

X. STATISTICAL COMMUNICATION THEORY

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A. INTRODUCTION

This group is interested in studying communication problems in the light of modern statistical concepts, aided by statistical methods and techniques, particularly correlation. These problems primarily concern the characteristics of stationary random processes, signal detection, and the Wiener theory of optimum linear systems.

Investigations are being made on (a) a method of Wiener for the characterization of a nonlinear system; (b) the measurement, calculation, and application of second-order correlation functions; (c) the effects of periodic sampling in the detection of a periodic signal in noise by correlation; and (d) the use of a logarithmic device in signal detection.

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B. SECOND-ORDER CORRELATION

The second-order autocorrelation function of a periodic or random function $f_1(t)$ defined as

$$\phi_{111}(\tau_1, \tau_2) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f_1(t) f_1(t + \tau_1) f_1(t + \tau_1 + \tau_2) dt$$

has been computed for a periodic wave consisting of the positive half cycles of a sinusoid as shown in Fig. X-1. The expression is

$$\begin{aligned} \phi_{111}(\tau_1, \tau_2) = \frac{1}{4\pi} \left\{ \left[1 + \cos(\tau_1 + \tau_2) \right] \cos \tau_2 - \left[\frac{2}{3} \cos^3(\tau_1 + \tau_2) - \cos(\tau_1 + \tau_2) - \frac{1}{3} \right] \cos(2\tau_1 + \tau_2) \right. \\ \left. + \frac{2}{3} \sin(2\tau_1 + \tau_2) \sin^3(\tau_1 + \tau_2) \right\} \end{aligned} \quad \begin{array}{l} \text{for } 0 \leq \tau_1 \leq \pi \\ 0 \leq \tau_2 \leq \pi \end{array}$$

This function is plotted in Fig. X-2.

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C. EFFECTS OF PERIODIC SAMPLING ON OUTPUT SIGNAL-TO-NOISE RATIO IN AUTOCORRELATION DETECTION

It is possible, by using the techniques of correlation analysis, to detect the presence of a periodic signal in random noise. When autocorrelation is performed electronically, and the input periodic signal is sinusoidal, it has been shown (1) that the

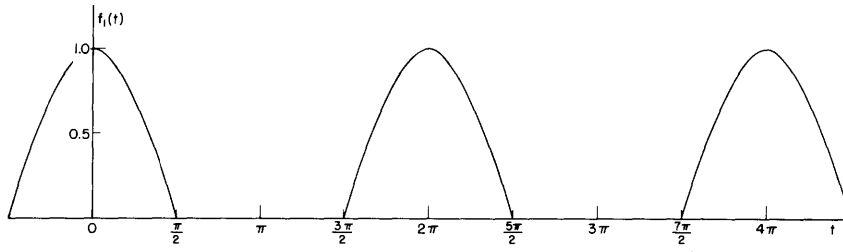


Fig. X-1

Periodic wave consisting of the positive half cycles of a sinusoid.

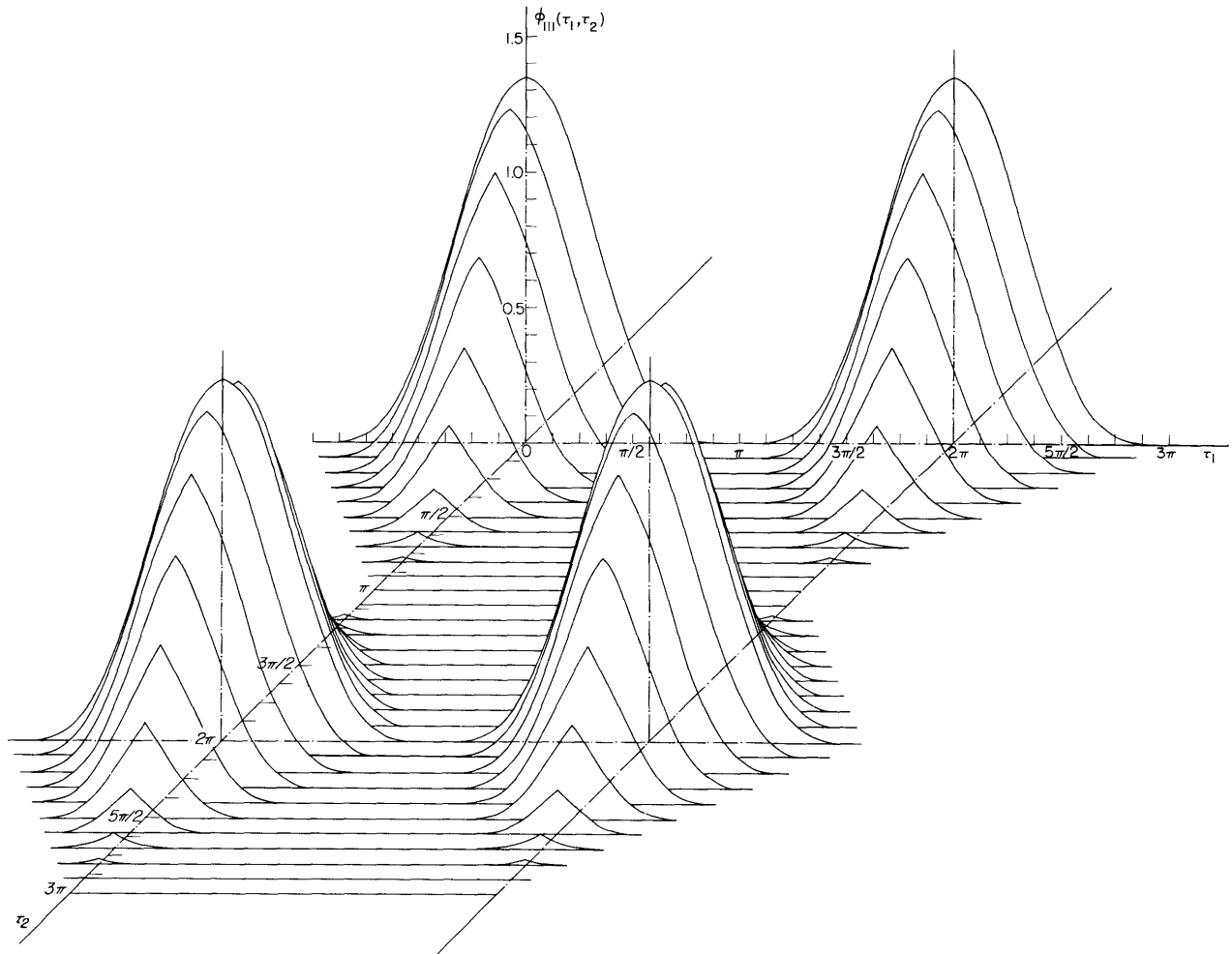


Fig. X-2

The second-order autocorrelation function $f_1(t)$.

output signal-to-noise ratio is given by

$$R_{oa} = 10 \log_{10} \frac{n}{1 + 4\rho_i^2 + 2\rho_i^4} \quad (1)$$

where n is the sample size, and ρ_i is the input noise-to-signal ratio. Equation 1 is exactly correct only for the case in which the sampling process performed by the machine is random.

Since periodic sampling is a more practical process, the equivalent of Eq. 1 has been re-derived for the case of periodic sampling. For this case, under the assumption that the initial sampling time is random, the output signal-to-noise ratio turns out to be given by

$$R'_{oa} = 10 \log_{10} \frac{n}{4\rho_i^2 + 2\rho_i^4 + B} \quad (2)$$

where all the symbols are used as defined above, and B is given by

$$B = \frac{1}{n} \frac{\sin^2 \left[2\pi n \frac{\omega_1}{\omega_0} \right]}{\sin^2 \left[2\pi \frac{\omega_1}{\omega_0} \right]} \quad (3)$$

where ω_1 is the radian frequency of the periodic signal at the input, and ω_0 is the radian sampling frequency of the machine. B is a highly oscillatory function bounded by

$$B_{\max} = n \quad (4)$$

which occurs for $\omega_1/\omega_0 = k/2$ where k is any integer, and

$$B_{\min} = 0 \quad (5)$$

which occurs for $\omega_1/\omega_0 = k/2n$ where k is any integer except $0, n, 2n, \dots$.

In the absence of other requirements, these results might serve as a criterion for deciding on a sampling frequency appropriate to a particular problem.

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References

1. Y. W. Lee, T. P. Cheatham, Jr., and J. B. Wiesner, Application of correlation analysis to the detection of periodic signals in noise, Proc. I.R.E. 38, 1165-1171 (1950).