L. D. Smullin
Prof. L. J. Chu
A. G. Barrett
A. Bers
D. L. Bobroff
C. Fried
H. A. Haus
A. J. Lichtenberg
H. Shelton

A. A ONE-DIMENSIONAL ELECTRON BEAM AS A FOUR-TERMINAL NETWORK

L. J. Chu (1) suggested the use of a kinetic power density in the small signal analysis of one-dimensional electron beams. The kinetic power density is defined by the product

$$S_k = \frac{1}{2} \operatorname{Re}\left[V_1 J_1^*\right] \tag{1}$$

where

$$V_1 \equiv -\frac{m}{e} v_o v_1$$

$m/e > 0$, v_o is the time average velocity, v_1 is the complex amplitude of the sinusoidal velocity modulation, and J_1 is the complex amplitude of the sinusoidal current density modulation.

The kinetic power is conserved in structures which do not extract rf power from the beam; for example, a velocity jump, an electron gun with a high external impedance, and the like. If the electron beam is coupled with a lossless external structure like a helix, then the sum of the kinetic power and the electromagnetic power in the external system is conserved.

It can be shown that the concept of the kinetic power and its properties, explained above, are applicable to any electron flow in which the time average current and velocity and their rf perturbations at any point are parallel (2). This situation is met, for instance, in a concentric, spherical, one-dimensional flow of electrons and, in general, if the electron trajectories follow the lines of an infinite magnetic field.

Let us consider a beam in the absence of an external circuit. A curvilinear coordinate system can be built around the electron trajectories. A simplified treatment results if it is assumed that the time average and rf parameters of the beam depend only upon the curvilinear coordinate along the electron trajectories, u^1 (using the notation of reference 3, p. 39). The voltage V_1 and the current density J_1 as a function of u^1 describe completely the rf behavior of the beam. It can be shown that a transformation of the dependent variables V_1 and J_1 and the use of the force equation and the continuity equation lead to a pair of equations similar to transmission-line equations.

$$\frac{dU}{d\left(h_1 u^1\right)} = \frac{1}{j\omega\epsilon} \frac{I}{h_2 h_3} \tag{2}$$

$$\frac{dI}{d\left(h_1 u^1\right)} = -j\omega\epsilon \frac{\omega_p^2}{v_o^2} h_2 h_3 U \tag{3}$$

where ω_p is the plasma frequency, in general a function of u^1, $U = V_1 \exp(j\theta)$, $I = J_1 h_2 h_3 \exp(j\theta)$ (I is closely related to the total rf current in the beam), θ is the transit angle,

$$\theta = \omega \int_0^{u^1} \frac{h_1 du^1}{v_o}$$

and h_1, h_2, h_3 are the parameters of the curvilinear coordinate system (3).

Thus, a section of the beam between two values of u^1 can be considered as a linear, passive, reciprocal, four-terminal network; therefore, the U's and I's at the two surfaces are related by a matrix transformation

$$\begin{pmatrix} U_2 \\ I_2 \end{pmatrix} = \begin{pmatrix} A & jB \\ -jC & D \end{pmatrix} \begin{pmatrix} U_1 \\ I_1 \end{pmatrix} \tag{4}$$

The coefficients A to D are all real and satisfy the reciprocity relation, $AD - BC = 1$.

A simple model of a beam with periodic scallops can be analyzed on this basis. Such a beam may be treated as a cascade of lossless, symmetrical, reciprocal, four-terminal networks. The complex phase function Γ of the system is given by

$$\cosh \Gamma = A = D$$

and its iterative impedance is

$$Z_i = \frac{1}{jC} (A^2 - 1)^{1/2}$$

Γ is either pure real or pure imaginary. Growing waves exist when $A > 1$. Then, the rf velocity, v_1, and the rf current density, J_1, increase exponentially. Their values at the n-th reference plane are

$$v_{1n} = \frac{e}{m} \frac{1}{v_o} \left[A_1 \exp(-\Gamma n) + A_2 \exp(+\Gamma n)\right] \exp(-j\theta_o n) \tag{5}$$

$$J_{1n} = \frac{1}{Z_i h_2 h_3} \left[A_1 \exp(-\Gamma n) - A_2 \exp(+\Gamma n)\right] \exp(-j\theta_o n) \tag{6}$$

θ_o is the transit angle in one network.

In a conventional cascade filter network the solution which grows with increasing n

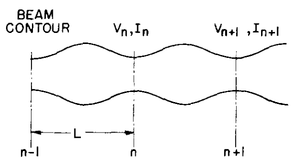

Fig. VII-1
A scalloped beam as a periodic network.

can be excited only by reflection from a termination. On the other hand, the only sensible boundary conditions in a beam problem are the velocity and current modulations at an input reference plane, say n = 0. The phenomenon of spatial growth is linked to particular input conditions and not to reflection from a termination.

The spatial periodicity which causes growth can be found from an analogy with a periodically loaded transmission line. The so-called attenuation bands of a loaded transmission line occur when the wavelength of the unloaded transmission line is equal to 2L/k, where L is the spatial period of the loading and k is an integer. Similarly, we have to expect that growing waves will be found on a scalloped electron beam (see Fig. VII-1) when the wavelength of the analog transmission line for the beam without scallops is equal to 2L/k, where k is an integer. The mentioned wavelength is given by

$$\lambda_p = 2\pi v_o/\omega_p$$

Growth occurs over a wider range of values of λ_p if the scallops are more pronounced.

B. Agdur,* H. A. Haus

References

1. L. J. Chu, The concept of kinetic power, IRE Conference on Electron Devices, Durham, N. H., June 1951.
2. P. Parzen, Research Report TW-20, New York University (May 1953).
3. J. A. Stratton, Electromagnetic theory (McGraw-Hill Book Company, Inc., New York, 1941).

B. A GENERALIZED THEORY OF NOISE IN ELECTRON BEAMS

In the past, different models for the noise input at the potential minimum of an electron gun have been used to compute the noise-current standing wave in an electron beam following the gun (1, 2). Assuming an input consisting of uncorrelated current and velocity modulations (3) a minimum noise figure of a traveling-wave tube has been found. To date, no analysis exists which would give an expression for the velocity and current fluctuations at a reference plane beyond the potential minimum at high frequencies. In the absence of such an analysis it is useful to obtain the most general expressions for the noise current in an electron beam which are not dependent upon a model of the noise

*On leave from Chalmers Institute of Technology, Gothenburg.

input at the potential minimum. The work outlined below leads to such expressions. The expression is given for the minimum noise figure of a traveling-wave tube obtainable by means of velocity jumps or related schemes. The following assumptions are made:

(a) The velocity spread of the beam at any reference plane is small compared to the average velocity of the beam. L. R. Walker (4) has shown that a treatment on a small-signal, single-velocity basis is then justified.

(b) The rf behavior of the beam is determined by the knowledge of the velocity and current modulation of the beam at any one reference plane. This includes one-dimensional beams and, approximately, a beam of finite dimensions in which only the two main space-charge waves are of importance.

(c) The direct-current density, the average velocity, v_o, the rf current density and velocity, are all parallel at any one point.

Assumptions (a) and (c) allow us to treat a section of the beam as a linear four-terminal network as outlined in section A. Four quantities can be defined at any one reference plane: $\Phi(\omega)$ is the self-power spectrum of the noise voltage (for the definition of the voltage see section A, Eq. 1); $\Psi(\omega)$ is the self-power spectrum of the noise current; $\Pi(\omega)$ is the real part of the cross-power spectrum of the noise voltage and noise current; and $\Lambda(\omega)$ is the imaginary part of the cross-power spectrum. All of these quantities are the Fourier transforms of the corresponding autocorrelation and crosscorrelation functions, respectively. Passing the beam through the gap of a cavity of high Q makes $\Psi(\omega)$ accessible to measurement.

The following relationships hold in a drift space: For a given Φ_1, Ψ_1, and Λ_1 at one reference plane (say at z = 0), Φ, Ψ, and Λ at a plane z are

$$\Phi = \cos^2 \frac{\omega_p z}{v_o} \Phi_1 + Z_o^2 \sin^2 \frac{\omega_p z}{v_o} \Psi_1 - \frac{1}{2} Z_o \sin \frac{2\omega_p z}{v_o} \Lambda_1 \tag{1}$$

$$\Psi = Y_o^2 \sin^2 \frac{\omega_p z}{v_o} \Phi_1 + \cos^2 \frac{\omega_p z}{v_o} \Psi_1 + \frac{1}{2} Y_o \sin \frac{2\omega_p z}{v_o} \Lambda_1 \tag{2}$$

$$\Lambda = \frac{1}{2} Z_o \sin \frac{2\omega_p z}{v_o} \left[Y_o^2 \Phi_1 - \Psi_1 \right] + \cos \frac{2\omega_p z}{v_o} \Lambda_1 \tag{3}$$

where

$$Z_o = \frac{1}{Y_o} = -2 \frac{V_o}{I_o} \frac{\omega_p}{\omega}$$

V_o is the potential of the beam, and I_o is the direct current in the beam.

The maximum of Ψ and the maximum of Φ lie a distance $\lambda_p/4 = v_o/8\pi\omega_p$ apart. The imaginary part of the cross-power spectrum, Λ, is zero at a maximum of Φ or Ψ.

These facts are familiar from conventional transmission-line theory. The product of the maximum and minimum of Ψ is given by

$$\Psi_{max}\Psi_{min} = Y_o^2\left[\Phi_1\Psi_1 - \Lambda_1^2\right] \tag{4}$$

When complete correlation exists between the voltage and current, we have

$$Y_o^2\left[\Phi_1\Psi_1 - \Lambda_1^2\right] = Y_o^2\,\Pi_1^2 \tag{5}$$

In general, however, Eq. 5 does not hold. Equation 4 is therefore a result that differs from the conventional transmission-line theory.

The effect of a lossless transformer (a velocity jump, the accelerating region in an electron gun, etc.) can be taken into account by a matrix transformation. The matrix given in Eq. 4 of section A applies in these cases. It can be shown that the expression $\Phi\Psi - \Lambda^2$, and the real part of the cross-power spectrum, Π, are invariant with respect to such a transformation. If it is assumed that only lossless transformers act upon the beam, and if the values of Φ, Ψ, and Λ at a plane beyond the potential minimum in the gun are indicated by a subscript "o," then the product $\Psi_{max}\Psi_{min}$ in a beam of "characteristic" admittance Y_o following the transformers is

$$\Psi_{max}\Psi_{min} = Y_o^2\left[\Phi_o\Psi_o - \Lambda_o^2\right] \tag{6}$$

Φ_o, Ψ_o, and Λ_o are determined by the noise-smoothing process in the region of the potential minimum.

An infinitesimal rf gap of external resistance, R, gives a $\left(\Phi_2\Psi_2 - \Lambda_2^2\right)$ at its output indicated by the subscript "2" in terms of Φ_1, Ψ_1, Λ_1, and Π_1 at its input.

$$\Phi_2\Psi_2 - \Lambda_2^2 = \Phi_1\Psi_1 - \Lambda_1^2 + R^2\Psi_1^2 - 2R\Pi_1\Psi_1$$

From the preceding arguments it follows that the noise-current power spectrum of a beam in a drift space is completely defined by, say, the position of Ψ_{max}, the values of Ψ_{max} and Ψ_{min}, and the real part of the cross-power spectrum, Π. With the assumption that only lossless transformers act upon the beam, the product $\Psi_{max}\Psi_{min}$ is directly determined by the character of the noise-smoothing process in the potential minimum region of the electron gun.

Measurements will be carried out in the near future in an attempt to find the parameters of this simplified theory.

A noise figure of a traveling-wave tube can be minimized by a proper choice of lossless transformers acting upon the beam. The minimum noise figure of the tube obtainable by this method is

$$F_{min} = 1 + \frac{4\pi}{kT} (4QC\, f_{max} f_{min})^{1/2} \left(\Phi_o \Psi_o - \Lambda_o^2\right)^{1/2} - \Pi_o$$

where f_{max} and f_{min} are the maximum and minimum of the function $f(QC, d, \delta z)$ defined by Watkins (2).

H. A. Haus

References

1. J. R. Pierce, Traveling-wave tubes (Van Nostrand, New York, 1950).
2. D. A. Watkins, Technical Report 31, Electronics Research Laboratory, Stanford University, March 15, 1951.
3. J. R. Pierce, Minimum noise figure of a traveling-wave tube (to be published in the Journal of Applied Physics).
4. L. R. Walker, private communication.

C. NOISE IN ELECTRON BEAMS

The sensitivity of our noise-measuring apparatus was considerably improved by putting into operation the synchronous detector mentioned in the Quarterly Progress Report of October 15, 1953. The gain in sensitivity was approximately 12 db, making possible accurate measurements of noise on confined-flow electron beams. The parallel Pierce gun used in these experiments is identical with the one employed for the pulsed-beam noise measurements described in the October report.

The noise measurements were performed first on steady-flow beams for various values of the magnetic field, beam-forming electrode bias, and cathode temperature. A typical curve of the rms noise current is shown in Fig. VII-2.

Varying the magnetic field (above a required minimum collimating field) had no effect on the noise within the observed range of 258-602 gauss; this was expected for a parallel-flow electron beam.

The noise-current curves for the same magnetic field and anode voltage values as those of Fig. VII-2, but with different biasing voltages on the beam-forming electrode in the range between ±6 volts, showed no essential change from that of Fig. VII-2, even though the resulting current variation was appreciable ($I_o = 2.1$ ma at $E_{cc} = -6$ volts and 3.9 ma at $E_{cc} = +6$ volts).

Increasing the cathode temperature above the minimum required to obtain space-charge-limited emission resulted in a slight increase of perveance which could be attributed to thermal expansion in the gun region. The noise curves were, again, strikingly similar to the curve shown in Fig. VII-2.

In all of these noise curves the maximum standing-wave ratio was approximately

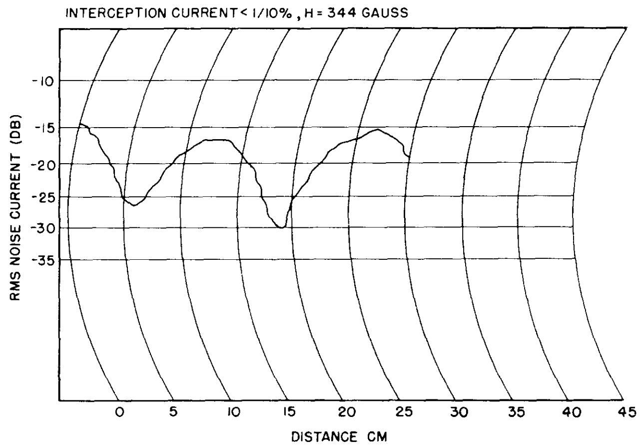

Fig. VII-2
RMS noise current for confined-flow, space-charge-limited, steady electron beam: V_o = 1500 volts; I_o = 2.8 ma; pressure ≈ 1.5×10^{-7} mm Hg.

14-15 db, with a first minimum of 26-27 db, and a deeper second minimum of 29-30 db below shot noise level. All measurements were well reproducible.

Attempts have been made to bridge the steady-current beam measurements just described and the pulsed measurements (with a relatively unneutralized beam) given in the Quarterly Progress Report of October 15, 1953; therefore, the long-pulse experiment given in detail in the Quarterly Progress Report, January 15, 1954, was repeated for confined-flow beams. Pulse lengths of 2.5 μsec, 25 μsec, 250 μsec, 2500 μsec, and 25,000 μsec were used at a 50-percent duty cycle. The resulting noise curves were again similar to the one shown in Fig. VII-2 and no growing noise was observed for any time delay of the gating pulse.

When the fixed, 1-μsec pulser was used, however, growing noise waves were observed, as reported in October. The only apparent difference is the fact that the fixed, 1-μsec pulse had a very short rise time, whereas the variable width pulse had a rise time of approximately 1.5 μsec. This was quite in contrast to the results obtained on convergent-flow beams, where a continuous transition could be observed between the growing noise wave and the noise standing wave as the time delay of the gating pulse was varied from zero to several hundred microseconds.

C. Fried

D. LOW-NOISE GUN DESIGN

The investigation of passive electron beam transducers, assumed to be described by the Rack-Llewellyn one-dimensional model and equations, has been continued.

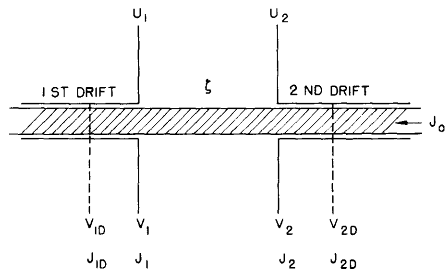

Fig. VII-3

Electron beam accelerating region with its preceding and following drifts.

u, dc electron velocity
v, electron velocity fluctuation
J, convection current density fluctuation
J_o, dc current density
ζ, Llewellyn-Peterson space-charge factor
δ, u_2/u_1

A general accelerating region, Fig. VII-3, has been analyzed for its noise-reduction properties. The noise is assumed to be determined entirely by the velocity fluctuations at the potential minimum in front of an emitting cathode. The noise-reduction factor of the region is defined as

$$r_1 = \left| \frac{v_{2D\,max}}{v_{1D\,max}} \right|^2$$

where $v_{1D\,max}$ and $v_{2D\,max}$ are, respectively, the maxima of the velocity fluctuations in the drift regions preceding and following the accelerating region.

The noise-reduction factor, r_1, was found to be a function of three variables: δ, ζ, and y_1^*.

$$r_1 = \frac{\left(Ay_1^* + B\right)^2 + C^2\left(Ey_1^* + 1\right)^2}{\frac{C^2}{\delta} y_1^{*2} + 1}$$

A, B, C, and E are all functions of δ and ζ only, while y_1^* is a function of the "input ratio" $|J_1/v_1|$, δ, and ζ, at a given frequency of operation, ω, and required dc current density, J_o.

The noise-reduction factor, r, of an electron gun with n such regions is given by $r = r_1 \times r_2 \times \ldots \times r_n$.

The important case of $\zeta = 0$ (approximated in regions with thin electron beams) yields

$$r_1 = \frac{1}{\delta^2} = \left(\frac{u_1}{u_2}\right)^2$$

This is the same as the noise reduction of a velocity jump (a zero transit-time accelerting region). The above result, however, was obtained for arbitrary transit time and "input ratio."

A. Bers

E. INTERNALLY COATED CATHODES

For a time, there existed doubt as to whether the total observed emission from the internally coated cathode came through the cathode hole, or whether less dense current emission was coming from other regions of the cathode activated by migrated, diffused, or evaporated, active material. Investigation of a hollow internally coated cathode with a 0.015-inch cathode hole by means of a movable anode containing a 0.005-inch hole behind which was a collector, showed no evidence of any emission except from the hole, at the temperatures below 1000°C at which we worked. The beam was quite hollow under most combinations of temperature, anode voltage, anode spacing, and axial magnetic field; although under certain combinations of the first three, the emission from the hole was observed to be almost uniform. Curves of retarding potential on the collector at different anode voltages and positions produced little immediately interpretable information, except that the problem is not one of electrons coming over a single potential barrier.

The tube in Fig. VII-4 was built for determining whether or not the enclosed cavity in back of the hole has anything to do with the emission properties of the cathode and for observing the effects of electric fields applied to the coated side of the cathode. This tube yielded similar anode I-V curves, as shown in Fig. VII-5. Here, as in all of the plots, we used five temperatures, ranging from T_1 (below 700°C) to T_5 (about 850°C); and we see that at low temperatures the same characteristics exist. Figure VII-6 shows the effect of negative fields at the coating. Figure VII-7 shows the effect of a positive field. Note that at low temperature a few volts on the plate will "rob" almost all of the electrons from the 500-volt anode. Figures VII-8 and VII-9 show the effect of an axial magnetic field. From these observations, the model represented in Fig. VII-10 was set up. Here we see the voltage due to space-charge that exists between the plate and the cathode. In the region of the hole, the deviation from the case of the infinite parallel planes is only estimated. The effect of this very steep "hill" is that electrons far from the hole, leaving from "B," will in most cases be returned to the cathode without

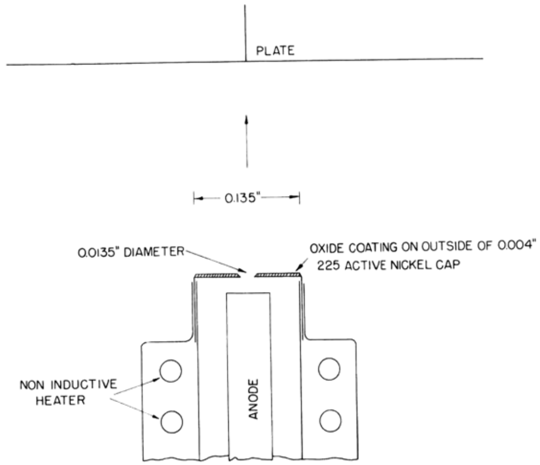

Fig. VII-4

Experimental tube.

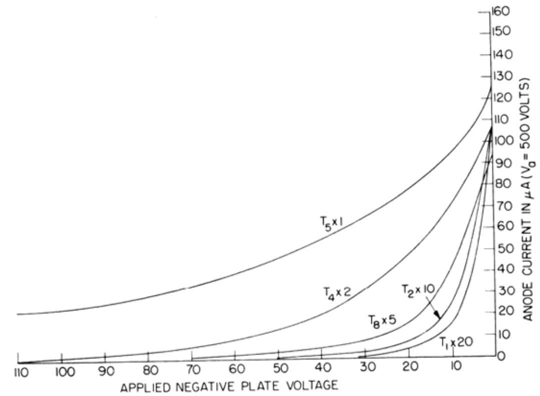

Fig. VII-6

Effect of negative field on current through hole.

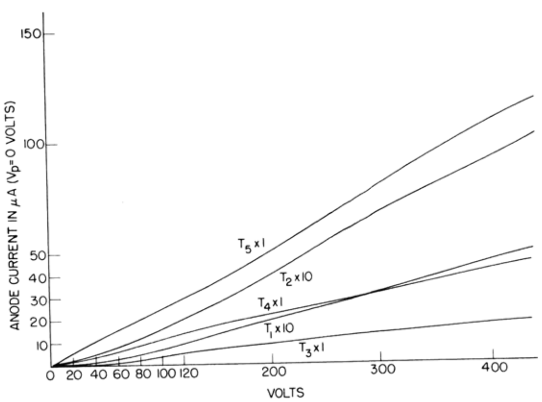

Fig. VII-5

Anode I-V characteristics.

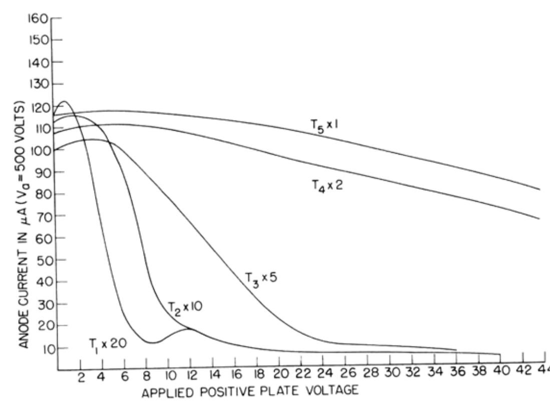

Fig. VII-7

Effect of positive field on current through hole.

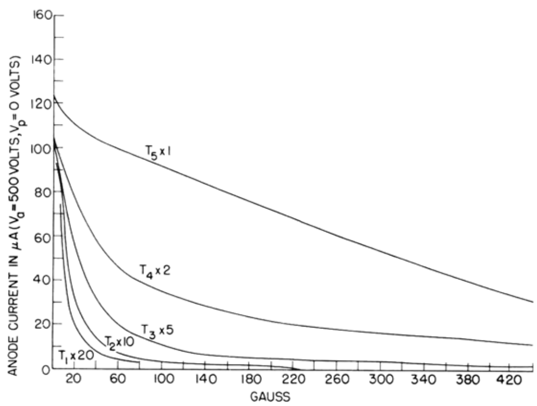

Fig. VII-8

Effect of axial magnetic field on current through hole.

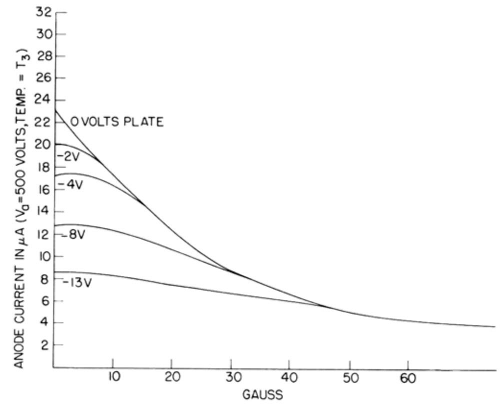

Fig. VII-9

Effect of combined axial magnetic field and negative field.

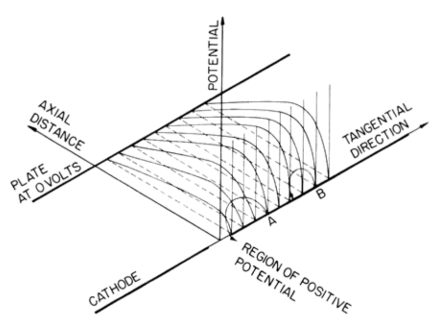

Fig. VII-10

Model explaining hollow cathode.

falling into the hole; while electrons leaving from "A," with normal or tangential velocities toward the hole, will pass through the hole. The effect of a negative voltage on the plate, then, is to steepen this "hill" and to reduce the distance from the hole from which electrons can originate. The axial magnetic field has a similar effect and, hence, the relation shown in Fig. VII-9. Positive voltages simply drain off some of the electrons that otherwise would have gone through the hole.

H. Shelton