STORAGE AND VERTICAL TRANSPORT OF STRATOSPHERIC MOMENTUM DURING THE IGY

by

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ABSTRACT

The size of the storage change of absolute zonal angular momentum in the lower stratosphere is compared with the vertical transport of momentum by standing and transient eddies through the seventy-five and forty millibar surfaces throughout the year. Only in winter do these eddies contribute appreciably in the correct direction to the change in storage (about one third). For the spring season a balance is sought between all terms of an expanded momentum equation, taken over an elevated volume. Meridional motions toward the south at speeds of zero to 5 cms per sec are possible if frictional transports are calculated from the eddy viscosity coefficients based on tungsten diffusion data and a momentum balance is assumed.

ACKNOWLEDGEMENTS

The author is grateful to Professor R. E. Newell for suggesting the subject and for much valuable advice in the course of the work. This paper, like many others, was possible because of calculating procedures developed by the M.I.T. Planetary Circulations Project under Professor V. P. Starr. Dr. A. A. Barnes prepared the computer program. Mr. Hess of the Air Force Cambridge Research Laboratory provided data upon which a portion of the paper is based.

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I. INTRODUCTION

Over the past decade the Planetary Circulations Project at the Massachusetts Institute of Technology has conducted extensive studies of the momentum and energy balances of the troposphere and lower stratosphere. The studies were diagnostic in nature, attempting to determine which processes are important in the operation of the general circulation of the atmosphere. The works concerned such matters as the transport of angular momentum, the flux of water vapor, and exchanges between various energy forms. Methods used in the studies depended on long term statistics from observational data applied to momentum and energy relationships well known in meteorology.

Meteorological observations taken as a whole gave sufficient coverage in time and space for the Northern Hemisphere to about the one hundred millibar level, to allow the studies to proceed. The accuracy of individual observations was limited and a statistical procedure using large amounts of data or some type of smoothing was necessary to eliminate noise.

Before the turn of the century Reynolds (1894) employed a procedure whereby velocity components could be expanded into mean values and departures therefrom, which has been exploited by investigators in recent times. Priestley (1949) and Starr and

White (1952), developed calculation schemes that allowed statistics from large amounts of data to be applied to the balance relationships for momentum and energy.

Conclusions about the tropospheric circulation from the studies altered long-standing concepts of the operation of the general circulation. Many papers were produced which demonstrate the large part played by horizontal-moving eddies in the momentum and energy balances, for example Starr and White (1954). These eddies act as a vehicle by which momentum may travel against its gradient (to maintain the mid-latitude westerlies). By contrast small-scale turbulent eddies dissipate or equalize velocity fields.

During recent years, data resources in the upper atmosphere have greatly increased. Sufficient observations have become available so that crude patterns of atmospheric parameters can be drawn up to the one-hundred kilometer level. The U. S. Weather Bureau and others have analyzed height contours for constant pressure levels in the lower stratosphere, to thirty millibars on a day-to-day basis; at higher levels one must rely on mean vertical patterns for seasonal extremes.

During the IGY period and thereafter quantities of data for the lower stratosphere became available. Studies by Barnes (1962) on the stratospheric energy balance and by Dickinson (1962) on momentum divergence were based on data from the IGY period. Our study, too, depends in large part on this material.

The purpose of the present work is to compare the vertical flux of absolute zonal angular momentum through a pressure surface taken near the transition zone between the troposphere and stratosphere with the change in storage of that quantity above the surface, taken as its increase or decrease over periods of the order of months. An attempt is made to evaluate **spring mean meridional motion** from the momentum balance in elevated volumes. Studies to the time of Jensen (1962) considered primarily volumes taken from the surface to the top of the troposphere, requiring only horizontal transports at a number of tropospheric levels. Vertical transports did not appear in the momentum balances for these volumes. The selection of the volume for our study requires that vertical transports be introduced. Two methods have been employed recently to compute vertical motions from observed data. Murakami (1960) used zonally averaged north-south flow in the continuity relationship starting at the surface and continuing to one-hundred millibars. The second method is to compute vertical velocities from a form of the first law of thermodynamics; this way, as developed by Barnes (1962), is employed here. The first procedure depends on having very accurate north-south velocities at low levels, if large errors are to be avoided higher in the atmosphere; the second is of somewhat limited value, due to our lack of data on non-adiabatic heating.

Our volume covers the polar cap north of twenty degrees north latitude and is based upon the seventy-five millibar surface.

Vertical transports are also computed for the pressure level at forty millibars, and are compared with those at seventy-five millibars. Non-adiabatic heating or cooling at either seventy-five or forty millibars would cause errors in our vertical transports. A recent paper by Manabe and Moller (1961) gives a computed distribution of annual mean rate of temperature change. A center of cooling at sixty-five degrees north latitude and seventeen kilometers altitude of six - tenths of a degree centigrade per day is the prominent feature affecting our vertical motions. Those for the seventy-five millibar surface at high latitudes will have the largest non-adiabatic term; low latitudes on both surfaces have small heating rates. Generally levels taken in the stratosphere have less non-adiabatic temperature changes than do levels in the troposphere.

The stratospheric flow on which our data is based is, from the point of view of location, different from the tropospheric flow in an important respect. In the stratosphere we are dealing with an integral system, which is remote from the surface where the dissipation is greatest. We will attempt an estimate of the drag placed on flow in the upper stratosphere and troposphere by the volume of our study, whose momentum balance we will evaluate.

II. MOMENTUM EQUATION

Definition of symbols

x = horizontal coordinate west-east

y = horizontal coordinate south-north

z = coordinate vertically, defined by hydrostatic relationship

p = coordinate vertically in pressure units

t = time

The coordinates rotate with earth and are orthogonal.

u, v, ω- time rates of change of x, y, P of fluid at a point.

Ω = rate of rotation of earth

g = acceleration due to gravity

Fx = retarding force due to friction in west-east direction

R = radius of earth

r = radius of a latitude circle

ρ = density of fluid, variable in space

M = ur $+ \Omega r^2$ absolute zonal angular momentum per unit mass

v = refers to a part of volume of stratosphere (x, y, P components)

v_z = (x, y, z Components)

ϕ = latitude

2, 1 subscripts that refer to end and start of time period

$K_{s_1}, K_{s_2}, K_{s_3}$ eddy viscosity coefficient, over the volume bottom and top surface

$(\overline{\quad})$ is time average; $\frac{1}{t_2 - t_1}\int_{t_1}^{t_2}(\quad)\,dt$

S_1, S_2, S_3 area of side boundary, area of bottom surface (constant P), area of top surface (constant P)

$(\;\;)'$ departure from time average, $(\;\;) - (\overline{\;\;\;})$

$[\;\;]$ is zonal average around a latitude circle, $\frac{1}{2\pi r}\int_0^{2\pi r}(\;\;)\,dx$.

$(\;\;)^*$ departure from zonal average, $(\;\;) - [\;\;]$

The forces affecting the zonal absolute momentum are friction and pressure gradient. The time rate of change of momentum for a unit mass of air may be expressed as;

$$\frac{dM}{dt} = -gr\frac{\partial z}{\partial x} + rF_x \qquad (1)$$

We may transform equation (1) in the following manner

$$\frac{dM}{dt} = \frac{\partial M}{\partial t} + u\frac{\partial M}{\partial x} + v\frac{\partial M}{\partial y} + w\frac{\partial M}{\partial p}$$

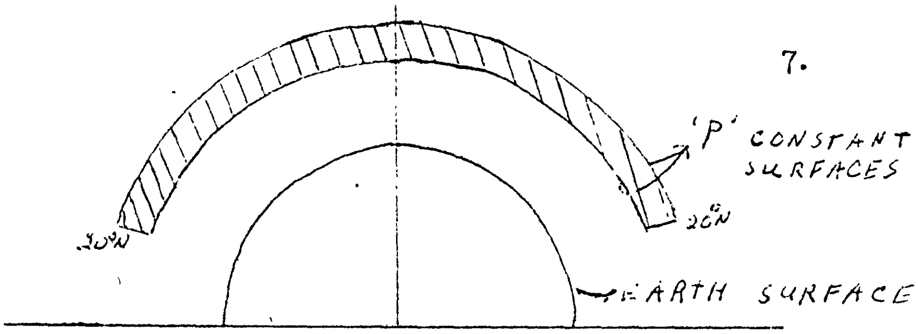
$$= \frac{\partial M}{\partial t} + \frac{\partial uM}{\partial x} + \frac{\partial vM}{\partial y} + \frac{\partial wM}{\partial p} - M\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial p}\right) \qquad (2)$$

The quantity in parenthesis in equation (2) vanishes by the equation of continuity of mass. Then equation (1) becomes

$$\frac{\partial M}{\partial t} = -\frac{\partial uM}{\partial x} - \frac{\partial vM}{\partial y} - \frac{\partial wM}{\partial p} - g\frac{\partial rz}{\partial x} + rF_x \qquad (3)$$

Thus the rate at which absolute zonal angular momentum changes at a point is determined by the convergence of the transport of momentum plus the external torques applied by pressure gradient and friction.

We form an integral of equation (3) for an elevated volume over the Northern Hemisphere north of twenty degrees latitude and located between two pressure surfaces in the lower portion of the stratosphere, as the cross-hatched space below.



Then equation (3) becomes

$$\int \frac{\partial M}{\partial t} dV = -\int \left(\frac{\partial uM}{\partial x} + \frac{\partial vM}{\partial y} + \frac{\partial wM}{\partial p} \right) dV - g \int \frac{\partial \gamma Z}{\partial x} dV + \int \gamma F_x dV \qquad (4)$$

We apply the definition of 'M' in equation (4) and change the terms as follows.

Since the time variable is independent of the spacial components, we may rearrange orders of integration and differentiation in the term on the left.

$$\int \frac{\partial M}{\partial t} dV = \int \frac{\partial u\gamma}{\partial t} dV = \frac{\partial}{\partial t} \int u\gamma dV \qquad (5)$$

We apply the divergence theorem to the relative wind terms of the divergence integral,

$$-\int \left(\frac{\partial uM}{\partial x} + \frac{\partial vM}{\partial y} + \frac{\partial wM}{\partial p} \right) dV = -\int \left(\frac{\partial \Omega r^2 u}{\partial x} + \frac{\partial \Omega r^2 v}{\partial y} + \frac{\partial \Omega r^2 w}{\partial p} \right) dV$$
$$+ \int uv\gamma dS_1 - \int uw\gamma dS_2 + \int uw\gamma dS_3 \qquad (6)$$

The first integral on the right is the divergence of momentum due to the earth's rotation within the volume; while the last three express the transport of momentum into the volume.

The friction term may be converted into a surface integral giving the torque on our volume by small scale, vertical eddies;

$$\int \gamma F_x dV = \int g\rho\gamma K \frac{\partial^2 u}{\partial z^2} dV_z$$
$$= -\int g\rho\gamma K_{S_2} \frac{\partial u}{\partial z} dS_2 + \int g\rho\gamma K_{S_3} \frac{\partial u}{\partial z} dS_3 \qquad (7)$$

The hydrostatic relationship is used to change the vertical coordinate, and only vertical wind shear is included in the frictional torque.

Substituting from equations (5) through (7) in (4)

$$\begin{aligned}\int u r\,dV = &-\int\left(\frac{\partial \Omega r^2 u}{\partial x}+\frac{\partial \Omega r^2 v}{\partial y}+\frac{\partial \Omega r^2 \omega}{\partial P}\right)dV+\int u v r\,ds_1\\ &-\int u\omega r\,ds_2+\int u\omega r\,ds_3-g\int\frac{\partial r z}{\partial x}dV-\int g\rho r K_{s_2}\frac{\partial u}{\partial z}ds_2+\int g\rho r K_{s_3}\frac{\partial u}{\partial z}ds_3\end{aligned}\tag{8}$$

Integrating the terms in equation (8) around a latitude circle for the 'x' coordinate (integration of terms differentiated in 'x' drop out) and between t_1 and t_2 for the time coordinate, using the definitions on page 5.

$$\begin{aligned}\iint([u]_2-[u]_1)r\,dy\,dP = &-\iint\frac{\partial \Omega r^2[\overline{uv}]}{\partial y}dy\,dP-\iint\frac{\partial \Omega r^2[\overline{u\omega}]}{\partial P}dy\,dP+\int_{s_1}[\overline{uv}]r\,dP\\ &-\int_{s_2}[\overline{u\omega}]r\,dy+\int_{s_3}[\overline{u\omega}]r\,dy-\int_{s_2}g\rho r K_{s_2}\frac{\partial[\bar{u}]}{\partial z}+\int_{s_3}g\rho r K_{s_3}\frac{\partial[\bar{u}]}{\partial z}dy\end{aligned}\tag{9}$$

We expand the terms for the divergence of momentum due to rotation of the earth;

$$\iint\frac{\partial \Omega r[\bar{\omega}]}{\partial P}dy\,dP = -\iint \Omega r^2\frac{\partial[\bar{v}]}{\partial y}dy\,dP+2\omega R\iint \sin\theta\cos\theta[\bar{v}]\,dy\,dP\tag{10}$$

and

$$-\iint\frac{\partial \Omega r[\bar{\omega}]}{\partial P}dy\,dP = \Omega\int_{s_3} r[\bar{\omega}]\,dy-\Omega\int_{s_2} r[\bar{\omega}]\,dy\tag{11}$$

Substitution of equations (10) and (11) in (9) and expansion of non-linear terms after the manner of Starr and White (1952) gives

$$\begin{aligned}\iint([u]_2-[u]_1)r\,dy\,dP = &\int_{s_3}([\bar{u}^*\bar{\omega}^*]+[\bar{u}\bar{\omega}])r\,dy-\int_{s_2}([\bar{u}^*\bar{\omega}^*]+[\bar{u}\bar{\omega}])r\,dy\\ &+\int_{s_1}([\bar{u}^*\bar{v}^*]+[\bar{u}\bar{v}])r\,dP+\int_{s_3}[\bar{\omega}][\bar{\omega}]r\,dy-\int_{s_2}[\bar{\omega}][\bar{\omega}]r\,dy\\ &+\int_{s_1}[\bar{\omega}][\bar{v}]r\,dP+\Omega\int_{s_3} r[\bar{\omega}]\,dy-\Omega\int_{s_2} r[\bar{\omega}]\,dy-\Omega\iint r^2\frac{\partial[\bar{v}]}{\partial y}dy\,dP\\ &+2\omega R\iint \sin\theta\cos\theta[\bar{v}]\,dy\,dP-gK_{s_2}\int_{s_2}\rho r\frac{\partial[\bar{u}]}{\partial z}dy+gK_{s_3}\int_{s_3}\rho r\frac{\partial[\bar{u}]}{\partial z}dy\end{aligned}\tag{12}$$

The left side of equation (12) is the change in storage of momentum within our volume over a period of time. On the right is the transport-in by horizontal and vertical eddies of relative momentum - plus the transport-in by horizontal and vertical mean motions of absolute momentum - plus torque due to the longitudinal variation of the meridional wind - plus torque due to coriolis effects - plus drags on the top and bottom surfaces by small-scale eddies.

III. DATA AND REDUCTION PROCEDURES

Information for the IGY period from over two hundred upper-air stations distributed over the northern hemisphere was reduced by the MIT Planetary Circulations Project for use in studying the conservation processes of the stratosphere. Data from July 1957 through June 1958 was involved. Barnes (1962) discusses the data distribution completely. Generally one may say that coverage was good in continental areas except for China; in other areas data amounts were limited, but usually sufficient.

Standard rawinsonde stations were the original data sources. Winds were calculated from tracks of vertical rising, spherical balloons. Tracking inaccuracies become great at low elevation angles and data is not reduced when the balloon is near the horizon; as a result our data is biased toward low wind speeds, mainly in winter. For vertical motion computations, wind and temperature data from the rawinsonde observations are required as described by Barnes (1962). Information on non-adiabatic heating is not available from rawinsonde data and limitation of terms that depend on vertical motion result. These are considered in section IV.

Grid point data was available initially for time averages and standard deviations of 'u' and 'ω'; these, together with a program to reduce the information to values of '$[\bar{u}][\bar{\omega}]$' and '$[\bar{u}^* \bar{\omega}^*]$' were part of earlier work by Dr. Barnes (1962). A program tape to calculate '$[\overline{u'\omega'}]$' at each station from tapes of rawinsonde data was assembled by Dr. Barnes and checked by the author. Data tapes were available from previous work for the periods July through September and October through December, 1957. Later, tapes for the periods January through March 1958 and April through June 1958 were cut. Within the four periods 00Z and 12Z data were reduced separately. Data from the one hundred and from the fifty millibar levels were used to compute information for the seventy-five millibar level. Likewise fifty and thirty millibar levels were used for the pressure level midway between. An LGP-30 high speed computer was used for these computations. Station values of '$\overline{u'\omega'}$' were plotted on polar projections. Maps for the two levels and two time periods from July 1957 through December 1957 were analyzed by the author; those for January 1958 through June 1958, by Mr. Dickinson. Values from a grid with five degrees spacing in latitude and longitude were averaged zonally to obtain '$[\overline{u'\omega'}]$'. Finally computation of the vertical transport integrals for standing and transient eddies was performed by the author.

Reduced data used in the integrations for horizontal transport terms due to eddies, in equation (12), were calculated by Mr. Oort in connection with previous studies under the Planetary Circulations Project.

The data and reduction procedures used in the time and space averaged 'ωs' that form the basis for computation of rotational and mean terms are described in section VI.

Values of $[\bar{u}]$ were taken from different sources depending on altitude. Below forty millibars work by Dickinson (1962) was used. Analyses were used at higher levels. For the IGY period, the United States Weather Bureau has analyzed constant pressure charts at thirty millibars. Height data was less accurate than that from lower levels. Among the methods used as aids in spacing of contours, a most important one was to obtain the gradient from the geostrophic approximation; continuity was observed between charts. To eliminate the possibility of including an analysis that is peculiar in some way or that was drawn from biased or inaccurate data on a particular day, the author compared the Weather Bureau maps used with those drawn independetly by Behr, et al, (1960) at the Free University of Berlin, for adjoining days. A third check was made by computing the absolute angular momentum from monthly time-mean analysis for 1955-1959 by Muench (1962). Geostrophic values of 'u' were taken from the time means. Except for the polar night breakdown period comparisons showed only small differences between analyses by different groups. Zonal average winds were taken from the Weather Bureau maps by the author, using averages around latitude circles. Results were integrated over height to form the storage term of absolute zonal angular momentum, independent of time.

Density was a necessary input to the friction term in equation (12). Summer values of density were used throughout the study. Up to thirty kilometers, values were taken from constant pressure maps at thirty and fifty millibars analyzed by Wege (1958), and averaged zonally by the author. Above thirty kilometers density inputs were zonal averages of Quiroz (1961), taken against latitude and altitude.

IV. STORAGE

Figure 1 gives the variation against time of absolute zonal angular momentum for our volume. Variations described here are those with a time scale of a month. The curve covers a year's time and shows the expected annual cycle.

The areas contained above and below the time axis are about balanced with the polar night breakdown appearing as a distorting feature that affects momentum storage in late winter and early spring.

The characteristic summer pattern establishes itself in June, with all latitudes showing negative momentum inputs. The increase in negative momentum going from June to the maximum shown in July is due mostly to considerably increased flow from the east at lower latitudes; although all latitudes show an increase. By contrast the loss of momentum going into August is more the result of a weakening of the system as a whole, while southerly latitudes maintain their flow comparatively well.

By September a weak polar vortex covers the latitudes down to about forty degress. South of forty degrees the momentum storage maintains its sign, but becomes less in absolute magnitude, into October. At the same time the positive contribution from latitudes north of forty degrees becomes somewhat stronger. During November the polar vortex controls the flow at nearly all latitudes; the highest storage is at forty degrees. Up to the January maximum, all latitudes increase their positive momentum. The contracting of the vortex with large vorticity increase commented on by Boville, et al, (1961) is reflected in our storages by the northward movement of the maximum to about fifty degrees - between November and January.

The breakdown of the polar night jet is evident in our curve for January and February. Comparison of our graph with an analysis of five-year average patterns by Muench (1962) shows that a breakdown in January is unusual; and that a momentum 150 X 10 gm cm^2 per sec higher is more typical of early February. After the collapse of the polar night jet our storages increase slightly from February into April; the total is fairly evenly distributed over the latitude range. Between April and July a reduction in momentum content takes place at an overall rate that is about equal, but opposite in sign, to the increase between July and January. The replacement of positive by negative momentum takes place rapidly in April then more slowly in May. First flow toward the west is established below forty degrees. An increase of easterly momentum in the south and a decrease of westerly momentum at high latitudes follows; in late May and June the pattern breaks over into a complete flow toward the west, with a decrease in momentum at a rate that is about equal to that during the polar night breakdown.

The shape of the curve in figure 1 supports the choice of quarterly time-periods chosen for this and previous studies under the Planetary Circulations Project, as intervals for integration purposes in transport and friction terms. Our four periods, matching the seasons, all fall between a positive or negative maximum on the curve and an adjacent crossing of the 'zero' value by momentum storage. Thus our periods bracket a time neither of stratospheric circulation reversal nor of large change in the rate of momentum storage.

TABLE I

Zonal angular momentum content in gr. cm^2 per sec for latitudes and times.

LAT	15 July	15 Aug	15 Sept	5 Oct	15 Oct	15 Nov	5 Jan	5 Feb	5 Mar	5 Apr	25 Apr	15 May	15 June
20	-148	-132	-86	-47	-39	-16	+23	0	+16	-8	-47	-86	-16
25	-136	-107	-71	-36	-29	+7	+43	+14	+21	0	-36	-57	-43
30	-116	-90	-58	-26	-13	+19	+52	+26	+19	+13	-26	-39	-71
35	-86	-58	-29	-6	-17	+35	+58	+35	+17	+23	-12	-12	-46
40	-61	-35	0	-10	+27	+45	+66	+35	+15	+30	+5	+5	-30
45	-46	-21	+9	-21	+20	+34	+76	+30	+17	+30	+13	+9	-21
50	-31	-10	+10	-28	+17	+24	+73	+24	+17	+24	+17	+10	-14
55	-25	-6	+6	-20	+20	+31	+76	+14	+14	+20	+14	+6	-11
60	-17	-4	+4	-11	+17	+32	+69	+6	+11	+17	+13	+4	-11
65	-14	-3	+5	-8	+15	+29	+47	+5	+6	+12	+8	+2	-8
70	-5	-1	+3	-5	+12	+23	+30	+4	+2	+7	+4	0	-4
75	-2	-1	+1	-3	+6	+10	+17	+3	+1	+3	+2	0	-1
80	-1	0	0	-1	+2	+4	+3	+1	0	+1	+1	0	0
TOT	-688	-468	-206	-8	+38	+277	+633	+197	+156	+172	-44	-158	-276

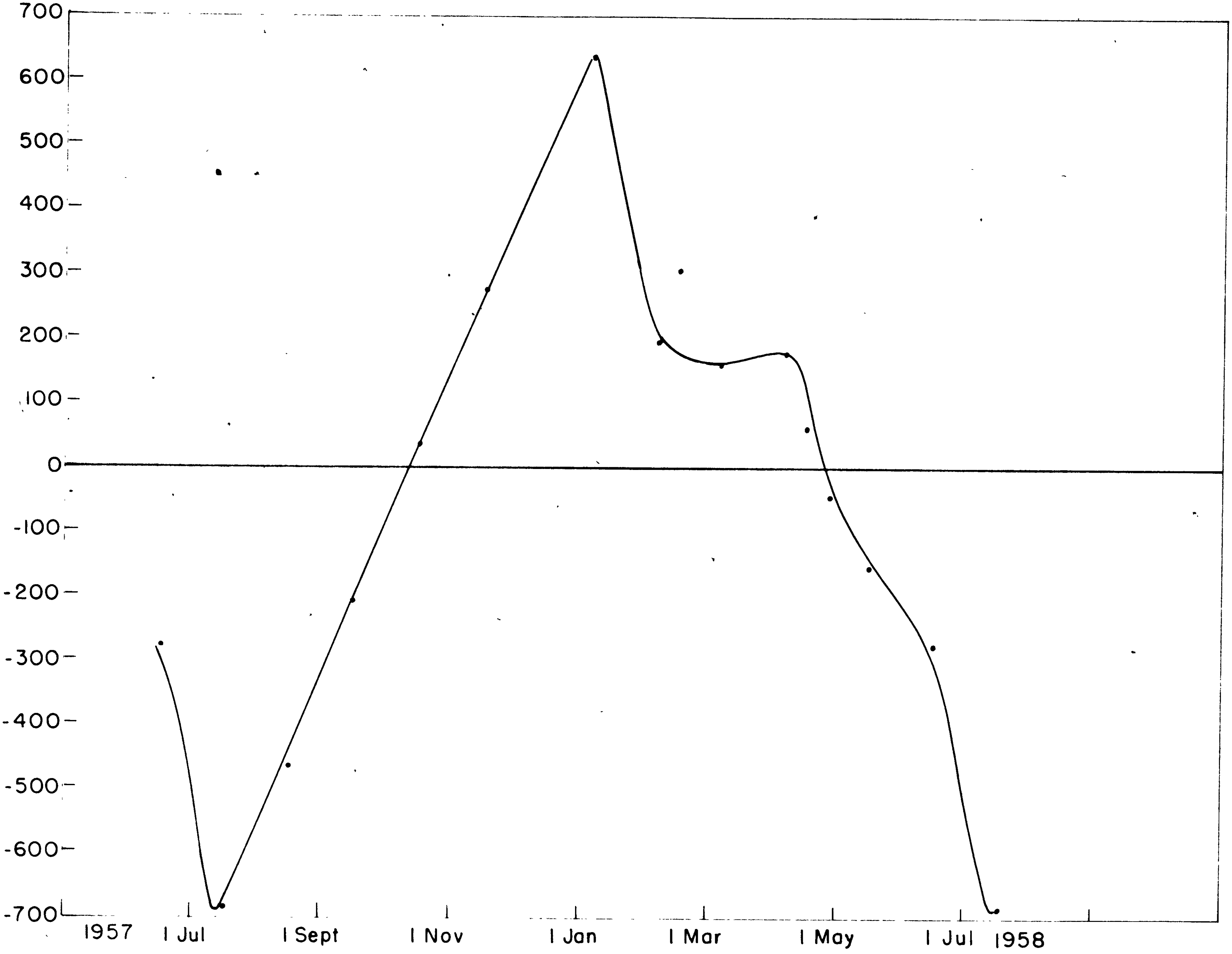


Fig. 1. Momentum Storage Against Time.

Grms cm^2 /sec x 10^{29}

V. VERTICAL EDDY TRANSPORTS

A. Discussion of Analyses

The appendix A is made up of analyses of time correlations of u' and w' for stations included in the study for the two, three-month periods between July and December 1957. Data is from one hundred and from fifty millibars for the seventy-five millibar analyses, from fifty and from thirty millibars for the analyses at forty millibars.

The reduction and analysis of the data separately for 00Z and 12Z provides us two results, for each time period and level. Provided daily variations may be ignored, we can compare the two results to study the adequacy of our data sample. Analyses from the summer period for 00Z and 12Z are very similar both as to location of centers and as to their strength. An exception is areas of sparse data over Siberia and China. In the fall period locations of centers continue to be about the same, but strengths show considerable variation. Analyses for the higher level do not differ appreciably more between 00Z and 12Z than those at the lower surface. Comparison of spacial averages reduced to give momentum transports for the two times show small differences in view of orders of magnitude of larger terms in the momentum balance. Average of departures from quarterly time-means by 00Z and 12Z analyses is 20 gm cm^2 per sec, while the maximum departure is

60 gm cm^2 per sec; storage changes from table II are as large as 670 gm cm^2 per sec.

Computation of vertical motions on which our station values are based included no non-adiabatic heating. According to Barnes (1962) mean values of the vertical motion are inaccurate due to non-adiabatic heating, but standing eddy terms using these vertical motions are reliable. Transient eddies based on adiabatic vertical motions seem dependable.

The analyses of time correlations contain many centers of action in particular latitude belts; however over much of the map little or no connection between time variations of u and ω is noted. Centers tend to be farther south in the summer than winter at both levels, occuring mostly between thirty and fifty degrees of latitude at the former time. In fall at the upper level centers are between forty and seventy degrees. At seventy-five millibars in the same period the belts between thirty and eighty degrees contain areas of high correlation.

The centers tend to occur along latitudes of strong winds, in the fall. A comparison of the forty millibar maps from the appendix with Synoptic Weather Maps by the Weather Bureau for thirty millibars in November indicates that, except for the North American continent where centers are generally absent, our systems lie along the strong flow areas of the polar night jet. Strongest correlations occur above the Alaska ridge and a weaker high in the North Atlantic, with downward motions in the Atlantic between Europe and Iceland and upward vertical flow near Alaska. Directions alternate going zonally along the jet. At seventy-five millibars centers are more widely distributed longitudinally. As at the higher level, one may trace the upper-level jet along our centers; but now others are present near thirty degrees latitude, probably in response to tropospheric flow. North America continues to be a quiet area; centers located north of the Alaska and North Atlantic highs are somewhat weaker than the corresponding features at forty millibars.

In summer the centers are generally weaker, particularly at seventy-five millibars. At the higher level, the only prominent features are at mid-latitudes over Asia. These patterns could not be associated easily with synoptic analyses.

B. Discussion of Zonal Averages

Computation of momentum transports were based on zonal averages of the eddy terms from our analyses for 00Z and 12Z. Plotted points in figures 2 through 9 are averages from the two times with the sign reversed so that positive portions of our curve show upward transports.

(1) April through June (figures 2 and 3).

The most striking feature of the transport by vertical eddies in the spring season is the small contribution by standing eddies. No other season has so slight a contribution by either transport. This effect may perhaps be looked upon as a failure of long waves from the troposphere to penetrate into the stratosphere during the time of reversal by the stratospheric circulation and when higher levels are moving in the opposite direction from those below.

At latitudes below forty-five degrees there are negative transports by transient eddies, at both levels. Their direction is proper to contribute to the establishment of the summer circulation from the east during the spring season.

In common with all other seasons, both terms show small contributions at northerly latitudes, due to the reduced radius in momentum computations. Like other seasons, where either eddy transport is large, the one at forty millibars is reduced as compared with that at seventy-five millibars by a factor of over one-to-two.

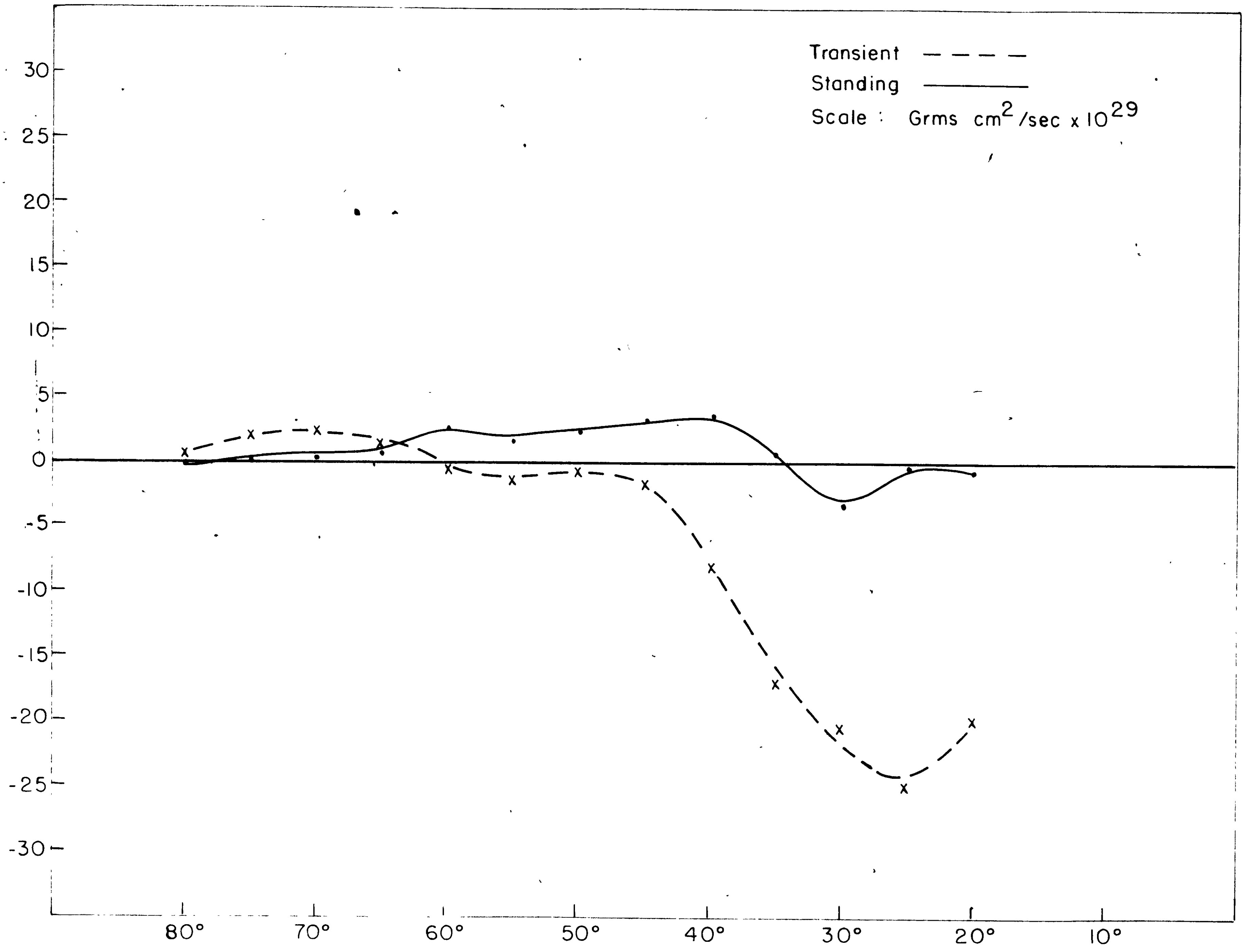


Fig. 2. Momentum Transport by Vertical Eddies.
Apr. - Jun. 1958
75mb Level

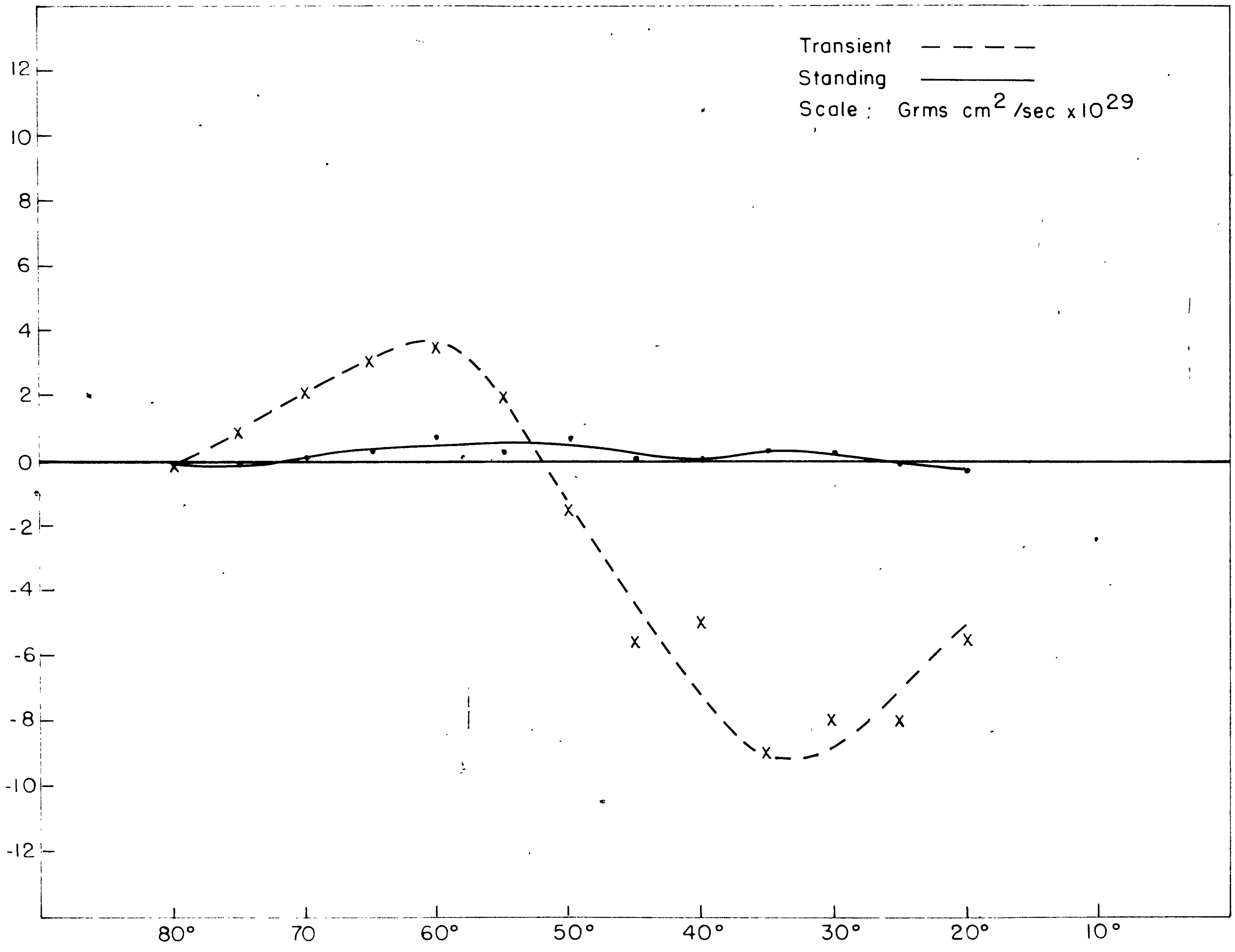


Fig.3. Momentum Transport by Vertical Eddies.
Apr – Jun 1958 40mb Level.

(2) July through September (figures 4 and 5).

The standing eddy continues to be small at the higher level, but the seventy-five millibar level at twenty-five degrees latitude has a noticeable negative transport—opposing the change in storage for the fall season.

Between the two levels the transient eddies are out of phase for the various latitudes. One would suspect that the seventy-five millibar level is influenced by tropospheric systems, while the forty millibar surface shows stratospheric effects. The positive transport at forty millibars supports the storage change for this season. From figure 1, the storage is negative for the fall quarter; a positive transport then requires a downward movement of negative momentum. Negative transport at twenty-five degrees latitude tend to continue flow from the east, as observed at lower latitudes.

(3) October through December (figures 6 and 7).

Transient eddies between the two levels continue to be out of phase. Amplitudes are about double those of the summer season and equal to those in the spring.

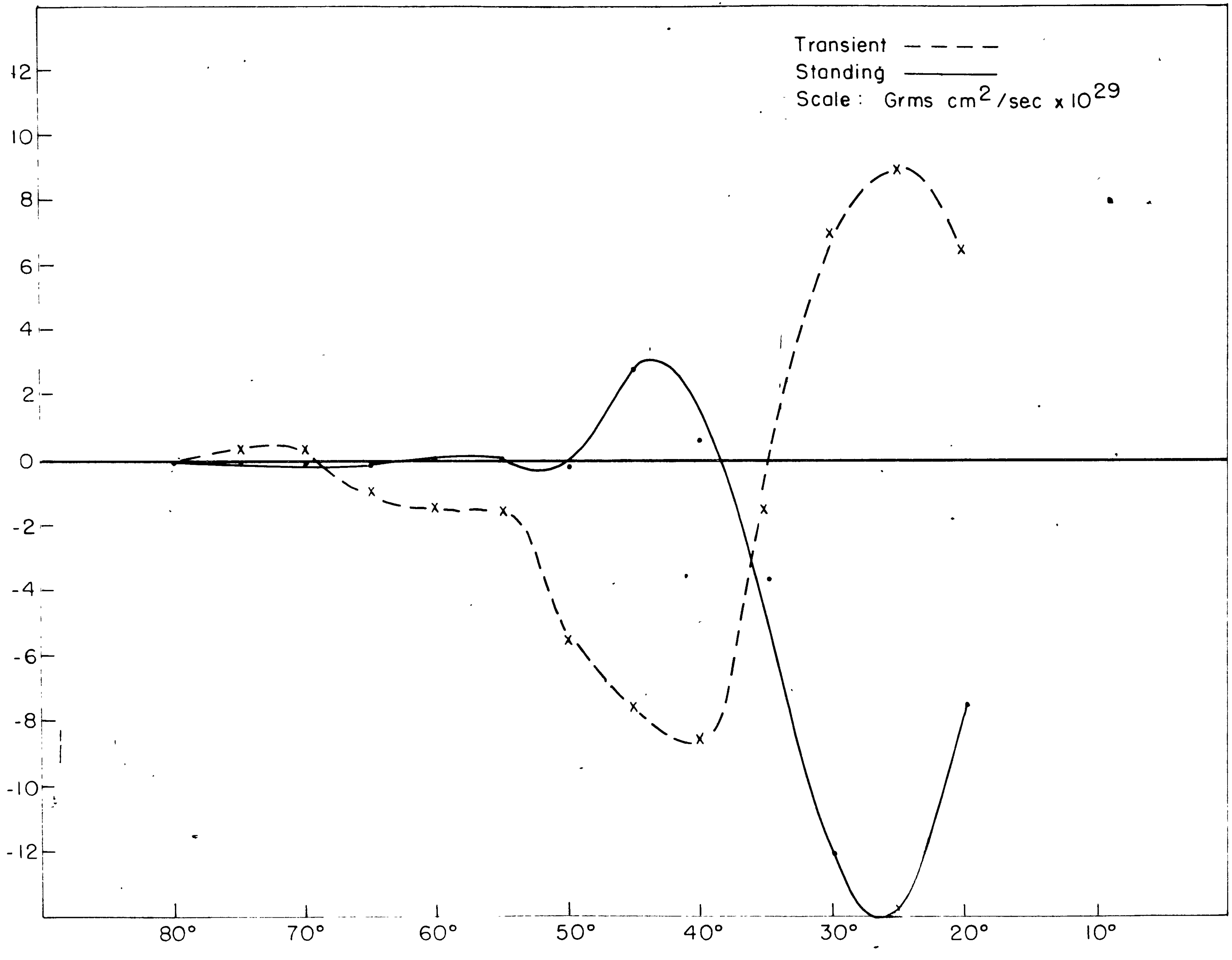


Fig. 4. Momentum Transport by Vertical Eddies.
Jul. - Sept. 1957
75mb Level.

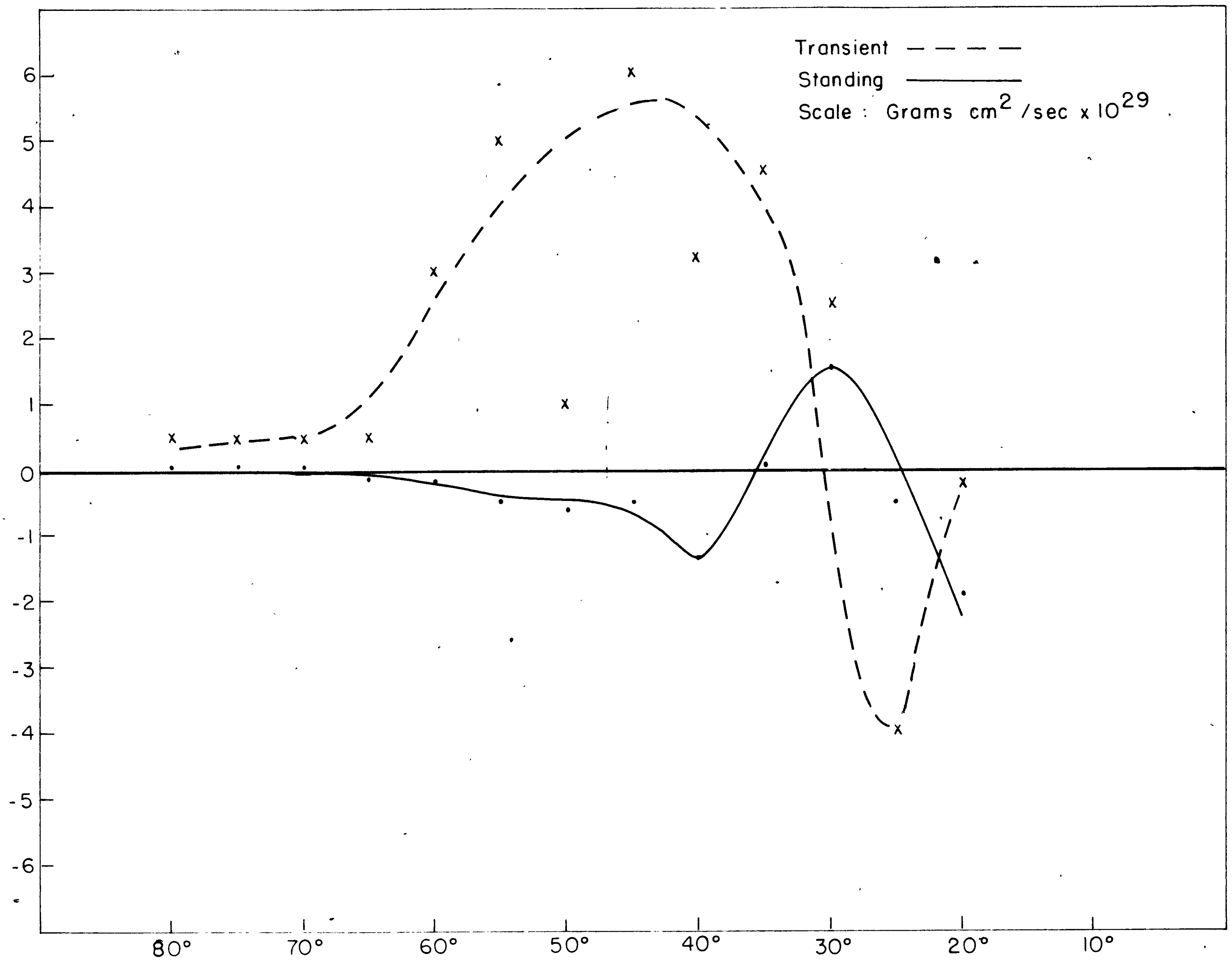


Fig. 5. Momentum Transport by Vertical Eddies
Jul.- Sept. 1957
40mb Level

Standing eddies contribute at both levels in the same direction at most latitudes. The positive transport by transient eddies at high latitudes supports the formation of the contracted polar night jet in late fall. However, the sum over latitudes for both eddies are in the opposite direction to the storage change.

(4) January through March (figures 8 and 9).

Both terms are in phase between the two levels. South of sixty degrees latitude, negative transports are of the proper direction to contribute to the loss of momentum throughout the winter. North of sixty degrees latitude the storage shows a strong drop early in the period, followed by a gain over February and March reflected in the slight increase in total storage over all latitudes north of twenty degrees, in figure 1. Our transient eddy north of sixty degrees latitude agrees with the sign of the storage term at those latitudes for most of the period.

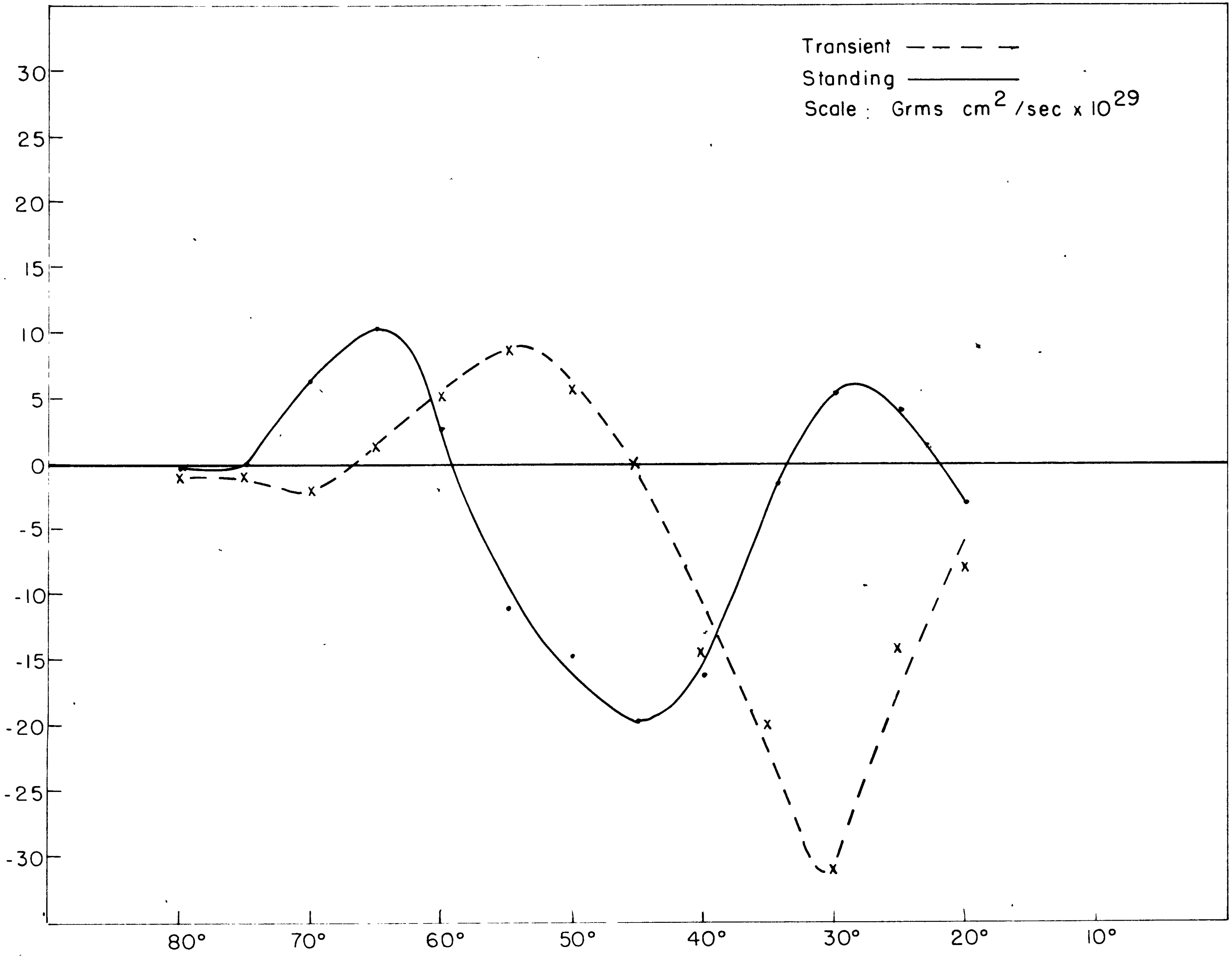


Fig. 6. Momentum Transport by Vertical Eddies.
Oct. - Dec. 1957
75mb Level.

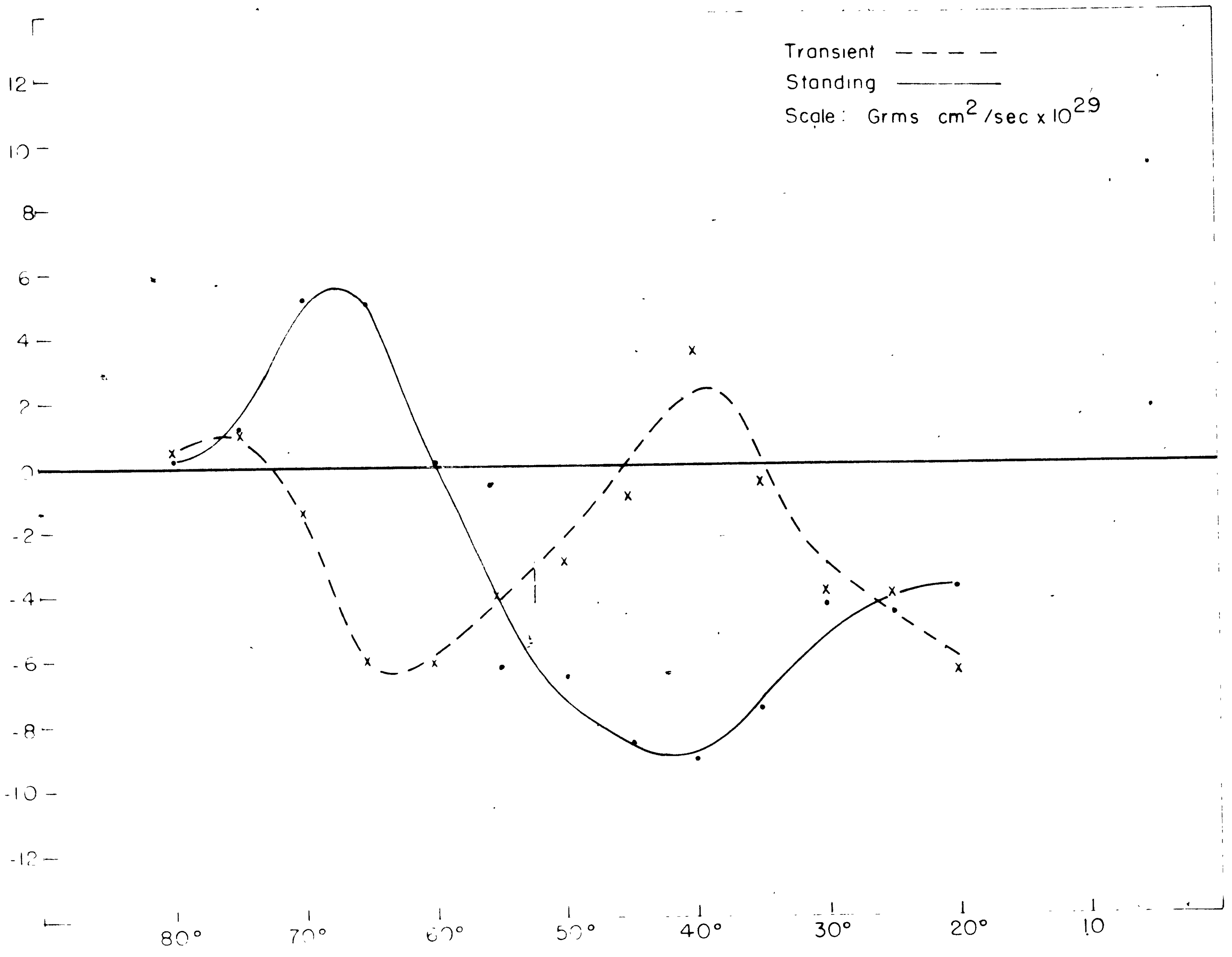


Fig. 7. Momentum Transport by Vertical Eddies.
Oct - Dec 1957 40mb Level.

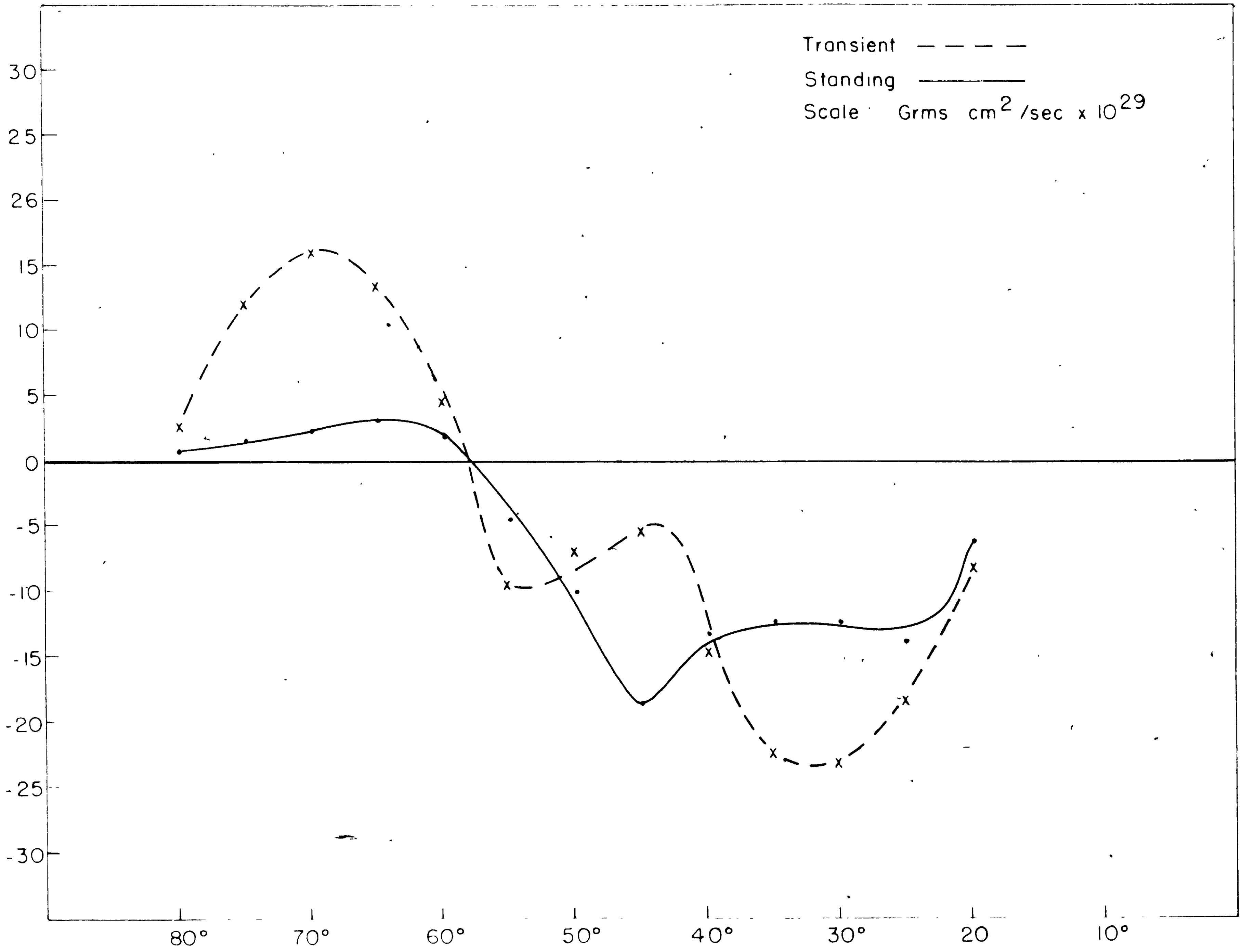


Fig. 8. Momentum Transport by Vertical Eddies.
Jan. - Mar. 1958
75mb Level.

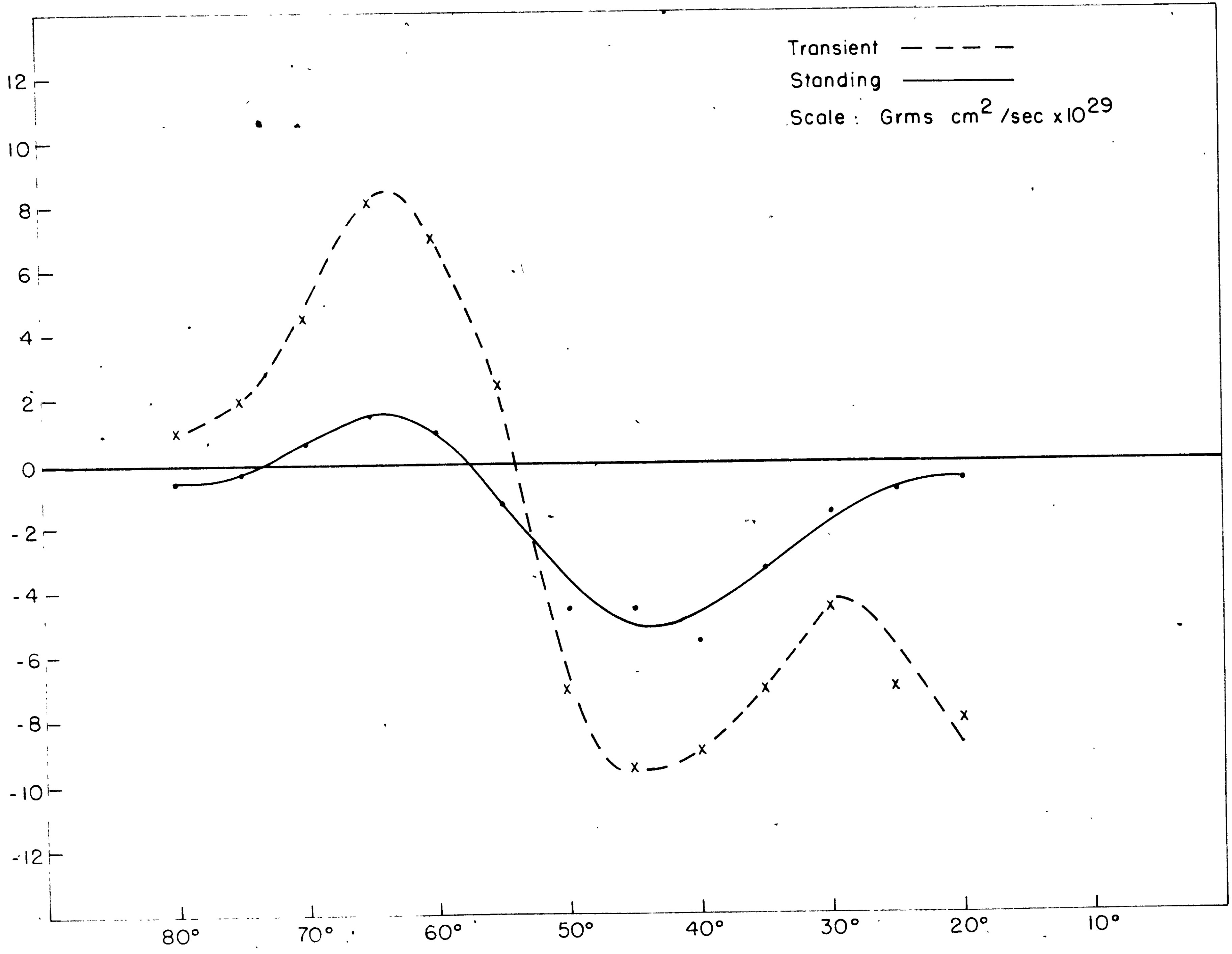


Fig. 9. Momentum Transport by Vertical Eddies.
Jan. - Mar. 1958
40mb Level.

C. Comparison of Eddy Transport with Change in Storage

TABLE II

		Standing Eddy		Transient Eddy	
		75mb	40mb	75mb	40mb
April - June	- 670	+ 9	+ 2	- 88	- 31
July - Sept.	+ 470	- 34	- 4	- 4	+ 34
Oct. - Dec.	+ 630	- 45	- 38	- 70	- 31
Jan. - March	- 430	- 85	- 20	- 60	- 30

Momentum by Periods (gms cm^2 per sec x10^{29})

Only in winter do the vertical eddies contribute appreciably to the stratospheric storage change; in that period about thirty-five per cent may be accounted for by vertical eddies at seventy-five millibars. During the increase in momentum storage in summer and fall, the vertical eddies at seventy-five millibars act oppositely from the storage change. The spring season is discussed more fully in section VII.

VI. MOMENTUM BALANCE IN THE SPRING SEASON

Figures in table II show that the transport by vertical eddies is not sufficient in size and in some cases is incorrect in sign to account for momentum storage changes observed in the lower stratosphere. Horizontal eddies have been studied extensively by Dickinson (1962). Terms of this type ($[\bar{u}^* \bar{v}^*]$ and $[\overline{u'v'}]$) are included in table III and fail to account for the storage change. In this section we will be concerned with terms in equation 12 that depend on mean motions. To this point our data sources have been sufficient in amount and representativeness to give reliable transports.

Mean vertical motions based on our data are inaccurate as noted in section V, and only estimates of its value are possible; its determination from the continuity relationship is difficult because values of the meridional component are not reliable.

Several ways of determining meridional means have been attempted, with varying results. Oort (1962) compiled space and time means from IGY data. Teweles (1963) considered friction and vertical divergence to be small;

he kept the storage change and horizontal divergence terms in his form of the equation of motion. Dickinson (1962) used the momentum equation with assumed values of friction. These three studies give varying values of meridional motion. Summer averages from Oort are generally several times larger at mid latitudes than corresponding figures for April and July by Teweles. Yearly averages by Dickinson, with a large friction coefficient introduced, show a south motion at twenty to thirty degrees latitude that is several times larger than that from either of the other studies; without friction Dickinson's figures become small and northward.

In a study of diurnal variations in atmospheric parameters Harris, Finger, and Teweles (1962) took advantage of adjacent periods when rawinsonde data from Terceira in the Azores contained first observations at 03Z, 09Z, 15Z, 21Z and then at 00Z, 06Z, 12Z, 18Z to display semidiurnal periods in the wind. Such a period in the light north-south component would cause an appreciable bias in time averages, such as those of Oort, when the variation is other than zero at standard observation time. The author investigated the use of tracks from constant level balloons as a method of gathering time-averaged data for wind calculations.

A. Meridional Motions from Constant Level Balloon Tracks

Mr. Hess of the Air Force Cambridge Research Laboratory is analyzing data from balloon flights made since the summer of 1956. The flights are for operational purposes; tracking data is a by-product. Mr. Hess consented to the use of his data in a portion of this study.

Flight profiles generally level-off at about thirty kilometers above the surface, during morning hours. The balloon remains at the established altitude until past noon. Over most of the afternoon loss of heat by radiation causes a gradual descent of about two kilometers. After sunset a rapid descent takes place, that is checked by release of ballast after a ten kilometer drop in altitude. The flight remains at about constant level throughout the night. After sunrise the balloon ascends by about ten kilometers; the rise is checked near the original level-off, by gas valving.

In recent months the development of constant pressure balloons have made possible flights free of day-to-night variations in height. Ballast and valving problems are reduced or eliminated; a flight of a month's duration has been accomplished at the twenty kilometer altitude.

Radio bearings by the Federal Communications Commission (FCC) are used to fix the balloon's position. For the winds used in the study, only class-A fixes were used in determining tracks; one such position has a possible error of twenty miles. Giles and Peterson (1958) compared FCC radio fixes with balloon position determined by ground photographs, in a study that verified the twenty mile accuracy figure.

For most of our data, tracks extended over a six-hour period for each wind calculation - either over all-day or all-night hours, so that a constant altitude applied. Generally smooth tracks could be drawn through several fixes for each wind. Exceptions to the six hour period were the fifty winds taken from constant pressure flight data. Their tracks were over twenty-four hour periods. Flight altitudes were taken from the planned profiles.

These balloons have varying shapes; diameters are as large as several hundred feet. Their response to short period variations in the flow would be poor. For time and space means of the wind, their tracks seem to provide reliable responses.

Nearly all of the tracks were over the United States between thirty and fifty degrees latitude. At best a third of the earth's longitudes were covered. The results can be considered only indications of zonal averages.

Time averages were taken over the three spring months. The author grouped wind readings as to altitude range, between eighteen and twenty-three kilometers and between twenty-three and thirty-two kilometers - without regard to latitude. Since many of the flights have large changes in altitude from day to night, our data would reflect any bias in the north-south flow that had this period in its variation. About one hundred fifteen readings were available for the lower levels; about seventy-five for the higher levels. Individual readings for north-south components varied considerably over a range of several meters per second. The average for eighteen to twenty-three kilometers was 5 cms per sec toward the south with a standard deviation of 256 cm per sec; that for twenty-three to thirty-three kilometers was 17 cms per sec toward the north with a standard deviation of 275 cm per sec.

Standing eddies in the long waves could cause a large error in our meridional motions, because of the limited longitudinal extent of the data. For wave number one, the one-third of a wave length covered by our flights could fall in entirely southward motions and over-estimate the southerly component. An examination of our $[\overline{u}^*\overline{\omega}^*]$ latitudinal variation in the spring (figures 2 and 3) reveals that the standing eddy is negligible. We cannot be sure whether the standing eddies do not exist, average-out zonally, or have no associated vertical motions.

Analyses by Oort (1962) of $[\overline{v}]$ for the spring show a number of small cells, with considerable alternation of centers of north and south motion. Our averages over one-third of the earth's longitudes cover what seems to be a weak standing wave.

From the foregoing, we may conclude that the true meridional motion is yet to be determined. Nevertheless we may use the figures as guidelines in a crude evaluation of the momentum budget, realizing that this part of the problem will require reexamination as more information becomes available. We should emphasize that the constant level balloon approach may prove a very convenient way to observe mean meridional motions.

B. Momentum Balance Equation, Spring Season.

Terms in equation 12, chapter II, are given in table III. The upper portion of the table contains storage and eddy terms from previous chapters, where the storage has been moved to the right hand side. In the lower portion several mean values of meridional motion are assumed and the necessary frictional transports are estimated, so that a balance may be made. Meridional motion for column 1 is -5 cms per sec; column 2, -2.2 cms per sec; column 3, 0 cms per sec. Mean vertical motions are assumed zero. Frictional transports at the top of each volume (surface S_3) are those necessary to complete a balance (all terms are on the same side of equation 12 and must add to zero.) Eddy viscosity coefficients, at the seventy-five millibar surface between 10^3 and 10^4 cms^2 per sec were considered possible; a figure of $10^{4.5}$ cms^2 per sec quoted by Newell (1962) from J. Spar as observed in tungsten diffusion was used as an upper limit. The value must be an upper limit because the observed spread of tungsten is the combined effect of motions on all scales. The vertical shear factor was taken from summer profiles by Batten (1961) modified to represent spring conditions, with a vertical scale of ten kilometers.

For conditions in column 1, 320 momentum units are observed to have been removed from our lower volume. Eddy vertical transports account for 110 units, but horizontal eddies put in nearly this amount. Our assumed meridional motion toward the south of 5 cms per sec acts through the Coriolis term to remove 950 units and leaves us overbalanced negatively by 620 units. Using a viscosity coefficient of 10^4 cms^2 per sec, we calculate an input of momentum of 1600 units by friction; thus our volume above must remove 930 units, which may be accomplished with a coefficient of 2 X 10^4. We observe that the upper volume looses 345 momentum units and that our southward motion acts through Coriolis torque to remove 810 units and leave us with too much momentum removal by 480 units. The 930 units input from below by friction leaves 450 units to be removed through ten millibars. A coefficient of eddy viscosity of $3x10^4$ cms^2 per sec at ten millibars would allow this balance.

Using a southward drift of 2.2 cms per sec, in column 2, the required frictional transports are much reduced, as compared with column 1.

TABLE III

Momentum Balance in Spring

Units 10^{29} gm cm^2 per sec

for column (1),$[\bar v]$=5 cms per sec; (2), 2.2 cms per sec;

(3),0 cms per sec

	75 to 40 mbs			40 to 10 mbs		
$-\iint([u]_2-[u]_1)v\,dy\,dP$	+320			+345		
$\int_{S_3}[\bar u^*\bar w^*]v\,dy$	0			0		
$\int_{S_3}[\overline{u'w'}]v\,dy$	-30			0		
$-\int_{S_2}[\bar u^*\bar w^*]v\,dy$	+10			0		
$-\int_{S_2}[\overline{u'w'}]v\,dy$	-90			+ 30		
$\int_{S_1}[\bar u^*\bar v^*]v\,dy$	0			- 10		
$\int_{S_1}[\overline{u'v'}]v\,dP$	+95			- 40		
	(1)	(2)	(3)	(1)	(2)	(3)
$\int_{S_3}[\bar u][\bar w]v\,dy$	0	0	0	0	0	0
$-\int_{S_2}[\bar u][\bar w]v\,dy$	0	0	0	0	0	0
$\int_{S_1}[\bar u][\bar v]v\,dP$	+ 25	+10	0	+ 5	0	0
$\Omega\int_{S_3}v[\bar w]dy$	0	0	0	0	0	0
$-\Omega\int_{S_2}v[\bar w]dy$	0	0	0	0	0	0
$-\Omega\iint v^2\frac{\partial[\bar v]}{\partial y}dy\,dP$	0	0	0	0	0	0
$2\Omega R\iint\sin\theta\cos\theta[\bar w]dy\,dP$	- 950	-425	0	- 810	-360	0
TOTAL	- 620	- 65	+305	-480	-35	+325
$-gK_{S_2}\int_{S_2}\rho v\frac{\partial[\bar u]}{\partial z}dy$	+1600	+160	+160	+930	+95	+465
$gK_{S_3}\int_{S_3}\rho v\frac{\partial[\bar u]}{\partial z}dy$	- 930	- 95	- 465	-450	- 60	-790

Again storage and eddy terms at the top of the table for the lower volume leave us with a required removal of 310 units; the Coriolis term again removes too much leaving 65 units to be supplied by friction. Using a coefficient of 10^3 cms^2 per sec at seventy-five millibars and one of $2x10^3$ cms^2 per sec at forty millibars gives a balance over our lower volume, with 95 units to be absorbed by the volume above. In the top volume, storage change and Coriolis torque are again the dominant terms; they leave us with a net removal required of 35 units. The 95 unit input from below by friction is off-set by these 35 units and a 60 unit removal is required by friction through the ten millibar surface, requiring a coefficient of $4x10^3$ cms^2 per sec.

The removal of all meridional motion, in column 3, leaves friction as the only significant term to off-set our observed storage changes. For the lower volume 305 units must be removed. A coefficient of 10^3 puts in 160 units through the lower surface and necessitates the removal of 465 units by the top volume. A coefficient of 10^4 is required at forty millibars. For the top volume, frictional removal at ten millibars must off-set the 325 units observed change plus the 465 unit input from below. A coefficient of 5×10^4 cms^2 per sec is required.

The conditions in column 2 seem the most easily obtained. A comparatively small eddy viscosity coefficient of 10^3 cms^2 per sec is required at seventy-five millibars with a gradual increase with height. Conditions in column 1 required a large coefficient at lower levels. For column 3 a steep gradient with height of eddy coefficient is required at our levels, similar to that given by Lettau (1951) for levels near the top of the stratosphere. Any non-zero mean vertical motions would change these comments about the balance. In view of Barnes' (1962) findings that mean adiabatic vertical motions are in the opposite direction to the expected mean diabatic vertical motions it is obviously not wise at the present time to make any other assumptions about the mean vertical motions than that they are zero.

The large frictional transports of opposite sign from the stratosphere and troposphere in table III point to a danger in the use of this part of the atmosphere as a base for dynamical studies. Effects sometimes act in opposite directions, partially off-setting each other. One would expect only to arrive at the difference of two large effects and to obtain an answer not representative of any major regime.

VII. DIVERGENCE BY VERTICAL EDDY TRANSPORTS FOR THE PERIOD JANUARY THROUGH JULY 1958

Dickinson (1962) computed divergence against latitude for vertical eddies - terms of the type $\frac{\partial}{\partial p}\left[\overline{u'w'} + \overline{u}^*\overline{w}^*\right]$, for January through June 1958. Those for the periods July through December 1957 are given in figure 10. Data used to calculate the divergence was that observed at the one hundred, fifty, and thirty millibar surfaces - north of twenty degrees latitude. The latitudinal variation given should be thought of as applying at the fifty millibar level.

Both the transient and standing eddy terms give convergence at lower latitudes, extending north to about fifty degrees for the former and to about sixty degrees for the latter term. Both are again convergent above about seventy degrees. Magnitudes are similar and show the same order as that given by Dickinson (1962) for the January to June 1958 period. Patterns are different in that the winter and spring seasons have divergence in the south and convergence in the far north. For the most part, the directions are controlled by the sign of transport terms below the fifty millibar surface. If the divergences are converted to momentum units, our lower latitude curves will be more heavily weighted than that in the north and a pattern of convergence in the summer and fall with divergence in the winter and spring would result. These directions are the proper ones to contribute to storage of momentum in the lower stratosphere.

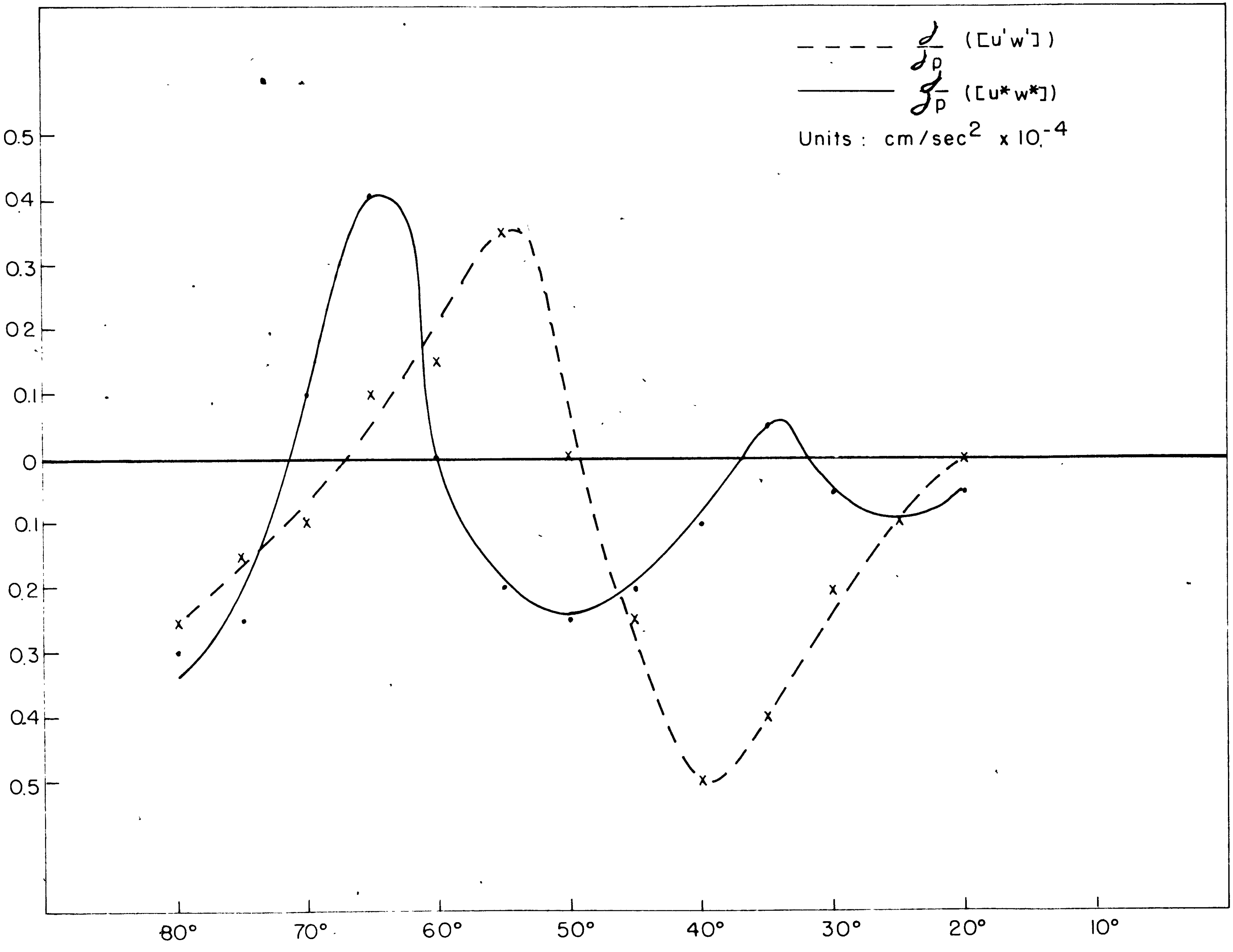


Fig. 10. Momentum Divergence Due to Vertical Eddies. Jul. – Dec. 1957.

VIII CONCLUSIONS

Only in fall and winter seasons do the standing vertical eddies at most latitudes contribute momentum transports comparable in size with the change in storage in the stratosphere - an indication that standing waves do not penetrate the easterly, stratospheric flow. Directions of these eddies as a function of latitude are very similar for the two seasons and levels. Thus our study supports conclusions of Boville (1961) for the 1958-59 season that long waves are present in the stratosphere in the cold seasons. During the summer and fall, seasons of increasing total stratospheric storage, the transient eddies are out-of-phase between forty and seventy-five millibars; the lower surface is most likely affected by travelling tropospheric systems, while the higher level experiences stratospheric conditions. For times when the storage is being reduced, in winter and spring, the transient eddies south of fifty to sixty degrees latitude act to remove momentum from the stratosphere. Eddy transports combined and summed over latitude contribute thrity-five per cent of stratospheric storage change in winter. In other seasons, their contribution is either in the reverse direction to the storage change or (as in spring) is small.

In the spring season, a momentum balance can be achieved with mean meridional drifts of 0, -2.2 cms per sec, and -5 cms per sec acting through Coriolis effects; frictional transports required agree with those that result when one uses eddy viscosity coefficients from tungsten diffusion. Both stratospheric and tropospheric systems may act through the frictional torques to influence dynamics in the lower stratosphere. However, these influences do not seem to act concurrently. The light southerly drift in the lower stratosphere is most likely the result of a very slow northward motion in the troposphere. Thus the levels of seventy-five to ten millibars may be influenced by major systems above or below.

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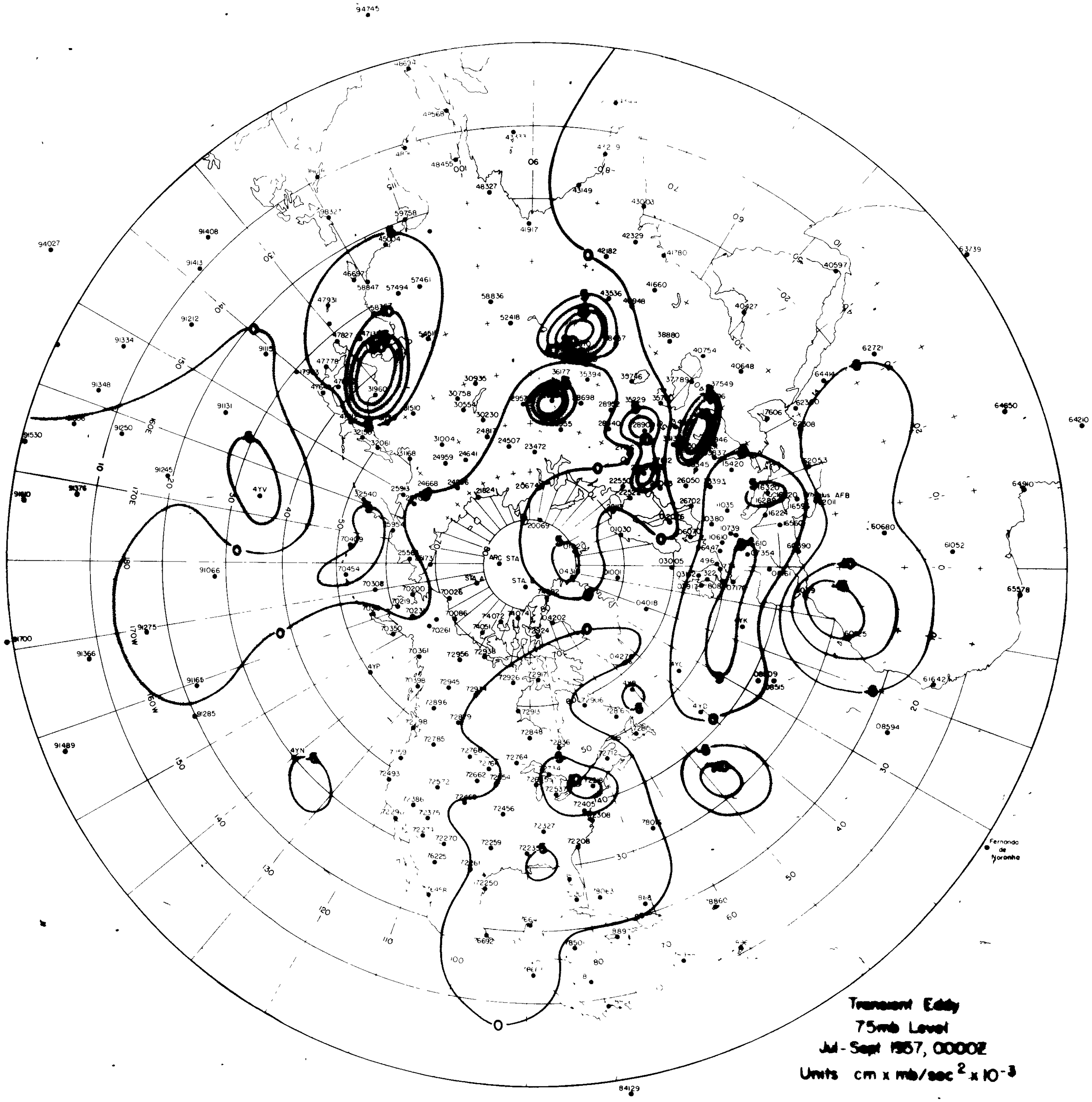
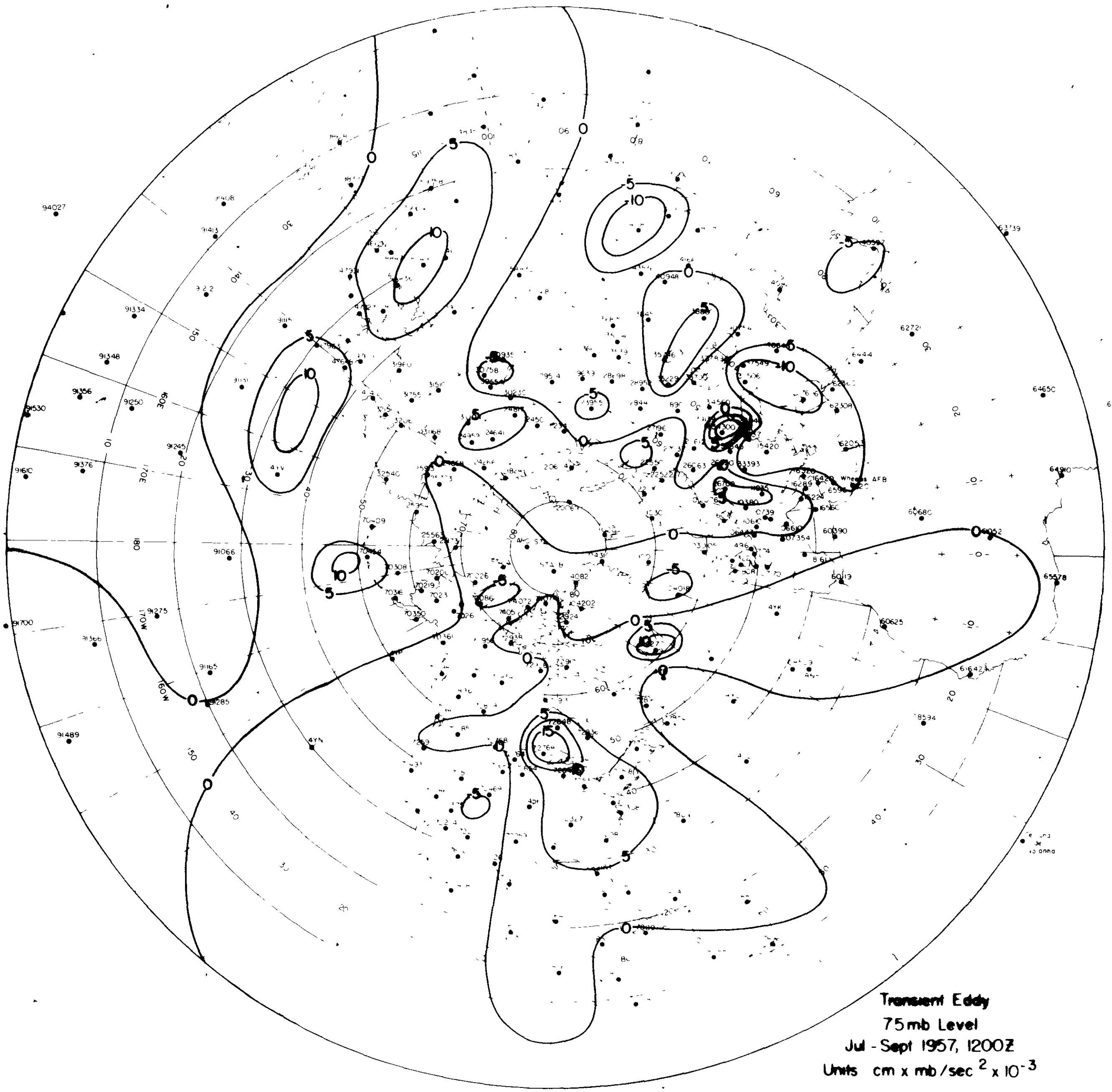
APPENDIX

FIGURES

TABLES



Transient Eddy
75mb Level
Jul-Sept 1957, 0000Z
Units cm x mb/sec^2 x 10^{-3}



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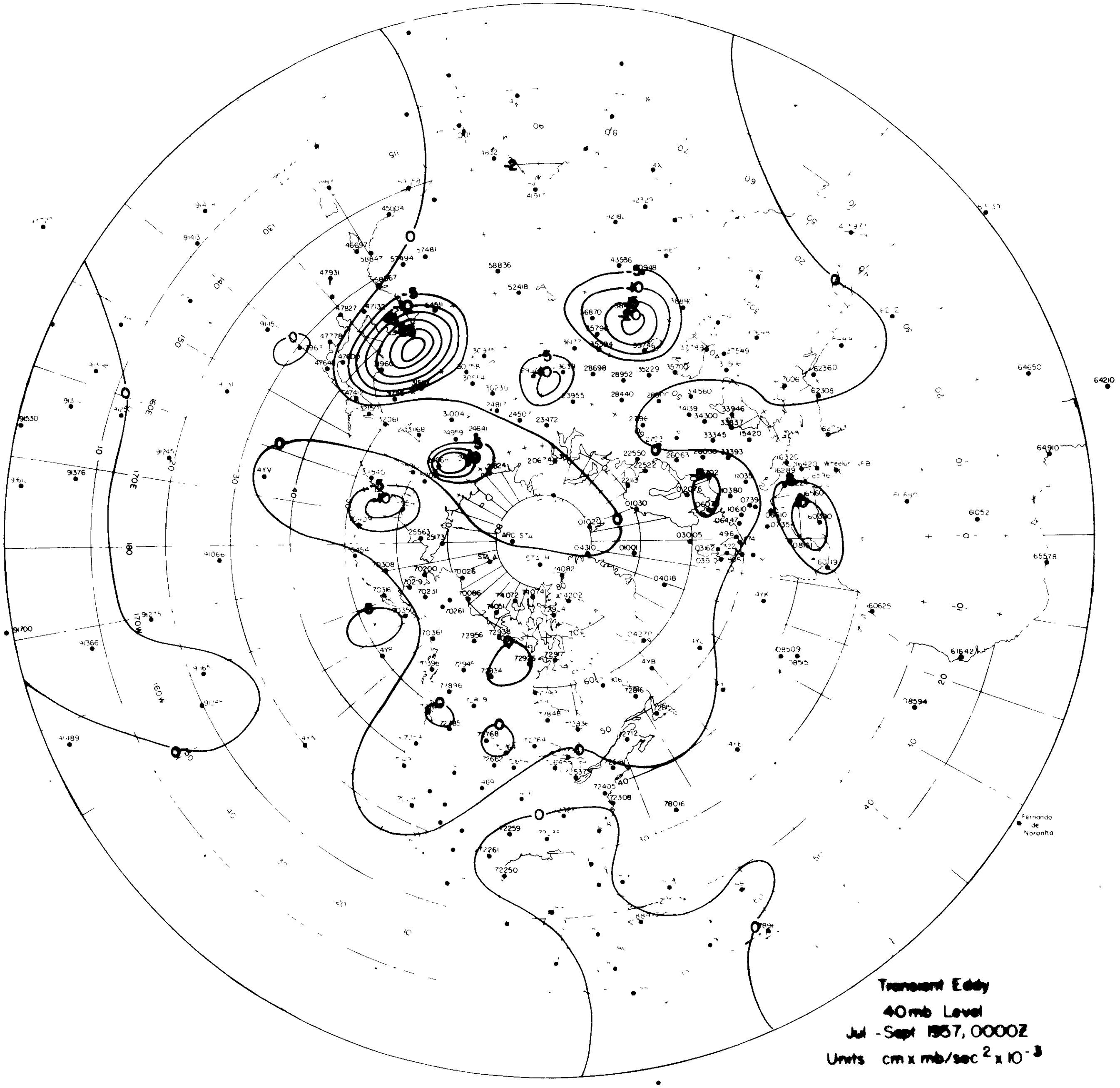
Jul - Sept 1957, 1200Z
Units cm x mb/sec ^{2} x 10^{-3}



Transient Eddy
40mb Level
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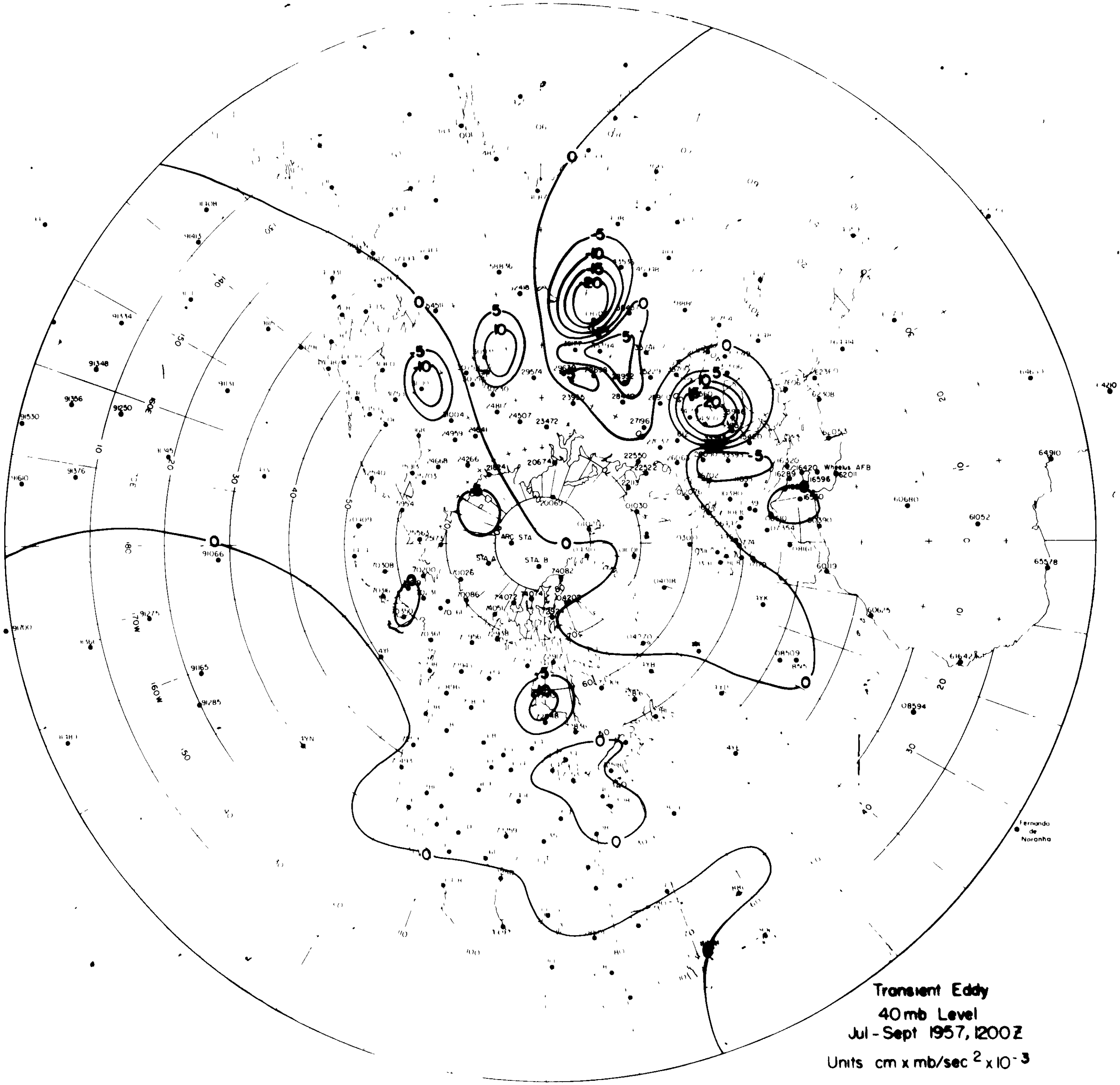
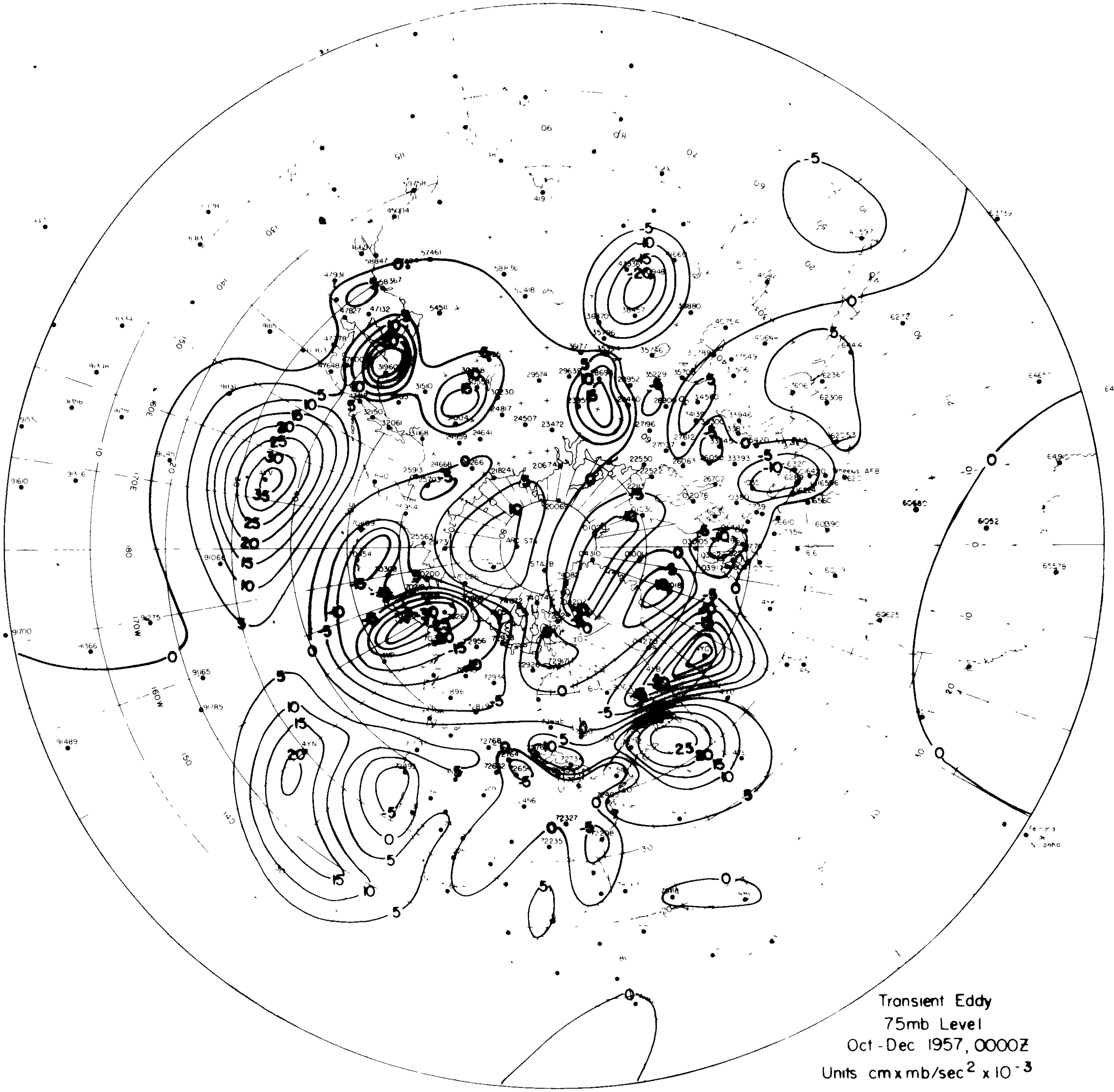
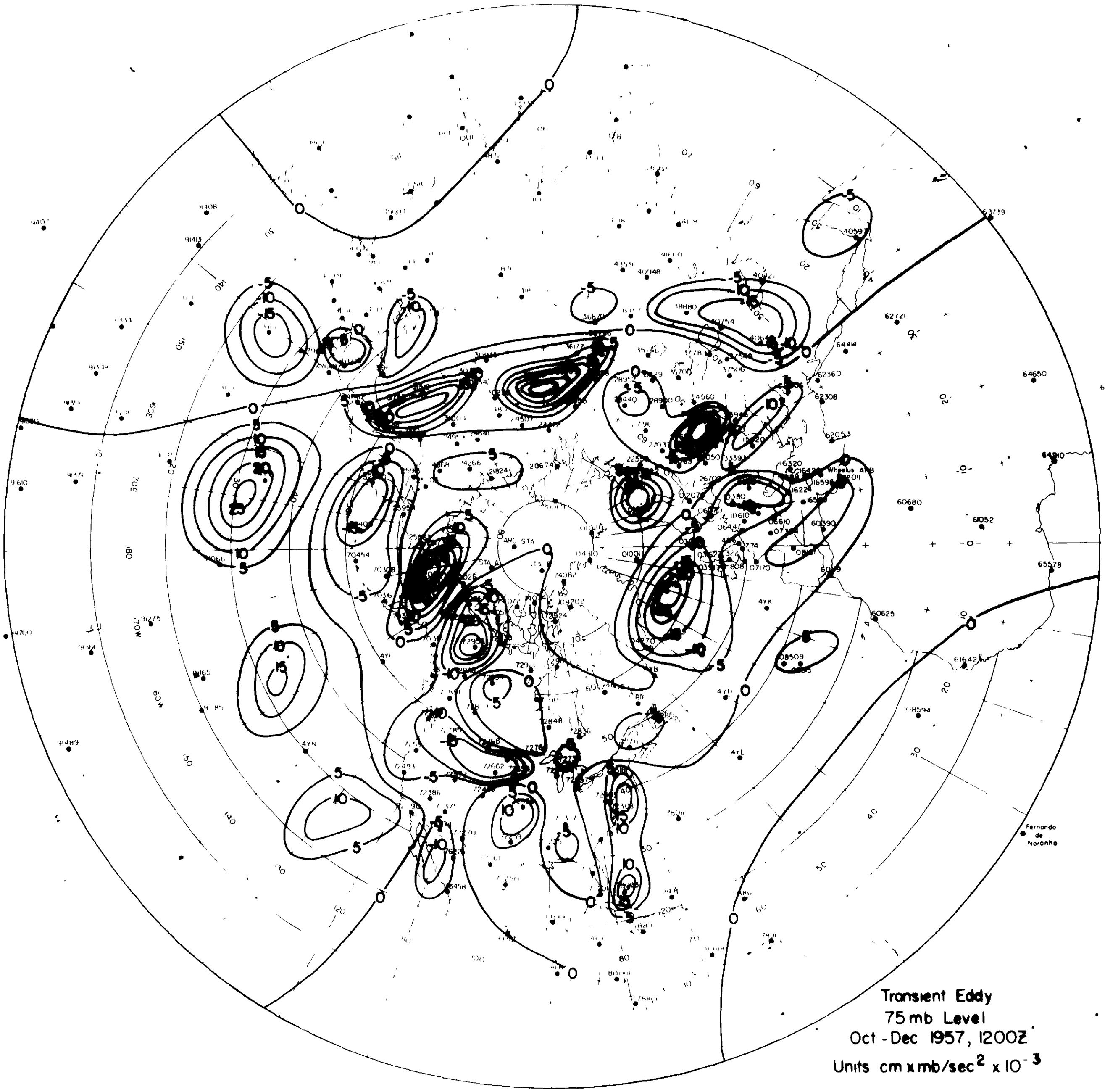
Units cm x mb/sec^2 x 10^{-3}



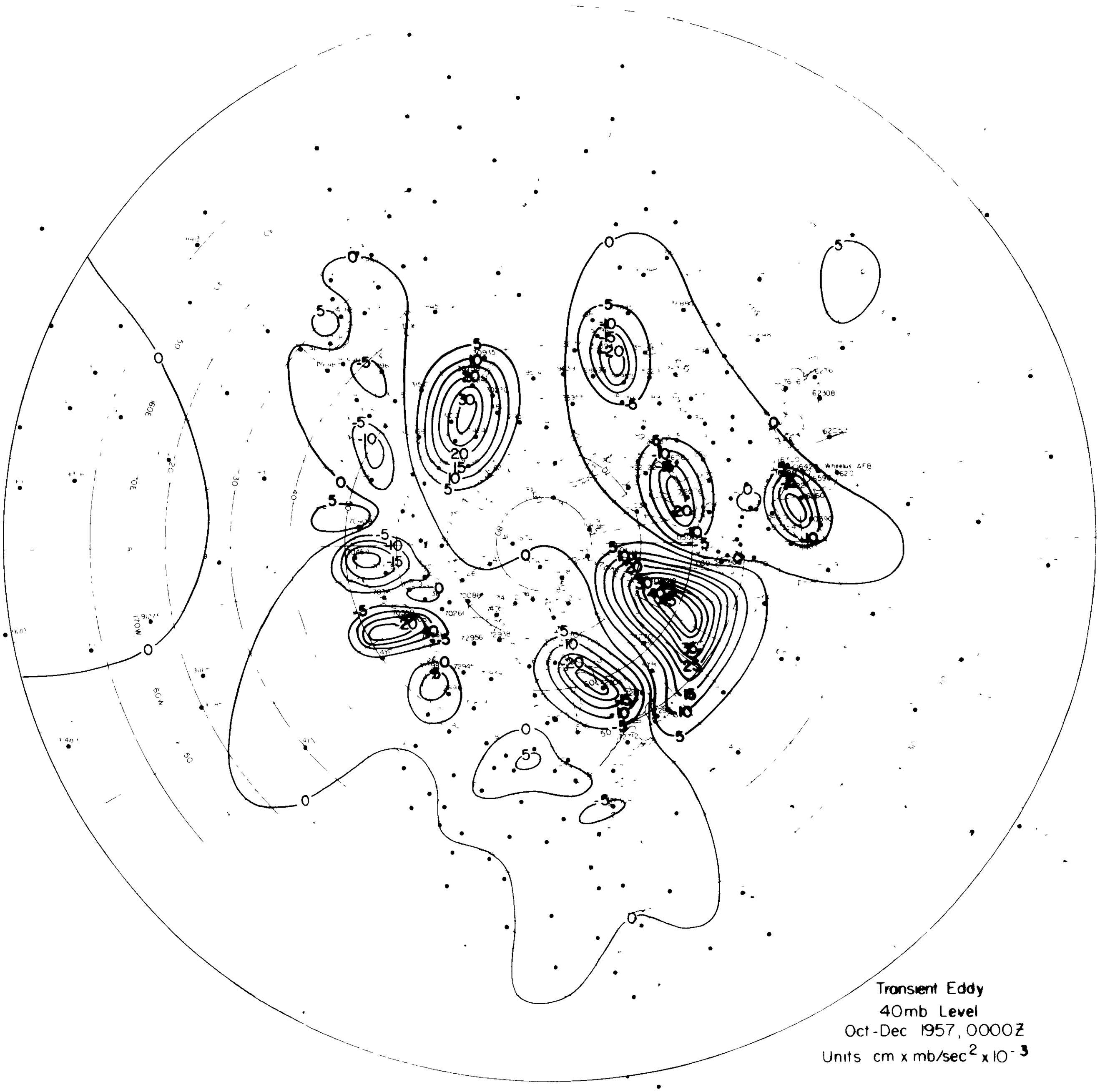
Transient Eddy
40 mb Level
Jul - Sept 1957, 1200Z
Units cm x mb/sec ^{2} x 10 $^{-3}$



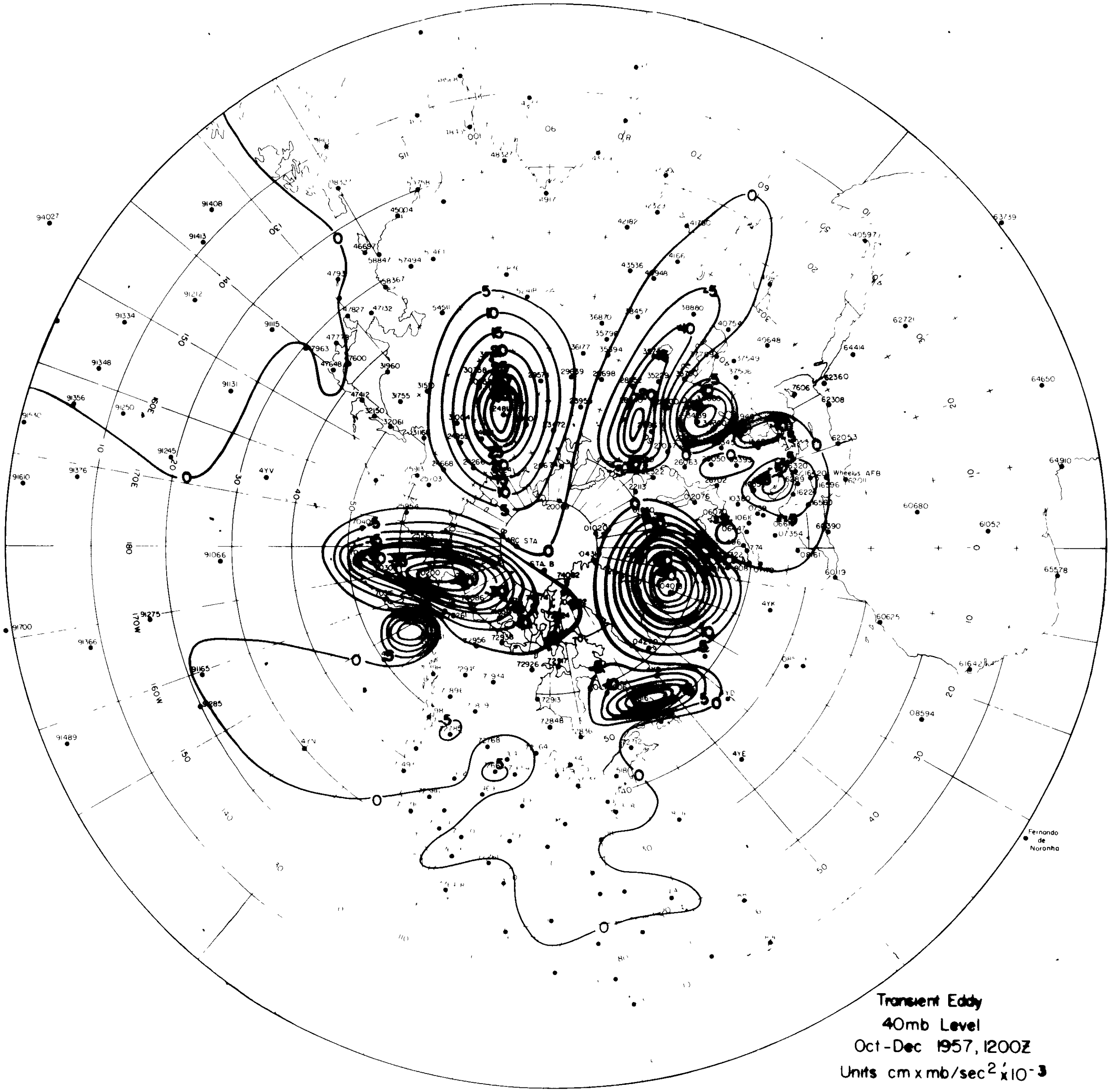
Transient Eddy
75mb Level
Oct-Dec 1957, 0000Z
Units cm x mb/sec^2 x 10^{-3}



Transient Eddy
75 mb Level
Oct - Dec 1957, 1200Z
Units cm x mb/sec^2 x 10^{-3}



Transient Eddy
40mb Level
Oct-Dec 1957, 0000Z
Units cm x mb/sec^2 x 10^{-3}



Transient Eddy
40mb Level
Oct-Dec 1957, 1200Z
Units cm x mb/sec^2 x10^{-3}