h_{-1}^2 P_z} = \frac{30}{\pi} \frac{1}{z'} \left[\frac{(\gamma_o a)^2 \tan\psi - h_o a}{ka(h_o a \tan\psi - 1)} I_1(\gamma_{-1}a) K_1(\gamma_{-1}a) \frac{\sin x}{x} \right]^2$$

$$\times \left[1 - \frac{I_0(\gamma_{-1}a) I_2(\gamma_{-1}a)}{I_1^2(\gamma_{-1}a)} \right] \tag{3}$$

A new parameter has been introduced, namely,

$$z' = \frac{P_z}{|I|^2} \frac{1}{\left(\frac{\mu}{\epsilon}\right)^{1/2}}.$$

This quantity measures the total axial power flow (the algebraic sum of the powers carried by all space harmonics) in terms of the current flowing on the tape. The last factor in Eq. 3 gives the reduction in impedance arising from the assumption that the beam fills the cross section as opposed to the assumption that all the current flows at radius a.

To carry out the calculation (Eq. 3) rigorously is indeed complicated for one must first solve the tape-helix boundary value problem for the fundamental propagation constant* h_o and proceed from there to a very tedious calculation of z'. Sensiper has carried out these two calculations for a narrow tape ($\delta/p = 1/10\pi$) 10° helix, and by substituting his results in Eq. 3, one obtains the values given in Table I. Also tabulated here is the phase velocity of the -1 space harmonic given by the relation

$$\frac{c}{v_{-1}} = \frac{h_{-1}a}{ka} = \frac{\left|h_o a - \cot \psi \right|}{ka} = \left| \frac{c}{v_o} - \frac{\cot \psi}{ka} \right| \tag{4}$$

where v_o is the phase velocity of the zeroth space harmonic.

Table I

Tape Helix Parameters from Sensiper

($\delta/p = 1/10\pi$, $\psi = 10°$)

ka	z'	$\dfrac{h_o a}{ka}$	$\dfrac{v_{-1}}{c}$	$(z_{c,-1})^{1/3}$
0.10	0.43	4.61	0.0192	0.93
0.20	0.375	5.32	0.0433	1.21
0.30	0.37	5.56	0.0747	1.43
0.40	0.37	5.72	0.118	1.69

Approximate Determination of Trends in Helix BWO Design

The results given in Table I can be put in a form more convenient for design purposes, but before this is done, the calculation (Eq. 3) will be carried out approximately

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*In the frequency range of interest, there may be as many as three distinct fundamental propagation constants, corresponding to three independent modes of propagation. Here the interest is confined to the often observed mode whose asymptotic zeroth space harmonic phase velocity is sin ψ .

for helices with pitch angles of 10°, 15°, and 20° to establish trends in the parameters.

It has been verified many times experimentally that in the frequency range of interest here the fundamental propagation constant h_0 for the wire helix is given quite accurately by the value calculated from the sheath model. The normalized characteristic impedance z' can be taken as being independent of frequency over the range of interest[*]. A representative value for this quantity can be deduced from attenuation measurements; for the attenuation constant, being by definition

$$a = \frac{1}{2} \frac{P_L}{P_z} = \frac{1}{4} \frac{1}{R} \left(\frac{P_z}{|I|^2} \right)^{-1} \tag{5}$$

where P_L = power dissipated per unit axial length, and R = effective resistance per unit axial length, measures the characteristic impedance when the effective resistance of the helical wire is known. In the case of round wire whose diameter is small as compared with helix pitch and diameter and the free space wavelength, the current can be taken as being uniformly distributed around the wire circumference and the effective resistance is easily deduced. Some results of this experimental technique for determining z' are given in Table II. The data are not extensive and are applicable to round-wire helices; however, taken with the data given by Sensiper for a very narrow tape helix, they lead the author to choose z' = 0.3 as a representative value for a tape helix whose width is somewhat greater than the value used in Sensiper's calculation. The result to be obtained, varying as the cube root of z', is quite insensitive to small errors in this quantity, so a great deal of precision is not necessary.

Table II

Values of z' Deduced from Attenuation Measurements

	ψ	ka	δ/p	$\delta/2a$	z'
(a)[†]	3.26°	0.112	0.16	0.028	0.29
(b)[††]	10.1°	0.175	0.15	0.085	0.21
(c)[††]	17.7°	0.175	0.085	0.085	0.22

[†]This case is calculated from data by R. W. Peter et al. (RCA Rev. 13, 568, 1952)
[††]These cases follow from the measurement of the Q of a helical-ring resonator by the author.

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[*]The dependence of z' on frequency as calculated by Sensiper is regarded as typical: as long as neither the zeroth nor the -1 space harmonic phase velocity is near the velocity of light, z' is nearly constant.

Carrying out the solution of Eq. 3 by using the sheath model solution for h_o and $z' = 0.3$ yields the values given in Table III; also tabulated is the quantity v_{-1}/c. At low frequencies the wavelength of the -1 space harmonic is determined predominantly by the helix pitch; thus, the calculation of v_{-1}/c is quite accurate even though the values for h_o substituted in Eq. 4 are approximate.

Table III

Approximate Helix Parameters

ka	$\left(z_{c,-1}\right)^{1/3}$			v_{-1}/c		
	10°	15°	20°	10°	15°	20°
0.10	0.86	0.93	0.99	0.019	0.028	0.039
0.20	1.09	1.16	1.18	0.042	0.063	0.085
0.30	1.37	1.42	1.40	0.072	0.107	0.142
0.40	1.69	1.71	1.70	0.114	0.167	0.221

Circuit Length and Beam Perveance for Start-Oscillation

The conditions for start-oscillation can now be determined from Eq. 1b,

$$CN = \frac{1}{2}\left(z_{c,-1}\right)^{1/3}\left(\frac{I_o}{V_o}\right)^{1/3}\left(\frac{L}{\lambda_o}\frac{c}{v_{-1}}\right) = 0.312. \tag{6}$$

Although Eq. 1a tells us that at oscillation there is a departure between the cold-phase velocity of the -1 space harmonic and the electron velocity, in typical cases one finds that the fractional departure is of the order of the quantity v_{-1}/c and is thus small. In Eq. 6 we can therefore substitute $eV_o = (1/2)mv_{-1}^2$. The number of parameters can be further reduced by relating the beam voltage and current by the perveance

$$P = 10^6 \frac{I_o}{V_o^{3/2}}.$$

The result becomes then

$$\frac{L}{\lambda_o}P^{1/3} = 7.82\left(\frac{v_{-1}}{c}\right)^{2/3}\left(z_{c,-1}\right)^{-1/3} \tag{7}$$

which gives the product of the length of the circuit (in free-space wavelengths) with the cube root of the perveance for start-oscillation in terms of the helix parameters. For a given pitch angle helix, the right side of Eq. 7 can be put in terms of ka through the values given in Table III, so that one finally has the design curves of Fig. VII-7. In
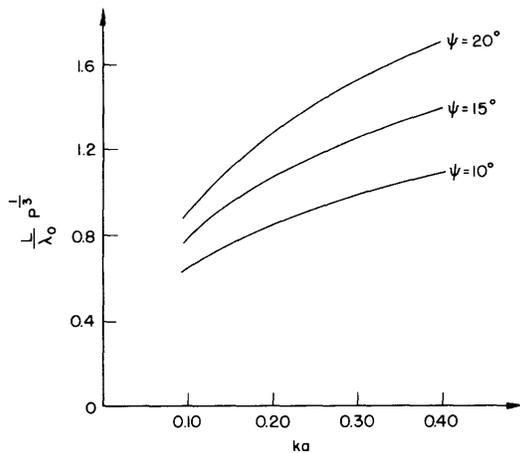
Fig. VII-7

Approximate start-oscillation
conditions for the tape-helix
backward wave oscillator.
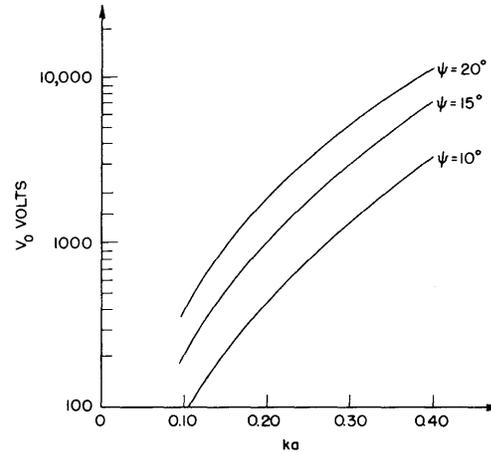
Fig. VII-8

Plot of beam voltage vs ka for
equality between the -1 space
harmonic and stream velocities.

Fig. VII-8, beam voltage vs oscillation frequency is plotted by assuming equality between the cold phase velocity of the -1 space harmonic and the electron velocity, which is an approximation but a rather good one.

Discussion of Results

One can readily establish from the curves of Fig. VII-7 that if the length-perveance product is sufficient to sustain oscillations at a given frequency, it will be more than sufficient at any higher frequency; thus, oscillators which are to cover a frequency band should be designed for the low-frequency extreme if the perveance is held constant. For a given frequency of oscillation (λ_o, k fixed) one notes that the minimum length-perveance product obtains for small helix radii. The general trend of smaller pitch angle helices resulting in smaller length-perveance products is apparent.

Since the interacting field decays rapidly with radial distance from the helix, for maximum coupling it is important that the beam come as close as possible to the helix wire. Throughout it has been tacitly assumed that the beam is in rectilinear flow within the helix. As a result of the $e^{j\theta}$ dependence of the interacting field, one finds that if the electrons themselves traverse a helical path and rotate about the cylinder axis with angular frequency ω_L, then the effective propagation constant of the interacting field is

$$h'_{-1} = \frac{h_{-1}}{1 \pm \omega_L/\omega}.$$

(8)

This effect influences BWO operation in regard to both starting conditions and frequency of oscillation and must be accounted for if ω_L/ω is not small.

Conclusions

Curves have been presented to show trends in helix BWO design. Approximations have been made in arriving at these curves so they should not be taken as being absolutely accurate. It is emphasized that the quantities z' and h_o which are necessary for the calculations are, either directly or indirectly, measurable in the laboratory. The presence of dielectric support for the helix has not been accounted for; with some additional complications, the procedure which has been followed could be applied to the case of a tape helix in a thick dielectric tube.

L. Stark

References

1. S. Sensiper: Electromagnetic Wave Propagation on Helical Conductors, Technical Report 194, Research Laboratory of Electronics, M.I.T. May, 1951

2. H. Heffner: Space Harmonic Interaction Tubes for Millimeter Waves, Quarterly Status Report No. 19, Electronics Research Laboratory, Stanford University, Palo Alto, California

3. Internally Coated Cathodes

Several new tubes of the diode type having internally coated cathodes have been tested. Figure VII-9 shows a saltshaker-like structure with the emission "surface" provided by ten holes 0.0157 inch in diameter.

The measured current was approximately 17 ma for a plate potential of 500 volts. The tube shown in Fig. VII-9 has been life-tested at approximately 13 ma $[\approx 1 \text{ amp/cm}^2$ beam density]. The duration of continuous test was about 750 hours with a total operating time of well over 800 hours. No decrease in the emission could be observed. As in the case of previously tested internally coated cathodes, this tube proved to be extremely temperature-sensitive. Currents as high as 80 to 100 ma could be easily obtained by increasing the temperature slightly. Further increase of current was prevented only by inadequate plate dissipation.

Tests with two multiple-hole tubes similar in structure to the one shown in Fig. VII-9 indicated that the current emission was roughly proportional to the number of holes. This result seemed to eliminate the possibility that the emission came from a monolayer of coating deposited on the outer surface of the cathode during the activation process. Beam shape testing experiments provided further evidence toward the validity of this statement. A single-hole cathode with a movable carbon film target as anode was used to
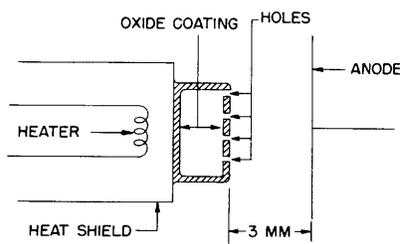
Fig. VII-9
Internally coated cathodes.

observe the shape of the electron beam. The resulting electron emission seemed to be concentrated in a thin beam having a diameter of the same order of magnitude as the hole.

Further experiments and calculation will be made in an effort to design a high-current-density gun structure for applications in microwave tubes.

C. Fried