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STOCHASTIC PROGRAMMING MODELS
FOR DEDICATED PORTFOLIO SELECTION

By

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INTRODUCTION

Dedicated portfolios, even when they are conservatively assembled and managed, are still subject to major uncertainties and risks. Uncertainties associated with interest rates, and their effect on bond prices, have received the greatest attention from financial planners and theorists. Other uncertainties include the cash requirements or liabilities of the dedicated portfolio, the possibility of bond re-calls or defaults due to bankruptcy, to name just a few. Joehnk (1983)[1] has a lengthier discussion of these and other risks associated with dedicated and other fixed income portfolio selection problems.

Many articles have appeared in the literature proposing immunization approaches that the dedicated portfolio manager can follow to try to control risk in the face of interest rate uncertainties. These approaches, often linked to duration measures (Fisher and Weil [1971][2], Biermag et al [1983][3]), are meant to guarantee that the portfolio's value does not fall below some target value at the end of a pre-specified horizon. Although duration was originally a theoretical development based on restrictive assumptions (MacCauley 1938)[4], some authors suggest that it possesses reasonable empirical properties even when these assumptions do not hold (Christensen, et al [1983][5]).

A development that parallels research on immunization is the recent introduction of optimization, or mathematical programming, models for dedicated portfolio selection. These are data driven models that allow financial managers to explicitly minimize the cost of assembling or re-asembling a portfolio to meet future requirements or liabilities, subject to constraints on the bonds' attributes. Duration may be included as an attribute to be constrained.

Most large brokerage houses now employ some type of mathematical programming model for dedicated portfolio selection. To date, these have been exclusively deterministic models that assume all data pertaining to the future, including interest rates, are known with certainty. Uncertainties are examined indirectly by sensitivity analyses, or by performing scenario tests on key parameters, such as interest rates, to see how the optimal solutions vary. While these approaches have their value, the analysis is incomplete because each model optimization assumes there is only one scenario of the future, and moreover that it will occur with certainty.

In this paper, we demonstrate how mathematical programming models for dedicated portfolio selection can be extended to explicitly treat uncertainties and risk. Specifically, we will develop stochastic programming with recourse models that allow a dedicated portfolio manager to consider simultaneously multiple scenarios of an uncertain future. The models calculate optimal contingency plans for each scenario which in turn are explicitly considered in the calculation of an optimal here-and-now purchasing (and/or selling) strategy for the portfolio. In effect, the here-and-now strategy is an optimal hedge against future uncertainties, taking into account the contingency plans that have been predetermined for each scenario.

Stochastic programming allows a direct and intuitive modeling of uncertainties. The results these models produce are not dependent upon assumptions about parallel shifts in the yield curve, or any other restrictive assumptions about underlying probability generating structures. Moreover, uncertainties are not limited to interest rates. Any combination of factors is allowed. All that is required is an objective or subjective forecast or assessment of the scenarios to be considered and their associated probabilities.

The modeling approach for dedicated portfolio selection that we propose here appears new. Bradley and Crane (1972)[6] report on a stochastic programming model for managing bank bond portfolios that is similar in spirit to ours, but which predates recent developments in fixed income and dedicated portfolio research. Moreover, our models are in part an outgrowth of large scale deterministic models for dedicated portfolio selection that are currently in active use by brokerage houses.

In the following section, we review deterministic mathematical programming models for dedicated portfolio selection. In the section after that, we extend the deterministic models to stochastic ones. We then illustrate the stochastic programming approach with a numerical dedicated portfolio selection problem. The paper concludes with remarks about future research.

DETERMINISTIC MATHEMATICAL PROGRAMMING MODELS

In this section, we review deterministic mathematical programming models for dedicated portfolio selection. The basic model we present determines a minimal cost purchasing strategy for bonds whose income streams are used to meet forecasted cash requirements or liabilities. After that, we discuss briefly extensions of the basic model. At the end of this section, we discuss practical implementations of these models. In the following section, the models are extended to stochastic models that explicitly treat uncertainties.

Basic Model

Indices:

j bonds ($j = 1, \ldots, N$)

t time periods ($t = 0, 1, \ldots, T$)

Parameters:

c_j = current market price for bond j (measured in dollars; par = \$100)

f_{jt} = cash flow produced by bond j at the end of period t (from coupons and principal repayment; measured in dollars)

L_t = cash requirement or liability to be paid at the end of period t (measured in millions of dollars)

q_j = conditional minimum purchase quantity of bond j (minimal quantity that must be purchased if any of bond j is purchased at all; measured in thousands of bonds)

Q_j = maximum allowable purchase of bond j (measured in millions of dollars)

a_t = reinvestment rate for period t (return at the end of period t of $1 invested at the end of period t-1)

Decision Variables:

x_j = quantity of bond j to be purchased here-and-now (measured in thousands of bonds)

s_t = cash surplus to be accumulated at the end of period t (measured in millions of dollars)

$$d_j = \begin{cases} 1 \text{ if bond j selected for the portfolio} \\ 0 \text{ otherwise} \end{cases}$$

The basic (deterministic) mathematical programming model for dedicated portfolio selection is the mixed integer program.

Minimize

$$\sum_{j=1}^{N} c_j x_j \quad + \quad s_0 \tag{1a}$$

subject to

$$\sum_{j=1}^{N} f_{jt} x_j \; + \; a_t s_{t-1} \; - \; s_t \; = \; L_t \qquad \text{for } t = 1, \ldots, T \tag{1b}$$

$$\left.\begin{aligned} x_j - q_j d_j &\geq 0 \\ x_j - Q_j d_j &\leq 0 \end{aligned}\right\} \text{ for } j = 1, \ldots, N \tag{1c}$$

$$x_j \geq 0, \quad s_t \geq 0, \quad d_j = 0 \text{ or } 1 \tag{1d}$$

The objective function (1a) in this model is the sum of bond purchases plus an initial cash investment s_0 that is guaranteed to return $a_0 s_0$ at the end of the first period in the planning horizon. An initial cash investment may be needed to cover cash requirements or liabilities in early periods (usually months) before coupons or principal repayments are realized in sufficient quantities.

The constraints (1b) are cash balance equations stating that cash from coupons and principal repayment, plus reinvestments of the cash surplus from the previous period, must at least equal the forecasted requirements or liabilities for the period. Any cash left over becomes the cash surplus that is re-invested for the current period. For conservative portfolio selection, a_t is sometimes taken to be small or even zero.

The constraints (1c) impose the logical condition that either $x_j = 0$ (when $d_j = 0$) or $q_j \le x_j \le Q_j$ (when $d_j = 1$). The imposition of these constraints is important to ensure that the model produces realistic, implementable answers since a fixed income portfolio manager will usually insist on a fairly large threshhold q_j for the purchase of each bond j.

Extensions

The basic model just described can be extended in a number of directions. First, we can easily add constraints on attributes; for example, average rating, average maturity, no more than 20% in utility bonds, etc. Such constraints can be expressed as linear inequalities of the form

$$\sum_{j=1}^{N} t_{ij} x_j \begin{cases} \le & b_i \\ = & b_i \\ \ge & b_i \end{cases}$$

where t_{ij} is the value of the attribute associated with bond j, and b_i is its target value. Attribute constraints may include one or more duration constraint.

Similar constraints may be added limiting the proportion of the portfolio invested in a specified industry to no more than or no less than a specified percentage. Zero-one constraints on the d_j can limit the number of individual bonds in the entire portfolio, or limit the number of bonds from a specified subset of the universe of bonds. The basic model can also be easily modified to impose the condition that the bond quantities be bought in integer lots between the limits q_j and Q_j.

Finally, the objective function in the model can be replaced by other objective functions. These might include maximizing yield, maximizing rating, and so on. In principle, any attribute can be designated as the objective function to be maximizied or minimized. Multi-objective optimization of the portfolio can be carried out by optimizing a weighted average of the different objective functions, or by the imposition of goal or target constraints on them.

The basic model and the extensions just described pertain to the single fund dedicated portfolio selection problem. The same constructs can be used to model multiple funds. For these problems, however, we must add constraints on flows between funds reflecting legal and management policies. We may also wish to employ a different objective function; for example, in the case of a construction fund, an objective function that maximizes the number of time periods whose requirements are covered by the fund.

Current Technology

The discussion above has focused on features of deterministic mathematical programming models currently employed by large brokerage houses to analyze dedicated portfolio selection problems. Before leaving this discussion, it is worthwhile discussing briefly the technology available for implementing efficient and effective planning systems embodying these models.

While optimization of large scale mathematical programming models presents a major challenge to the practitioner, commercial codes such as MIP/370 offered by IBM can generally be relied upon to efficiently extract an optimal solution from a model, once it has been generated. A bigger obstacle to the successful use of these models is flexible and efficient model generation from the appropriate data base. We have developed a proprietary package, called LOGS, for this purpose. LOGS contains a Descriptive Modeling Language that greatly facilitates model generation by breaking a large dedicated portfolio selection problem down into its primtiive elements which can be described separately and then systematically assembled into a global optimization model. Although LOGS was originally developed as a wrapper for MIP/370, it can be combined with virtually any MIP package. The reader is referred to Brown et al (1986)[7] for more details.

Specialized optimization routines are desirable for large dedicated portfolio selection problems involving liabilities in the hundreds of millions or even billions of dollars, several hundred liability periods, and 2000 or more bonds. We have developed LP approximations to reduce the number of bonds that must be considered, and layered branch-and-bound searches based on decision variable priorities that greatly speed up model optimization. Even the largest models can be optimized in less than 20 CPU minutes, usually in less than 10 CPU minutes, on a mainframe computer.

By contrast, many smaller companies and financial institutions wish to solve smaller dedicated portfolio selection problems arising in pension planning, municipal construction project financing, and so on. Developments over the past five years make it possible to completely model and optimize these problems on microcomputers (Resource Management Systems [1986][8]). A hybrid technology in which the micro is connected to a mainframe containing bond price information, and which can perform large scale model optimization if the need arises, is an attractive configuration.

STOCHASTIC PROGRAMMING MODELS

The models discussed in the previous section assume that all data pertaining to dedicated portfolio planning is known with certainty. In other words, the models consider a single, deterministic scenario of the future. We relax this assumption by explicitly modeling multiple scenarios of the future, each with an associated probability of occurrence. Although only one of these scenarios will actually occur, our analysis proceeds by computing optimal contingency plans for all of them. This information is then used to compute a here-and-now bond selection strategy that optimally hedges against the contingency plans.

The type of model just described is called stochastic programming with recourse (see Wagner (1969)[9], Bienstock and Shapiro (1984))[10]. Stochastic programming has been extensively studied as a theoretical construct in the operations research literature, but infrequently applied until this time. However, computer technologies, such as those discussed at the end of the previous section, have progressed to the point that the approach is viable for practical real-world problems.

In the discussion that follows, we present examples of stochastic programming models for dedicated portfolio selection. The examples are small, but not trivial, and were chosen to illustrate the types of questions that can be asked and answered by the models. The examples are merely selections from a large family of possible models for analyzing dedicated portfolio problems. A numerical example of our models is discussed in the following section.

EXTENSIONS TO THE BASIC MODEL

We begin with a discussion of the new indices, parameters and variables that we need to add to the deterministic model of the previous section.

New Indices:

k scenarios of the uncertain future ($k = 1, 2, \ldots, K$)

t^* last time period when all parameters known with certainty

New Parameters:

L_{kt} = cash liability to be met in period t under scenario k

a_{kt} = reinvestment rate for period t

p_k = probability that scenario k will occur

v_{kt} = discount factor for cash flows in period t under scenario k

New Decision Variables:

M_{kt} = additional cash required in period t under scenario k

With this background, we can state our basic stochastic programming model.

Minimize

$$\sum_{j=1}^{N} c_j x_j + \sum_{k=1}^{K} \sum_{t=t^*+1}^{T} p_k v_{kt} M_{kt} + s_0 \tag{2a}$$

subject to

$$\sum_{j=1}^{N} f_{jt} x_j + a_t s_{t-1} - s_t = L_t \tag{2b}$$

for $t = 1, 2, \ldots, t^*$

For $k = 1, 2, \ldots, K$

$$\sum_{j=1}^{N} f_{jt} x_j + a_{kt} s_{k,t-1} - s_{kt} + M_{kt} = L_{kt} \tag{2c}$$

for $t = t^*+1, \ldots, T$

$$\left.\begin{array}{l} x_j - q_j d_j \geq 0 \\ x_j - Q_j d_j \leq 0 \end{array}\right\} \text{ for } j = 1, \ldots, N \tag{2d}$$

$$x_j \geq 0, \quad s_t \geq 0, \quad s_{kt} \geq 0, \quad M_{kt} \geq 0, \quad d_j = 0 \text{ or } 1 \tag{2e}$$

The objective function (2a) includes the here-and-now costs

$$\sum_{j=1}^{N} c_j x_j + s_0$$

as before. To these we add the expected discounted costs

$$\sum_{k=1}^{K} \sum_{t=t^*+1}^{T} p_k v_{kt} M_{kt}$$

of the additional cash M_{kt} that will be needed in later periods to meet liabilities. The liability constraints (2b) have precisely the same form as the liability constraints (1b) in the deterministic model. This is because we assume that liabilities and interest rates for the first t^* periods are known with certainty.

For each scenario k, the constraints (2c) for $t=t^*+1,...,T$ are the probabilistic liability constraints to be satisfied. Unlike the deterministic case, we cannot be sure that the liabilities L_{kt} can be met by cash flows from bond investments and reinvestment of cash surpluses. For this reason, we have included the recourse variables M_{kt} representing cash that must be added in period t to meet the liabilities under scenario k.

The reader should note that reinvestments of cash surpluses in each period are now dependent on the scenario rate of return a_{kt}. This number should be directly derived from the risk free rate of interest i_{kt} for that period by the relation $a_{kt} = 1 + i_{kt}$. Moreover, we compute the discount factor

$$v_{kt} = 1/(\prod_{t=1}^{T} (1+i_{kt}))$$

Note also that, unlike the deterministic model where the analyst may feel obliged to assume conservatively low reinvestment rates, in our stochastic programming model, the forecasted rates a_{kt} can and should be used in each scenario.

Incorporating Call Options

A major concern of financial planners during times of high interest rates is the possibility that bonds with call options will in fact be called when interest rates decline sufficiently. Thus, if a low interest rate scenario actually comes to pass, the planner may suddenly be faced with large amounts of surplus cash that need to be reinvested, but at lower yields than would have been received from the coupons of the called bond.

The basic model described above can be readily extended to incorporate the uncertainties associated with call options. The analysis permits the planner to decide here and now whether the risk associated with a call option inhibits the purchase of the bond.

For expositional simplicity, we assume that only one bond, say bond j = j1, has a call option. Moreover, we assume that the call option will be realistically exercised only under one scenario, say scenario K which we can assume to be the low cost scenario. Again for expositional simplicity, we assume that bond j1 will be called during this scenario in period $t^{**} \geq t^{*}+1$ if it is ever to be called. We let r_{j1}, t^{**} denote the premium rate at which the bond is called in this period.

Finally, we let p_{K0}, p_{K1} and denote the probabilities associated with the new scenarios; K0 corresponds to "the bond is never called"; K1 corresponds to "the bond is called in period t^{**}". Since these scenarios are a refinement of scenario K in the basic model, we have

$$p_{K0} + p_{K1} = p_K$$

Extending the basic model to incorporate the possibility of a call on bond j1 in period t^{**} is relatively straightforward. The objective function (2a) is the same except the summation involving K is extended to include the two new scenarios replacing scenario K.

The constraints of our modell remain unchanged except constraints (2c) pertaining to scenario k which must be split into two new scenarios starting in period t^{**}. In particular, we have for k = K0

$$\sum_{j=1}^{N} f_{jt}x_j + a_{Kt}s_{K0,t-1} - s_{K0t} + M_{K0t} = L_{Kt} \qquad \text{for } t = t^{**},\ldots,T$$

and for k = K1

$$\sum_{j=1}^{N} f_{jt}x_j + r_{j1,t^{**}}x_j + a_{Kt}s_{K1,t^{**}-1} - s_{K1,t^{**}} + M_{K1,t^{**}} = L_{Kt}$$

and $j \neq j1$

and

$$\sum_{j=1}^{N} f_{jt}x_j + a_{Kt}s_{K1,t-1} - s_{K1t} + M_{K1t} = L_{Kt}$$

for $t=t^{**}+1,\ldots,T$

where

$$s_{K0,t^{**}-1} = s_{K1,t^{**}-1} = s_{Kt}$$

Note that the linkage from scenario K to the two new scenarios K0 and K1 is via the cash surplus term $s_{K,t^{**}-1}$.

Cost vs. Risk Tradeoff

The pension manager or financial officer responsible for the dedicated portfolio is at risk in the face of the uncertainties because he may be asked at some future time to put more money into the portfolio to cover its liabilities. The manager can control this risk by placing a constraint of the form

$$\sum_{t=t^{**}+1,}^{T} M_{kt} \leq R \qquad (4)$$

on all the scenarios k. Thus, under any eventuality (scenario), the maximal amount of additional money that may be required to be put into the portfolio is R.

If R is very large, the additional amount paid in under all scenarios will be less than R. Even if R is relatively small, R may not be reached exactly due to the lumpiness inherent in our bond selection. For certain values of R, the constraints (2d) may lead to an optimal portfolio in which constraint (4) is not satisfied exactly, although it does affect the portfolio's selection. At the extreme, with R = 0, the manager takes no risk and selects a portfolio here-and-now that guarantees that no additional money will be required to meet liabilities.

In a specific application, the manager may have a clear choice for R based on judgmental or political considerations. In other circumstances, the choice may be less clear. In such an instance, we can optimize the model parametrically to obtain an expected cost vs. risk curve analogous to the return vs. risk curve of modern portfolio theory (e.g. Alexander & Francis [1986])[11]. The manager must then use personal judgment in deciding where on the curve to be in structuring the fixed income portfolio. An example of a cost vs. risk curve is provided in the following section where we present a numerical example of the models.

Stochastic Programming Methodologies

We complete this section with three comments about stochastic programming models and methods for optimizing them as they relate to the dedicated portfolio selection models just discussed. First, the reader may have noticed that the uncertainties about interest rates were abruptly resolved at the start of period t^*+1, after which we knew with certainty which of the three scenarios obtained for the remainder of our planning horizon. This was clearly a gross oversimplification of the way in which forecasts about interest rates are determined and evolve. The simplification can be justified, at least in part, by the fact that we are mainly concerned with an optimal here-and-now portfolio selection. Thus, we may be willing to accept the simplification because it nevertheless provides us with useful information about future contingency plans upon which we base our here-and-now hedging against the uncertainties.

On the other hand, if we decide that the simplification is overdrawn, we can choose to extend the stochastic programming model to one with more stages during which the interest rates evolve in a probabilistic manner. Figure 1 contains a tree depicting such a multi-stage evolution of interest rates.

In any event, whether or not we are willing to accept the simplication inherent in the model described above, that model can be viewed mathematically as an approximation to more complex models in which the scenarios are played out in more detail. The interest rate scenarios that we know with certainty starting in period t^*+1 in the above model are the mathematical expectations of probabilistic interest rate forecasts stretching out until the end of our planning horizon. In short, the model developed above is but one in a series of stochastic programming approximations that we could develop for analyzing the effect of interest rate uncertainties on dedicated portfolio selection strategies. More research is required to understand the nature of these approximations and how best to apply them to dedicated portfolio selection problems.

Secondly, we have not addressed in our model development how one proceeds from a descriptive forecasting model of interest rates and liabilities to scenarios in a stochastic programming model. Space does not allow us to discuss these techniques, although they are obviously important to our model construction. It suffices to say that effective techniques for extracting scenarios from forecasting models do exist and could be readily implemented in a planning system embodying the stochastic programming models. The reader is referred to Hiller (1986)[12] for details.

Finally, we mention that stochastic programming with recourse models are well structured for the application of decomposition methods. This means that very large models could be broken down into manageable sub-models that in turn could be systematically reassembled. The sub-models in this case are the linear programs for determining contingency plans for each scenario, plus a master mixed integer programming sub-model for determining the here-and-now bond selection strategy. A decomposition approach based on this methdology was successfully implemented by Bienstock and Shapiro (1984)[13].

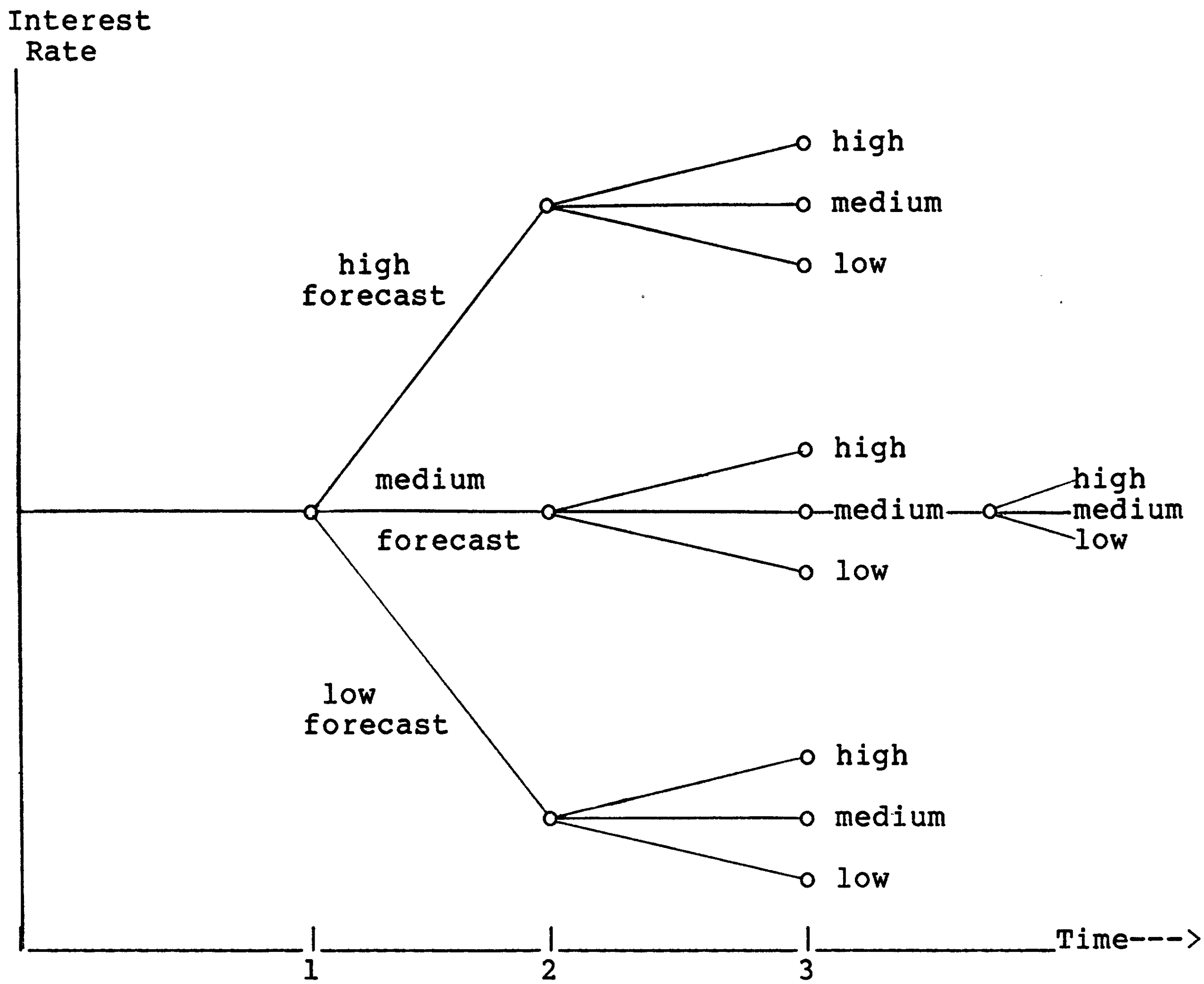

FIGURE 1: Multi-stage Interest Rate Forecasting

ILLUSTRATIVE EXAMPLE

We illustrate the stochastic programming models discussed in the previous section with a numerical example. The example involves the 8 bonds displayed in Table 1 and the liability stream displayed in Table 2. For simplicity, we have assumed the liabilities are known with certainty and that it is uncertainty aboaut interest rates that drives the analysis. Notice that bond 8 has a call option starting in year 4. A premium of 3.2 points will be given if the bond is called at the end of year 4; the premium declines linearly until the maturity date at the end of year 12.

Interest rate uncertainties and the possibility that bond 8 will be called are the two stochastic elements that we wish to explicitly model. Table 3 lists the three interest rate scenarios that we will consider. For expositional simplicity, we assume that the only scenario under which bond 8 will be called is scenario 3. Moreover, the only period in which it may be called is period 7; we assess a probability of .5 that it will be called in that period.

Given this description of the uncertainties, our 15 year planning horizon is broken into three stages: the first stage consisting of the first three years during which all data is known with certainty; the second stage consisting of years 4, 5, 6 during which we know which interest scenario has occurred but do not know whether or not bond 8 will be called; the third stage

BOND DATA

TABLE 1

BOND NUMBER	PRICE	COUPON	MATURITY (years)
1	95.375	.085	5
2	100.125	.09	12
3	103.50	.0925	15
4	099.00	.105	3
5	091.00	.075	6
6	089.00	.07	8
7	088.00	.0675	11
8*	119.50	.12	12

NOTES:

All bonds have maximum purchase quantity equal to 50,000, conditional minimum purchase quantity equal to 10,000.

* Bond callable starting in year 4.

STOCHASTIC PROGRAMMING EXAMPLE

LIABILITY STREAM

TABLE 2

YEAR	LIABILITY ($ x 10^6)
0	10
1	11
2	12
3	14
4	15
5	17
6	19
7	20
8	22
9	24
10	26
11	29
12	31
13	33
14	36
15	40

STOCHASTIC PROGRAMMING EXAMPLE

INTEREST RATE SCENARIOS

TABLE 3

YEAR	SCENARIO 1	SCENARIO 2	SCENARIO 3
1	.08	.08	.08
2	.08	.08	.08
3	.08	.08	.080
4	.085	.08	.075
5	.09	.08	.07
6	.095	.08	.065
7	.10	.08	.06
8	.105	.08	.06
9	.11	.08	.06
10	.11	.08	.06
11	.11	.08	.06
12	.11	.08	.06
13	.11	.08	.06
14	.11	.08	.06
15	.11	.08	.06
PROBABILITY OF OCCURRENCE	0.20	0.50	0.30

consisting of years 7 through 15 during which all the uncertainties have been revealed (interest rates and bond calls). Contingency actions of putting more money into the portfolio are allowed in both the second and the third stages. The different scenarios are depicted graphically in Figure 2. Note that the example actually involves four scenarios: high interest rate, medium interest rate, low interest rate/bond 8 called, low interest rate/bond 8 not called.

We made a number of runs with our stochastic programming model varying the risk permitted during the second and third stages. Recall that risk is measured by the amount of additional money that must be put into the dedicated portfolio to cover shortfalls in income. The results are listed in Table 4.

At zero risk, that is, when a portfolio must be selected here-and-now that guarantees an income stream covering all scenarios, a spread of bonds not including the callable bond is optimal. At relatively small levels of risk, the callable bond becomes somewhat attractive as a alternative to the lower yielding bonds 6 and 7.

With risk unconstrained (we actually set the risk parameter at 500), the optimal choice is to eschew bond 8 and select instead the higher yield bond 4, despite the fact that it matures early. The optimal contingency plan under the low interest scenario is then to pump a total of $97.85M into the portfolio towards the end of the planning horizon.

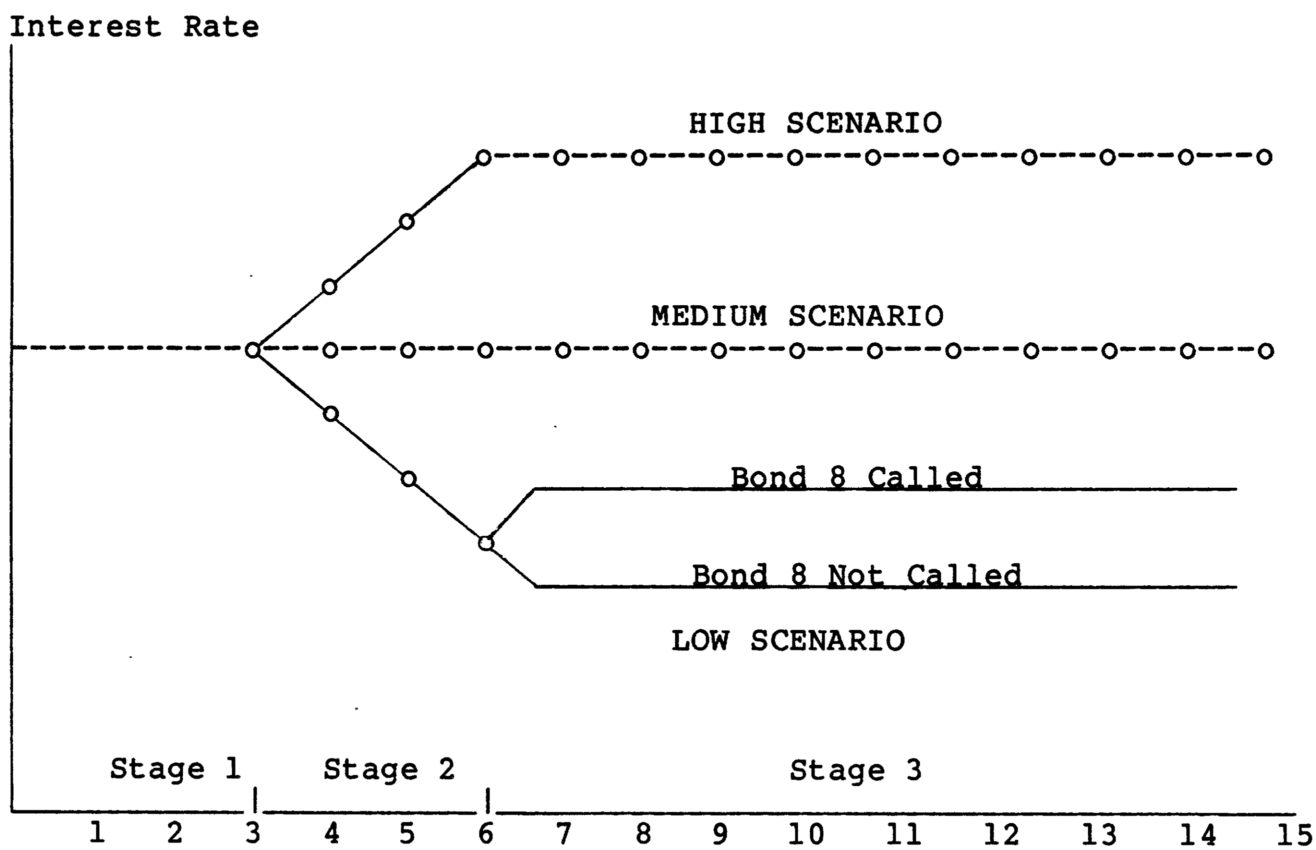

FIGURE 2: Stochastic Programming Example Scenario Descriptions

STOCHASTIC PROGRAMMING EXAMPLE

OPTIMAL BOND SELECTION STRATEGIES

TABLE 4

RISK	0	5	10	20	40	500 (97.85)
BOND						
1	0	0	0	0	0	0
2	50.0	50.0	50.0	50.0	50.0	50.0
3	36.6	36.6	36.6	36.6	23.9	0
4	0	0	0	0	10.0	38.7
5	10.0	42.4	50.0	50.0	50.0	50.0
6	32.5	42.3	0	0	0	0
7	46.4	0	0	0	0	0
8	0	0	24.8	22.2	21.0	0
Here-and-Now Cost	176.80	167.00	163.28	160.17	155.29	133.88
Future Expected Cost	0	8.19	11.28	13.78	18.60	39.92
TOTAL COST	176.80	175.19	174.53	173.95	173.89	173.76

Bond 5 appears attractive at all levels of risk because it matures at the end of year 6 when the added cash can be used to meet rising liability payments, and when there is a likelihood that it can be profitably re-invested. A summary of the risk binding scenarios is given in Table 5.

Figure 3 is a plot of expected cost versus risk. Since this plot has a "knee" at approximately $20M risk, we can conclude that the financial manager has three distinct qualitative choices available to him in the face of uncertainties. He can be very risk averse and choose the conservative portfolio associated with zero risk. Second, he can be moderately conservative (moderately risky) and include the high yielding but callable bond 8, but not include the high yield, early maturing bond 4. He must then be prepared to pump more money into the portfolio in later years, especially under the low interest rate scenario. Third, he can be less conservative (more risky), and choose bond 4 over bond 8, and be prepared in the low interest rate scenario to pump a relatively large amount of additional cash into the portfolio in years 11, 13, and 14.

Finally, Table 6 lists the results of portfolio optimization when we replaced the stochastic model by two version of its expected value model. In version one, we replaced all of the cash flows by their average or expected cash flows. Specifically, the cash flow from bond 8 was taken to be its average over the four scenarios; the reinvestment rates after period 3 were taken

STOCHASTIC PROGRAMMING EXAMPLE

BINDING SCENARIOS

TABLE 5

Risk $\leq$ 5	Low Interest Scenario	
		$5M added in Period 4
Risk $\leq$ 10	Low Interest/Bond Called Scenario	
		$4.24M added in Period 4 $5.76M added in Period 7
Risk $\leq$ 20	Low Interest/Bond Called Scenario*	
		$12.33M added in Period 9 $ 7.67M added in Period 14
Risk $\leq$ 40	Low Interest/Bond Called Scenario	
		$40M added in Period 14
Risk Unbounded	Low Interest Scenario	
		$16.0M added in Period 11 $ 8.9M added in Period 13 $73.7M added in Period 14

* Low Interest/Bond Not Called Scenario

$8.87M added in Period 11
$5.83M added in Period 14

Medium Interest Rate Scenario

$4.69M added in Period 11
$3.89M added in Period 14

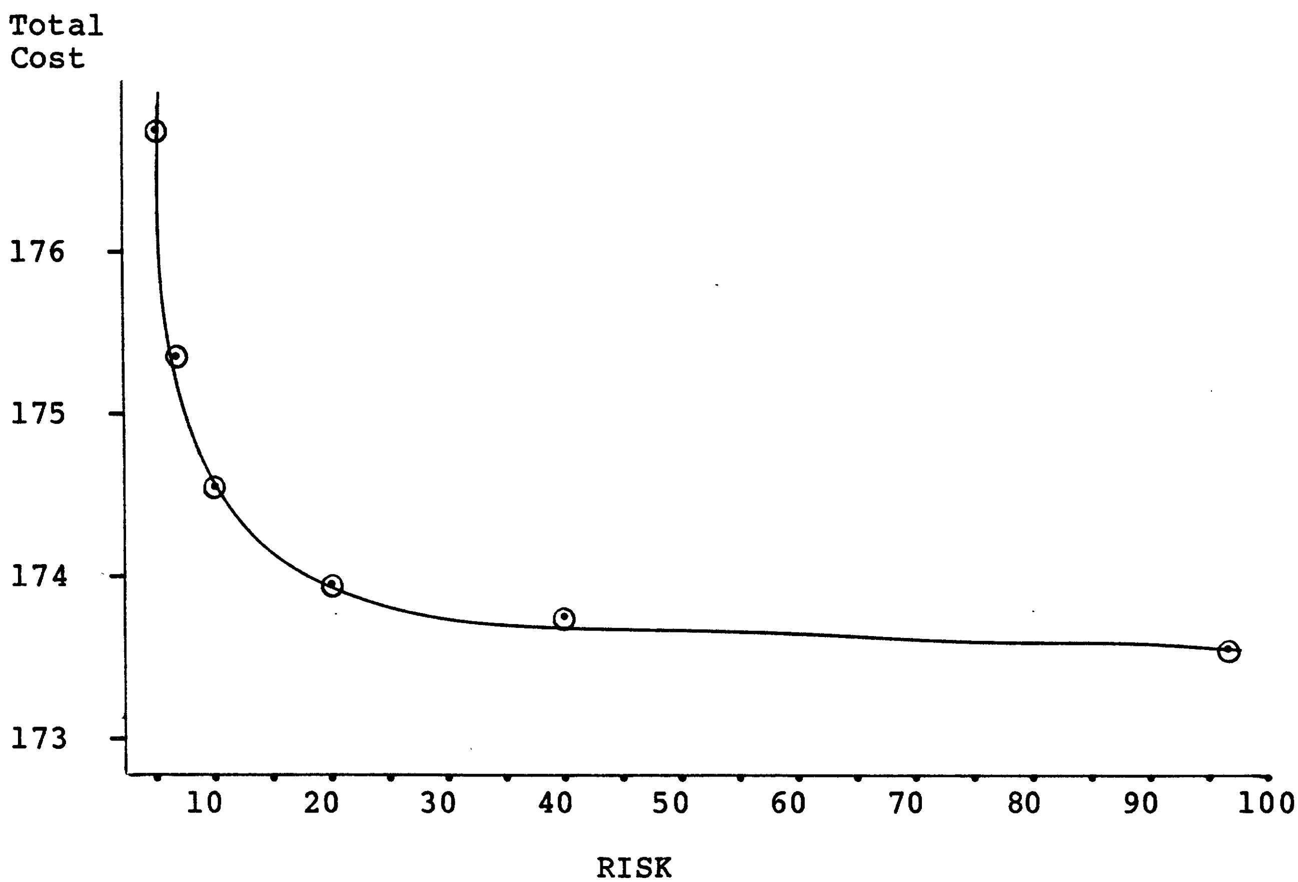

FIGURE 3: **Total Expected Cost vs. Risk**
Stochastic Programming Model

STOCHASTIC PROGRAMMING EXAMPLE

AVERAGE CASES

OPTIMAL BOND SELECTION STRATEGIES

TABLE 6

BOND	AVERAGE REINVESTMENT RATE	CONSERVATIVE REINVESTMENT RATE
1	0	0
2	50.0	33.9
3	0	36.6
4	10.0	0
5	47.9	0
6	0	21.7
7	0	16.4
8	50.0	50.0
TOTAL COST	173.09	175.35

to be the average of the three interest rate scenario reinvestment rates. In version two, we chose the least favorable scenario (that is, the low interest rate scenario) as the one on which to base reinvestments. As the reader can see, the expected value models produced significantly different results than the stochastic models - for example, the callable bond 8 looks much more attractive in the deterministic, expected value models than it does in the stochastic models.

CONCLUSIONS AND AREAS OF FUTURE RESEARCH

The stochastic programming with recourse models that we have presented in this paper offer the portfolio manager new perspectives on selecting bonds for the portfolio. By explicitly describing all relevant scenarios of the future, and analyzing them from a global perspective, the manager can avoid unduly biasing the decision in favor of one scenario over another. The extremely conservative approach taken by some managers that reinvestments of cash surpluses will not yield any interest income can clearly be avoided. In any event, the manager can use the stochastic programming models to explicity evaluate the tradeoff between money invested here-and-now and expectations about additional money that will be needed to meet future liabilities.

Many straightforward extensions of the basic models are possible. Uncertainties about bond defaults, or mortgage backed securities, for example, could be included in our models. We are currently developing a stochastic programming model related to the ones described above for fixed income problems, such as bond selection, for mutual funds, where the goal is to maximize capital gains rather than to minimize the costs of meeting liabilities. We expect to complete a companion paper about that model in the near future.

At the technical modeling level, basic and applied research needs to be performed to evaluate the varied stochastic programming model approximations possible for dedicated and fixed income portfolio selection. As we pointed out, our main interest in modeling future uncertainties is to determine an optimal, or demonstrably good, here-and-now bond selection strategy. The impact of more or less detail about future uncertainties on the here-and-now strategy is not yet well understood. As the numerical examples illustrated, however, even the coarsest descriptions of future uncertainties allow a much richer analysis of the portfolio planning problems than purely deterministic models.

Finally, we look forward to a practical test of our new modeling ideas. Plans are underway to implement a large-scale version of the basic model. We hope to report on the results of this experiment in the near future.

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