XIX. NETWORK SYNTHESIS

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A. SYNTHESIS OF RC NETWORKS IN THE TIME DOMAIN

1. Introduction. The significance of eliminating inductors wherever possible in the design of passive lumped networks has given considerable stimulus to the problem of design of RC networks. In 1949, E. A. Guillemin established certain results of theoretical and practical significance (1). His main result consists of the following:

"... Except for a constant loss, any transfer characteristic may be approximated arbitrarily closely by means of a linear passive network containing only R's and C's, and its physical realization is always possible in an unbalanced form suitable for operation between conventional vacuum tubes."

The reader undoubtedly realizes the practical importance of this statement, which more or less puts RC networks on an equal basis with RLC networks. In mathematical terms Guillemin specifically shows that any transfer characteristic of a two-terminal pair passive RLC network with a finite number of lumped elements could be "approximated" by the transfer characteristic of a two-terminal pair passive RC network with lumped elements.

$$\left| Z_{12_{RLC}}(j\omega) \right|^2 \approx \left| Z_{12_{RC}}(j\omega) \right|^2$$

(Evidently in Guillemin's statement one must exclude the case in which $|Z_{12(RLC)}(j\omega)|^2$ becomes infinitely large. However, this case is of no importance in practice. It should be noted, too, that Guillemin's statement can be verified by the use of several different approximation techniques such as those given in references 2 and 3.)

2. Purpose. The object of this investigation is to show:

"If f(t) is the response of an RLC two-terminal pair, passive network with a finite number of lumped elements to a unit impulse applied at time t = 0, then f(t), except for a constant scale factor, can be 'approximated' by the response of a passive two-terminal pair RC network with lumped elements to a unit impulse applied at time t = 0. The tolerance of the approximation can be made as small as possible, but then one must allow a more complex RC network." A method for finding the desired RC transfer function is described. Once such a transfer function is obtained, one may consider the existing synthesis procedures for its realization.

3. A Useful Transformation. Let f(t) be the response of an RLC finite, linear, passive, lumped-parameter network to a unit impulse $u_0(t)$. Then f(t) is a continuous transcendental function on the positive real axis of the time. Also

$$f(t) \equiv 0, \quad t < 0 \tag{1}$$

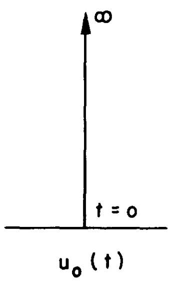

Fig. XIX-1
Unit impulse.

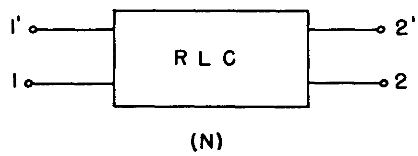

Fig. XIX-2
Network with finite linear lumped passive elements.

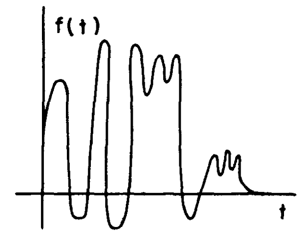

Fig. XIX-3
Assumed response of N to $u_o(t)$.

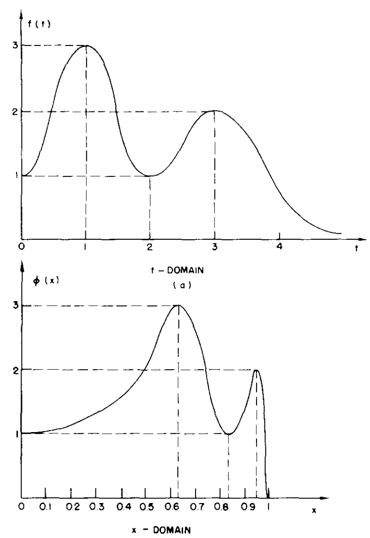

Fig. XIX-4
The transformation of the time domain into the x-domain by Eq. 2.

We may conveniently use the following transformation:

$$x = 1 - \epsilon^{-a_o t}, \quad t \geqslant 0 \tag{2}$$

where a_o is a positive constant. (See Figs. XIX-1, XIX-2, XIX-3.)

To each point of the positive real axis of the t-domain (Fig. XIX-4), there corresponds one point of the positive real axis of the x-domain. The positive real axis of the t-domain is transformed into a segment (0, 1) of the x-axis.

$$\left.\begin{array}{lll} t = 0 & \text{corresponds to} & x = 0 \\ t = \infty & \text{corresponds to} & x = 1 \end{array}\right\} \quad (3)$$

This changing of the time scale will give us a new function $\phi(x)$, instead of $f(t)$, such that if x_k corresponds to an instant t_k then $\phi(x_k) = f(t_k)$. The merit of this transformation will be better understood in the following sections.

4. Weierstrass' Approximation Theorem. This approximation theorem is well known (4, 5). It states:

If $\phi(x)$ is a continuous function in (a, b) and ϵ is a given positive number, there exists a polynomial $P(x)$ such that

$$|\phi(x) - P(x)| < \epsilon; \quad a \leqslant x \leqslant b \quad (4)$$

Since $f(t)$ and hence $\phi(x)$ are continuous functions, within the intervals with which we are concerned, we may apply inequality 4 by letting $a = 0$ and $b = 1$. Thus a polynomial

$$P(x) = a_0 + a_1 x + a_2 x^2 + \ldots a_n x^n \quad (5)$$

could be found to approximate $\phi(x)$ in the sense of inequality 4 as closely as desired.

5. Determination of an RC Transfer Function. There are a great number of approximation procedures known and used in engineering problems. We are not considering here a discussion of approximation techniques. It is fairly simple to select, for instance, a number of points on the graph of $\phi(x)$ and determine the coefficients of a polynomial $P(x)$ passing through those points. Now $P(x)$ should be transformed in a time response $P(t)$ by the use of Eq. 2.

$$P(t) = a_0 + a_1 \left(1 - \epsilon^{-a_o t}\right) + a_2 \left(1 - \epsilon^{-a_o t}\right)^2 + \ldots a_n \left(1 - \epsilon^{-a_o t}\right)^n \quad (6)$$

or

$$P(t) = A_0 + A_1 \epsilon^{-a_o t} + A_2 \epsilon^{-2a_o t} + \ldots A_n \epsilon^{-na_o t} \quad (7)$$

Now, using the Laplace transformation

$$F(s) = \int_0^\infty \epsilon^{(-st)} f(t)\, dt \quad (8)$$

one easily derives the transform of each term of Eq. 7, and finally

$$P(s) = \frac{A_0}{s} + \frac{A_1}{s + a_o} + \frac{A_2}{s + 2a_o} + \ldots \frac{A_n}{s + na_o} \quad (9)$$

The coefficients A_k are real, otherwise arbitrary. Equation 9 describes an RC transfer function whose poles are preassigned on the negative real axis of the frequency domain. This procedure is summarized in the following section.

6. Synthesis Procedure Summarized.

Step 1. Using transformation 2, graphically or analytically determine $\phi(x)$ from a given f(t).

Step 2. Using existing approximation techniques, determine a polynomial P(x) such that $|\phi(x) - P(x)| < \epsilon$ for $0 \leqslant x \leqslant 1$.

Step 3. Using Eq. 2, transform P(x) into a time function P(t).

Step 4. Using the familiar Laplace transformation of Eq. 8, find P(s), the Laplace transform of P(t).

Step 5. Use existing RC synthesis procedures such as those given in references 6 and 7 to obtain a network realization.

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References

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