A QUASI-GEOSTROPHIC DIAGNOSTIC INVESTIGATION OF A TROPICAL STORM

by

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ABSTRACT

Large-scale diagnostic vertical velocities as determined by a quasi-geostrophic, ten-level model were computed for tropical storm Debbie of September 1965. An isogon-isotach analysis of the horizontal wind field was used in order to calculate a height field suitable for input into the geostrophic model. Results indicate that it is possible to describe accurately tropical motions using the data network in the vicinity of the Gulf of Mexico and the Caribbean Sea. Computations were made with and without latent heating. The calculated vertical velocities qualitatively agree with the cloudiness shown in Tiros nephs but are not quantitatively consistent with reported rainfall rates. It appears that an interaction between cyclone- and cumulus-scale motions occurs in the region of maximum large-scale, upward vertical velocities thus producing an area of enhanced upward motion in this region.

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INTRODUCTION

Multilevel dynamic models of the atmosphere have been used recently to calculate diagnostic vertical velocities in an effort to determine the relationship between vertical motion and cloudiness and precipitation. Studies by Barr and Lawrence (1964) and Sanders (1965) have shown that horizontal motions are equally as important as vertical motions in accounting for observed cloud systems in middle latitudes. Indeed, in order to adequately represent cloudiness by vertical motions of multilevel models, the availability of moisture and allowance for horizontal changes in static stability must be included.

The main difficulty in associating upward vertical motion with the cloudiness shown in Tiros satellite pictures is the fact that cloud elements move with a spectrum of horizontal velocities. While it is true that the large-scale vertical motion pattern and its movement in the flow pattern determine the location of the large-scale cloud mass, horizontal winds frequently advect saturated parcels of air into descent regions and unsaturated air into ascent regions. If the moist air in descent regions has not had sufficient time to evaporate and the unsaturated air in the ascent region has not condensed, a lack of correlation between the dynamically-calculated vertical velocities and the cloud pictures will be found. Agreement between cloudiness and vertical motion is best in the presence of abundant moisture and in the early stages of cyclone formation before the horizontal motions have fully developed. In lower latitudes where horizontal velocities are smaller and warm ocean surfaces provide a readily available source of moisture, it is anticipated that Tiros pictures might be more representative of actual vertical motions.

With the exception of monsoon circulations and hurricanes the horizontal gradients of geopotential height and temperature are much smaller in the tropics than the corresponding gradients at middle latitudes. Because of the smallness of these gradients it is not intuitively apparent that differential vorticity and thermal advections on isobaric surfaces play a prominent role in the production of rising motion. In tropical regions the Rossby number, which is the ratio of the horizontal relative accelerations to the Coriolis acceleration, approaches one and it can no longer be regarded as much less than one in the scaling of the geostrophic equations. This means that for the same velocity and length scales the relative importance of non-geostrophic effects is increasing near the equator. For these reasons it is not known *a priori* whether the quasi-geostrophic model will give qualitatively acceptable results in tropical regions. It is the purpose of this study to determine how well diagnostic results of a ten level quasi-geostrophic model with incorporation of latent heating are correlated with cloud cover as seen by Tiros weather satellites in the tropics. Tropical storm Debbie of September 1965 was chosen for analysis because a depression was wanted for the study which had closed isobars at the surface but which was not so intense as to invalidate the geostrophic assumption.

THE DIAGNOSTIC MODEL

The large scale motions of middle latitudes are nearly in geostrophic and hydrostatic balance and are well represented by quasi-geostrophic models. A diagnostic, geostrophic equation for calculating the large-scale vertical motion field and a geostrophic height tendency equation can be obtained by combining hydrodynamic and thermodynamic equations appropriately scaled for middle latitude flow where the Rossby number is small. The form of the vorticity equation in x, y, p, t-coordinates consistent with this type of scaling is

$$\nabla^2 \frac{\partial \Phi}{\partial t} = -f_0 \vec{V} \cdot \nabla \eta + f_0^2 \frac{\partial \omega}{\partial p} \tag{1}$$

where Φ is geopotential, η is the absolute vorticity, f_0 is a constant value of the Coriolis parameter chosen at the center of the grid, and $\omega \equiv \frac{dp}{dt}$ represents the vertical motion. Similar scaling on the thermodynamic equation results in

$$\frac{\partial}{\partial p} \frac{\partial \Phi}{\partial t} = -\vec{V} \cdot \nabla \frac{\partial \Phi}{\partial p} - \sigma \omega - H \tag{2}$$

where $\sigma \equiv \frac{\partial \Phi}{\partial p} \frac{\partial \ln \theta}{\partial p}$ (a function only of pressure) is a measure of static stability and $H \equiv \frac{R}{C_p p} \frac{dQ}{dt}$ is the diabatic heating. By combining these equations so as to eliminate $\frac{\partial \Phi}{\partial t}$, a diagnostic ω- equation results.

$$\left[\nabla^2 + \frac{f_0^2}{\sigma}\frac{\partial^2}{\partial p^2}\right]\omega = -\frac{f_0}{\sigma}\frac{\partial}{\partial p}(\bar{V}\cdot\nabla\eta) + \frac{1}{\sigma}\nabla^2\left(-\bar{V}\cdot\nabla\frac{\partial\Phi}{\partial p}\right) - \frac{1}{\sigma}\nabla^2 H \qquad (3)$$

To the extent that vertical motions are sinusoidal the left side of this equation is negatively correlated with ω itself. The resulting baroclinic vertical motions can be partitioned in order to determine the unique contributions of the synoptic mechanisms which serve as forcing functions for the large scale ascending or descending motions. The first term on the right side of equation (3) is the variation of absolute vorticity with height. An upward increase of vorticity advection with elevation results in ascent and an upward decrease contributes to descent. The second term is the horizontal Laplacian of thickness advection. Maxima of warm advection tend to produce ascent and cold advection, descent. The remaining term is the diabatic heating effect. Ascent is found in regions of maximum heating and descent near regions of maximum cooling.

A simple parameterization of latent heating is used in this study so that the diabatic heating term will be included in equation (3). Phillips' (1963) scale analysis of the thermodynamic equation shows that the maximum amount of latent heat release consistent with the geostrophic assumption is associated with two centimeters of precipitation over a period of twenty-four hours and over synoptic scale distances. Although precipitation rates associated with tropical storm Debbie cannot be accurately determined because of its track over ocean areas, it is felt that this maximum rate may well have been exceeded by a factor of two or three in certain localized areas around the storm. In these areas the quasi-geostrophic theory is not strictly valid; but it is hoped that these large rainfall rates are associated with small-scale features and that the large-scale flow is well represented by the geostrophic model.

Appropriate boundary conditions are that ω vanish at the top of the atmosphere and at the lateral boundaries of the computational grid. At the lowest level of input, the 1000 mb surface, it is assumed that

$$\omega_{10} = \rho_{10}\left[\frac{\partial \Phi}{\partial t} - \vec{V}_{10}\cdot\nabla\Phi_S - \frac{g}{f_0} C_D |\vec{V}_S| \nabla\times\vec{V}_S\right] \tag{4}$$

where S subscripts refer to values at the surface of the earth, 10 subscripts to 1000 mb, and C_D is the drag coefficient. These terms are respectively the contributions due to local pressure change, horizontal motion over sloping terrain, and frictional divergence within the surface friction layer.

The height tendency equation is

$$\left[\nabla^2 + f_0^2 \frac{\partial}{\partial p}\frac{1}{\sigma}\frac{\partial}{\partial p}\right]\frac{\partial \Phi}{\partial t} = -f_0\vec{V}\cdot\nabla\eta + f_0^2\frac{\partial}{\partial p}\left(-\frac{1}{\sigma}\vec{V}\cdot\nabla\frac{\partial \Phi}{\partial p}\right) - \frac{f_0^2}{\sigma}\frac{\partial H}{\partial p} \tag{5}$$

Again to the extent that $\frac{\partial \Phi}{\partial t}$ has a sinusoidal distribution, the differential operator and $\frac{\partial \Phi}{\partial t}$ itself are negatively correlated. The terms in equation (5) represent respectively the effects of absolute vorticity advection, differential temperature advection in the vertical, and differential diabatic heating in the vertical. Assuming a sinusoidal distribution, equation (5) states that cyclonic vorticity advection, warm advection above and cold advection below, and an upward increase of diabatic heating at a given point in the atmosphere tend to produce height falls at that point while their opposites tend to produce height rises. An appropriate boundary condition is that $\frac{\partial \Phi}{\partial t}$ vanish at the lateral boundaries.

The input data are values of ground elevation (taken from Berkofsky and Bertoni's (1955) smoothed topography) and of calculated heights at the mandatory pressure levels up to 50 mb excluding 200 and 100 mb. The base map is a Lambert conformal projection with a mesh length of 165.1 km at 30° and 60° and the computational grid is 32 by 24 points. Values of σ are obtained from a horizontally averaged sounding.

HEIGHT CALCULATIONS

The vertical structure of the low latitude wind field is not as coherently organized as the vertical structure of middle latitudes. Wind speeds are lighter and wind directions are quite variable in the vertical. At middle and high latitudes the wind field is usually established indirectly from an analysis of the height field on a constant pressure surface. These conventional middle latitude techniques do not accurately describe the synoptic-scale motions of the tropics since the horizontal gradients of height are of the same order of magnitude as the error involved in reporting the observed heights. The horizontal field of motion is most accurately described by the winds themselves and an isogon-isotach analysis of the wind field represents the flow better than a direct analysis of the observed heights; from this analysis it is possible to deduce a height field which is consistent with the observed winds.

It is well known from the Helmholtz theorem that a two-dimensional vector field can be partitioned into non-divergent and irrotational components. In particular applying this theorem to the horizontal wind field

$$\vec{V} = \hat{k} \times \nabla \psi + \nabla \chi \tag{6}$$

where $\hat{k}$ is the unit vertical vector, ψ is the horizontal stream function ($\hat{k} \times \nabla \psi$ is non divergent), and χ is the horizontal velocity potential ($\nabla \chi$ is irrotational). The manner in which the observed wind is partitioned between ψ and χ depends on the choice of boundary conditions. Since the goal of this analysis is to calculate a height field from the wind field for input into a quasi-geostrophic ten-level model which is non-divergent, the wind field was partitioned so as to minimize the kinetic energy of the divergent part of the wind field. If the vertical component of the curl of equation (6) is taken, the result is

$$\nabla^2 \psi = \hat{k} \cdot \nabla \times \vec{V} = \zeta \tag{7}$$

Thus the horizontal Laplacian of the streamfunction equals the vertical component of the relative vorticity (ζ) of the wind field. This equation is a Poisson equation which can be solved by numerical techniques if ζ is computed from the observed wind field and ψ or its normal derivative is specified on the boundary of the computational grid. Taking the component of the wind field parallel to the boundary

$$V_s = \frac{\partial}{\partial n} \psi + \frac{\partial}{\partial s} \chi \tag{8}$$

where n and s are in the normal and tangential directions: n positive in the outward direction and s positive in the counter-clockwise direction along the boundary. The normal derivative of ψ, therefore, is specified in terms of one known quantity V_s and one unknown quantity $\frac{\partial \chi}{\partial s}$. As stated above it is desired to decompose the wind so that the kinetic energy associated with the velocity potential is minimized. The boundary conditions which accomplish this are

$$\frac{\partial}{\partial n} \psi = V_s \qquad \frac{\partial}{\partial s} \chi = 0 \tag{9}$$

These boundary conditions also associate the nondivergent, irrotational component of the vector field with the streamfunction. Equation (7) may now be solved for ψ by the Liebmann overrelaxation process and a "pseudoheight" field may be calculated by using

$$\tilde{z} = \frac{f_o}{g} \psi + C \tag{10}$$

where g is gravity and C is an arbitrary constant. Even though the absolute magnitude of $\tilde{z}$ is not precisely determined, this height field is suitable for input into the ten level model because only the horizontal gradient of geopotential is needed in equations (3) and (5).

METHOD OF ANALYSIS

Although the network of stations reporting upper air data in the Caribbean area is more dense than in other tropical regions, the data is still quite sparse compared to that available in middle latitudes. In order to insure that the reported winds were as accurate as possible time sections of horizontal winds from radiosonde and pibal reports were plotted for each of the stations listed in table 1. This was done for 00Z and 12Z data of September 25 to September 28th. These time sections provided a means for correcting inconsistencies both in time and in the vertical, for subjectively smoothing the wind data in order to eliminate small-scale features of the wind field, and for interpolating the 12Z winds at those Mexican stations which report only at 00Z.

The resulting wind data was plotted and isogon-isotach analyses were carried out at each of the input levels required by the ten-level model from the 25th to the 28th at 12Z. 12Z was chosen for the time of analysis because it was closer to the time of the Tiros satellite passage. As stated previously the ten-level model has its lowest level of input at 1000 mb. Since the bottom boundary condition on the

ω-equation includes frictional effects and since the geostrophic assumption is valid only above the frictional boundary layer, the 1000-mb winds do not represent frictionless flow suitable for the lowest input level. A study of the time sections revealed that the 2000-foot winds were relatively free from frictional influence and yet resembled closely the 1000-mb flow; therefore, the 2000-foot wind field was substituted for the 1000-mb field. In regions where the surface was above this level the flow at upper levels was used as a guide in the 2000-foot analysis. At all levels an effort was made for vertical consistency while still fitting the analysis to the

Table 1

72201	Key West
76225	Chihuahua, Mexico
76256	Guaymas
76394	Monterrey
76644	Merida
76679	Mexico City
76692	Veracruz
78016	Bermuda
78063	Grand Bahama
78076	Eleuthera Island, Bahamas
78118	Turks Island
78367	Guantanamo
78384	Grand Cayman
78397	Kingston, Jamaica
78501	Swan Island
78526	Puerto Rico
78806	Balboa, Canal Zone
78861	Antigua
78866	St. Maarten
78897	Guadelupe
78954	Barbados
78967	Trinidad
78988	Curacao
80001	San Andres
80222	Bogota, Columbia

observed winds. In the northeast and southwest corners of the grid where upper air data was lacking the flow was analysed in the simplest possible manner so as not to create any large relative vorticities which might incorrectly influence ensuing computations at interior points. Reconnaissance data provided by the National Hurricane Research Laboratory was used where applicable in the vicinity of the depression.

A nine point, two pass smoother-unsmoother was used in order to eliminate unwanted small-scale motions. Computations were made when the smoother-unsmoother was applied to the vorticity field before calculation of the ψ field and in a second case to the ψ field itself after relaxation of unsmoothed vorticities. Very little difference was noted between the resulting ψ fields and the former method was used throughout the computations.

It should be noted that the resulting height field was not always consistent in the vertical. In those regions of weak flow near singular points in the isogon-isotach analysis where the irrotational component was more representative of the actual wind field than the non-divergent component, displacements of singular points from the analysed location sometimes were as great as three grid distances.

The pseudoheight field as determined by equation (10) was used as input for the ten-level model. Since input heights were limited to positive values and three decimal digits, the constant in equation (10) was chosen in each case so that the lowest height was identically zero. This permitted input heights in whole feet from the lowest input level up to 400 mb and in tens of feet at the remaining levels.

ÆCORPORATION OF LATENT HEAT

Latent heat plays a very important role not only in influencing the long-term motions of the general circulation but also in the short-term synoptic-scale motions. Although the main source of energy for a developing cyclone is the eddy available potential energy resulting from horizontal temperature gradients, recent studies have shown that diabatic heating resulting from condensation is an important process in the intensification and strengthening of large-scale middle-latitude disturbances and in accelerating the movement of cyclones in the lower troposphere. In tropical latitudes small-scale convection processes are the principle mechanism through which latent heat is released. Convective cells act as a heat source for organized tropical depressions thus providing for the maintenance of these depressions against frictional dissipation at the surface.

Many attempts have been made to parameterize the influence of heating due to microscale convective activities on the large-scale tropical motions. The present method closely parallels those proposals of Ooyama (1963) and Charney and Eliassen (1964). It is assumed that a frictional boundary layer exists which is full of saturated water vapor. Areas of convergence in the boundary layer give rise to ascending currents which advect water vapor upward into the column of air above. The water vapor transferred through the top of the friction layer is assumed to condense immediately and simultaneously warm the air column over the area of convergence. The heating is parameterized in the following way.

Let $\omega_{fric} =$ the frictional vertical velocity

and $$H = \frac{R}{c_p p} \frac{dQ}{dt} \tag{11}$$

where $$\frac{dQ}{dt} = \text{heating per unit mass} \tag{12}$$

where H represents the diabatic heating, L is the latent heat of condensation, q_s is the saturation specific humidity and p_s is the surface pressure. $\frac{dQ}{dt}$ represents the heat released per unit mass to the column of air above by an upward flux of saturated air from the friction layer. If σ is now averaged from the bottom to the top layer

Then $$\sigma = \frac{\partial \Phi}{\partial p} \frac{\partial \ln \theta}{\partial p} = \frac{\Delta \theta}{\theta} \frac{1}{\Delta p} \frac{RT}{p} \tag{13}$$

and $$\frac{H}{\sigma} = -\frac{\theta}{\Delta \theta} \frac{\Delta p}{T c_p} \frac{L q_s \omega_{fric}/g}{p_s/g} \tag{14}$$

let $$\eta = \frac{L q_s}{c_p T} \frac{\theta}{\Delta \theta} \tag{15}$$

therefore $$\frac{H}{\sigma} = -\frac{\Delta p}{p_s} \eta \, \omega_{fric} \tag{16}$$

where $\frac{\Delta p}{p_s} \eta = 0$ for $\omega_{fric} > 0$ and $\frac{\Delta p}{p_s} \eta = 3$ for $\omega_{fric} < 0$. With the introduction of the diabatic heating term the thermodynamic equation becomes

$$\frac{\partial}{\partial t}\frac{\partial \Phi}{\partial p} = -\vec{V}\cdot\nabla\frac{\partial \Phi}{\partial p} - \sigma\omega + \sigma\frac{\Delta p}{p_s}\eta\,\omega_{frict} \qquad (17)$$

It was suggested by Charney that ω_{frict} be determined by use of the formula

$$\omega_{frict} = -\frac{g}{f_0}\hat{k}\cdot\nabla\times\left(\rho_s C_D|\vec{V}_s|\vec{V}_s\right) \qquad (18)$$

where S subscripts refer to the surface and C_D is the drag coefficient. This is done rather than assume some average value for $|\vec{V}_s|$ and have ω_{frict} directly proportional to the vorticity at the lowest layer. In this way the present work deviates slightly from the method used by Ooyama and Nitta. This approach is a simpler method than that used by Charney and Eliassen since the horizontal transport of water vapor above the friction layer was not considered.

TROPICAL STORM DEBBIE

Debbie was first observed by reconnaissance aircraft on September 24, 1965, to the west of Swan Island as part of an easterly wave which was slowly moving through the western Caribbean. At that time a tropical depression with lowest surface pressure of 1003 mb, maximum winds of 20 knots, and no well-defined circulation was found. Debbie progressed on a northwestward course crossing the northeastern tip of the Yucatan Peninsula on the 26th and emerging into the Gulf of Mexico somewhat weaker than before as shown by an increase in minimum surface pressure to 1007 mb. During the 26th and 27th the storm's course became more northerly and forward speed increased slightly. Early on the 27th maximum low-level winds increased to 30 knots but no organization was apparent and highest winds and shower activity were for the most part confined to the east and northeast of the storm, similar to the previous days. On the 28th Debbie's course became northeastward, her forward speed increased, and her surface pressure decreased to 1004 mb indicating slight intensification of the depression. Some activity was beginning to be noticed in the northwest quadrant with low level winds of 25 knots while winds to the northeast reached 45 knots.

From the 25th to the 28th Debbie's center was never well defined and there was no strong coupling between activities in the lower and upper troposphere. The main feature of the depression was a region of strong southerly and southeasterly flow (20 to 40 knots) to the east of the storm center. It was in this region that reconnaissance aircraft reported cloudiness and moderate to heavy rain and Tiros pictures indicated solid overcast.

The dominant feature of the upper troposphere on the 25th of September was a very sharp trough over the eastern United States. Debbie was located several hundred miles west and south of the trough line under a region of relatively weak flow aloft. As this trough moved further eastward height rises began to occur over the southeastern United States and eastern half of the Gulf of Mexico. Consequently weak anticyclonic circulation appeared at 250 mb on the 26th to the north of Debbie. By the 27th a new upper level trough had moved into the eastern United States; at higher latitudes it was located over the eastern Great Lakes region and at lower latitudes over Texas. The anticyclonic region aloft intensified, moved eastward, and was located just east of Debbie as the tropical storm moved into the Gulf. A day later that part of the trough which had been over Texas on the 27th phased in with a new higher latitude trough as it moved into the central United States. Debbie, which was very close to Louisiana and just east of the upper level trough axis on the 28th, was becoming more directly influenced by the middle latitude flow and soon lost its tropical characteristics as it moved over land.

RESULTS

Vertical velocities and height tendencies were computed for each day from September 25th to September 28th at 12 Z. Since the important features of the calculations are essentially the same for each day, only results for September 25th will be discussed in detail. On this day Tiros X passed almost directly over the disturbed area of cloudiness associated with Debbie thus providing excellent satellite coverage. Also, Debbie's center was located further south than the other days which were analysed; it is felt, therefore, that the results of this day provide a better means for determining the degree to which the geostrophic model is capable of representing tropical motions. Because the center of the tropical storm was not well defined, reconnaissance aircraft could not accurately position the storm's location. Figure 1 represents the approximate course followed by Debbie. The surface map for September 25th at 12 Z is shown in figure 2; the low near the western coast of Mexico is associated with hurricane Hazel. Since the hurricane was located almost on the western boundary of the computational grid, the results for this region are not necessarily realistic because the height tendencies and vertical velocities were set identically equal to zero on the boundary.

Analyses of the "pseudoheight" field derived from equation (10) have shown that these height fields are quite representative of the observed winds. As a check of the method a wind field was calculated from the "pseudoheight" field. It was found that these winds were systematically weaker than the observed winds but retained at least 80% of the speed and normally differed in direction from the observed winds by less than 15°. Examples of the "pseudoheight" field at 850 mb, 500 mb, and 250 mb are given in figures 3 - 5. The close agreement between the derived height field and the observed winds in the lower, middle, and upper troposphere is revealed in these figures. As stated in the introduction it is expected that Tiros pictures will be quite representative of the actual vertical motions in the tropics; but because of the relatively small geopotential height and temperature gradients it is not known if the meteorological network of upper air stations is sufficiently dense to adequately represent these gradients or if the diagnostic vertical motions computed from the geostrophic model will compare favorably with atmospheric motions. Results for each of the days in the study indicate that the location of the maximum upward motion and maximum downward motion are well correlated with the cloudy and clear regions respectively in the Tiros pictures. A sketch of the Tiros neph for September 25th is shown in figure 6; it should be noted that the satellite picture was taken several hours after the meteorological observation time. The adiabatic vertical motion at 800 mb is shown in figure 7 and at 600 mb in figure 9. The correspondence between maximum upward motion and cloudiness is clearly evident. The magnitude of the maximum upward vertical motion is on the order of a half a centimeter per second: a value which is characteristic of the adiabatic calculations. Simple calculations made on a pseudoadiabatic chart assuming 1) that the rate of condensation equals the rate of precipitation, 2) that a column of air 500 mb in the vertical is being lifted, and 3) that no divergence occurs in the lifted column show that vertical motion of this size is more than an order of magnitude smaller than that required to produce rainfall rates in excess of one inch per day.

With the introduction of latent heating parameterized as discussed previously, a diabatic heating term is added to the ω-equation. This heat source may be varied in the vertical by changing the coefficient associated with the diabatic term. For the purposes of this study the heating has been distributed in the following manner.

Level	Coefficient
900 mb	.275
800 mb	.275
700 mb	.225
600 mb	.150
500 mb	.075

At all other levels only adiabatic motions have been considered. Figures 8 and 10 show the vertical motion at 800 mb and 600 mb when the diabatic term is included. The magnitude of the ascent regions has been increased by a factor of two or three with maximum ascent in excess of 1-1/2 cm/sec. In this case the maximum rainfall rate calculated by the method mentioned above is on the order of a half inch per day: a magnitude which is still much smaller than typical observed rates.

If it is assumed that the upward motion is moist adiabatic, the thermodynamic equation for these motions is

$$\frac{dQ}{dt} = -L\frac{dw_s}{dp}\omega \qquad (19)$$

where w_s is the saturation mixing ratio, $\frac{dw_s}{dp}$ is taken along a moist adiabat, and ω includes diabatic and adiabatic effects. This equation offers a means of checking the validity of the 10-level model calculations with the diabatic term included. When the heating that is specified by equation (12) is substituted into equation (19), it is found that the resulting ω's are smaller by a factor of three or four than those derived by the 10-level model. This implies that the heating specified in the model should be larger in order to account for the larger 10-level ω's ; but, if the heating is increased then the 10-level ω's will be correspondingly larger. Because the heating model used here is a very simple *ad hoc* representation of the actual latent heating in the atmosphere, it is unlikely that a repeated iteration process involving equation (19) and the 10-level ω's would converge. Nevertheless, even this simple type of diabatic heating has produced a qualitative improvement of the ω field. In order to improve the consistency of the results a parameterization of the heating is needed which also considers motions above the friction layer. It should be noted that the structure of the tropical atmosphere is such that the process of condensation is closely associated with cumulus-scale motions. A parameterization of latent heating in a large-scale model for use in the tropics is, therefore, a means of representing the dynamics of the small-scale motions in terms of the large-scale motions. When the present model for latent heating is used, the large-scale upward motion calculated by the geostrophic model is well correlated with the cloudy region in the Tiros picture but not quantitatively consistent with the upward motion which seems necessary in order to produce the observed rainfall; it appears, therefore, that these large-scale motions are acting so as to produce an area where small-scale updrafts account for a major portion of the precipitation. The cumulus- and cyclone-scales are thus cooperating to produce a region of enhanced upward motion. A similar conclusion concerning the interaction between large- and small-scale motions was reached by Charney and Eliassen (1963) in their discussion of the tropical hurricane.

The vertical motion field in the upper troposphere at 400 mb and 200 mb is shown in figures 11 and 12; no latent heating was specified above 500 mb so that these ω's represent only adiabatic forcing.

In order to determine more precisely the correlation between the computed ω's and the cloud cover, tables were made in which the 10-level ω at each grid point was categorized according to the cloud cover in the Tiros nephanalysis which corresponded to it. No attempt was made to adjust the cloud position to the meteorological observation time of 12Z and the outer two rows of grid points were not included because of the dependence of the ω's at these points on the boundary conditions. The sum of the results at 800 mb, 700 mb, 600 mb, and 500 mb are shown in table 2 for the adiabatically computed ω's and in table 3 for the ω's computed with the addition of the diabatic term. These tables show that where there was cloud in the nephanalysis, upward motion was computed almost all of the time. The disappointing fact is that upward motion is calculated in two regions where one might have expected subsidence from the nephanalysis. The first of these is in the western Caribbean south of the region of heavy cloud where surface observations indicate the presence of clouds in agreement with the negative ω's while the neph shows a region which is mostly clear. The second of these regions is to the northwest of the frontal cloudiness in the western Atlantic where there is a direct transition from heavy cloudiness to clear sky in the nephanalysis. The maximum computed ascent is located in the frontal zone and the maximum computed descent is in the clear region but the southeastward displacement of the front in the four hours between the map time and the satellite observation time resulted in negative ω's being computed in a region corresponding to a clear area in the neph.

It is apparent from an inspection of the two tables that there is very little difference between the adiabatically determined ω's and

TABLE 2. Adiabatic vertical velocities in units of 10^{-4} mb/sec

Cloud amounts are

Open (O)	0 - 20% coverage
Mostly open (MOP)	20 - 50% coverage
Mostly covered (MCO)	50 - 80% coverage
Covered (C)	80 - 100% coverage

Numbers are the totals for calculations at 800 mb, 700 mb, 600 mb and 500 mb where the ω's are categorized according to the neph classification at the same point.

	$\omega \geq +5$	$+5 > \omega > 0$	$0 \geq \omega > -5$	$-5 \geq \omega$
O	49	39	30	18
MOP	10	104	266	80
MCO	0	0	43	37
C	0	5	119	220

TABLE 3. Vertical velocities with diabatic term. Units are the same as above

	$\omega \geq +5$	$+5 > \omega > 0$	$0 \geq \omega > -5$	$-5 \geq \omega$
O	51	34	26	25
MOP	11	91	266	92
MCO	0	0	32	48
C	0	3	102	239

those determined with the diabatic term. The main difference is that with the addition of the diabatic term the magnitude of the maximum updrafts has been increased.

Since a good correlation between the 10-level ω's and the Tiros neph exists even though the cloud position in the neph was not adjusted to 12Z, the region of maximum updraft to the east of Debbie and the saturated air associated with the ascent must have moved slowly and at approximately the same speed. In addition the horizontal velocities relative to the storm's motion must have been small enough so that the region of heavy cloud was found near the dynamically forced updraft rather than being advected into regions of weak updraft or subsidence as frequently happens in middle latitudes. Thus, although horizontal motions are equally as important as vertical motions in accounting for observed cloud systems in middle latitudes, in the tropics the large-scale vertical motions seem to determine the location of the cloud cover and the horizontal motions are normally of secondary importance.

From studies of middle latitude observations it is known that isotherms and height contours are nearly parallel in the middle and upper troposphere. The thickness advection is not small, however, because the rapid upward increase of wind speed compensates for the small angle between the isotherms and height contours. In a similar manner the magnitude of the differential vorticity advection increases up to the level of the jet stream core. Thus in the middle and upper troposphere the vorticity advection term in the ω -equation is generally comparable to the temperature advection term. In the lower portions of the atmosphere the wind field has a more pronounced component of flow normal to the temperature field. Temperature advections, therefore, remain large while vorticity advections are more weakly developed because of the smaller horizontal wind speeds. Thus temperature advections play the major role in the production of vertical motion in the lower troposphere at middle latitudes. In this lower latitude study the magnitude of the thermally induced ω's is greater than that due to the vorticity term in the lower levels of the atmosphere. At higher levels the magnitudes of the forcing functions in the ω -equation are nearly equal. It seems strange that the influence of the vorticity term is not smaller in the middle troposphere where the horizontal velocities tend to reach a minimum; this is probably a phenomenon peculiar to tropical storm Debbie and unrepresentative of average tropical motions. Also to be noted is the fact that at all levels there seems to be a tendency for the vertical motions due to temperature advections to have a sign opposite from those resulting from vorticity advections.

The height tendencies offer a further check on the accuracy of the calculations. At all levels the tendencies seem to be consistent with the observed atmospheric motions. In particular the height tendencies in tens of feet per twelve hours calculated at 950 mb can be compared with the three hour pressure tendencies in tenths of a millibar as given by the regular synoptic reports since these two quantities are of approximately the same magnitude. Although the number of reported tendencies is not sufficient to permit a thorough analysis of the isallobars the surface pressure tendencies and the calculated height tendencies (see figure 13) are consistent both in sign and magnitude. This is quite remarkable since the magnitude of the calculated ω's is much smaller than the actual vertical motions.

Petterssen's[1] formula for the speed of a low center was applied to Debbie's position on the 25th of September. A speed of 10 km/hour in a west north west direction was derived; this compares favorably with an actual motion of 8 km/hour toward the northwest and indicates that the formula is applicable to surface lows in the tropics when an accurate surface pressure analysis is available.

CONCLUSIONS

Although the upper air data network in the Caribbean region is more dense than in most other tropical regions, large areas are present in the Caribbean, the Gulf of Mexico, and the western Atlantic where no meteorological observations are taken; as a result it was not known whether this network was sufficient to permit an accurate analysis of the geopotential gradients for use in a quasi-geostrophic model. The realistic results of the model indicate that by using the horizontal wind field as initial data a "pseudoheight" field may be calculated which is representative of the existing tropical motions. The vertical motions computed adiabatically from the 10-level model agree qualitatively but not quantitatively with typical observed rainfall rates.

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[1] Petterssen, Sverre: Weather Analysis and Forecasting, Volume I. McGraw-Hill Book Company, Inc., New York, 1956, p. 49.

The addition of a diabatic latent heating term has increased the magnitude of the upward vertical motion but not enough to account for the rainfall rates. Admittedly the parameterization of latent heating is quite simple but the incorporation of even this simple model of the heating has improved the results. Since the large-scale vertical motions are smaller by a factor of three or four than those assumed to be required to produce the observed rainfall, it seems apparent that in the region of prominent updraft around tropical storm Debbie the cyclone-scale motions are interacting with the cumulus-scale motions in such a manner that the vertical motion is enhanced. The Tiros nephs have been used as a means of verifying the results of the 10-level model; however, since the 10-level and the cloudiness revealed by the satellite pictures agree qualitatively, based on the four days considered in this study the Tiros pictures appear to be an excellent representation of the large-scale vertical motion field in the tropics.

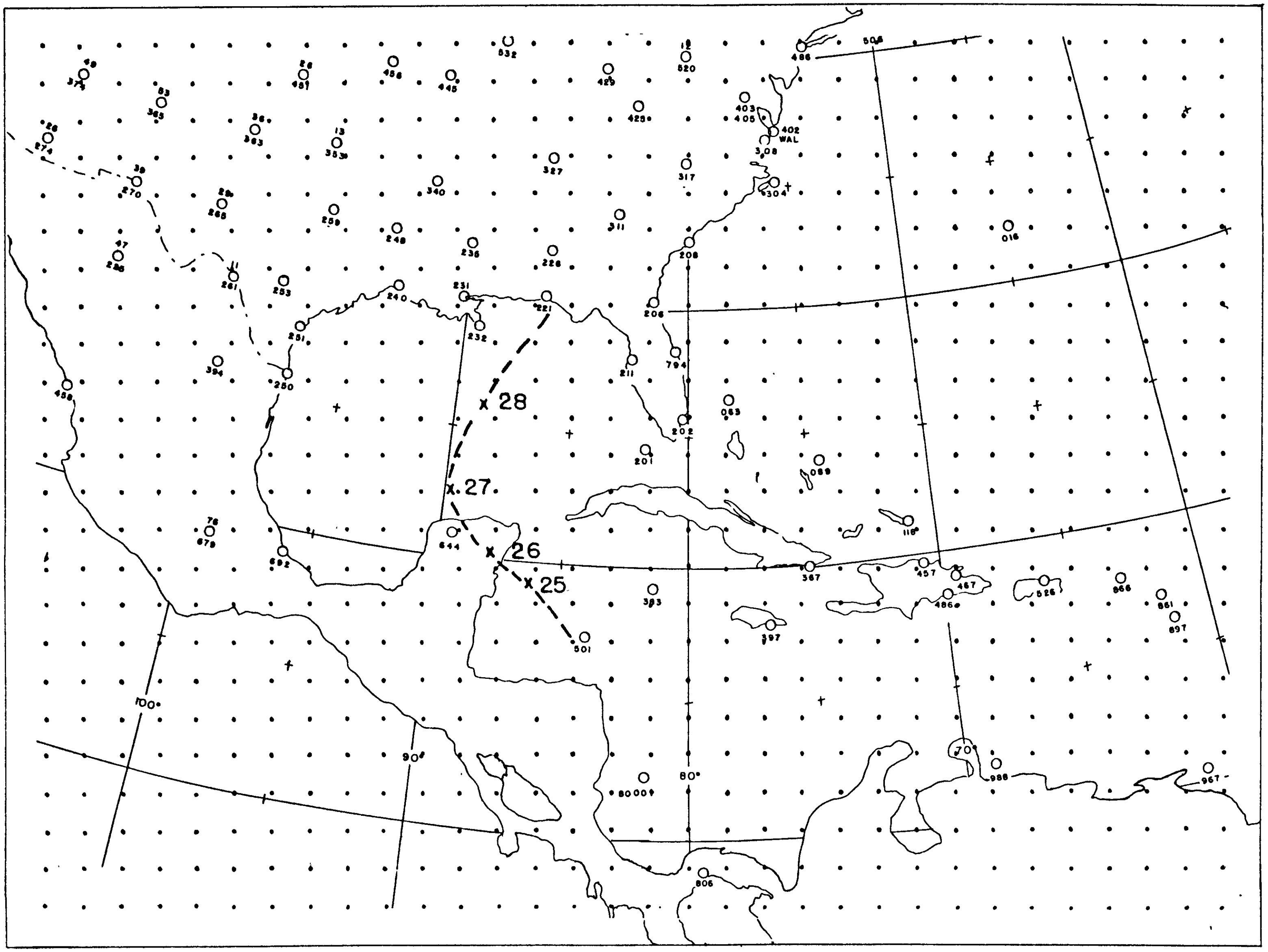

Figure 1. Approximate course followed by tropical storm Debbie, x's mark the 1200 GMT positions from September 25th to 28th.

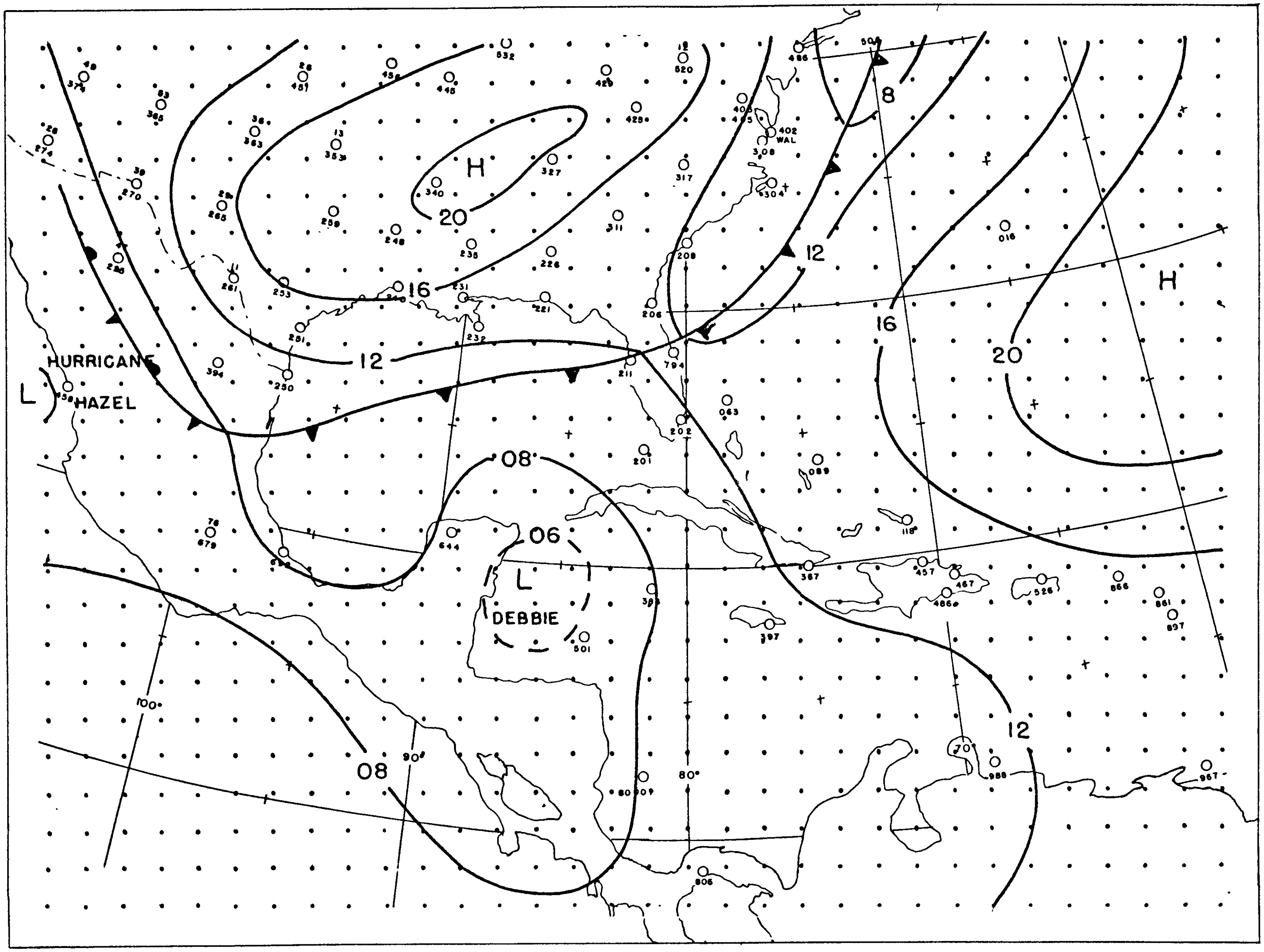

Figure 2. Surface pressure map 1200 GMT 25 September 1965.

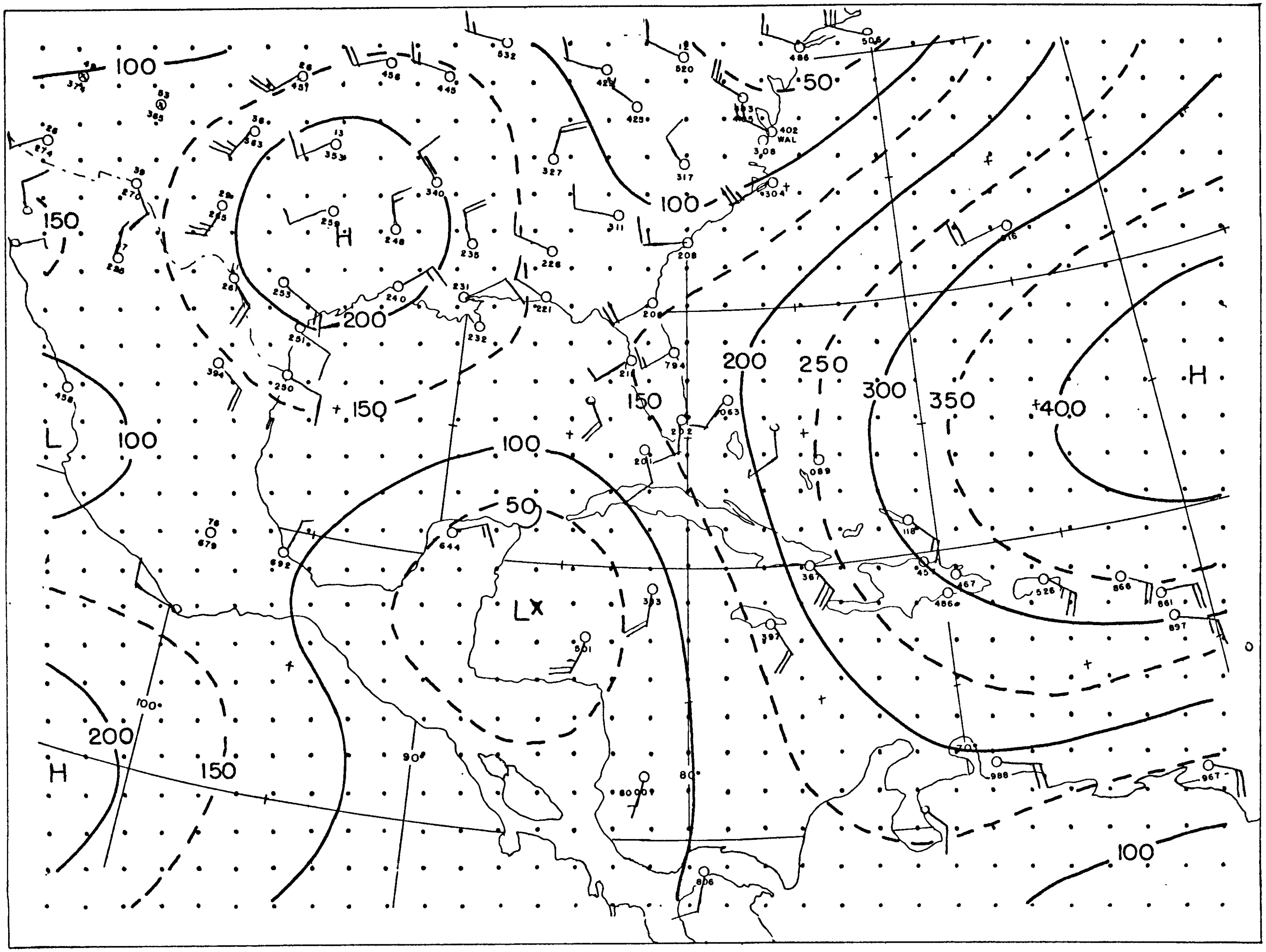

Figure 3. 850 mb pseudoheight field 1200 GMT 25 September 1965. Heights are in whole feet, × indicates grid point at which the height equals zero.

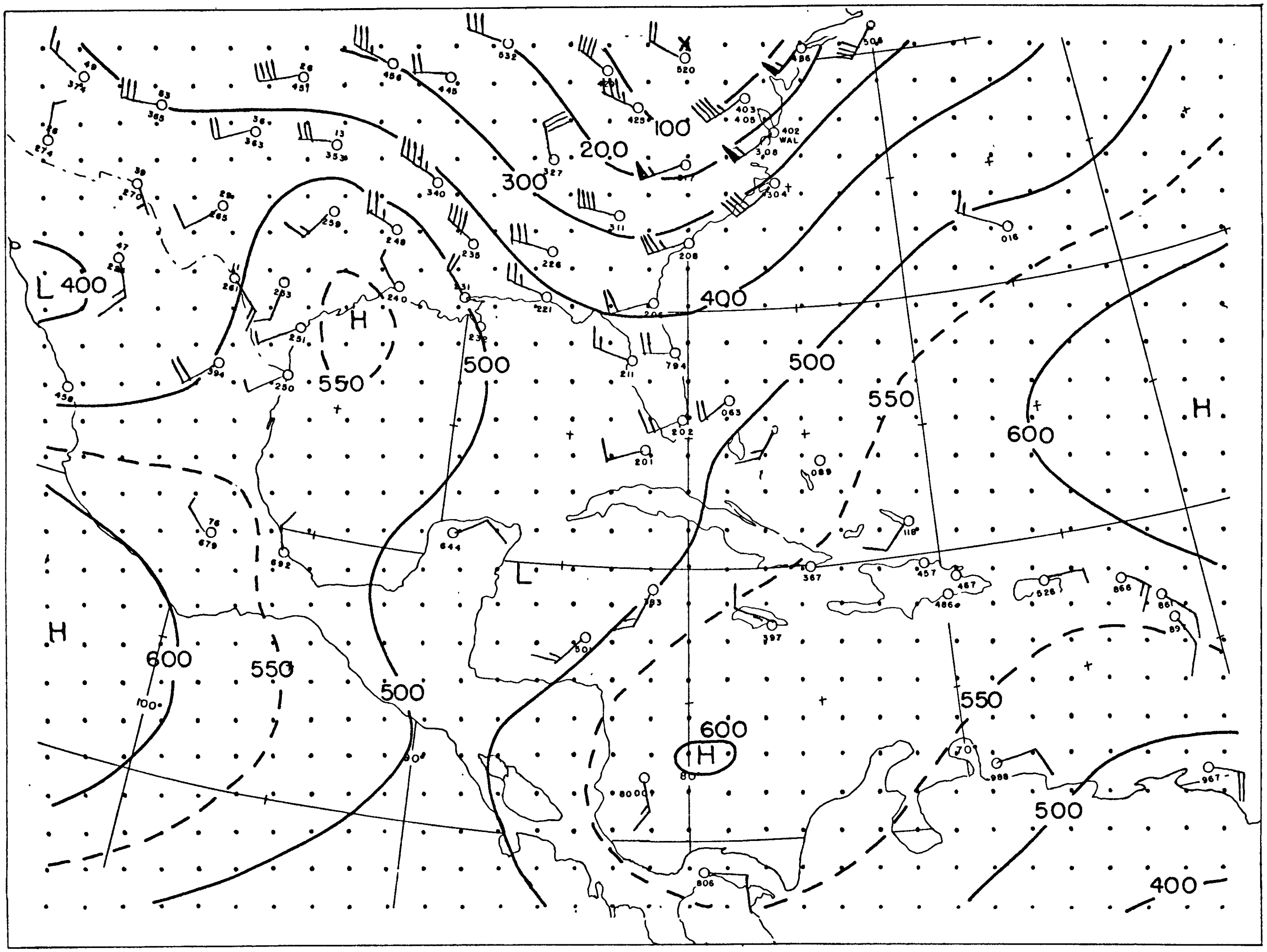

Figure 4. 500 mb pseudoheight field 1200 GMT 25 September 1965. Heights are in whole feet, × indicates grid point at which the height equals zero.

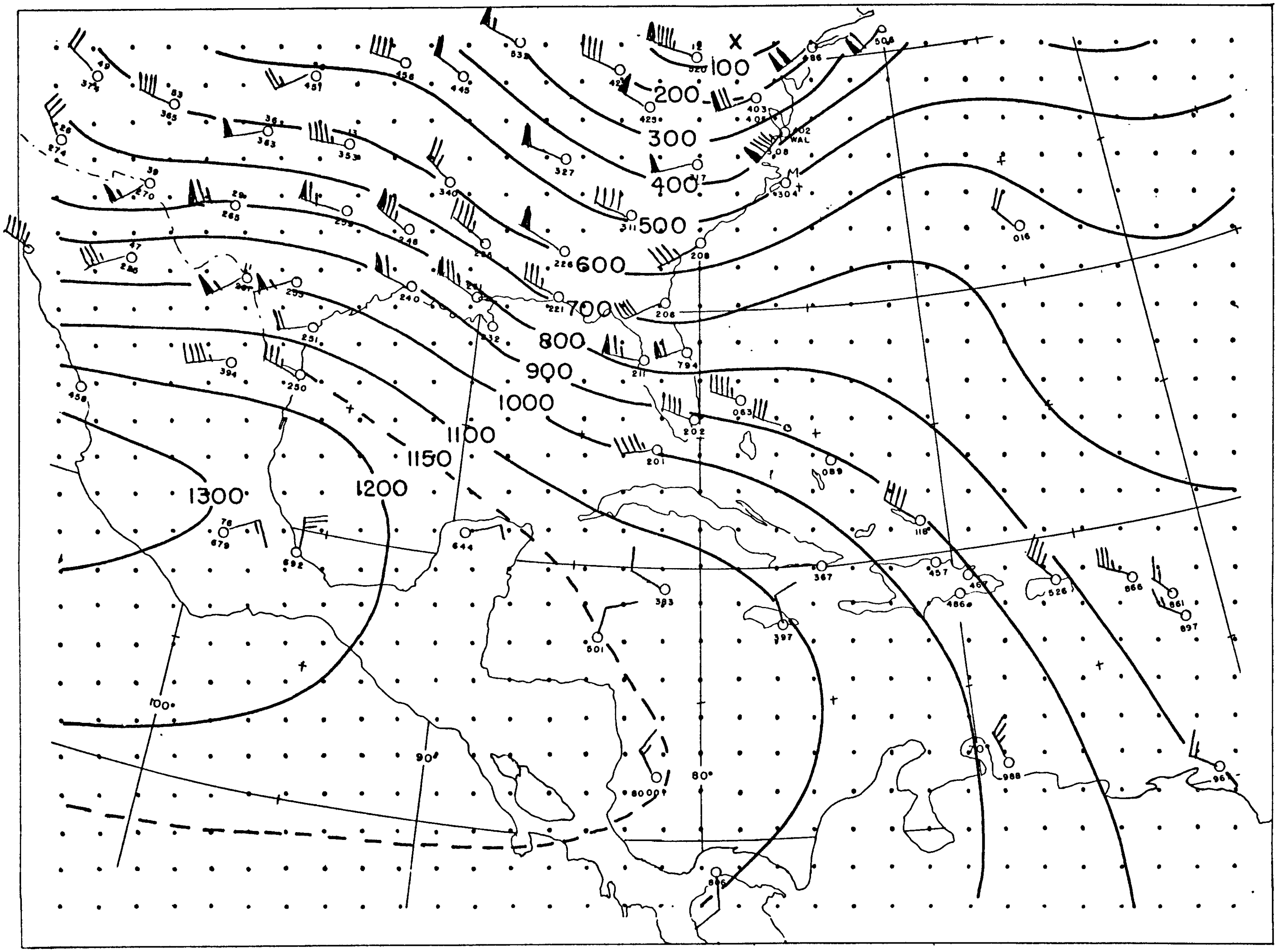

Figure 5. 250 mb pseudoheight field 1200 GMT 25 September 1965. Heights are in whole feet, × indicates grid point at which the height equals zero.

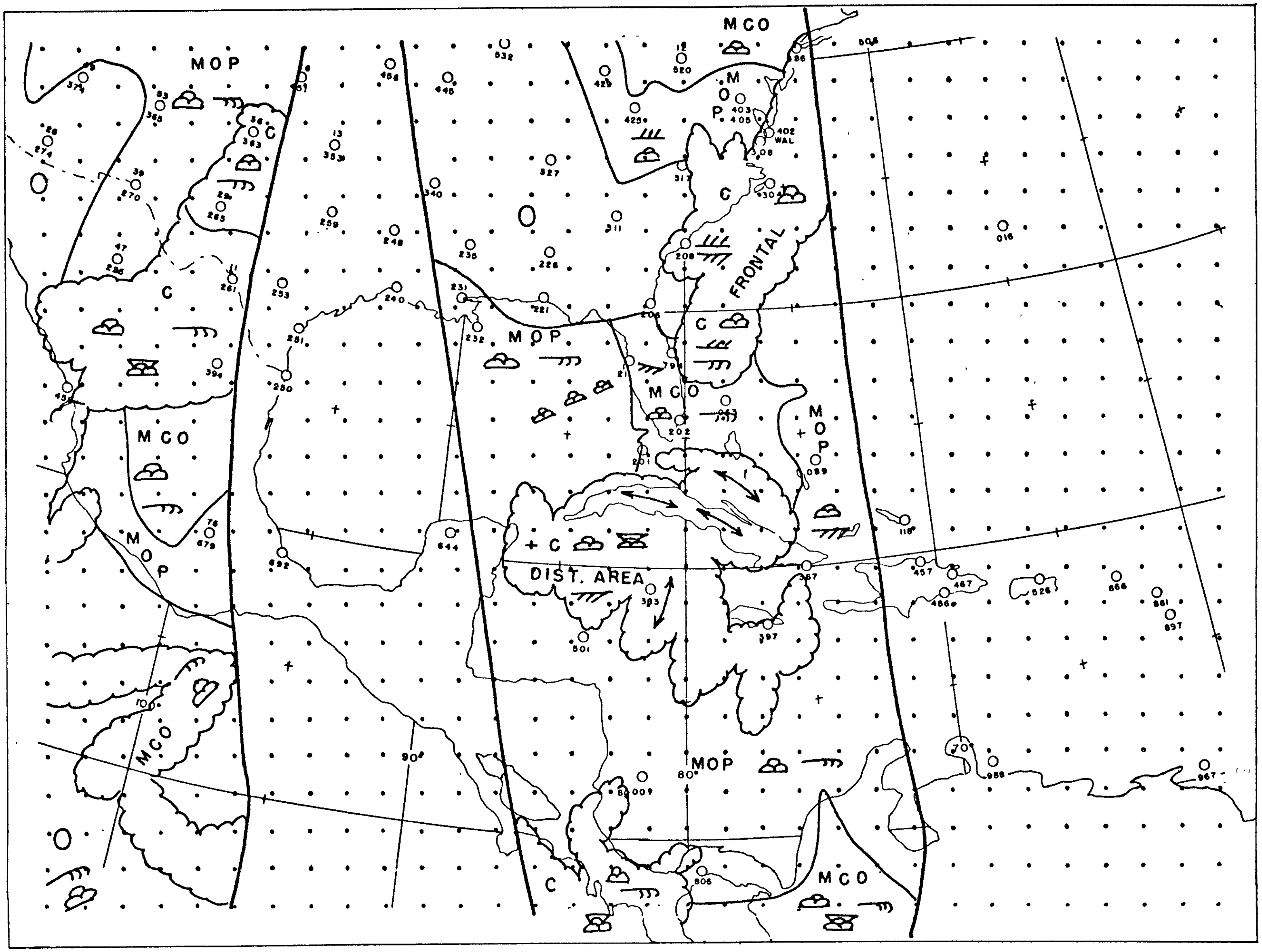

Figure 6. Sketch of Tiros X nephanalyses for 25 September 1965. The analyses on the right is for orbit 1222 at 1619 GMT and the analysis on the left is for orbit 1223 at 1800 GMT.

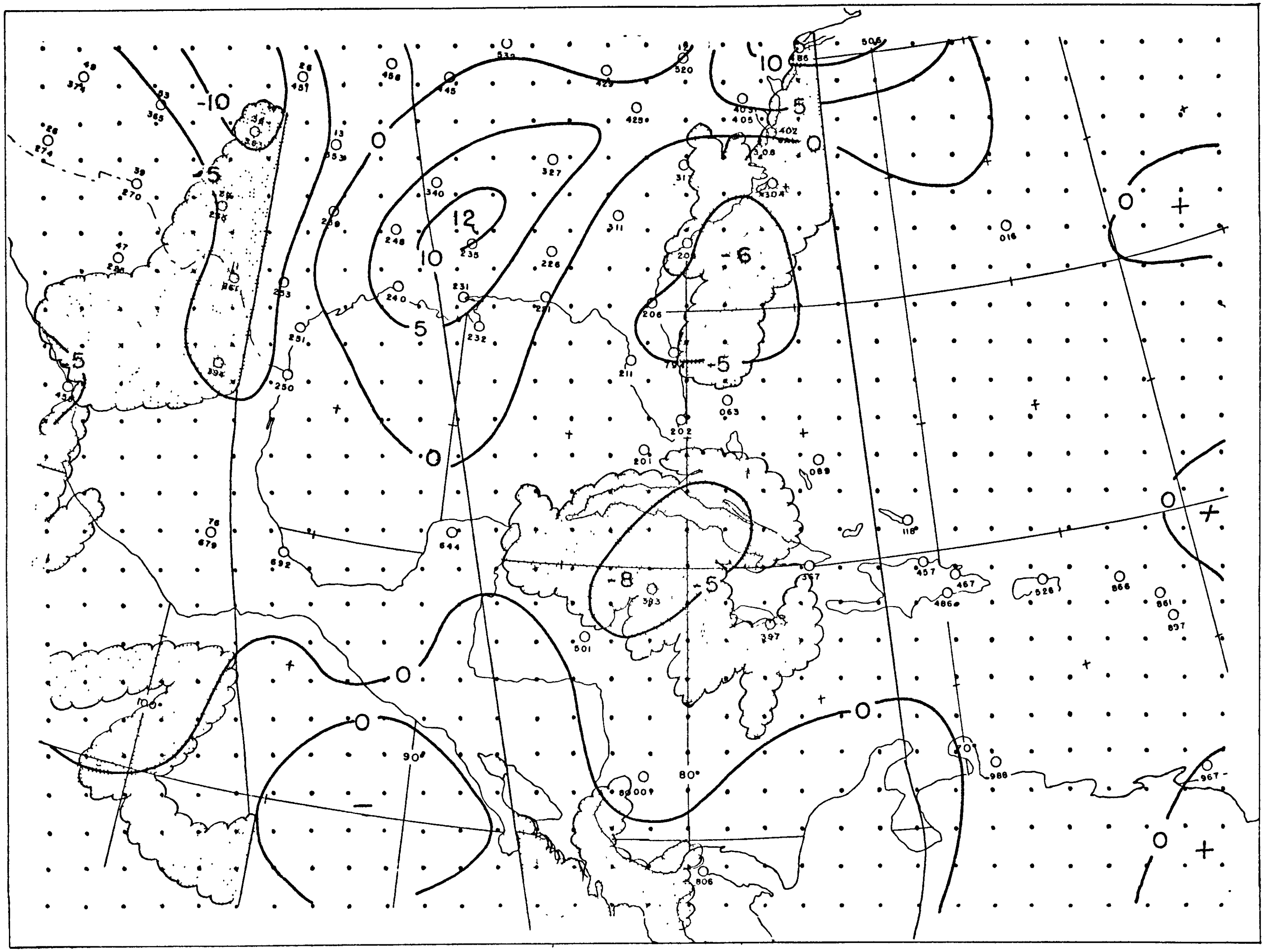

Figure 7. 800-mb adiabatic vertical motion 1200 GMT 25 September 1965. Units on all vertical motion fields are 10^{-4} mb sec^{-1} and shaded areas represent regions of major cloud systems taken from nephanalyses.

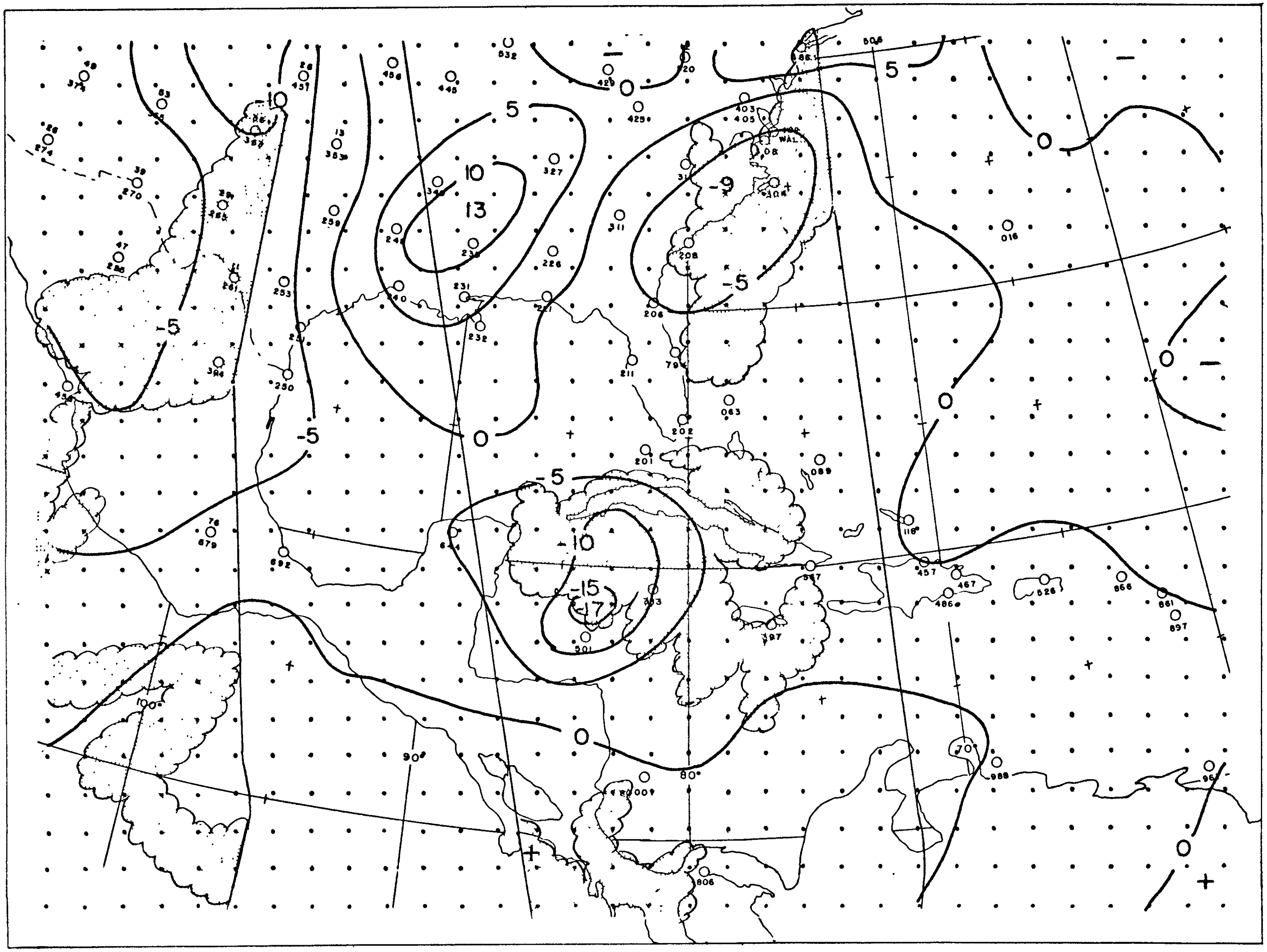

Figure 8. 800-mb total vertical motion 1200 GMT 25 September 1965.

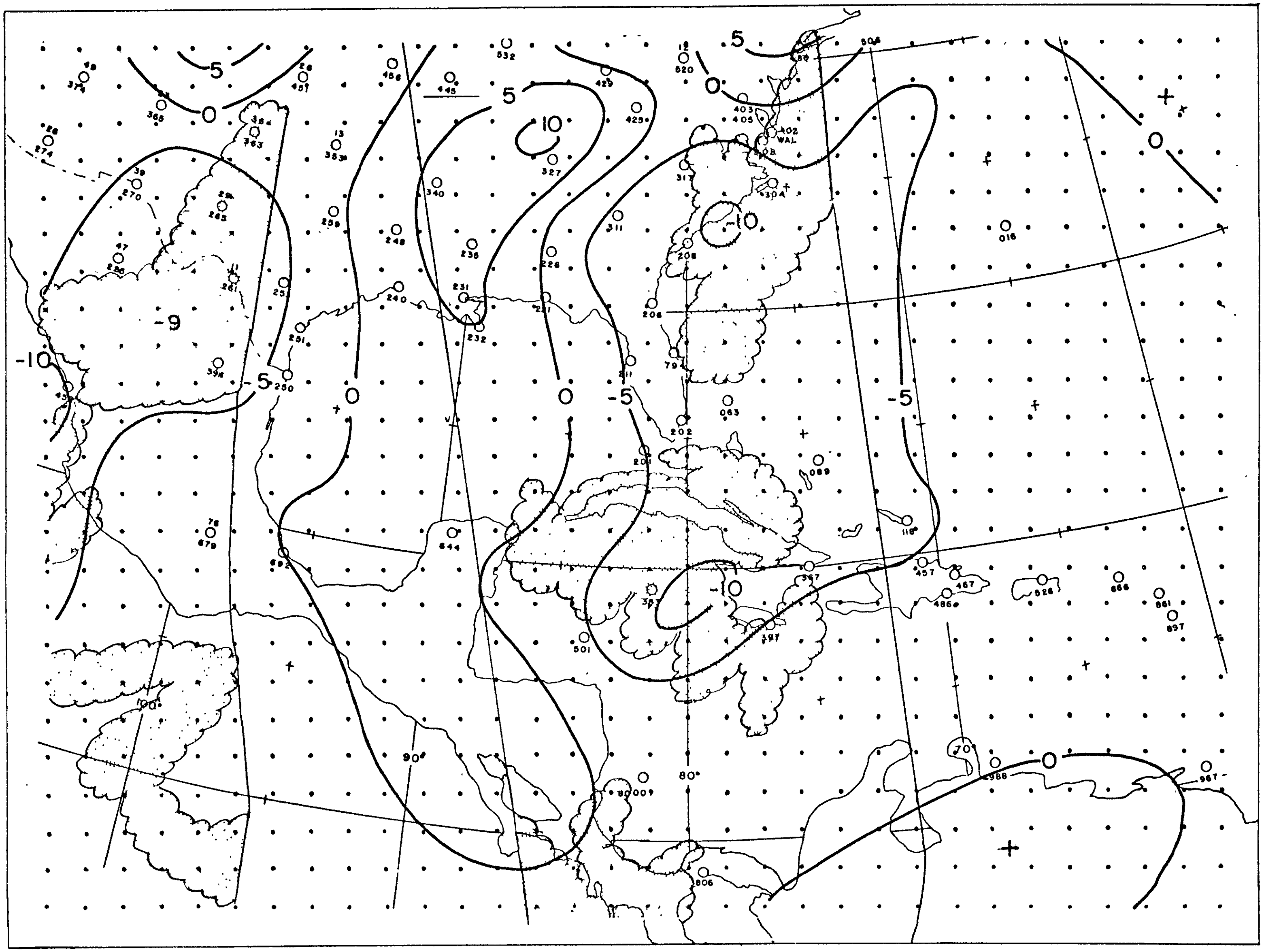

Figure 9. 600-mb adiabatic vertical motion 1200 GMT 25 September 1965.

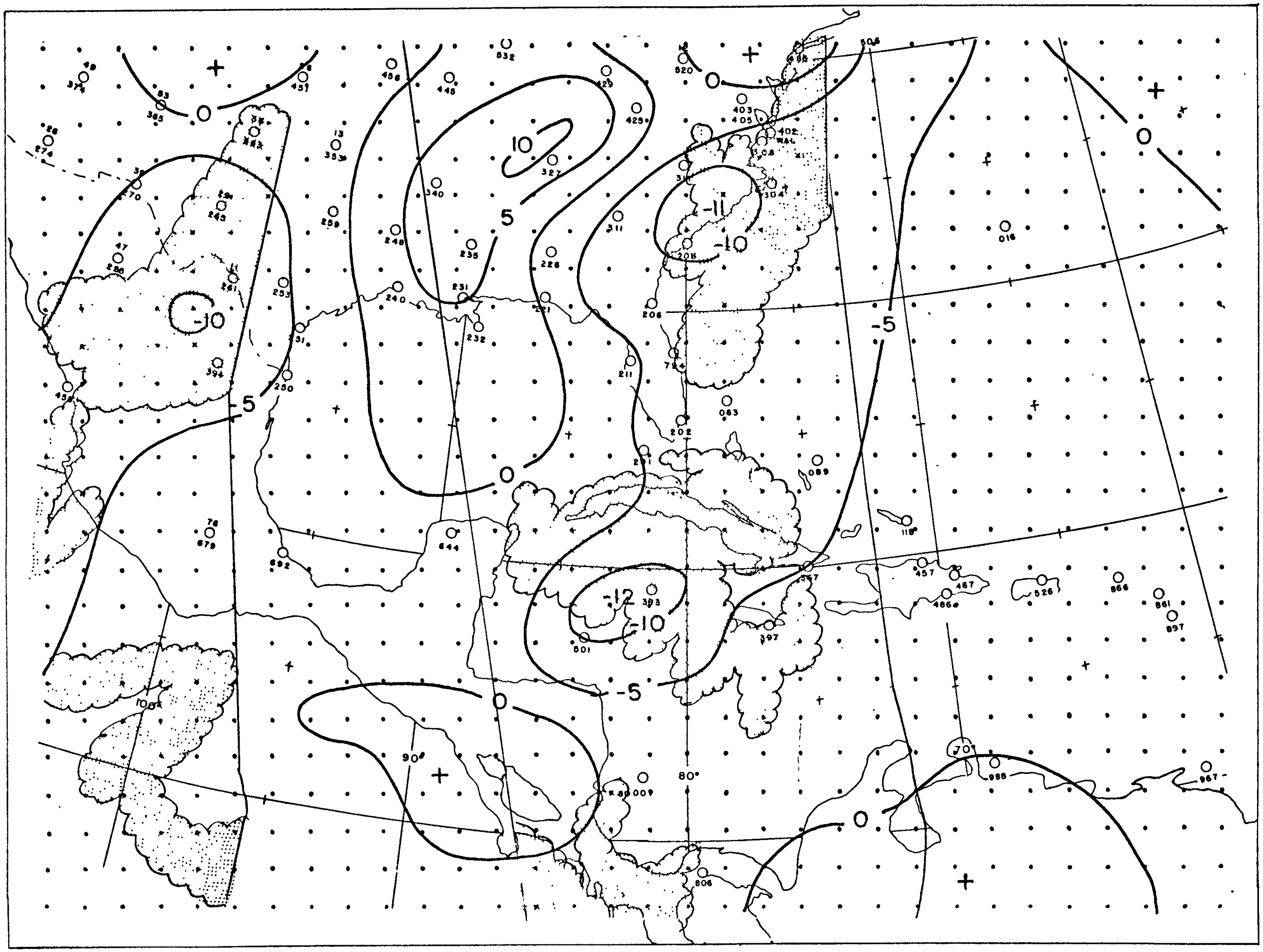

Figure 10. 600-mb total vertical motion 1200 GMT 25 September 1965.

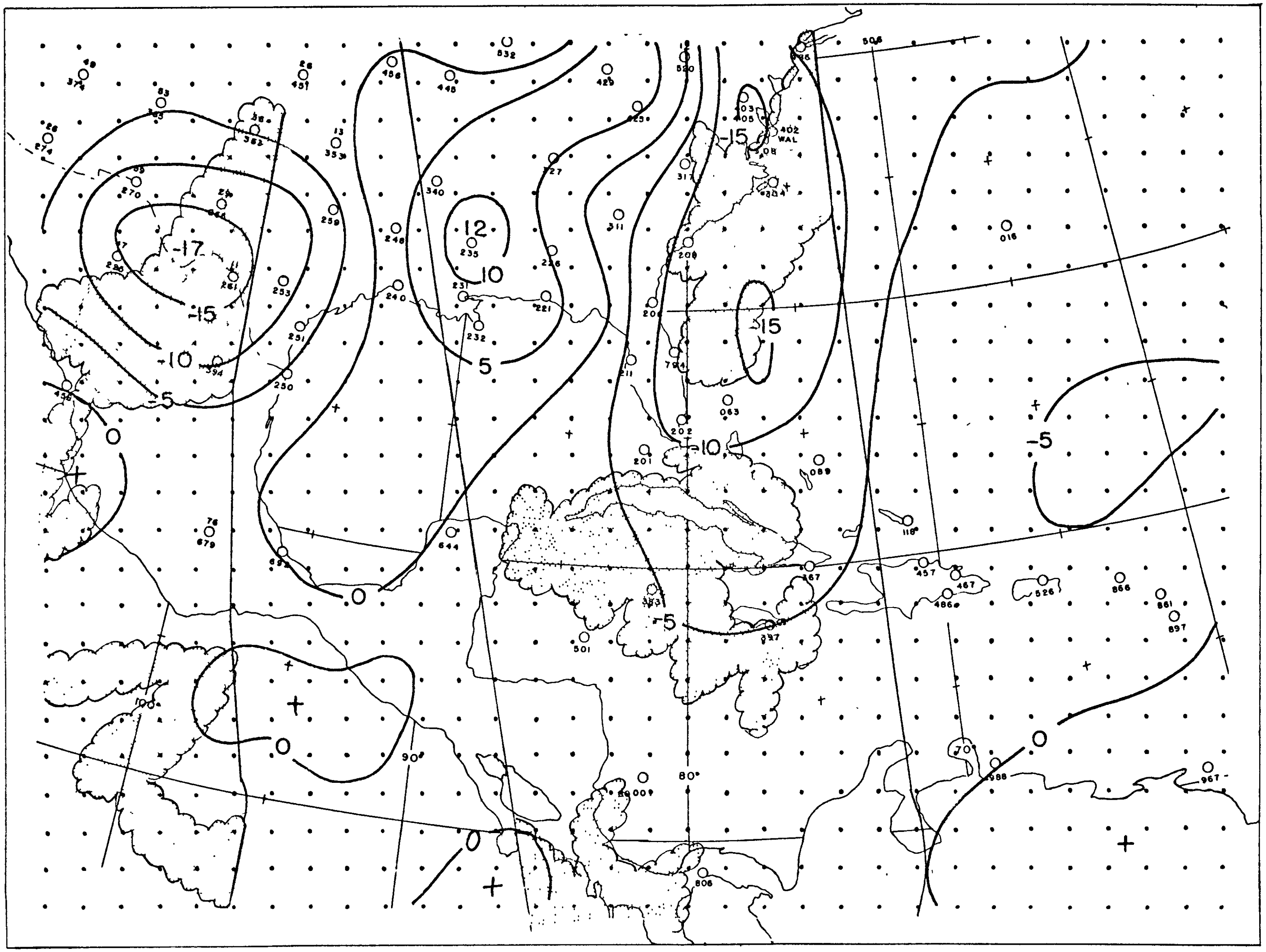

Figure 11. 400-mb total vertical motion 1200 GMT 25 September 1965.

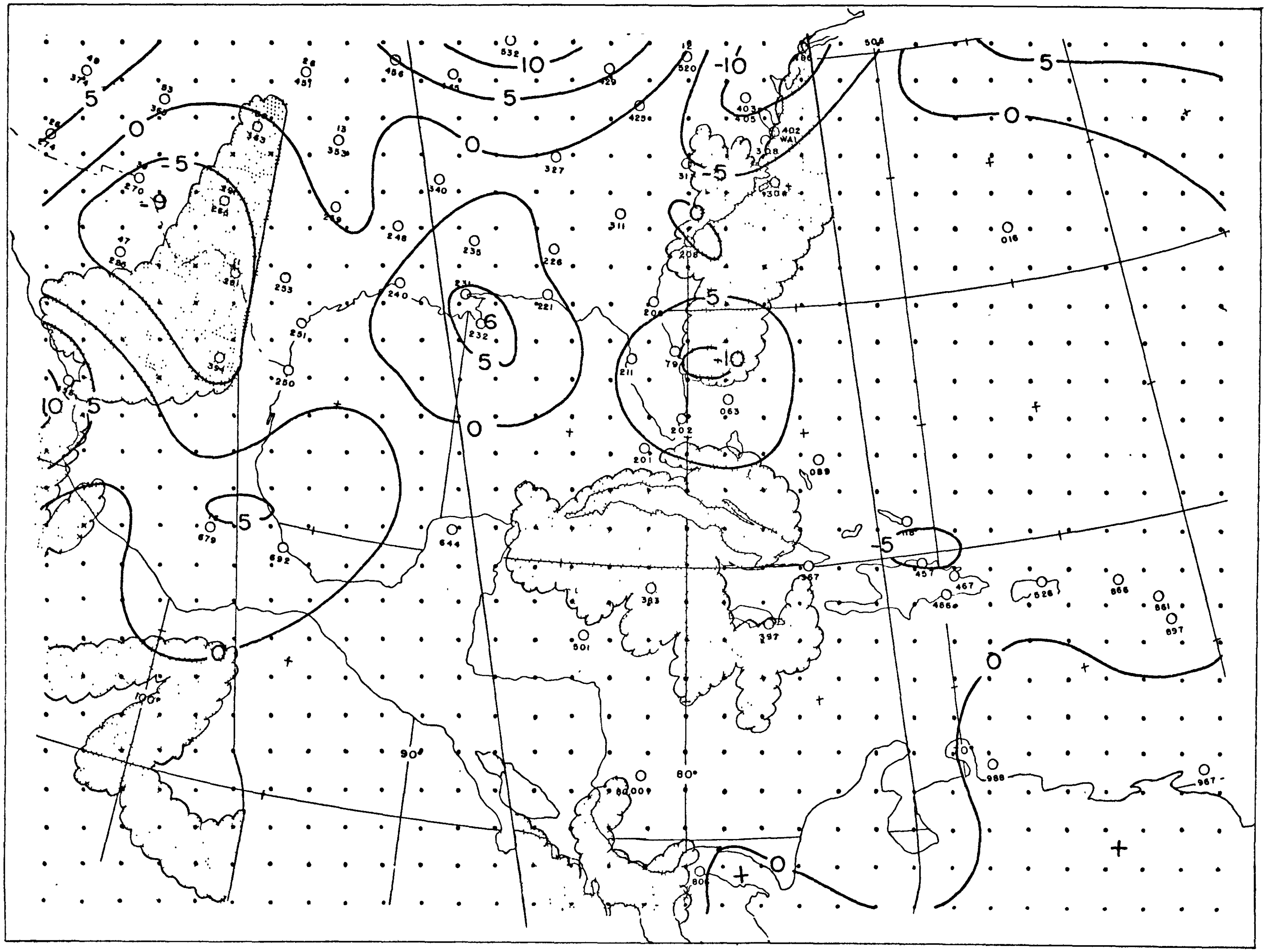

Figure 12. 200-mb total vertical motion 1200 GMT 25 September 1965.

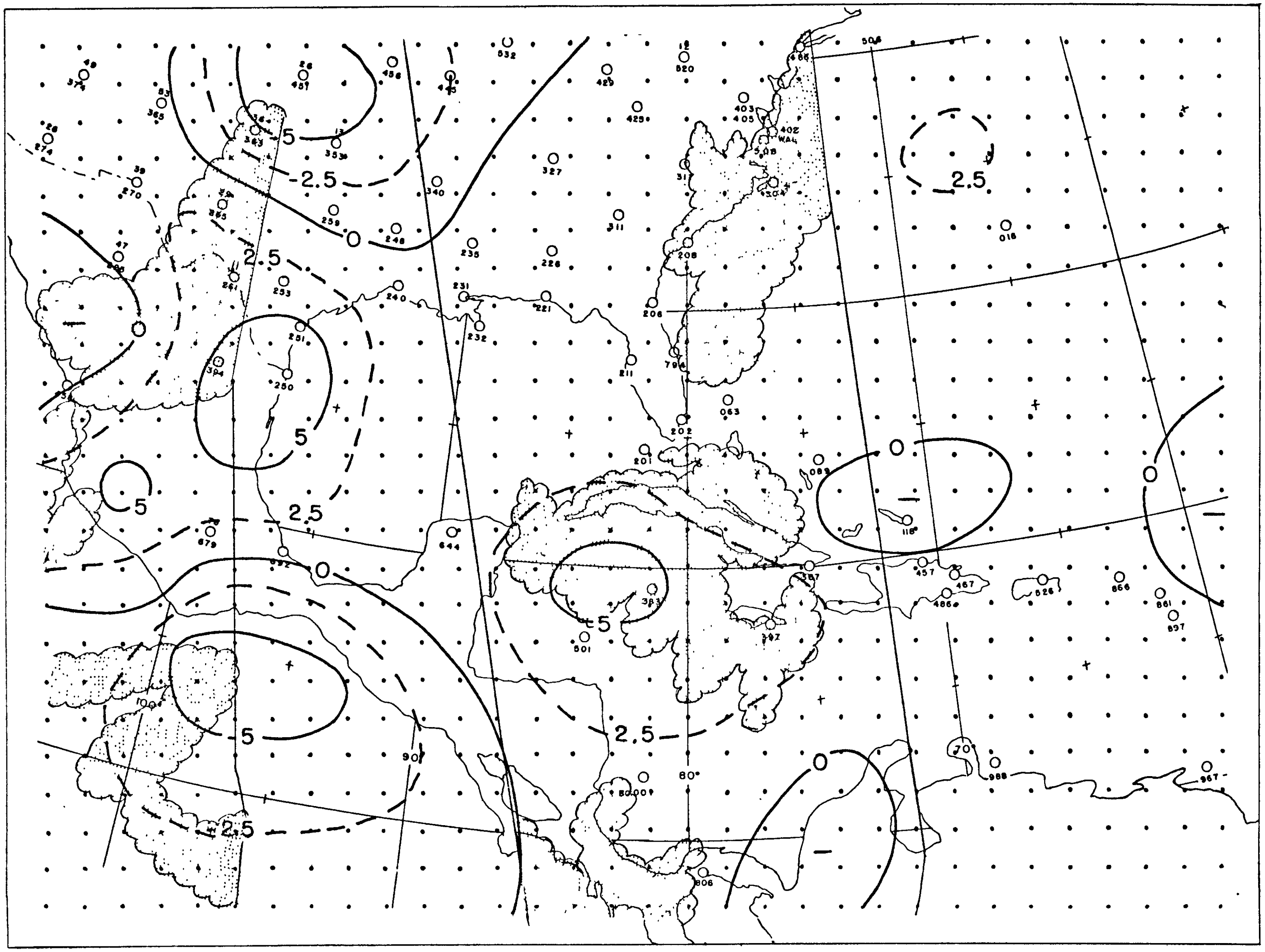

Figure 13. 950-mb height tendencies 1200 GMT 25 September 1965. Units are in tens of feet per twelve hours.

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