Propagation of Gravitons in the Shock Wave Geometry

by

Ruitian Lang

Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of

Bachelor of Science in Physics

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 2009

Author . . . .
Department of Physics
May 8, 2009

Certified by . . . .
Hong Liu
Associate Professor
Thesis Supervisor

Accepted by . . . .
David E. Pritchard
Senior Thesis Coordinator, Department of Physics
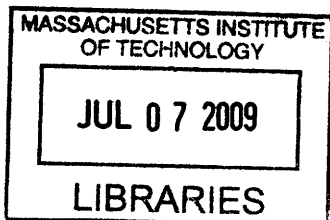
Propagation of Gravitons in the Shock Wave Geometry

by

Ruitian Lang

Submitted to the Department of Physics
on May 8, 2009, in partial fulfillment of the
requirements for the degree of
Bachelor of Science in Physics

Abstract

In this thesis, I study propagation of gravitons in the shock wave geometry in the context of the AdS/CFT correspondence, with the goal to uncover some constraint on the supergravity action in the AdS space. In studying the shock wave geometry in an anti-deSitter (AdS) space, I find that the functional form of the shock wave metric does not receive α' correction, but the wave profile does. Then I study the propagation of gravitons in the shock wave geometry and show that the wave function has a finite jump at the shock wave frontier, and this corresponds to a shift in position of the graviton in the semi-classical picture.

Thesis Supervisor: Hong Liu
Title: Associate Professor

Acknowledgments

Contents

List of Figures

List of Tables

Chapter 1

Introduction

The AdS/CFT correspondence proposed by J. Maldacena[4] states that a certain type of string theory on the anti de-Sitter space (the AdS space) is equivalent to the conformal field theory (CFT) on the boundary of the AdS space. Roughly speaking, the strong-coupling and large N (to be explained later) limit of the CFT corresponds to the weak coupling, or classical, limit of the string theory, which reduces to a supergravity theory. This correspondence turns out to be important in understanding both the string theory and the conformal field theory.

A conformal field theory is invariant under all the transformations under which the standard Minkowski metric $\eta_{\mu\nu}$ becomes $h(x)\eta_{\mu\nu}$ for some (space-time dependent) scalar factor h. If $h(x) = 1$ everywhere, then the transformation becomes a Poincare transformation, but conformal transformations allow for arbitrary scalar factors. An example of conformal field theory in $4d$ is the $\mathcal{N} = 4$ supersymmetric $U(N)$ Yang-Mills theory with the following Lagrangian:

$$\mathcal{L} = \frac{1}{g_{YM}^2}\text{Tr}[F^2 + (D\phi)^2 + \bar{\chi}\gamma^\mu D_\mu\chi + \sum_{IJ}[\phi^I\phi^J]^2 + \bar{\chi}\Gamma^I\phi^I\chi] + \theta\text{Tr}(F \wedge F), \quad (1.0.1)$$

where χ denotes four Majorana spinor fields and ϕ^I, $I = 1, ..., 6$ denotes six scalar fields.

A d-dimensional AdS space, denoted by AdS_d, can be defined as a hypersurface in a $(d+1)$-dimensional flat space with signature $(-, -, +, ..., +)$ given by the quadratic

13

equation

$$-T_1^2 - T_2^2 + X_1^2 + ... + X_{d-1}^2 = -L^2. \tag{1.0.2}$$

After choosing the following coordinates

$$r = X_{d-1} - T_2, x_0 = T_1 L/y, \text{ and } x_i = X_i L/y \text{for } i = 1, 2, d-2,$$

the metric of the AdS space can be written as

$$ds^2 = (\frac{r^2}{L^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{L^2}{r^2}dr^2). \tag{1.0.3}$$

Now the AdS/CFT correspondence can be stated as follows[1]:

Proposition 1.0.4. *The Type IIB string theory on $AdS_5 \times S^5$ with common radius L of AdS_5 and the five-dimensional sphere S^5, string coupling constant g_s and 5-form flux $\int_{S^5} F_5^+ = N$ is equivalent to the 4-dimensional $\mathcal{N} = 4$ supersymmetric conformal $U(N)$ Yang Mills theory with coupling constant g_{YM} with the following identification*

$$\lambda = g_{YM}^2 N = g_s N. \tag{1.0.5}$$

The parameters g_s and N are related to the important parameter α', which is the square of the typical string length, as follows:

$$g_s N = \frac{L^4}{\alpha'^2}. \tag{1.0.6}$$

This proposition remains a conjecture, but there has now existed a vast literature to support it. The precise meaning of "equivalence" is as follows. For any field ϕ in $AdS_5 \times S^5$, there exists an operator $\mathcal{O}$ in the conformal field theory, such that the partition function of ϕ in the string theory is equal to the moment generating functional of $\mathcal{O}$ in the conformal field theory:

$$\int \mathcal{D}\phi \exp(-\mathcal{S}[\phi]) = \langle \exp \int d^4x \phi_0(x)\mathcal{O}(x) \rangle_{CFT}, \tag{1.0.7}$$

where $\mathcal{S}[\phi]$ is the action and ϕ_0 is the boundary value of ϕ. The left hand side is the analogy of the partition function $\sum_{x,p} e^{-\beta H(x,p)}$ in statistical mechanics, except that the sum over the phase space is replaced by a path integral. The right hand side is the expectation of the exponential, and the derivatives of its logarithm with respect to $\phi_0(x_1), ..., \phi_0(x_n)$ gives the n-point correlation function $\langle \mathcal{O}(x_1)...\mathcal{O}(x_n) \rangle$.

An interesting aspect of the correspondence is the strong coupling limit of the field theory corresponds to the weak coupling limit of the string theory. For simplicity, consider the case where N is large. In fact, string theory is organized as a Taylor expansion in the parameters $\alpha'/L^2 = 1/\sqrt{4\pi\lambda}$ and g_s, so when λ is large, the string theory is dominated by the lowest orders in α'/L^2. On the other hand, the Feynman diagram expansion in the conformal field theory is an expansion in $1/N$ and λ, so this limit is the strong coupling limit of the field theory. Similarly, the weak coupling limit of the conformal field theory (the limit $\lambda \to 0$) corresponds to the strong coupling limit of the string theory. More details about this duality will be given in Section 2.

In the large λ and large N limit, the contribution with the lowest power of α', namely the classical contribution, dominates, so any quantum field ϕ reduces to its classical value that solves its equation of motion. For questions in which we are interested, it is enough to focus on the fields in the AdS space and ignore the S^5 part in this limit. Therefore, as $\lambda \to \infty$ the theory reduces to a supergravity theory in AdS_5 with the Einstein-Hilbert Lagrangian $R - 2\Lambda$, where Λ is the cosmological constant. For large but finite λ, one can use the external field method to write down the effective classical action of the theory, in which the quantum correction enters as higher order terms in α'. However, α', as the square of the typical string length, has energy dimension -2, and the only ways to construct scalars with higher positive dimensions are to take product of curvature tensors (each has dimension $+2$) and to take covariant derivatives. Therefore, the α' corrections must be terms that contain higher power in the curvature tensor and/or its derivatives. The goal of this paper is to discover a way to obtain some constraint on these α' corrections.

In general, people know little about the specific form of the α' correction to the action. However, useful constraints on the correction can be obtained. Indeed, M

Brigante, H Liu *et al* find that in the black-hole metric of the *AdS* space with the Gauss-Bonnet α' correction

$$\Delta \mathcal{L} = \frac{\lambda_{GB}}{2} L^2 (R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}), \qquad (1.0.8)$$

the unitarity bound of the CFT is violated when $\lambda_{GB} > 9/100$[3]. They obtain this result by studying the propagation of gravitons in the AdS theory near its boundary. The graviton field corresponds to the energy-momentum tensor operator on the CFT side, and the values of the operator at two points with space-like correlation must be uncorrelated. However, it is shown in [3] that when $\lambda_{GB} > 9/100$, two points with space-like separation on the boundary can be connected by a trajectory of a graviton, which violates causality.

However, the constraint on the α' correction, such as the contraint on the λ_{GB} in Eq. (1.0.8), should be a fundamental constraint on the theory, and thus independent of specific models. Therefore, more or tighter constraints might be obtained by studying other specific geometries. In this paper, the shock wave geometry and the propagation of gravitons are considered. The shock wave geometry is a geometry that looks like the original AdS geometry Eq. (1.0.3) everywhere except at the shock wave frontier $x_1 = -t$, where the wave profile is given by a function f. Therefore, the wave propagates in the speed of light and scatters graviton when it meets it. The scattering will change the trajectory of the graviton and potentially make the trajectory connect two points on the boundary with space-like separation, when the scattering is sufficiently strong and in the right direction.

This paper is organized as follows. In Section 2, we briefly review the basics of the AdS/CFT correspondence. In Section 3, we study the shock wave geometry in a general d-dimensional AdS space. The propagation of gravitons will be the subject of Section 4, where we will restrict to AdS_5, for which the correspondence was proposed.

Chapter 2

Background

This section contains a brief review of the background in conformal field theories, string theories and the AdS/CFT correspondence, mainly following [4] and [1].

2.1 Conformal Field Theories and the Supersymmetric Yang-Mills theory

In some sense, quantum field theories are defined by their symmetries. The commonly used relativistic quantum field theories are invariant under the Poincare group, the group that preserves the Minkowski metric. It turns out that one can also construct quantum field theories that are invariant under all the transformations that preserve the angles between any two vectors, known as conformal transformations. A quantum field theory that is invariant under all conformal transformations is called a conformal field theory.

Intuitively, the chief additional symmetry that the conformal symmetry group has is scale transformations. We will see that this is indeed the case, at least at the classical level, following [5]. The general definition of the energy-momentum tensor $T^{\mu\nu}$ is

$$T^{\mu\nu} = \frac{\delta \mathcal{S}_M}{\delta g_{\mu\nu}}, \tag{2.1.1}$$

where $\mathcal{S}_M$ is the matter action. Under an infinitesimal coordinate transformation

$\tilde{x}^{\mu} = x^{\mu} + \xi^{\mu}(x)$, we have

$$
\begin{aligned}
g_{\mu\nu}(x)dx^{\mu}dx^{\nu} &= \tilde{g}_{\mu\nu}(\tilde{x})d\tilde{x}^{\mu}d\tilde{x}^{\nu} \\
&= (\tilde{g}_{\mu\nu}(x) + \partial_{\lambda}\tilde{g}_{\mu\nu}\xi^{\lambda})(dx^{\mu} + \partial_{\rho}\xi^{\mu}dx^{\rho})(dx^{\nu} + \partial_{\delta}\xi^{\nu}dx^{\delta}) \\
&= \tilde{g}_{\mu\nu}(x)dx^{\mu}dx^{\nu} + (\partial_{\lambda}g_{\mu\nu}\xi^{\lambda} + g_{\lambda\nu}\partial_{\mu}\xi^{\lambda} + g_{\mu\lambda}\partial_{\nu}\xi^{\lambda})dx^{\mu}dx^{\nu} + \mathcal{O}(|\xi|^2) \\
&= \tilde{g}_{\mu\nu}(x)dx^{\mu}dx^{\nu} + [(\partial_{\lambda}g_{\mu\nu} - \partial_{\mu}g_{\lambda\nu} - \partial_{\nu}g_{\mu\lambda})\xi^{\lambda} + \partial_{\mu}\xi_{\nu} + \partial_{\nu}\xi_{\mu}]dx^{\mu}dx^{\nu} + \mathcal{O}(|\xi|^2) \\
&= \tilde{g}_{\mu\nu}(x)dx^{\mu}dx^{\nu} + (-2\Gamma^{\lambda}{}_{\mu\nu}\xi_{\lambda} + \partial_{\nu}\xi_{\mu} + \partial_{\mu}\xi_{\nu})dx^{\mu}dx^{\nu} + \mathcal{O}(|\xi|^2) \\
&= (\tilde{g}_{\mu\nu}(x) + \nabla_{\mu}\xi_{\nu} + \nabla_{\nu}\xi_{\mu})dx^{\mu}dx^{\nu} + \mathcal{O}(|\xi|^2).
\end{aligned}
$$

Therefore,

$$
\delta g_{\mu\nu} = -\nabla_{\mu}\xi_{\nu} - \nabla_{\nu}\xi_{\mu}. \tag{2.1.2}
$$

Now for a scale transformation, $\xi^{\mu} = \delta\lambda x^{\mu}$, so $\delta g_{\mu\nu} = -2\delta\lambda g_{\mu\nu}$. Hence, the action is scale invariant when

$$
T^{\mu}{}_{\mu} = T^{\mu\nu}g_{\mu\nu} = 0. \tag{2.1.3}
$$

A general infinitesimal conformal transformation satisfies $\delta g_{\mu\nu} = h(x)g_{\mu\nu}$ for some function h, so the action is invariant under this transformation as long as Eq. (2.1.3) is satisfied.

However, quantum mechanics may bring in anomalies so that a conformally invariant classical theory may not be conformally invariant as a quantum theory. The reason is that during the renormalization process the parameters in the action may acquire anomalous dimensions, and transform nontrivially under scaling. Therefore, the proper requirement for a quantum theory to be conformally invariant is that the renormalization group (RG) flows of the parameters are zero.

The AdS/CFT correspondence was originally proposed and has been more extensively studied for a particular conformal field theory, namely the $\mathcal{N} = 4$ $U(N)$ supersymmetric Yang Mills theory in 4 dimensions. A supersymmetry is a symmetry whose currents are fermionic. Therefore, supersymmetries relate bosonic and fermionic fields together [8]. Supersymmetries are compatible with gauge symmetries, so one can have a gauge theory (Yang Mills theory) that also have supersymmetries.

Such theories are called super Yang Mills theories, and are labeled by the number of supersymmetry generators $\mathcal{N}$ and the gauge group.

In the $\mathcal{N} = 4$ $U(N)$ super Yang Mills theory, we have a gauge field A_μ, six scalars ϕ^I, $I = 1, ..., 6$ and four fermions χ_i, $i = 1, 2, 3, 4$. Here we have suppressed the gauge indices, with the understanding that all the scalars and spinors live in the adjoint representation of the gauge group. The Lagrangian of the theory is [5]

$$\mathcal{L} = \frac{1}{g_{YM}^2} \text{Tr}[F^2 + (D\phi)^2 + \bar{\chi}\gamma^\mu D_\mu \chi + \sum_{IJ}[\phi^I \phi^J]^2 + \bar{\chi}\Gamma^I \phi^I \chi] + \theta \text{Tr}(F \wedge F). \quad (2.1.4)$$

The last term is a topological term, meaning that its integral over the bulk only depends on the topological properties of the space. For simplicity, we assume that $\theta = 0$ from now on. Here, g_{YM} is the coupling constant of the theory. It can be shown that the theory has an $SU(4)$ symmetry. The simply connected Lie group $SU(4)$ is the universal covering group of $SO(6)$, so the group has two special representations, the vector representation of $SU(4)$ and the vector representation of $SO(6)$. Here, ψ_i lives in the vector representation of $SU(4)$ and ϕ^I lives in the vector representation of $SO(6)$. These symmetries do not commute with the supersymmetries and are called the "R-symmetries".

It is easy to see that the Lagrangian Eq. (2.1.4) is conformally invariant, since the only parameter g_{YM} is dimensionless. Furthermore, it is shown in [1] that the RG flows are indeed zero. Therefore, this theory is conformally invariant. Two important parameters of the theory is the coupling constant g_{YM} and the "size" of the gauge group N, whose roles will be seen later in this chapter.

2.2   String Theory and D3 branes

String theory is a promising theory that can potentially unifies gravity and quantum field theory. The basic objects of string theory are strings. The time evolution of a string forms a two-dimensional surface, known as a world sheet. Therefore, a string theory involves two geometries, the world sheet geometry and the space-time

geometry. In order to include fermions in the theory, it is necessary to combind string theory and supersymmetry, which leads to the so-called superstring theory.

In general, the string theory amplitudes are organized as an expansion over a parameter α', which is the square of the Planck length $l_P = \sqrt{\hbar G / c^3} \approx 1.62 \times 10^{-35}$m. In the limit $\alpha' \to 0$, the quantum corrections can be neglected, and the theory reduces to a classical theory. The classical theory is governed by an action, in which the dynamic variables are tensor fields, so it is a supergravity theory. For example, the $D = 11$ supergravity theory has the following action[1]

$$\mathcal{S}_{11} = \frac{1}{2\kappa_1 1^2} \int [\sqrt{G}(R_G - \frac{1}{2}|F_4|^2) - \frac{1}{6}A_3 \wedge F_4 \wedge F_4] + \text{fermions}, \qquad (2.2.1)$$

where $G_{\mu\nu}$ is the metric of the space-time, R_G is the scalar curvature corresponding to this metric, A_3 is a 3-form (or an antisymmetric tensor of rank 3), and $F_4 = dA_3$. In general, the action of a supergravity theory contains curvature tensors of the geometry as well as some differential forms built from p-forms A_p and its exterior differential. One might naively think that we need to consider antisymmetric tensors of all ranks, but this is not necessary since the $(D - p)-$forms and the $p-$ forms are related by the Hodge star operator[7]

$$(*\eta)_{i_1...i_{n-k}} = \frac{1}{k!}\eta_{j_1...j_k}\sqrt{g}\epsilon_{j_1...j_k i_1...i_{n-k}}, \qquad (2.2.2)$$

where η is any $k-$ form and $\epsilon_{j_1...j_k i_1...i_{n-k}}$ is the totally antisymmetric tensor. The physical significance of the Hodge operator can be illustrated with an example. Consider the electromagnetism in 4 dimensions. The field tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is a 2-form, and is related to $\vec{E}$ and $\vec{B}$ through

$$
\begin{aligned}
E_i &= F_{0i}; \\
B_i &= \frac{1}{2}\epsilon_{ijk}F_{jk}.
\end{aligned}
$$

Now by Eq. (2.2.2), $(*F)_{0i} = \epsilon_{ijk}F_{jk}/2$ and $(*F)_{jk} = \epsilon_{ijk}F_{0i}$. Hence,

$$\begin{aligned} B_i &= (*F)_{0i}; \\ E_i &= \frac{1}{2}\epsilon_{ijk}(*F)_{jk}. \end{aligned}$$

Therefore, the Hodge star operator interchanges the electric and magnetic fields, and thus it is also called the magnetic dual operator. The magnetic dual relates field tensors of rank p to those of rank $D - p$ and thus the gauge tensors A or rank p to those of rank $(D - p - 2)$. Therefore, it suffices to consider the gauge tensors up to rank $(D - 2)/2$. The superstring theory that concerns us is the Type IIB superstring theory, which lives in $D = 10$. Therefore, for a 5-form field strength F_5, its magnetic dual is also a 5-form, so it makes sense to talk about self-dual form F_5, or the form A_4 from which it is derived.

An important family of solutions to supergravity are the $p-$branes. A $p-$brane is a solution to supergravity with a nontrivial $(p + 1)-$gauge form A_{p+1} living on it. However, we know from differential geometry that $(p + 1)-$dimensional sheets can be paired with $(p + 1)-$forms through integral

$$(\Sigma_{p+1}, A_{p+1}) \mapsto \int_{\Sigma_{p+1}} A_{p+1}. \tag{2.2.3}$$

Hence, a $p-$brane is geometrically a $(p+1)-$dimensional sheet with Minkowski metric on it, and p is the space dimension of the brane. Transversal to the brane is a $(D - p - 1)$ dimensional space, and it can be shown that the solution can always be chosen so that the transversal space has the full rotational symmetry $SO(D - p - 1)$ [1]. The brane itself has the generalized Poincare symmetry to $(p + 1)-$ dimensions, namely $\mathbb{R}^{p+1} \times SO(1, p)$, where $\mathbb{R}^{p+1}$ parametrizes the translational symmetries and $SO(1, p)$ is the group of rotations and boosts that leave the $(p + 1)-$ dimensional Minkowski metric fixed. In summary, the full symmetry group of a $p-$brane is $\mathbb{R}^{p+1} \times SO(1, p) \times SO(D - p - 1)$. By this symmetry, the most general form of the metric is

thus[1]

$$ds^2 = H(\vec{y})^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu + H(\vec{y})^{1/2}\delta_{\mu\nu}dy^\mu dy^\nu, \qquad (2.2.4)$$

where x is the coordinate on the brane and y is the coordinate on the transversal space. The function H can be obtained by solving the equation of motion derived from the supergravity action.

One needs a further refinement of the concept of branes. Consider a space with N parallel branes in it. The excitations on the branes correspond to strings that origins from one brane and ends at another, allowing the two branes to be identical. To find the possible vibrational modes of a string, one needs to specify its boundary condition at the two end points, which depends in turn on the type of the branes. The type that concerns us is called the D branes, at which the boundary conditions are Dirichlet conditions, namely, the two ends of the strings are fixed. A D brane that has a nontrivial gauge $(p+1)-$form A_{p+1} will be called a Dp brane. According to which branes a string begins and ends with, there are N^2 types of strings.

The strings among the branes have positive tension. Therefore, a string has energy roughly equal to the product of the tension and its length. It is thus important to distinguish between the strings that begin and end at the same brane and those that connect two different branes. For the former type, the length of the string can be shrunk to zero continuously, meaning that these strings can have arbitrarily low energy. In contrast, the minimal length of a string between two different branes is the distance between the two branes, and thus the energy of these strings has a positive lower bound. In terms of the energy spectrum, the strings that begin and end at the same brane are massless, and those connecting different branes have positive mass. However, the mass of strings between two branes approach zero when the distance between these two branes approach zero. In the limit case that all the branes coincide in space, all strings become massless.

More details of the brane dynamics have been worked out, for example in [6]. It turns out that the effective action of the strings among N $D3$ branes contains N^2 gauge fields corresponding to moving the ends of the strings along and among the

branes, and these gauge fields generate a $U(N)$ gauge group. When all the branes coincide, all the gauge bosons remain massless, and the system has the full $U(N)$ gauge symmetry. However, as mentioned above, when the branes are pulled apart, some strings acquire masses. Consequently, the corresponding gauge bosons in the low energy effective theory also acquire masses, as through the Higgs mechanism in the conventional quantum field theory when gauge symmetries are simultaneously broken. Therefore, the separation of branes correspond to simultaneous broken of the gauge symmetry, and in the case when all the branes are separated, only the $U(1)^N$ gauge symmetries remains unbroken, one $U(1)$ for each brane.

2.3   The AdS/CFT Correspondence

As seen in the previous subsection, the low energy limit of the dynamics of N coincident $D3$ branes is a $U(N)$ gauge theory on the branes. (Note that the $D3$ brane is 4-dimensional and has the Minkowski metric.) Furthermore, as [1] points out, the gauge theory also has $\mathcal{N} = 4$ supersymmetries and is conformally invariant, and for N $D3$ branes, the gauge theory that is obtained in this way is precisely the $\mathcal{N} = 4$ $U(N)$ super Yang Mills theory introduced in Section 1.1. Hence, it should correspond to some supergravity theory, and it remains to work out the geometry of the N coincident $D3$ branes.

The metric of the $D3$ branes obeys the general form Eq. (2.3), and the function $H(\vec{y})$ has a simple form[1]

$$H(\vec{y}) = 1 + \frac{L^4}{y^4}, \tag{2.3.1}$$

where

$$L^4 = 4\pi g_s N \alpha'^2, \tag{2.3.2}$$

and g_s is the string coupling constant. Note that L^4 is proportional to N, the number of branes. The metric can be further simplified in the limit $N \to \infty$: substituting Eq. into Eq. and using the spherical coordinate on the transverse space, we obtain

that, for $L \gg y$,

$$ds^2 = (\frac{y^2}{L^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{L^2}{y^2}dy^2) + L^2 d\Omega_5^2, \qquad (2.3.3)$$

where $d\Omega_5^2$ is the spherical metric on the intersection of the transverse space with constant y. Now we recognize the first two terms as the AdS metric Eq. (1.0.3). Note that the $D = 4$ Minkowski metric is restored as $y \to \infty$ with the substitution $\tilde{x}_\mu = yx_\mu$, which corresponds to the intersection of the locus Eq. (1.0.2) with the infinity hyperplane

$$-T_1^2 - T_2^2 + X_1^2 + ... + X_4^2 = 0. \qquad (2.3.4)$$

The infinity hyperplane of the 6-dimensional space has no scale, so it can be viewed as a projective space. From this perspective, the above equation becomes a compactification of the Minkowski space [9]: generically $T_2 + X_4 \neq 0$ so we can set that equal to 1, and then $T_2 - X_4$ can be solved from the equation, leaving the independent coordinates (T_1, X_1, X_2, X_3) with a Minkowski metric. This shows that the Minkowski space is an affine chart of the quadric surface Eq. (2.3.4), and adding the points with $T_2 + X_4 = 0$ to the affine piece makes the surface compact. Therefore, the surface Eq. (2.3.4) is the space in which the supersymmetric gauge theory lives. In summary, the $D3$ geometry becomes a product $AdS_5 \times S^5$, where the radius of both the AdS_5 and the five-dimensional sphere S^5 is L, in the limit $L \gg y$.

The problem with replacing the $D3$-metric with the $AdS_5 \times S_5$ metric is that the important parameter N, the number of branes, disappears if we view L as an independent parameter, as we will do in later sections. However, N can be restored as a topological quantity as follows. By the magnetic duality, for the Type IIB superstring theory which live in $D = 10$, the gauge forms of rank $(p+1)$ are dual to the gauge forms of rank $(D - p - 3)$, so for a $D3$ brane, the gauge form is dual to itself. The gauge field and the field strength are denoted by A_4^+ and $F_5^+ = dA_4^+$, respectively. The integral $\int_{S^5} F_5^+$ is well-defined, and each $D3$ brane contributes an equal share. Therefore, the integral is proportional to N. It is shown in [1] that the integral is exactly N.

Given the $\mathcal{N} = 4 \, U(N)$ super Yang Mills theory in 4-dimensional Minkowski space

and the string theory on $AdS_5 \times S^5$, there is a hint that the two theories should be equivalent, namely they have the same symmetry. The super Yang Mills theory has a conformal symmetry in the Minkowski space, which is the same as the symmetry of the quadric surface Eq. (2.3.4), namely $SO(2,4)$. Also, the theory has an $SO(6)$ R-symmetry as mentioned in Section 1.1, so the symmetry group is $SO(2,4) \times SO(6)$. On the other hand, the AdS_5 has the $SO(2,4)$ symmetry as can be seen from its definition Eq. (3.1.1), and the symmetry of S^5 is $SO(6)$. Hence, the two theories have the same ordinary symmetries. With some computation in string theory, one can also show that they have the same supersymmetries [1]. Now that they have the same symmetry, it is reasonable to guess that these two theories are equivalent, and the result was given in Proposition 1.0.4 and Eq. (1.0.7).

The equivalence of these two theories are best understood in the 't Hooft limit $N \to \infty$, while keeping the parameter λ fixed. The parameter λ was defined in Eq. (1.0.5). In the string theory, we have

$$\lambda = g_s N = \frac{L^4}{4\pi\alpha'^2}, \tag{2.3.5}$$

where the second equality follows from Eq. (2.3.2). Hence, the limit $N \to \infty$ with λ fixed corresponds to the limit $g_s \to 0$. In the gauge theory, we have

$$\lambda = g_{YM}^2 N. \tag{2.3.6}$$

The physical significance is as follows [4]. Since the matter fields live in the adjoint representation of $U(N)$, they can be viewed as $N \times N$ matrices, and the general form of the matter Lagrangian is as follows

$$\mathcal{L} \sim \frac{1}{g^2}\text{Tr}[(\partial M)^2 + M^2 + M^3 + ...].$$

In terms of the Feynman diagrams, the propagator is proportional to g^2, each vertex contains a factor of g^{-2}, and each closed loop indicates taking traces of the matrices and thus produces a factor of N. Let f, e, v be the number of faces (or loops), edges

and vertices in a Feynman diagrams, respectively. Then the contribution from the diagram is proportional to

$$(g^2)^{e-v} N^f = N^{f-e+v} \lambda^{e-v}. \tag{2.3.7}$$

Now the exponent of N is a topological quantity known as the Euler characteristic, and for a closed surface with genus h, the Euler characteristic equals $2 - 2h$. Therefore, the limit $N \to \infty$ with fixed λ corresponds to the contribution only from graphs with $h = 0$, or planar graphs. Note also that each vertex is associated with more than 2 edges, so $e - v$ increases with the number of vertices, and thus higher order Feynman diagrams have higher power in λ. This discussion clarifies the idea of duality mentioned in Chapter 1: the strong coupling limit of the field theory $\lambda \to \infty$ corresponds to the limit $\alpha' \to 0$, the weak coupling limit of the string theory.

To apply the AdS/CFT correspondence, one needs to find the operator $\mathcal{O}$ in Eq. (1.0.7), given the field ϕ. In general, this is nontrivial. However, in some special cases, it is possible to guess the operator $\mathcal{O}$. As an example, consider a scalar field in the large λ limit, whose equation of motion is $\Box \phi - m^2 \phi = 0$. The discussion below follows [5]. In the AdS part of the metric Eq. (2.3.3),

$$
\begin{aligned}
\Box \phi &= \frac{1}{\sqrt{g}} \partial_a (g^{ab} \sqrt{g}) \partial_b \phi \\
&= \frac{L^3}{y^3} \frac{\partial}{\partial y} \Big(\frac{y^2}{L^2} \frac{y^3}{L^3} \frac{\partial \phi}{\partial y} \Big) + \eta^{\mu\nu} \frac{L^2}{y^2} \frac{\partial^2 \phi}{\partial x^\mu \partial x^\nu} \\
&= \frac{1}{y^3 L^2} \frac{\partial}{\partial y} \Big(y^5 \frac{\partial \phi}{\partial y} \Big) + \eta^{\mu\nu} \frac{L^2}{y^2} \frac{\partial^2 \phi}{\partial x^\mu \partial x^\nu}.
\end{aligned}
$$

Hence, in the Fourier space for the Minkowski part (parametrized by x^μ), the equation of motion becomes

$$\frac{1}{y^3 L^2} \frac{\partial}{\partial y} \Big(y^5 \frac{\partial \phi}{\partial y} \Big) + \frac{L^2}{y^2} p^2 \phi - m^2 \phi = 0. \tag{2.3.8}$$

As $y \to \infty$, the second term vanishes, and the boundary behavior of the solution is

given by $\phi(y) = y^\alpha$, where α solves the index equation

$$\alpha(\alpha + 4) - m^2 L^2 = 0.$$

The roots are

$$\alpha_\pm = -2 \pm \sqrt{4 + m^2 L^2}. \tag{2.3.9}$$

Clearly, the branch y^{α_+} dominates. However, $\phi(y)$ diverges as $y \to \infty$, so one needs to renormalize the boundary value $\phi_0(y)$ through

$$\phi(x, y)|_{y=Y} = Y^{\alpha_+} \phi_0^r(x), \tag{2.3.10}$$

where "r" means renormalized. The metric Eq. (2.3.3) is invariant under the conformal transformation $x \to \lambda x$ and $y \to \lambda^{-1} y$, which restricts to the standard conformal transformation on the boundary. Hence, in this transformation, $\phi(x, y)$ does not scale, so the renormalized boundary value $\phi_0^r(x)$ scale as λ^{α_+}, and thus the operator $\mathcal{O}$ has mass dimension

$$\Delta = 4 + \alpha_+ = 2 + \sqrt{4 + m^2 L^2}. \tag{2.3.11}$$

Therefore, the massless scalar field corresponds to an operator $\mathcal{O}$ with mass dimension 4. The massless scalar may appear in the metric as $\delta ds^2 = \frac{y^2}{L^2} \phi_{\mu\nu} dx^\mu dx^\nu$ where the indices μ and ν are not summed over. This type of scalar fields are called gravitons. One can show that the equation of motion of $\phi_{\mu\nu}$ is precisely $\Box \phi_{\mu\nu} = 0$. (We will demonstrate a particular case in Section 3.) Therefore, $\phi_\mu \nu$ corresponds to an operator $\mathcal{O}^{\mu\nu}$ with mass dimension 4 in the conformal field theory. Hence, one can guess that $\mathcal{O}^{\mu\nu} = T^{\mu\nu}$, the stress tensor [5]. Therefore, the study of graviton propagation is equivalent to the study of the correlation function of the stress tensor in the conformal field theory.

Chapter 3

The Shock Wave Geometry

In this section, the AdS space and the shock wave on it are studied from a geometric perspective.

3.1 The AdS metric

A general d-dimensional AdS space can be embedded in a $(d+1)$-dimensional flat space with signature $(-1, -1, 1, ..., 1)$ as follows:

$$-T_1^2 - T_2^2 + X_1^2 + ... + X_{d-1}^2 = -L^2, \tag{3.1.1}$$

where L is a measure of the curvature of the AdS space, and is determined, through the equation of motion, by the cosmological constant λ. The computation is made easy by the following choice of coordinate system of the AdS space,

$$y_0 = \frac{T_1}{L + T_2}; \tag{3.1.2}$$

$$y_i = \frac{X_i}{L + T_2} \text{ for } i = 1, ..., d - 1. \tag{3.1.3}$$

By the substitution principle, the metric can be written as

$$ds^2 = \frac{4L^2 \eta_{\mu\nu}}{(1 - \eta_{\alpha\beta} y^\alpha y^\beta)^2} dy^\mu dy^\nu, \tag{3.1.4}$$

where $\eta_{\mu\nu}$ is the Minkowski metric. One can work out the curvature tensor, the Ricci tensor and the scalar curvature, and the results are

$$R_{\mu\nu\rho\lambda} = (g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda})L^{-2}; \tag{3.1.5}$$

$$R_{\mu\nu} = (1-d)g_{\mu\nu}L^{-2}; \tag{3.1.6}$$

$$R = d(1-d)L^{-2}. \tag{3.1.7}$$

It is clear then that this metric solves the Einstein equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\Lambda g_{\mu\nu} \tag{3.1.8}$$

with

$$\Lambda = -\frac{(d-1)(d-2)}{2}L^{-2}. \tag{3.1.9}$$

This equation can be used to determine L. In fact, the functional form Eq. (3.1.4) can solve the equation of motion derived from actions with α' corrections:

$$\mathcal{S} = \int d^d y \sqrt{g}(R - 2\Lambda) + \text{higher order terms,}$$

The proof proceeds as follows. The curvature tensor Eq. (3.1.5) can be written entirely in terms of the metric tensor, so covariant derivatives of the curvature tensor produced by variational calculus all vanish and any powers and contractions of the curvature tensor and the metric tensor contain only $\delta g_{\mu\nu}$ and contractions of metric tensors, with appropriate powers of L. However, all such contractions must reduce to a constant multiple of $g^{\mu\nu}\delta g_{\mu\nu}$. Therefore, substituting Eq. (3.1.4) into the most general form of the equation of motion leads to

$$p(L^2)g^{\mu\nu} = \Lambda g^{\mu\nu}, \tag{3.1.10}$$

where $p(L^2)$ is a polynomial in L^2. We see that Eq. (3.1.4) solves the equation of motion as long as $p(L^2) = \Lambda$.

The above argument shows that with α' corrections the functional form Eq. (3.1.4)

remains a solution to the equation of motion, but the value of L may be modified by those corrections. As an example, we will work out a particular case explicitly which will be important later in the paper. Consider the following action

$$S = \int d^d y \sqrt{g} [R - 2\Lambda + \lambda(R^2 - 4R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta})]. \qquad (3.1.11)$$

This particular form of α' correction is called the Gauss-Bonnet term. By the symmetry of the curvature and Ricci tensors, the variation of the action can be written as

$$\delta S = \int d^d y \sqrt{g} (\frac{1}{2} g^{\mu\nu} \delta g_{\mu\nu})[R - 2\Lambda + \lambda(R^2 - 4R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta})] +$$
$$\sqrt{g}[\delta R + 2\lambda(R\delta R - 4R_\alpha{}^\beta \delta R_\beta{}^\alpha + R_{\alpha\beta}{}^{\gamma\delta}\delta R^{\alpha\beta}{}_{\gamma\delta})].$$

Substituting in Eqs. (3.1.5), (3.1.6) and (3.1.7), we obtain that

$$\begin{aligned}
\delta S &= \int d^d y \sqrt{g} (\frac{1}{2} g^{\mu\nu} \delta g_{\mu\nu}) L^{-2} \{d(1-d) - 2\Lambda L^2 + \lambda L^{-2}[d^2(d-1)^2 - 4(d-1)^2 d + 2d(d-1)]\} + \\
&\quad \sqrt{g} \{1 + 2\lambda L^{-2}[d(1-d) - 4(1-d) - 2]\}\delta R \\
&= L^{-2} \int d^d y \sqrt{g} g^{\mu\nu} \delta g_{\mu\nu} \{-\Lambda L^2 + \frac{1}{2}d(1-d) + \\
&\quad \frac{\lambda}{2L^2}d(d-1)(d^2 - 5d + 6) - (d-1)[-1 + 2\lambda L^{-2}(d^2 - 5d + 6)]\} \\
&= L^{-2} \int d^d y \sqrt{g} g^{\mu\nu} \delta g_{\mu\nu} [-\Lambda L^2 - \frac{1}{2}(d-2)(d-1) + \frac{\lambda}{2L^2}(d-4)(d-1)(d^2 - 5d + 6)].
\end{aligned}$$

Therefore,

$$\Lambda = -\frac{(d-2)(d-1)}{2L^2} + \lambda\frac{(d-1)(d-2)(d-3)(d-4)}{2L^4}. \qquad (3.1.12)$$

3.2 The Shock Wave Metric

As mentioned before, a shock wave is a wave that travels with the speed of light and affects matters only near its frontier $y_1 = -y_0$. Therefore, the metric in the shock wave geometry should be the same as the AdS metric away from the frontier. Then

for the shock wave to be nontrivial, it must be a delta function centered at the frontier $\delta(y_0 + y_1)$. However, the profile of the shock wave, namely a multiplicative factor of the delta function cannot be determined by the general consideration and must be solved from the equation of motion.

By the above consideration, the shock wave metric can be defined with the coordinate system introduced in Eqs. (3.1.2) and (3.1.3) as:

$$ds^2 = \frac{4L^2\eta_{\mu\nu}}{(1 - \eta_{\alpha\beta}y^\alpha y^\beta)^2}dy^\mu dy^\nu + \frac{L^2\delta(y_+)f(\vec{y})}{1 - \eta_{\alpha\beta}y^\alpha y^\beta}dy_+^2, \tag{3.2.1}$$

where $y_\pm = y_0 \pm y_1$, $\vec{y}$ is an abbreviation of $y_2, ..., y_{d-1}$ and $f(\vec{y})$ is a scalar function to be determined by the equation of motion. This metric is the same as the AdS metric away from the shock wave frontier $y_+ = 0$. At the frontier, however, there will be a jump specified by the function f. To further simplify our notation, let

$$l_\mu = \partial_\mu y_+, \tag{3.2.2}$$

so $l_0 = l_1 = 1$ and $l_i = 0$ for $i > 1$. Now the shock wave metric (3.2.1) can be written as $ds^2 = g_{\mu\nu}dy^\mu dy^\nu$ where

$$g_{\mu\nu} = \frac{4L^2\eta_{\mu\nu}}{(1 - \eta_{\alpha\beta}y^\alpha y^\beta)^2} + \frac{L^2 F(y)}{1 - \eta_{\alpha\beta}y^\alpha y^\beta}l_\mu l_\nu, \tag{3.2.3}$$

where

$$F(y) = \delta(y_+)f(\vec{y}).$$

Clearly, the contravariant counterpart of the above metric is

$$g^{\mu\nu} = \frac{1}{4L^2}(1 - \eta_{\alpha\beta}y^\alpha y^\beta)^2\eta^{\mu\nu} - \frac{1}{16L^2}(1 - \eta_{\alpha\beta}y^\alpha y^\beta)^3\eta^{\mu\rho}\eta^{\nu\lambda}Fl_\rho l_\lambda. \tag{3.2.4}$$

Carrying out the calculation of curvature tensors, actions and equations of motion by hand is tedious. Fortunately, these tensor computation can be done by computer routines. Such a computation can be implemented as algebraic computation based

on the following set of symbols, up to renaming the indices:

$$\mathcal{X} = \{\eta_{\mu\nu}, \eta^{\mu\nu}, \delta^\mu_\nu, y^\mu, l_\mu, F, F_{,\mu_1\mu_2\ldots}, d, \sqrt{g}\}, \tag{3.2.5}$$

where d is the dimension of the AdS space. Some of the important relations among these symbols are restated in the following proposition:

Proposition 3.2.6. *The symbols in $\mathcal{X}$ satisfy the following properties:*

$$\begin{aligned}
\eta^{\mu\nu} l_\mu F_{,\nu\lambda_1\lambda_2} &= 0; \\
\eta^{\mu\nu} l_\mu l_\nu &= 0; \\
l_\mu y^\mu F &= 0.
\end{aligned}$$

Proof. The first identity holds since F is independent of y_-. The second identity follows from the definition of l_μ. The third identity holds since F contains a factor of $\delta(y_+)$. $\qquad\square$

We denote by $\mathbb{F}[\mathcal{X}]$ the set of all polynomials of symbols in $\mathcal{X}$, in which repeated indices are summed over and every monomial has the same free indices. Here $\mathbb{F}$ denotes an arbitrary coefficient field (with characteristic 0), which may be the rationals, the reals or the complexes. A rational expression on $\mathcal{X}$ is defined as the ratio of two elements of $\mathbb{F}[\mathcal{X}]$. We denote the set of rational expression on $\mathcal{X}$ by $\mathbb{F}(\mathcal{X})$. A new relation among symbols in $\mathcal{X}$ can be stated using the above notations:

Proposition 3.2.7.

$$\int d^d y\, l_\mu y^\mu F_{,\lambda_1\ldots\lambda_n} \Xi = -\int d^d y \sum_{k=1}^n l_{\lambda_k} F_{,\lambda_1\ldots\hat{\lambda_k}\ldots\lambda_n} \Xi,$$

for any $\Xi \in \mathbb{F}(\mathcal{X})$.

Proof. Integration by parts yields that

$$\int d^d y\, l_\mu y^\mu F_{,\lambda_1\ldots\lambda_n} \Xi = (-1)^n \int d^d y \sum_{k=1}^n l_{\lambda_k} F \Xi_{,\lambda_1\ldots\hat{\lambda_k}\ldots\lambda_n} + (-1)^n \int d^d y\, l_\mu y^\mu F \Xi_{,\lambda_1\ldots\lambda_n}.$$

By Proposition 3.2.6, the second term on the right hand side vanishes, and applying integration by parts to the first term completes the proof. $\qquad\square$

A nice property of $\mathbb{F}[\mathcal{X}]$ is that it is closed under differentiation. Indeed, this is trivial except for $\sqrt{g}$, but in the shockwave geometry $\partial_\mu \sqrt{g} = 2d\eta_{\mu\nu}y^\nu \sqrt{g}$. It follows that $\mathbb{F}(\mathcal{X})$ is also closed under differentiation. Now, apart from the constant L, the metric $g_{\mu\nu}$ and its contravariant counterpart $g^{\mu\nu}$ are elements of $\mathbb{R}(\mathcal{X})$. Therefore, all tensors constructed from the metric through differentiation and contraction are elements of $\mathbb{R}(\mathcal{X})$. In particular, the curvature tensor, the Lagrangian. Further more, the equation of motion is obtained from variational calculus and integration by parts, so it can be written as

$$\Pi_{\mu\nu} = \Lambda g_{\mu\nu}, \tag{3.2.8}$$

where $\Pi_{\mu\nu}$ is a tensor constructed from the metric and independent of Λ, and thus $\Pi_{\mu\nu} \in \mathbb{R}(\mathcal{X})$. With the above algebraic preparation, the equation of motion Eq. (3.2.8) can be computed with computer programs in a straight forward way. Moreover, the algebraic properties also constrain the general form of the equation of motion.

Theorem 3.2.9. *Assume that the Lagrangian is a polynomial in the curvature and metric tensors. Then the equation of motion Eq. (3.2.8) is equivalent to Eq. (3.1.10), the equation of motion without the shock wave, and a linear differential equation of the function f in Eq. (3.2.1).*

Proof. We write $\Pi_{\mu\nu} = \Pi_{\mu\nu}^{(0)} + \Pi_{\mu\nu}^{(1)}$, where $\Pi_{\mu\nu}^{(0)}$ is independent of f, and $\Pi_{\mu\nu}^{(1)} = 0$ when $f = 0$. Since all the contribution to $\Pi_{\mu\nu}$ from the shock wave part of the metric contains f, $\Pi_{\mu\nu}^{(0)}$ should be the same as the contribution from the AdS metric, namely $p(L^2)g_{\mu\nu}$ in (3.1.10). On the other hand, $\Pi_{\mu\nu}^{(1)}$ is produced from the integral $\int d^d y \sqrt{g} \Pi_{\mu\nu}^{(1)} \delta g^{\mu\nu}$. One can simplify $\Pi_{\mu\nu}^{(1)}$ so that in each term in the numerator, the number of symbols that appear in the term is minimal. In particular, $\Pi_{\mu\nu}^{(1)}$ contains no δ symbols, and $l_\rho y^\rho$ cannot appear simultaneously with derivatives of f since in such cases the term can be further reduced by using Proposition 3.2.7. Now suppose a term in $\Pi_{\mu\nu}^{(1)}$ contains l_ρ for some index $\rho \neq \mu, \nu$. Then l_ρ either contracts with a

y^ρ in which case the term vanishes by Proposition 3.2.6, or contracts with $\eta^{\rho\lambda}$ for some λ, in which case $\eta^{\rho\lambda}$ cannot contract with another η symbol by irreducibility, so it contracts with l_λ or $F_{,\lambda...}$, but both contractions are zero. Therefore, a nontrivial reduced term in $\Pi^{(1)}_{\mu\nu}$ cannot contain any l_ρ for $\rho \neq \mu, \nu$. Now each term must contain f or its derivative, so it contains at least two l symbols. By the above argument, the two l symbols must be l_μ and l_ν and there cannot any other l symbols. Therefore,

$$\Pi^{(1)}_{\mu\nu} = l_\mu l_\nu Df(\vec{y})\delta(y_+),$$

where D is some linear operator. Therefore, the equation of motion Eq. (3.2.8) can be written as

$$p(L^2)g_{\mu\nu} + l_\mu l_\nu Df(\vec{y})\delta(y_+) = \Lambda g_{\mu\nu}.$$

Clearly, the above equation is equivalent to Eq. (3.1.10) and the linear differential equation $Df = 0$. □

Under the Einstein-Hilbert action, $\Pi_{\mu\nu} = \frac{1}{2}g_{\mu\nu}R - R_{\mu\nu}$. The computer routine yields the following equation of motion

$$\nabla^2 f - (d-2)f = 0, \tag{3.2.10}$$

where the Laplacian in the transverse space is defined as

$$\nabla^2 F = \frac{1}{4}(1 - \eta_{\alpha\beta}y^\alpha y^\beta)^2 \eta^{\mu\nu} F_{,\mu\nu} + (\frac{1}{2}d - 2)(1 - \eta_{\alpha\beta}y^\alpha y^\beta)y^\mu F_{,\mu} \tag{3.2.11}$$

This recovers the equation of motion given by other authors [2].

Horowitz argues in [2] that the equation of motion Eq. (3.2.10) should not receive α' corrections, which does not follow from Theorem 3.2.9. In fact, the equation of motion does become more complicated as the α' corrections are included, and it may also contain higher order derivatives. To illustrate this, we will show that the fourth order derivative of f appears in the equation of motion when we include the correction $\alpha R_{\mu\nu}R^{\mu\nu}$.

We will need the following expression for the Ricci curvature in a general (pseudo)Riemannian manifold:

$$R_{ab} = -\frac{1}{2}g^{fc}g^{de}g_{bc,d}g_{ae,f} + \frac{1}{2}g^{fd}g^{ce}g_{bc,d}g_{ae,f} + \frac{1}{4}g^{ec}g^{df}g_{cd,a}g_{ef,b} - \frac{1}{4}g^{fc}g^{de}g_{ba,c}g_{de,f} + \frac{1}{4}g^{fc}g^{de}g_{ac,b}g_{de,f}$$

$$+\frac{1}{4}g^{fc}g^{de}g_{bc,a}g_{de,f} - \frac{1}{2}g^{cd}g_{cd,ba} + \frac{1}{2}g^{ec}g^{df}g_{cd,e}g_{ba,f} - \frac{1}{2}g^{dc}g_{ba,cd} - \frac{1}{2}g^{ec}g^{df}g_{cd,e}g_{af,b} + \frac{1}{2}g^{dc}g_{ac,bd}$$

$$-\frac{1}{2}g^{ec}g^{df}g_{cd,e}g_{bf,a} + \frac{1}{2}g^{dc}g_{bc,ad},$$

and the Ricci tensor in this Shock Wave geometry:

$$R_{ab} = -\frac{1}{4}dl_bl_aF + l_bl_aF_{,c}y^c - \frac{1}{8}\eta^{dc}l_bl_a(1-\eta_{\alpha\beta}y^\alpha y^\beta)^1 F_{,cd} - \frac{1}{2}dl_bl_a(1-\eta_{\alpha\beta}y^\alpha y^\beta)^{-1}F + l_al_bF$$

$$-4d\eta_{ab}(1-\eta_{\alpha\beta}y^\alpha y^\beta)^{-2} + 4\eta_{ab}(1-\eta_{\alpha\beta}y^\alpha y^\beta)^{-2} - \frac{1}{4}dl_al_bF_{,c}y^c.$$

Example 3.2.12. *The 4th order derivative of F appears on the LHS of the EOM when a correction term $\alpha R_{ab}R^{ab}$ to the Lagrangian is included, where α is a constant.*

Proof. The correction to the Lagrangian can be rewritten as $\alpha R_{ac}g^{bc}R_{bd}g^{ad}$, so its contribution to the variation of the action is

$$\int d^dy\sqrt{g}(\frac{1}{2}g^{cd}\delta g_{cd}\alpha R_{ab}R^{ab} + 2\alpha R_{ac}g^{bc}\delta(R_{bd}g^{ad})).$$

The fourth order derivative of F can appear only if the variation operator hits a second derivative of the metric and the result is multiplied by the only term containing the second derivative of F in the Ricci tensor. However, the term in the Ricci tensor containing $F_{,\mu\nu}$ also contains 2 l_μ's, so the other factors in the same term cannot contain any more l_μ's by our previous remark. Therefore, the terms containing the fourth derivatives of F are contained in the following (where we have written $(1-\eta_{\alpha\beta}y^\alpha y^\beta)$ as $(1-y^2)$ for simplicity):

$$\int d^dy 2\alpha\sqrt{g}(-\frac{1}{8}l_al_c\eta^{ef}(1-y^2)F_{,ef})(\frac{1}{4}(1-y^2)^2\eta^{bc})g^{da}g^{gh}\frac{1}{2}\delta(-g_{bd,gh} + g_{bg,hd} + g_{dg,hb} - g_{gh,bd})$$

$$= \int d^dy\frac{\alpha}{512}\sqrt{g}(1-y^2)^7l_al_c\eta^{ef}F_{,ef}\eta^{bc}\eta^{da}\eta^{gh}\delta(g_{bd,gh} - g_{bg,hd} - g_{dg,hb} + g_{gh,bd}).$$

36

After integrating by parts, to obtain the fourth order derivatives of F, the derivatives must hit $F_{,ef}$, but a derivative of F is annihilated by $l_\mu \eta^{\mu\nu}$ if ν is a direction in which the derivative is taken, so the only nontrivial contribution comes from $g_{bd,gh}$. Therefore, the desired term is

$$\int d^d y \frac{\alpha}{512} \sqrt{g} (1 - y^2)^7 l_a l_c \eta^{ef} \eta^{gh} F_{,efgh} \eta^{da} \eta^{bc} \delta_{bd}.$$

$\square$

Nevertheless, the equation of motion Eq. (3.2.10) remains the same when the α' correction is of the Gauss-Bonnet form, as can be shown by computer programs. This result will be important in studying the propagation of gravitons in the shock wave geometry.

Chapter 4

Gravitons in the Shock Wave Geometry

4.1 Action with a Graviton Field

In what follows, we will work in the particular case $d = 5$; in other words, the AdS space is 5-dimensional. The study of gravitons is most convenient in the coordinate that we used in Chapter 1:

$$r = X_{d-1} - T_2, t = T_1 L/r, \text{ and } x_i = X_i L/r,$$

for $i = 1, ..., d - 2$. Whenever we write $x_i x_i$, $x_i dx_i$ or $dx_i dx_i$, the summation over i from 1 to $d - 2$ is always implied unless otherwise indicated.

Since

$$(-t^2 + x_i x_i)r^2 L^{-2} = -T_1^2 + \sum_{i=1}^{d-2} X_i^2 = -L^2 + T^2 - X_{d-1}^2 = -L^2 - r(T_2 + X_{d-1}),$$

we have

$$X_{d-1} = -\frac{L^2}{2r} - (-t^2 + x_i x_i)\frac{r}{2L^2} + \frac{r}{2}; \text{ and}$$

$$T_2 = -\frac{L^2}{2r} - (-t^2 + x_i x_i)\frac{r}{2L^2} - \frac{r}{2}.$$

By the substitution rule, the AdS metric becomes

$$ds_0^2 = -dT_1^2 - dT_2^2 + \sum_{i=1}^{d-1} dX_i^2 = \frac{r^2}{L^2}(-dt^2 + dx_i dx_i) + \frac{L^2}{r^2} dr^2, \qquad (4.1.1)$$

which recovers Eq. (1.0.3). To compute the shockwave correction, we note that

$$
\begin{aligned}
1 - \eta_{\alpha\beta} y^\alpha y^\beta &= 1 - \frac{-T_1^2 + \sum_{i=1}^{d-1} X_i^2}{L + T_2} = \frac{2L}{L + T_2}; \text{ and} \\
y_+ &= \frac{x_+ r}{L(L + T_2)}.
\end{aligned}
$$

Therefore,

$$\frac{L^2 \delta(y_+) f}{1 - \eta_{\alpha\beta} y^\alpha y^\beta} dy_+^2 = \frac{1}{2} r \delta(x_+) f dx_+^2, \qquad (4.1.2)$$

where we have for convenience introduced $x_\pm = t \pm x_1$. Combining (4.1.1) and (4.1.2), we have the full shockwave metric

$$ds^2 = \frac{r^2}{L^2}(-dt^2 + dx_i dx_i) + \frac{L^2}{r^2} dr^2 + \frac{1}{2} r \delta(x_+) f dx_+^2. \qquad (4.1.3)$$

Furthermore,

$$\frac{\partial(L + T_2)}{\partial x_-} = -\frac{r}{2L^2} x_+,$$

so $\delta(x_+) \partial y_i / \partial x_- = 0$ for all $i > 1$ and thus by the chain rule

$$\frac{\delta(x_+) \partial f}{\partial x_-} = \delta(x_+) \sum_{i=2}^{d-1} \frac{\partial y_i}{\partial x_-} \frac{\partial f}{\partial y_i} = 0.$$

The EOM of f in this coordinate system can be obtained by computing the Ricci tensor using RGTC, or by performing the change of coordinate from Eq. (3.2.10), and the result is

$$r^4 \frac{\partial^2 f}{\partial r^2} + 3r^3 \frac{\partial f}{\partial r} - 3r^2 f + L^4 \left(\frac{\partial^2 f}{\partial x_2^2} + \frac{\partial^2 f}{\partial x_3^2} \right) = 0. \qquad (4.1.4)$$

Now we are ready to write down the action including the shock wave and the graviton fields. The graviton field ϕ, assumed to be independent of x_2 and x_3 for

simplicity, enters the metric as

$$ds^2_{graviton} = \frac{r^2}{L^2}(2\phi(r,t,x_1)dx_2 dx_3).$$

The action with the Gauss-Bonnet term Eq. (3.1.11) can now be written as

$$
\begin{aligned}
S &= \int d^5x \sqrt{g}(\mathcal{L}_1 + \lambda \mathcal{L}_2) + \mathcal{O}[\phi^4]; \\
\sqrt{g} &= \frac{r^3}{L^3}(1 - \frac{1}{2}\phi^2); \\
\mathcal{L}_1 &= -\frac{8}{L^2} + \frac{1}{2L^2 r^2}[L^4 \frac{\partial \phi}{\partial x_+}\frac{\partial \phi}{\partial x_-} - r^4(\frac{\partial \phi}{\partial r})^2 + \frac{1}{2r}L^6 f\delta(x_+)(\frac{\partial \phi}{\partial x_-})^2 - 8r^2\phi^2]; \\
\mathcal{L}_2 &= \frac{96}{L^4} + \frac{2}{L^4 r^2}[-L^4 \frac{\partial \phi}{\partial x_+}\frac{\partial \phi}{\partial x_-} + r^4(\frac{\partial \phi}{\partial r})^2 + 24r^2\phi^2 \\
&\quad -\frac{L^6}{4r}(\frac{\partial \phi}{\partial x_-})^2(f + r\frac{\partial f}{\partial r} + r^2\frac{\partial^2 f}{\partial r^2})\delta(x_+)],
\end{aligned}
$$

where we have substituted in (3.1.12) as we are only interested in the dynamics of ϕ.

4.2   The Equation of Motion and Its Solution

By taking the variation of S with respect to ϕ, we find the equation of motion as

$$2(1-2\tilde{\lambda})(L^4 r\frac{\partial^2 \phi}{\partial x_+ \partial x_-} - 5r^4\frac{\partial \phi}{\partial r} - r^5\frac{\partial^2 \phi}{\partial r^2}) + L^6\delta(x_+)\frac{\partial^2 \phi}{\partial x_-^2}[f - \tilde{\lambda}(f + r\frac{\partial f}{\partial r} + r^2\frac{\partial^2 f}{\partial r^2})] = 0,$$

$$(4.2.1)$$

where $\tilde{\lambda} = 2\lambda/L^2$. We can see that the EOM receives no corrections from the Gauss-Bonnet Lagrangian away from the shock wave frontier.

However, this is not the whole story. By looking at other components of the Einstein tensor, we find other equations to be satisfied by ϕ. Three independent

equations are obtained in this way:

$$[\phi + \tilde{\lambda}(2\phi + 3r\frac{\partial\phi}{\partial r} + r^2\frac{\partial^2\phi}{\partial r^2})]\frac{\partial^2 f}{\partial x_2 \partial x_3}\delta(x_+) = 0;$$

$$(1 - 2\tilde{\lambda})\frac{\partial\phi}{\partial x_-}\frac{\partial f}{\partial x_3}\delta(x_+) - r\tilde{\lambda}\frac{\partial^2\phi}{\partial x_-^2}(\frac{\partial f}{\partial x_3} - r\frac{\partial^2 f}{\partial x_3 \partial r})\delta(x_+) = 0;$$

$$(1 - 2\tilde{\lambda})\frac{\partial\phi}{\partial x_-}\frac{\partial f}{\partial x_2}\delta(x_+) - r\tilde{\lambda}\frac{\partial^2\phi}{\partial x_-^2}(\frac{\partial f}{\partial x_2} - r\frac{\partial^2 f}{\partial x_2 \partial r})\delta(x_+) = 0.$$

All the above equations are only concerned with the value of ϕ and its derivatives at the shock wave frontier. However, the behavior of ϕ there has already be prescribed in (4.2.1), so in general, no non-trivial solutions exist. This is because we assume that ϕ is independent of x_2 and x_3. Therefore, the non-existence of solution precisely tells us that when the shock depends on x_2 and x_3, the graviton field is forced to be x_2, x_3-dependent. However, all the three equations become trivial when f is independent of x_2 and x_3, which will be the only case that concerns us.

Away from the shock wave frontier $x_+ = 0$, the equation of motion of the graviton is relatively simple and independent of $\tilde{\lambda}$:

$$L^4\frac{\partial^2\phi}{\partial x_+ \partial x_-} - 5r^3\frac{\partial\phi}{\partial r} - r^4\frac{\partial^2\phi}{\partial r^2} = 0. \tag{4.2.2}$$

One can solve this equation in the Fourier space in t and x_1:

$$\phi(r, t, x_1) = \frac{1}{4\pi^2}\int dk d\omega \phi_{\omega,k}(r)e^{ikx_1 - i\omega t}$$

. Then different modes decouple and one obtains

$$r^4\frac{\partial^2\phi_{\omega,k}}{\partial r^2} + 5r^3\frac{\partial\phi_{\omega,k}}{\partial r} + (\omega^2 - k^2)L^4\phi_{\omega,k} = 0. \tag{4.2.3}$$

The solutions are the Bessel functions. Specifically, for $\omega^2 - k^2 < 0$, the solutions are the modified Bessel functions $r^{-2}I_2(L^2\sqrt{k^2 - \omega^2}/r)$ and $r^{-2}K_2(L^2\sqrt{k^2 - \omega^2}/r)$. However, $I_2(L^2\sqrt{k^2 - \omega^2}/r)$ blows up exponentially as $r \to 0$, so only the K_2 solution

is sensible. The asymptotic behavior of the solution is

$$\phi(r) \sim r^{-\frac{3}{2}} \sqrt{\frac{\pi}{2L^2\sqrt{k^2-\omega^2}}} \exp(-L^2\sqrt{k^2-\omega^2}/r), \text{ for } r \ll L^2\sqrt{k^2-\omega^2};$$

$$\phi(r) \sim \frac{2}{L^4(k^2-\omega^2)}, \text{ for } r \gg L^2\sqrt{k^2-\omega^2}.$$

Therefore, the solution decays exponentially with $1/r$ and approaches a constant near the boundary. This shows that when $\omega^2 - k^2 < 0$, the graviton cannot propagate in the bulk of the AdS space.

For $\omega^2 - k^2 > 0$, the solutions to Eq. (4.2.3) can be expressed as the superposition of the Hankel functions $r^{-2}H_2^{(1)}(L^2\sqrt{\omega^2-k^2}/r)$ and $r^{-2}H_2^{(2)}(L^2\sqrt{\omega^2-k^2}/r)$. For $r \ll L^2\sqrt{\omega^2-k^2}$, their asymptotic behaviors are

$$r^{-2}H_2^{(1)}(L^2\sqrt{\omega^2-k^2}/r) \sim r^{-\frac{3}{2}} \sqrt{\frac{\pi}{2L^2\sqrt{\omega^2-k^2}}} \exp(iL^2\sqrt{\omega^2-k^2}/r); \quad (4.2.4)$$

$$r^{-2}H_2^{(2)}(L^2\sqrt{\omega^2-k^2}/r) \sim r^{-\frac{3}{2}} \sqrt{\frac{\pi}{2L^2\sqrt{\omega^2-k^2}}} \exp(-iL^2\sqrt{\omega^2-k^2}/r). \quad (4.2.5)$$

The factor $r^{-\frac{3}{2}}$ is dictated by the conservation of energy, since in the AdS space $\sqrt{g} \propto r^3$. It can be seen that $H_2^{(1)}$ corresponds to a wave that leaves the boundary, and $H_2^{(2)}$ corresponds to a wave that returns to the boundary. Near the boundary, both solutions approach a constant.

Now we turn to the study of the scattering between the graviton and the shock wave at $x_+ = 0$. For simplicity, we will focus on the plane wave limit, the limit where $\omega^2 - k^2$ is large, from now on. Then the asymptotic behaviors Eqs. (4.2.4) and (4.2.5) applies everywhere except for a small region very close to the boundary. The jump condition at the frontier can be derived from Eq. (4.2.1):

$$2(1-2\tilde{\lambda})(r\frac{\partial\phi}{\partial x_-})|_{x_+=0-}^{x_+=0+} + L^2\frac{\partial^2\phi}{\partial x_-^2}|_{x_+=0}[f - \tilde{\lambda}(f + r\frac{\partial f}{\partial r} + r^2\frac{\partial^2 f}{\partial r^2})] = 0, \quad (4.2.6)$$

with the understanding that the average over $x_+ = 0+$ and $x_+ = 0-$ is taken in the

second term. For the simplicity of presentation, we define

$$A(r) = \frac{L^2[f - \tilde{\lambda}(f + r\frac{\partial f}{\partial r} + r^2\frac{\partial^2 f}{\partial r^2})]}{2(1 - 2\tilde{\lambda})r}. \tag{4.2.7}$$

Then Eq. (4.2.6) can be written as

$$(\frac{\partial \phi}{\partial x_-})|_{x_+=0-}^{x_+=0+} = -A(r)\frac{\partial^2 \phi}{\partial x_-^2}|_{x_+=0} \tag{4.2.8}$$

In the typical scattering picture, we assume a form of incidental gravitational wave in the half space $x_+ < 0$, and determine the scattered wave in the half space $x_+ > 0$ from the jump condition Eq. (4.2.8). The gravitional waves propagating in the bulk of a half space behaves like Eqs. (4.2.4) and (4.2.5). In general, this scattering problem can be solved as follows. We take a Fourier expansion of ϕ on both half spaces $x_+ < 0$ and $x_+ > 0$ and match their values at $x_+ = 0$:

$$\psi(r, t, x_1) = \int \frac{d\omega dk}{(2\pi)^2}(c_+^{(1)}e^{i\frac{L^2\sqrt{\omega^2-k^2}}{r}} + c_+^{(2)}e^{-i\frac{L^2\sqrt{\omega^2-k^2}}{r}})r^{-3/2}e^{ikx_1-i\omega t}, \text{ for } x_+ > 0; \text{ and}$$

$$\psi(r, t, x_1) = \int \frac{d\omega dk}{(2\pi)^2}(c_-^{(1)}e^{i\frac{L^2\sqrt{\omega^2-k^2}}{r}} + c_-^{(2)}e^{-i\frac{L^2\sqrt{\omega^2-k^2}}{r}})r^{-3/2}e^{ikx_1-i\omega t}$$

Substituting these into Eq. (4.2.8), we obtain that

$$\int \frac{d\omega dk}{(2\pi)^2}(\omega + k)[(c_+^{(1)} - c_-^{(1)})e^{i\frac{L^2\sqrt{\omega^2-k^2}}{r}} + (c_+^{(2)} - c_-^{(2)})e^{-i\frac{L^2\sqrt{\omega^2-k^2}}{r}}]e^{i(k+\omega)x_1} =$$

$$-\int \frac{d\omega dk}{(2\pi)^2}(\omega + k)^2\frac{A(r)}{2}[(c_+^{(1)} + c_-^{(1)})e^{i\frac{L^2\sqrt{\omega^2-k^2}}{r}} + (c_+^{(2)} + c_-^{(2)})e^{-i\frac{L^2\sqrt{\omega^2-k^2}}{r}}]e^{i(k+\omega)x_1}$$

Notice that due to the constraint $x_1 + t = 0$, the Fourier factor only depends on $\omega + k$. An important implication of the above equation is that $k - \omega$ may change across the shock wave frontier, though $(k + \omega)$ is still conserved. The goal is to solve c_+ from the above equation in terms of c_-. This can be done by taking the Fourier transform over $1/r$ on both sides, but the Fourier transform of the function $A(r)$, depending on $\tilde{\lambda}$ and the shock wave profile f, can be complicated, so the result can also be cumbersome. One can obtain a better physical picture by using the WKB approximation and thus

44

transforming the wave problem into a geometric problem.

4.3 The Geodesic Picture

In order to derive the WKB approximation, we first need to introduce an effective metric in which we can write Eq. (4.2.2) in a covariant form. The effective metric is

$$ds^2 = \frac{r^2}{L^2}dr^2 - \frac{r^6}{L^6}(dt^2 - dx_1{}^2), \qquad (4.3.1)$$

and we denote the effective metric tensor by $\tilde{g}_{\mu\nu}$. It is easy to verify that Eq. (4.2.2) now can be written as

$$\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\tilde{\nabla}_\nu\phi = 0, \qquad (4.3.2)$$

where $\tilde{\nabla}$ denotes the covariant derivatives in the effective metric.

In the WKB approximation, we write $\phi = \rho e^{i\Theta}$, where ρ and Θ are real, and identify $\tilde{g}^{\mu\nu}\tilde{\nabla}_\mu\Theta$ with dx^ν/ds[3]. Then we have

$$\frac{dt}{ds} = \frac{L^6}{r^6}\omega \qquad (4.3.3)$$

$$\frac{dx_1}{ds} = \frac{L^6}{r^6}k \qquad (4.3.4)$$

in our earlier notation, and ω and k are conserved away from the shock wave frontier. From Eq. (4.3.2) we see that

$$(\frac{dr}{ds})^2 = \frac{L^4}{r^4}(\omega^2 - k^2). \qquad (4.3.5)$$

Note that the right hand side is positive since we have shown that the Fourier components with $\omega^2 - k^2 < 0$ cannot propagate in the bulk of the AdS space.

Now we study the singularity at the shock wave frontier. For this purpose, we write

$$\phi(r, t, x_1) = \int \frac{dx_-}{2\pi}\psi_{k_+,r}(x_+)e^{ik_+x_-}, \qquad (4.3.6)$$

where $k_\pm = k \pm \omega$. Now the jump condition Eq. (4.2.8) can be written as

$$\frac{\partial \psi_{k_+,r}(x_+)}{\partial x_+} = ik_+ A(r)\delta(x_+)\psi_{k_+,r}(x_+), \qquad (4.3.7)$$

where $A(r)$ is defined in Eq. (4.2.7). Now this equation holds for every k_+ and r, so we may regard $\psi_{k_+,r}$ as a function of only one variable, x_+.

Applying the Fourier transform on both sides of Eq. (4.3.7), we obtain that

$$ik_- \hat{\psi}_{k_+,r}(k_-) = ik_+ A(r)\psi_{k_+,r}(x_+ = 0), \qquad (4.3.8)$$

where $\hat{\psi}$ denotes the Fourier transform of ψ. This equation only applies to large k_- since we are focusing on the behavior of $\psi_{k_+,r}$ near $x_+ = 0$. Now the right hand side is a constant, so $\hat{\psi}_{k_+,r}(k_-) \sim 1/k_-$ for large k_-. In mathematics, if a function f over $\mathbb{R}$ satisfies

$$\int dk(1+k^2)^l |\hat{f}(k)|^2 < \infty,$$

then f is said to be an element of the Sobolev space $H_l(\mathbb{R})$. From this definition, we know that $\psi_{k_+,r}$, as a function of x_+, is an element of $H_0(\mathbb{R})$. However, if we define

$$\Psi_{k_+,r}(x_+) = \int^{x_+} d\xi \psi_{k_+,r}(\xi), \qquad (4.3.9)$$

then $\Psi_{k_+,r} \in H_1(\mathbb{R})$. By the Sobolev lemma, elements of $H_1(\mathbb{R})$ represent continuous functions, so $\Psi_{k_+,r}$ is continuous, and $\psi_{k_+,r}$ may have a jump in x_+ at the shock wave frontier, and it does according to Eq. (4.3.7).

Knowing that $\psi_{k_+,r}$ only has a finite jump at $x_+ = 0$, Eq. (4.3.7) is not hard to solve. In fact, we have

$$\psi_{k_+,r}(x_+ = 0+) = \frac{2 + iA(r)k_+}{2 - iA(r)k_+}\psi_{k_+,r}(x_+ = 0-). \qquad (4.3.10)$$

The inverse Fourier transform of the prefactor is

$$\int \frac{dk_+}{2\pi} \frac{2 + iA(r)k_+}{2 - iA(r)k_+} e^{-ik_+ x_-} = -\theta(A(r)x_-)\frac{4}{A(r)}e^{-\frac{2x_-}{A(r)}},$$

where $\theta(\cdot)$ is the step function. The wave function ϕ can thus be written as a convolution:

$$\phi(r, x_-, x_+ = 0+) = -2\pi \int d\xi \theta(A(r)\xi) \frac{4}{A(r)} e^{-\frac{2\xi}{A(r)}} \phi(r, x_- - \xi, x_+ = 0-). \quad (4.3.11)$$

In particular, when $\phi(r, t, x_1 - \xi)$ is a Gaussian wave packet centered at some $z(r)$

$$\phi(r, x_-, x_+ = 0-) = C(r) e^{-\frac{(x_- - z(r))^2}{2\sigma^2}} \quad (4.3.12)$$

for some arbitrary amplitude $C(r)$, the convolution Eq. (4.3.11)

$$\phi(r, x_-, x_+ = 0+) = -\frac{1}{2\pi} \int_0^\infty d\xi \frac{4}{|A(r)|} e^{-\frac{2\xi}{|A(r)|}} e^{-\frac{(x_- - \text{sgn}(A(r))\xi - z(r))^2}{2\sigma^2}}. \quad (4.3.13)$$

The integral is an error function, whose peak cannot be solved in a closed form. However, from the integrand one can see that the center of the wave packet will be shifted, and the direction of the shift depends on the sign of $A(r)$: it shifts to a greater x_- when $A(r)$ is positive and a smaller value when $A(r)$ is negative.

Chapter 5

Conclusion

In this paper, we study the shock wave geometry in the context of the AdS/CFT correspondence, and the propagation of gravitons in this geometry. Though a constraint on the α' correction to the supergravity action is not obtained, several important aspects of the physics are unveiled.

First of all, we find that the shock wave geometry Eq. (3.2.1) is always a solution to the equation of motion of the supergravity theory no matter what α' correction is included in the action, as long as the shock wave profile f is chosen appropriately. Moreover, the equation of f derived from the supergravity action is always a linear partial differential equation of f, though the specific form of the differential operator may depend on the α' correction. Also, when the α' correction is present, the relation between the curvature of the AdS space, measured by L^{-1}, and the cosmological constant may change. Finally, by explicit computation, we show that if the α' correction is of the Gauss-Bonnet form Eq. (3.1.11), then the α' correction does not enter the equation of f.

Secondly, we show in Eq. (4.2.3) that the propagation of the graviton away from the shock wave frontier receives no correction from the Gauss-Bonnet term in the supergravity action, but the scattering between the graviton and the shock wave does depend on the α' correction. The wave function of the graviton is a combination of Bessel functions. In the Fourier space, only components of the graviton field with $\omega > k$ can propagate in the bulk of the AdS space, and its propagation is simple

harmonic in $1/r$ when the graviton is sufficiently far from the boundary, as shown in Eqs. (4.2.4) and (4.2.5). Finally, the scattering with the shock wave shifts the position of the graviton according to Eq. (4.3.13), and changes the "integral of motion" $k - \omega$, which would remain constant without the shock wave.

Bibliography

[1] E. D'Hoker and D. Z. Freedman, arXiv:hep-th/0201253v2.

[2] G. Horowitz and N. Itzhaki, arXiv:hep-th/9901012v2.

[3] M. Brigante, H. Liu, *et al*, arXiv:0802.3318v2.

[4] J. Maldacena, arXiv:hep-th/9711200v3.

[5] J. Maldacena, arXiv:hep-th/0309246v5.

[6] J. Polchinski, arXiv:hep-th/9611050v2.

[7] F. Warner, *Foundations of Differentiable Manifolds and Lie Groups*, GTM94, Springer Press, 1983.

[8] S. Weinberg, *The Quantum Theory of Fields III*, Cambridge University Press, 2000.

[9] E. Witten, arXiv:hep-th/9802150v2.