

XXI. NETWORK SYNTHESIS

Prof. E. A. Guillemin
Dr. M. V. Cerrillo

Dr. F. M. Reza
E. F. Bolinder
R. A. Pucel

M. Strieby
P. M. Lewis

A. RESEARCH OBJECTIVES

Problems being worked upon by members of this group are directly or indirectly concerned with providing better means for handling the synthesis of networks for various specified response characteristics. As is well known, the process of synthesis consists essentially of two parts: (a) the solution to an approximation problem which yields a realizable rational function of the complex frequency satisfying stated requirements regarding desired response and acceptable tolerances; (b) the construction of a network schematic that exactly realizes the rational response function determined in part (a).

The approximation problem assumes one of two general aspects according to whether the desired response is specified as a function of frequency or as a function of time. When specified as a function of frequency, the solution to the approximation problem involves only the consideration of behavior patterns with frequency as the independent variable. When the desired response is specified as a time function, the problem of converting these data to equivalent specifications in terms of frequency arises, and brings with it the necessity of relating tolerances in the frequency function to tolerances in the associated time function.

These added considerations make the solution to the approximation problem extremely difficult, since the relation between the time and frequency domains is mathematically intricate and computationally tedious to deal with. Hence much consideration is given to providing a better understanding of these relationships alone, for it is felt that all other aspects of the approximation problem in time-domain synthesis will readily be clarified once a suitable means for correlating the time and frequency domains is at hand.

Although this problem has been pursued for some time, and numerous aspects have been clarified and some methods of synthesis have been devised which exploit these results, the summation of all this work does not yet constitute a satisfactory solution to the initial difficulty. This fact indicates the mathematically profound character of the problem. It will eventually be solved only through continued stubborn attack upon every promising approach that ingenuity can reveal.

In the construction of a network schematic when an appropriate rational function is given, a number of specific problems that are not yet adequately solved from a practical point of view are also being studied. One of these is concerned with devising better methods of finding networks for driving-point impedances that will avoid mutual inductive coupling and if possible provide means for accommodating incidental loss. These

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investigations are specifically concerned with improving upon the Brune, the Bott and Duffin, and the Darlington procedures, which are the currently known methods of dealing with this problem.

Another problem in this category is concerned with RC network synthesis; specifically with finding a computationally simple way of carrying out the procedure of B. J. Dasher (Technical Report No. 215) in order to make this scheme more nearly a practical reality. It is felt in this connection that RC networks are of considerable practical potential value and that their extensive use is hampered only by the fact that presently available methods of design are computationally too involved and cumbersome to appeal to the engineer who is the prospective user and who ought to benefit from the results of the kind of research reported here.

These are the major themes pursued by this research group. Numerous minor topics that inject themselves collaterally, directly or indirectly add their bit to a better understanding of the network picture as a whole.

E. A. Guillemin

B. REGARDING A RATIONAL APPROXIMATION TO e^{-S}

Before the advent of time-domain synthesis methods, this problem was described as the design of a linear phase shift network, which could be realized in terms of either an allpass network having a constant amplitude response, or as a lowpass filter having an essentially linear phase characteristic in its passband. The allpass network in such a design provides a phase characteristic with nearly constant slope up to some specifiable frequency, beyond which the phase curve approaches a horizontal asymptote.

In the light of our present more complete knowledge of the relation between transient (time domain) response and frequency response it seems worthwhile to reassess these older methods of approach. As will be seen, it turns out that these methods, if properly handled, can be used successfully as means for obtaining rational approximants to the delay factor e^{-S} that compete in quality very favorably with any of the time-domain synthesis procedures more recently developed for dealing with this same problem, and require far less disagreeable computational labor in carrying them out.

The oldest form of delay network involves an unbalanced ladder network having inductances in its series branches and capacitances in its shunt branches. When terminated in a resistive load, such a network has a transfer impedance of the form

$$Z_{12}(s) = \frac{1}{Q(s)} = \frac{1}{m + n} \quad (1)$$

where $Q(s)$ is a Hurwitz polynomial, and m and n are, respectively, its even and odd parts. The real and imaginary parts of this impedance for $s=j\omega$ are readily seen to be given by

$$\operatorname{Re}\left[Z_{12}(j\omega)\right] = \left[\frac{m}{m^2 - n^2}\right]_{s=j\omega} \quad (2)$$

and

$$\operatorname{Imag}\left[Z_{12}(j\omega)\right] = \left[\frac{-n}{m^2 - n^2}\right]_{s=j\omega} \quad (3)$$

The real and imaginary parts of $e^{-j\omega}$, on the other hand, are $\cos \omega$ and $-\sin \omega$; hence the approximation problem may be stated by writing

$$\left[\frac{m}{m^2 - n^2}\right]_{s=j\omega} \approx \cos \omega \quad (4)$$

$$\left[\frac{-n}{m^2 - n^2}\right]_{s=j\omega} \approx \sin \omega \quad (5)$$

A particularly simple way of obtaining an approximant over a frequency range $0 \leq \omega \leq \omega_c$ is to let the zeros of m and n throughout this range coincide, respectively, with the zeros of $\cos \omega$ and $\sin \omega$. Since the even and odd parts of a Hurwitz polynomial have simple zeros alternating on the j -axis, this procedure is sure to yield a polynomial $Q(s)$ that is Hurwitz, and hence the transfer impedance, Eq. 1, is realizable. Since all of its zeros lie at infinity, a realization in the form of the simple unbalanced ladder described above is assured. Let us see what kind of time-domain properties this design has.

To this end it is necessary that we determine the way in which the magnitude of Z_{12} (or of its real or imaginary parts) behaves with regard to frequency. Suppose we choose a total of r zeros for m or n and let

$$\text{the zeros of } m \text{ fall at } 1, 3, 5, \dots, 2r-1 \quad (6)$$

$$\text{the zeros of } n \text{ fall at } 0, 2, 4, \dots, 2r$$

Since by Eqs. 1, 2, and 3

$$\operatorname{Re}\left[Z_{12}(j\omega)\right] = \left[\frac{1}{m} \frac{1}{1 + (n^2/m^2)}\right]_{s=j\omega} \quad (7)$$

$$\operatorname{Imag}\left[Z_{12}(j\omega)\right] = \left[\frac{1}{n} \frac{1}{1 + (m^2/n^2)}\right]_{s=j\omega} \quad (8)$$

$$|Z_{12}(j\omega)|^2 = \left[\frac{1}{m^2} \frac{1}{1 + (n^2/m^2)}\right]_{s=j\omega} = \left[\frac{1}{n^2} \frac{1}{1 + (m^2/n^2)}\right]_{s=j\omega} \quad (9)$$

it becomes clear that the magnitude behaviors of the envelope of the real or imaginary part, or that of the magnitude, are all practically coincident, and that a good approximation to this magnitude behavior may be had through simply computing the values of

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$1/m$ at the zeros of n , or the values of $1/n$ at the zeros of m . A smooth curve passed through either of these sets of values is the envelope for the real or imaginary part, and at the same time the magnitude of Z_{12} with an approximation that becomes better with an increase in the total number of zeros r .

Suppose we compute m at the zeros of n . We have

$$\left. \begin{aligned} m(0) &= [1 \cdot 3 \cdot 5 \dots (2r - 1)] [1 \cdot 3 \cdot 5 \dots (2r - 1)] \\ m(2) &= [1 \cdot 3 \cdot 5 \dots (2r + 1)] [1 \cdot 3 \cdot 5 \dots (2r - 3)] \\ m(4) &= [1 \cdot 3 \cdot 5 \dots (2r + 3)] [1 \cdot 3 \cdot 5 \dots (2r - 5)] \\ &\dots\dots\dots \\ m(2r - 2) &= [1 \cdot 3 \cdot 5 \dots (2r - 1 + 2r - 1)] [1] \end{aligned} \right\} \quad (10)$$

If we let $f(\omega)$ be the envelope function, then we get

$$\begin{aligned} \frac{f(2)}{f(0)} &= \frac{2r - 1}{2r + 1}, \quad \frac{f(4)}{f(2)} = \frac{2r - 3}{2r + 3}, \quad \frac{f(6)}{f(4)} = \frac{2r - 5}{2r + 5}, \dots \\ \frac{f(2r - 2)}{f(2r - 4)} &= \frac{2r - (2r - 3)}{2r + (2r - 3)} = \frac{3}{4r - 3} \end{aligned} \quad (11)$$

As a numerical illustration, these ratios yield for $f(\omega)$ the following results:

For $r = 5$ and the points $\omega = 0, 2, 4, 6, 8$

$$f = 1, 0.818, 0.440, 0.147, 0.026 \quad (12)$$

For $r = 10$ and the points $\omega = 0, 2, 4, \dots, 18$

$$f = 1, 0.904, 0.668, 0.401, 0.193, 0.0564, 0.0164, 0.00347, 0.000496, 0.00004 \quad (13)$$

From plots of these functions or from the analytic relations, Eq. 11, one can see that the magnitude or envelope function has nearly gaussian behavior over most of the pertinent frequency range and hence the impulse response of a network having this kind of a transfer function will also be very nearly a gaussian pulse. Hence we need not fear that the response will contain objectionable oscillations.

However, we want the response to be a rather narrow gaussian pulse if the approximation of $Z_{12}(s)$ to e^{-st_0} is to be a good one, and so we think of choosing larger and larger values of r . In this connection we now observe that this design is uneconomical in the sense that it apparently wastes poles. In the above envelope values for $r = 10$ we note that beyond about the sixth zero of m or of n the amplitude response is so low that the rest of the zeros are doing practically no good; yet we cannot simply omit them since we then revert to a lower value of r .

We have, for simplicity, placed zeros at $\omega = 0, 1, 2, \dots$, since we can subsequently

change the spacing (or zero density along the j -axis) at will. This density controls the amount of delay t_0 desired. The narrowness of the resulting gaussian pulse in the time domain is then controlled by choosing r sufficiently large. If we stick to this kind of design, the law of diminishing returns soon makes itself felt and we anticipate poor efficiency from the standpoint of getting a given quality of result with a rational Z_{12} of lowest degree. A partial reason for this poor efficiency may be seen in the fact that the gaussian type of frequency behavior is ultraconservative with regard to ripple suppression in the time domain.

So long as we keep r relatively small (say $r = 5$), we get a rather efficient result, and we can make the width of our pulse in the time domain as small as we please by stretching the frequency scale. As we do so, the amount of delay decreases. Let us, therefore, imagine that we cure this difficulty by cascading the network having this Z_{12} with an appropriate length of lossless transmission line (properly terminated) that increases the delay by any desired amount and leaves the pulse shape of the over-all impulse response unchanged. Since the magnitude of the over-all transfer impedance is the magnitude of Z_{12} , and since this function has negligible values beyond some finite frequency, we see that although the lossless transmission line provides a linear phase characteristic out to infinite frequency, only a finite portion of this perfect behavior is utilized and the rest is wasted. This fact tells us that we don't really need a lossless transmission line in order to get practically the same result. All we need is a network that provides an essentially linear phase and flat amplitude response up to a frequency where the magnitude of Z_{12} has fallen to negligible values (1 percent or so).

The transfer function for this network is readily found. It has the form

$$Z_{12}^*(s) = \frac{P(-s)}{P(s)} = \frac{m_1 - n_1}{m_1 + n_1} \quad (14)$$

The real and imaginary parts are

$$\operatorname{Re} \left[Z_{12}^*(j\omega) \right] = \left[\frac{m_1^2 + n_1^2}{m_1^2 - n_1^2} \right]_{s=j\omega} \quad (15)$$

and

$$\operatorname{Imag} \left[Z_{12}^*(j\omega) \right] = \left[\frac{-m_1 n_1}{m_1^2 - n_1^2} \right]_{s=j\omega} \quad (16)$$

The simplest procedure here is to consider only the imaginary part and use as an approximation criterion

$$\left[\frac{m_1 n_1}{m_1^2 - n_1^2} \right] \approx \sin \omega \quad (17)$$

Choosing alternate zeros for m_1 and n_1 (so that $P(s)$ becomes a Hurwitz polynomial) and letting these coincide with the zeros of $\sin \omega$ over the appropriate frequency range, fixes m_1 and n_1 and hence $P(s)$ completely.

It is easy to satisfy one's self that the real part, Eq. 15, is thereby also approximated; or we can use an alternate simple procedure to approximate the function, Eq. 15, from its known zeros. In either case we determine the rational function Z_{12}^* which, multiplied by an appropriate Z_{12} , yields a desired solution to the approximation problem originally stated in which the amount of delay, the width of the resulting time-domain pulse, and to some extent its shape, are separately controllable through an appropriate choice of the total number of poles and their allotment to the component functions Z_{12} and Z_{12}^* .

E. A. Guillemin