XVI. SWITCHING CONTROL SYSTEMS

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There is a general algebraic method that permits the design of switching circuits to perform according to given specifications (1). For any reasonable set of specifications, it is possible to obtain several appropriate switching circuits.

Often, the design object is to obtain the circuit which, while satisfying the specifications, uses the fewest number of switch contacts or relay springs. The algebraic design method is a great help toward realizing this object; but, since this method always results in a series-parallel type of network, it does not take advantage of possible contact economies that nonseries-parallel (bridge or nonplanar) networks might afford. This report describes a preliminary investigation of switching circuits of the bridge type.

DEFINITIONS

(1) A bridge circuit is defined as a circuit that contains at least one bilateral branch.

(2) A bilateral branch is defined as a branch that contains at least one switch contact and is traversed in both directions in tracing out all possible paths from input to output terminals of the circuit containing the branch. A path is considered to exist even if the occurrence of two complementary transmissions in series prevents signal flow under normal conditions. In other words, the conditions for a branch to be bilateral depend only on the topological properties of the associated network and as such can be determined from the graph (2) of the network.

(3) A bilateral transmission is the transmission function associated with a bilateral branch.

(4) A bridge transmission is any transmission corresponding to a bridge circuit.

(5) A simple bridge circuit is a bridge circuit in which neither the bilateral transmission nor any of its components appear elsewhere in the circuit.

Theorem 1. Any bridge circuit may be realized in the form shown in Fig. XVI-1, where B is the bilateral transmission of one of the bilateral branches.

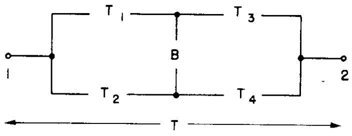

Fig. XVI-1
General form of a bridge circuit.

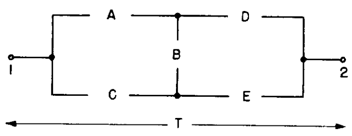

Fig. XVI-2
General form of a simple bridge circuit having bilateral transmission B.

Proof: Any bridge circuit must contain at least one bilateral branch. Let the associated bilateral transmission be denoted by B, and the branch terminals by 3 and 4. Denote the input and output terminals of the circuit by 1 and 2, respectively. By definition there must be a path from 1 to 2, traversing B from 3 to 4; thus there must be a path from 1 to 3 and from 4 to 2. Denote the corresponding transmissions by T_1 and T_4. Similarly a path from 1 to 2 traversing B from 4 to 3 must exist, and hence there must also be transmissions T_2 from 1 to 4, and T_3 from 3 to 2.

Corollary 1. The transmission of any bridge circuit containing a bilateral transmission B can be written in the form:

$$T = B(T_1T_4 + T_2T_3) + T_1T_3 + T_2T_4$$

where $T_1T_3 \not\equiv 0$ or $T_2T_4 \not\equiv 0$, and $BT_2T_3 \not\equiv 0$ or $BT_1T_4 \not\equiv 0$.

(If $T_1T_3 \equiv T_2T_4 \equiv 0$, then $T = B(T_1T_4 + T_2T_3)$, and T may be realized as economically in series-parallel form (without the need for a bilateral transmission B) as in bridge circuit form. Therefore, the requirement that either T_1T_3 or $T_2T_4 \not\equiv 0$ will henceforth be assumed. If $BT_2T_3 \equiv 0$ and $BT_1T_4 \equiv 0$, B is unnecessary; therefore the requirement that $BT_2T_3 \not\equiv 0$ or $BT_1T_4 \not\equiv 0$ will also be assumed.)

Theorem 2. If T is the transmission of a simple bridge circuit with bilateral transmission B, T can be written in the form

$$T = AD + CE + B(AE + CD)$$

where $BAE \not\equiv 0$, or $BCD \not\equiv 0$, AD or $CE \not\equiv 0$, and A, C, D, E are not functions of B.

The corresponding circuit is shown in Fig. XVI-2.

We now proceed to determine under what conditions a transmission $T = f + Bg$ can be realized as a simple bridge circuit with bilateral transmission B.

Expanding A, C, D, E in a standard sum:

$$A = A_0\, X_1'X_2'\ldots X_n' + A_1\, X_1'X_2'\ldots X_n + \ldots$$

$$C = C_0\, X_1'X_2'\ldots X_n' + C_1\, X_1'X_2'\ldots X_n + \ldots$$

$$D = D_0\, X_1'X_2'\ldots X_n' + D_1\, X_1'X_2'\ldots X_n + \ldots$$

$$E = E_0\, X_1'X_2'\ldots X_n' + E_1\, X_1'X_2'\ldots X_n + \ldots$$

$$AD = A_0D_0\, X_1'X_2'\ldots X_n' + A_1D_1\, X_1'X_2'\ldots X_n + \ldots$$

$$CE = C_0E_0\, X_1'X_2'\ldots X_n' + C_1E_1\, X_1'X_2'\ldots X_n + \ldots$$

$$AD + CE = (A_0D_0 + C_0E_0)\, X_1'X_2'\ldots X_n' + (A_1D_1 + C_1E_1)\, X_1'X_2'\ldots X_n + \ldots$$

$$f = f_0\, X_1'X_2'\ldots X_n' + f_1\, X_1'X_2'\ldots X_n + \ldots$$

$$g = g_0\, X_1'X_2'\ldots X_n' + g_1\, X_1'X_2'\ldots X_n + \ldots$$

It is assumed that T has been simplified by eliminating any terms which appear in both f and g through use of the theorem $X_1^*X_2^*\ldots X_n^* + \left(X_1^*X_2^*\ldots X_n^*\right)B = X_1^*X_2^*\ldots X_n^*$. (The notation X_1^* is used where either X_1 or X_1' may apply.) Thus the situation that $f_i = 1$, and $g_i = 1$ will not occur. In other words, we have either $f_i = g_i'$ or $f_i = 0$ and $g_i = 0$. However, whenever we have $f_i = 1$, g_i may also be set equal to 1 in order to simplify the resulting circuit without changing T. Thus it is sufficient to consider the three possibilities: $f_i = 0$, $g_i = 0$; $f_i = 0$, $g_i = 1$; and $f_i = 0$, $g_i = \phi$, where ϕ signifies that either a 0 or 1 may be assumed to apply.

Now we assume

$$T = f + Bg = AD + CE + B(AE + CD)$$

therefore

$$f = AD + CE, \quad g = AE + CD$$

Equating coefficients of like products $\left(X_1^*X_2^*\ldots X_n^*\right)$ yields

$$A_iD_i + C_iE_i = f_i$$

$$A_iE_i + C_iD_i = g_i$$

(1) Assume $f_i = g_i = 0$

$$\begin{aligned} A_iD_i + C_iE_i &= 0 \qquad \text{either } A_i = C_i = 0 \\ A_iE_i + C_iD_i &= 0 \qquad \text{or } D_i = E_i = 0 \end{aligned}$$

(2) Assume $f_i = 1$, $g_i = 0$

$$\begin{aligned} A_iD_i + C_iE_i &= 1 \qquad \text{either } A_i = D_i = 1 \\ A_iE_i + C_iD_i &= \phi \qquad \text{or } C_i = E_i = 1 \end{aligned}$$

(3) Assume $f_i = 0$, $g_i = 1$

$$\begin{aligned} A_iD_i + C_iE_i &= 0 \qquad \text{either } A_i = E_i = 1,\ C_i = D_i = 0 \\ A_iE_i + C_iD_i &= 1 \qquad \text{or } C_i = D_i = 1,\ A_i = E_i = 0 \end{aligned}$$

Consider the requirement $BAE \neq 0$ or $BCD \neq 0$. This requires $AE \neq 0$ or $CD \neq 0$. But $AE = A_0E_0\, X_1'X_2'\ldots X_n' + A_1E_1\, X_1'X_2'\ldots X_n + \ldots$. So that, if $AE \neq 0$, then $A_i = E_i = 1$

for at least one i. Similar reasoning for $BCD \neq 0$ leads to the requirement that $A_i = E_i = 1$ or $C_i = D_i = 1$ for at least one i.

Now consider the requirement that AD or $CE \neq 0$. This in turn leads to the requirement that $A_i = D_i = 1$ or $C_i = E_i = 1$ for at least one i.

In summary, we require that

$$\left.\begin{array}{ll} \text{(i)} & A_i = E_i = 1 \text{ or } C_i = D_i = 1 \\ \text{(ii)} & A_i = D_i = 1 \text{ or } C_i = E_i = 1 \end{array}\right\} \text{ for at least one } i$$

Study of cases 1, 2, and 3 shows that (i) requires that $g_i \neq 0$, and (ii) requires that $f_i \neq 0$.

Theorem 3. If $T = f + gB$ where f and g are not functions of B and $f \neq 0$ and $g \neq 0$, then T may be realized as a simple bridge circuit which contains B only as the bilateral transmission.

Theorem 4. If a transmission T is expanded in a standard sum:

$$T = T_0 X_1'X_2' \ldots X_n' + T_1 X_1'X_2' \ldots X_n + \ldots$$

and if whenever $T_i = 1$ for i even (odd) $T_i + 1 = 1$ $(T_i - 1 = 1)$ also (for at least one such i), and if $T_i = 1$ for i odd (even) with $T_i - 1 = 0$ $(T_i + 1 = 0)$ (for at least one such i), then T can be realized as a simple bridge circuit having $X_n(X_n')$ as the bilateral transmission.

Example 1.

$$T = xy + x'w + z(w+y)$$

If we let $f = xy + x'w$, $g = w + y$, it is clear that T can be realized as a simple bridge circuit with z bilateral. For the order wxy, $f_i = 1$ for i = 3, 4, 5, 7 and $g_i = 1$ for 1, 6. Thus for i = 1, 6

$$A_1 = E_1 = 1, \quad C_1 = D_1 = 0$$

$$A_6 = E_6 = 0, \quad C_6 = D_6 = 1$$

for i = 3, 4, 5, 7

$$A_i = D_i = 1 \text{ and } C_i = \phi,\ E_i = \phi \text{ or } A_i = \phi,\ D_i = \phi, \text{ and } C_i = E_i = 1$$

for i = 0, 2

$$A_i = C_i = 0 \text{ and } D_i = \phi,\ E_i = \phi \text{ or } D_i = E_i = 0,\ A_i = \phi,\ C_i = \phi$$

Figure XVI-3 shows how the A, C, D, E functions may be simultaneously minimized by the use of Karnaugh charts in accordance with the above constraints. The first row of tables shows the assignment for the $g_i = 1$ case; the second row, the assignment for the $g_i = f_i = 0$ case; the third row, the assignment for the $f_i = 1$ case; and the final row,

	A				C				D				E			
x	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
y w	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	0
0	0	1 1	3	2	0	1 0	3	2	0	1 0	3	2	0	1 1	3	2
1	4	5	7	6 0	4	5	7	6 1	4	5	7	6 1	4	5	7	6 0

0	0 0	1 1	3	2 0	0 0	1 0	3	2 0	0 φ	1 0	3	2 φ	0 φ	1 1	3	2 φ
1	4	5	7	6 0	4	5	7	6 1	4	5	7	6 1	4	5	7	6 0

0	0 0	1 1	3 1	2 0	0 0	1 0	3 φ	2 0	0 φ	1 0	3 1	2 φ	0 φ	1 1	3 φ	2 φ
1	4 φ	5 φ	7 1	6 0	4 1	5 1	7 φ	6 1	4 φ	5 φ	7 1	6 1	4 1	5 1	7 φ	6 0

0	0 0	1 1	3 1	2 0	0 0	1 0	3 0	2 0	0 0	1 0	3 1	2 1	0 1	1 1	3 0	2 0
1	4 0	5 1	7 1	6 0	4 1	5 1	7 1	6 1	4 0	5 0	7 1	6 1	4 1	5 1	7 0	6 0
	A = y				C = w				D = x				E = x'			

Fig. XVI-3

Karnaugh charts used to minimize A, C, D, and E, simultaneously.

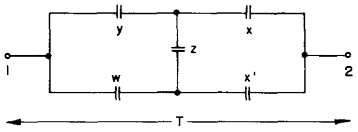

Fig. XVI-4

Bridge circuit realization of transmission T = xy + x'w + z(w+y).

the selection of the φ condition used in order to complete the specification.

The resulting bridge circuit is shown in Fig. XVI-4.

Another method by which this circuit could have been realized will be illustrated. By Theorem 2, if z is the bilateral transmission of a simple bridge circuit having transmission T then T can be written: T = AD + CE + z(AE + CD). We now expand the transmission T being considered about z, simplify, and equate the two expressions. Thus we have

$$T = yx + wx' + z(y+w) = AD + CE + B(AE + CD)$$

where B = z, and yx + x'w = AD + CE. Set

$$yx = AD; \; y = A, \; x = D$$

$$wx' = CE; \; w = C, \; x' = E$$

Now form AE + CD = yx' + wx. This clearly is not equal to w + y, but when the complete expression for T is formed

$$T = xy + x'w + z(xw + x'y)$$

and it is noted that

$$w(x' + xz) = w(x' + z) \text{ and } y(x + x'z) = y(x+z)$$

it is evident that T = xy + x'w + z(xw + x'y) does equal xy + x'w + z(w+y), as required. This assignment of A, B, C, and D leads to the circuit shown in Fig. XVI-4. Another possible assignment would have been

A = X, D = y, E = w, C = x'

This leads to the circuit of Fig. XVI-4 with terminals 1 and 2 interchanged.

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References

1. S. H. Caldwell, Teaching Notes, Courses 6.567 and 6.568, M.I.T.
2. E. A. Guillemin, Introductory Circuit Theory (John Wiley and Sons, Inc., New York, 1953) p. 5.