Progress Towards Strong Coupling between Collisionally Blockaded atoms and an Optical Cavity

by

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Abstract

In this thesis, I present the motivation and design for an experiment in which to probe the interactions between an atom and a single photon light field. I discuss the relevant technical considerations and many of the physical phenomena that can be tested with this particular experimental system. In particular, I dicuss the design, implementation, and control of a high finesse cavity overlapped with a 2μm dipole trap. The use of a relatively long (14mm) cavity grants good optical access, permitting the implementation of a dipole trap centered on the cavity, and small enough to capture only a single atom in the collisional blockade regime.

Thesis Supervisor: Vladan Vuletić
Title: Lester Wolfe Associate Professor of Physics

Acknowledgments

This thesis is the culmination of four years of education at MIT. In the process I have come a very long way, but in everything I did I never had to take a single step alone. I would like to thank Professor Vladan Vuletic, who believed in me enough to allow me to undergo this fantastic experience. My education in atomic physics has been wonderful, and it would not have been possible without the patience and help of so many good people. Marko Cetina, Andrew Griew, Ian Leroux, and Monika Schleier-Smith have all been sources of inspiration in their different approaches to life and science. I would like to thank Jonathan Simon and Haruka Tanji especially as two people who have changed my perspective on every aspect of science and its practice. I have grown immensely under their care, and they have tirelessly helped me develop both as a person and as a scientist.

Additionally, there are my friends outside of lab who have kept me sane and helped me draw my diagrams. Without their words of support and encouragement I am not sure I would have come this far. Finally, without my family none of this would have been worth doing. Their devotion and love keep me faithful to myself.

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Chapter 1

Introduction

Understanding the interaction of light with matter has been a principal focus of physics over the past century. The desire to explore questions of how photons affect matter has given rise to numerous breakthroughs in our understanding of fundamental physics. By exploring and studying phenomena on this level, physicists have been able to revise and extend fundamental theories, resulting in the development of increasingly powerful tools for describing the physical world. An example of this is the experimental work by Willis Lamb and Robert Retherford in measuring the $^2S_{\frac{1}{2}} - ^2P_{\frac{1}{2}}$ splitting in hydrogen that led directly to the formulation of quantum electrodynamics [12]. Experiments in atomic physics have been successful offering supporting evidence for fundamental theories. For example, experiments in atomic physics have been done to experimentally verify the theoretical predictions for the value of the electron g-factor to thirteen decimal places [23]. Perhaps the reason that the investigation of light-matter phenomena has been so fruitful in discovering, verifying, and measuring physical phenomena to such high precision is the simplicity of the physical systems under consideration.

In modern experiments, lasers are used to trap atoms in a hermetically sealed chamber under ultra high vacuum conditions. This setup allows for the study of an atomic system effectively decoupled from the environment. By reducing the temperature to near absolute zero, very weak couplings and small energy splittings can be resolved and employed. The atoms are confined near their motional ground state, which minimizes Doppler broadening. Decreasing inhomogenous broadening as caused by the Doppler effect is especicially important if the atoms are in an optical cavity, where the cavity resonance will be broadened by such energetic atoms. This broadening weakens the coupling between the cavity and the atoms, and makes it more difficult to perform experiments.

By permitting fine control over the internal and external degrees of freedom of atoms, atomic physics has become one of the primary avenues for exploring quantum optics. Anti-bunched light has been generated by single ions and atoms scattering laser beams tuned on resonance [10]. Also, quadrature squeezed light has been generated by atomic ensembles [1]. The existence of light with non-classical properties is another demonstration of the quantized nature of the electromagnetic field. Atomic physics has become, in many ways, a workspace for the modelling of more complicated physical systems. Because of how clean and isolated the experimental system is, information about the state of the system can be aquired in such a way that specific phenomena are clearly visible. That is, the system parameters can be chosen in such a way as to isolate the phenomena, and decouple them from other possible occurrences. Also, schemes can be designed to take advantage of atomic level structure by creating coherence between the levels. Such experiments can be used to study non-linearities in the atom light interactions. One of the most important kinds of experiments that are performed in atomic physics are those that test the basic machinery of quantum mechanics by determining how entangled a system is, and quantifying the degree of entanglement to show that classical limits are violated. Experimental results of correlations violating Bell's inequality continue to provide evidence in support of fundamental quantum mechanical principles, validating the belief in a non-classical world. Creating entanglement between photons is an important step in developing schemes in quantum communications, and creating coherence among atoms in ensembles is necessary for creating efficient quantum memory.

The experiments described above are limited in efficiency by the numerical aperture of the lens used to detect the photons emitted by the atoms. By using a cavity, the effective solid angle of the detection system may be increased to nearly 4π, and in this sense the cavity is like a high numerical aperture lens. Once the atom is trapped and cooled in the cavity, any photon released by the atom will be preferentially scattered into the cavity mode. This is important in improving the efficiency with which photons can be written into and read out of the atom. This feature of the experimental setup is what allows for experiments in quantum optics to show non-linear behavior of the atom field interaction. Without the cavity, collecting photons emitted by the atom would be a far more difficult task, as it would be necessary to detect over the entire 4π steradians of free space. Increasing the state detection efficiency is important to quantum information processing and will be necessary for the implementation of effective error correcting algorithms. The likelihood that the atom will scatter into the cavity mode is given by the cooperativity η. Taking this description from ray optics as the sole interpretation would forgo any discussion of electrodynamics. The real magic of experiments in the study of cavity quantum quantum electrodynamics is in the ability to cleanly observe effects caused by the interaction of atoms with single photons.

Traditionally, cavities were designed very short under the belief that the single quantum Rabi frequency g should be much larger than the cavity linewidth κ or the atom linewidth γ [18]. Under certain conditions, however, the only parameter that needs to be considered is the cooperativity $\eta = \frac{4g^2}{\kappa\gamma}$, and in this case the classical description of the cooperativity is appropriate. Under these conditions a longer cavity can be used without penalty, as long as η is large.

Experiments in atomic physics are renowned for their precision. Hundreds of electronic components must work harmoniously in an effort to control the experimental equipment. Stability in the electronic and optical signals is necessary to an extreme degree. Given the sheer number of components, having noise or instability in a large proportion of them will make running the experiment unfeasible. Thus, all the elements must be constructed to perform in a conservative and steady manner. Consider the lasers, which to have a frequency resolvable by the atom must have a smaller linewidth. The linewidth of cesium is 5.2 MHz, and the lasers are controlled to a linewidth of 1MHz. This level of control is very impressive as for an 852nm beam, its frequency is controlled to 1 part in 10^9. In this thesis, I will give an overview of the physics involved in trapping, cooling, and interacting with atoms in an optical cavity. This high cooperativity cavity is long enough to allow optical access for the use of a magneto optical trap and a dipole trap to confine the atoms. I will examine the technical considerations of building and controlling a system to capture only single atoms, and I will describe some of the important effects and phenomena observed in this regime.

Chapter 2

Trapping and Cooling and Cooperating

2.1 Cooling and Trapping of Atoms

The use of laser cooling and trapping of atoms has become a central part of atomic physics, and is one of the most important tools in the study of light matter interactions. Confining atoms spatially and cooling them to incredibly low temperatures, is a an effective preparation to studying them. By trapping the atoms, it is possible to access them in a precise manner, in such a way that all the theoretical machinery developed in the field of atomic physics is applicable. Experimenting on the atoms under these conditions allows for accurate calculations regarding the behavior of the atoms. Additionally, cooling the atoms is necessary for a successful implementation of any quantum information scheme in order to maintain coherence. In quantum information processing, long coherence times are essential to realize high fidelity writing and reading of information to and from the atoms, which will act as the qubits of the quantum computer. For there to be any hope of attaining the dream of constructing a quantum computer, fields like atomic physics will have to develop technical solutions to these problems. Decoherence occurs when the quantum system becomes entangled with the environment irreversibly. One way this can occur is through collisions with other atoms in the chamber. To minimize the possibility of decoherence by collisions with hot ambient atoms, the experiment is conducted in a vacuum chamber held at 10^{-9} torr. In addition to maximizing the coherence time by reducing interactions with the environment or other atoms, having cold atoms is necessary for successful atom-photon interactions. Cooling the atoms will diminish the doppler broadening when probing or interacting with them, increasing the precision and efficiency of the experiment by suppressing decoherence. In a many atom experiment where collective atomic excitations are being created, movement of the atoms will decohere the collective excitation, limiting the lifetime of the quantum memory [32].

All the calculations of atom-field interaction that make predictions of these physical processes so accurate operate under the dipole approximation. This approximation asserts that as long as the wavelength of the light is much longer than the size of the atom, any field the atom experiences will effectively be independent of spatial coordinates. For cesium with a characteristic radius of 2.65Å immersed in a laser of wavelength 852 nm, the field will change at most by .004 of its maximum value over the width of the cesium atom. Under the assumption that the atom is moving slowly enough that it is always subject to the dipole approximation the atoms will experience only a single time dependent field strength. Motion of the atoms causes inhomogenous broadening of the resonant peak, and must be avoided to maintain efficiency and resolution in the process under consideration. In order to minimize this effect in the experiment, and consequently to work under a set of conditions that allows the experiment to correspond well to theory, the atom's movement is preferred to be be much less than the wavelength of the field it is in.

2.1.1 Optical Molasses

In the simplest picture of laser cooling, a two level atom is in the field of two counterpropogating single frequency laser beams. This picture is accurate as long as the linewidth of the laser is much smaller than the linewidth of the atomic excited state. As long as the two levels differ in their orbital angular momentum quantum number by ± 1, the laser field will couple the two levels of the atom at the Rabi frequency $\Omega = d \cdot E$, where d is the dipole moment of the atom and E is the electric field.

Photons will be absorbed by the atom, and subsequently released by spontaneous emission. In a low intensity field where the Rabi frequency $\Omega \ll \Gamma$ (the linewidth of the atom), the rate of stimulated emission will be much smaller than the rate of spontaneous emission. Suppressing the rate of stimulated emission is important as this process will lead to fluctuating dipole force heating, which can heat than recoil heating. Photons scattered by stimulated emission will always carry momentum in the direction from which they came. Spontaneous emission scatters light randomly, and this process will cool the atoms overall. The rate at which an atom travelling with a velocity $\mathbf{v}$ scatters photons from a laser whose frequency is detuned from the atomic resonance by Δ is given by

$$\gamma_{p\pm} = \frac{\Gamma\,\Omega^2}{\Gamma^2 + 2\Omega^2 + 4(\Delta \pm \mathbf{k}\cdot\mathbf{v})^2} \tag{2.1}$$

The signs corresponds to two counterpropagating beams of equal intensity and detuning, each travelling in opposite directions. $\mathbf{k}\cdot\mathbf{v}$ accounts for the detuning caused by the velocity of the atom with respect to the propogation of the beam. This term represents the Doppler shift the atom sees moving relative to a light field with nominal wavelength $\lambda = \frac{2\pi}{k}$. The total force the atom will feel from the two beams is $\hbar k(\gamma_{p+} - \gamma_{p-})$, and thus Doppler cooling results in the atom experiencing a retarding force. A stationary atom will feel no force from the beams, as it will be equally detuned from both. A moving atom, however, will be less detuned from the beam moving against it, and thus will scatter more light from that beam. For a low intensity field, and for small values of $|kv|$ such that terms of order $|kv|^4$ and higher are neglected, the atom will experience a force

$$\mathbf{F} = \frac{\hbar k \Gamma \Omega^2 \Delta}{(\frac{\Gamma^2}{4} + \Delta^2)^2}\mathbf{k}\cdot\mathbf{v} \tag{2.2}$$

For an atom whose velocity is parallel to the beam, the expression takes the form $F = -\alpha v$, where $\alpha > 0$ when $\Delta < 0$. A retarding force proportional to the velocity is characteristic of the dissipative force felt by a particle moving through a viscous fluid at low Reynolds number. Thus, the term 'optical molasses' is apt in describing this phenomena, as can be seen in Fig. (2-1). An important characteristic of Doppler cooling is that the atom will experience an impulse upon absorption and emission of a photon. Since the scattering will be preferential for the beam going against the movement of the atom, the change in momentum from an absorption will be directed in such a way as to slow the atom. The subsequent emission will be random, and imparts no net momentum. Any scattering event will impart an energy $E_{recoil} = \frac{\hbar^2 k^2}{2m}$ onto the atom from the discretized nature of light. The result of this heating is that Doppler cooling has a lower limit on the temperature to which it can cool the gas. For an atom with momentum P_i, the total power felt by the atom is the sum of a cooling rate and a heating rate

$$\frac{dE}{dt} = P_i \frac{\hbar k}{m}(\gamma_{p+} - \gamma_{p-}) + 2\frac{\hbar^2 k^2}{2m}(\gamma_{p+} + \gamma_{p-}) \tag{2.3}$$

solving for the steady state energy $E_{ss} = \frac{P_i v}{2}$ by setting the expression to 0 yields

$$E_{ss} = \frac{\hbar \Gamma}{8} \frac{1 + (\frac{2\Delta}{\Gamma})^2}{\frac{2\Delta}{\Gamma}} \tag{2.4}$$

This function of Δ has a minimum at $\Delta = \frac{\Gamma}{2}$, for which $E_{ss} = \frac{\hbar\Gamma}{2}$. Setting this equal to $k_B T$ gives a measure of the temperature of the gas. Thus, Doppler cooling can bring the gas to a temperature no lower than $T_D = \frac{\hbar\Gamma}{2k_B}$. An intuitive way to see this is to note that the linewidth of the atom gives a measure of the range of velocities over which it can scatter light, and a wider spread of velocities indicates a higher temperature. An atom with a larger linewidth will scatter light at more frequencies, as the laser will only be able to resolve the atom as well as the linewidth. A higher scattering rate will result in higher heating from the recoil energy, and will result in a higher steady state temperature of the atom. Thus, it is important that the linewidth of the laser is much smaller than the linewidth of the atomic excited state, as otherwise the heating rate would be too high. For cesium, the Doppler limit is 125μK, which is very cold by our daily experiences, but not cold enough for us to reliably perform quantum information processing using the atoms. Additionally, since the atoms are only under the influence of a velocity dependent drag force, they will diffuse and will not be confined spatially in this scheme.

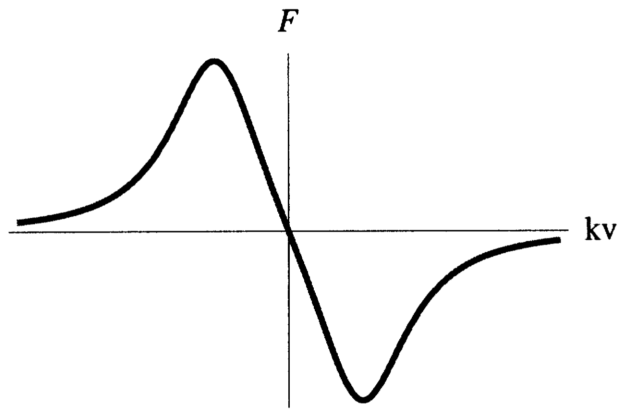

Figure 2-1: The force on a two level atom from two counterpropagating laser beams. Near 0 phase shift, the force is linear in the velocity, similar to the force a particle would experience travelling through a viscous fluid. At high velocities, the light the atom sees will be far detuned from the atomic resonance, and atoms will feel no pressure from the light.

2.1.2 Magneto Optical Trap

In order to confine atoms in position space as well as in momentum space, we must implement a spatially dependent potential. A magneto optical trap will reduce the temperature of the atoms to T_D as well as confine them spatially. The Zeeman effect will split the energies of the magnetic sublevels of an angular momentum state by adding an energy $\mu_B B m_f$, to first order in the strong field limit. By placing the 0 of a linear field at the trap location, atoms on either side of the trap will experience a splitting of the magnetic sublevels but in opposite directions, such that the $m_f > 0$ states will be raised on one side, and the $m_f < 0$ states will be raised on the other, as in Fig. (2-2).

Selection rules dictate that a π polarized dipole field can only couple atomic states in an absorptive transition where $\Delta l = \pm 1$ and $\Delta m_f = 0$, and a σ_+ and σ_- polarized dipole field can only couple states where $\Delta l = \pm 1$ and $\Delta m_f = \pm 1$. The use of optical molasses with π polarized light is insufficient to contain the atoms spatially, as an atom will couple to the counterpropogating beams in the same way independent of its position. $\sigma_{\pm}$ polarized light can be used to circumvent this problem, as σ_{+} light couples to the $\Delta m_f = 1$ transition, and σ_{-} light couples to the $\Delta m_f = -1$ transition. In this scheme, two circularly polarized beams are used to couple the excited states to the ground state, to ensure that the atoms always feels a restoring force as it drifts away from the trap. Said another way, the atoms will preferentially interact with the beam that pushes it towards the center of the trap. By having each of the two counterpropogating beams in the optical molasses be a different helicity of circularly polarized light, it is possible to couple the two beams to different transitions, and then use the magnetic field to have the appropriate transition be on-resonance to force the atoms back to the center. If the frequency of the beams is detuned from the transition, then as the atoms move into a region of positive (negative) magnetic field, the $\Delta m_f = -1$ ($\Delta m_f = +1$) will lower in energy, thus be less detuned, and will thus scatter more than the other transition. This will push the atoms back towards the center of the trap.

2.1.3 Polarization Gradient Cooling

In the early days of laser cooling, researchers were surprised to observe that the atoms cooled by counterpropogating beams reached temperatures as much as 40 times lower than the theoretical Doppler temperature T_D. While it was not originally known what effect was responsible for this result, several theoretical models have been developed to describe the effects of polarization gradient cooling (PGC) in accounting for sub-Doppler cooling. By considering only two levels of the atom interacting with one mode of the light field, obtaining the result that T_D is a lower limit is rigorous. Therefore, it is necessary to consider other states of the atom to explain why sub-Doppler cooling is possible from a simple optical molasses cooling. In PGC two counterpropating beams with orthogonal linear polarization are incident upon the atoms. If the beams have equal intensities then the amplitude of the field the atom sees in the dipole

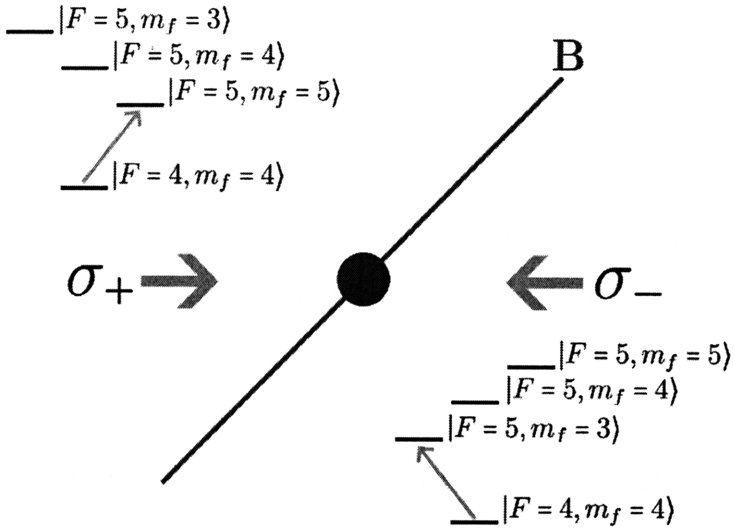

Figure 2-2: This schematic of the operation of a one dimensional MOT shows the linear magnetic field (blue) superimposed on the trapped atom where the field is 0. σ_+ and σ_- light enter from either side of the trap, and the corresponding splitting of the $|F = 5\rangle$ magnetic sublevels is shown. The red arrows show the coupling that takes place on either side of the field trap, pushing the atoms to the center.

approximation is

$$\mathbf{E} = E_o e^{i(\omega t + kz)}\hat{x} + E_o e^{i(\omega t - kz)}\hat{y} = 2E_o e^{i\omega t}(e^{kz}\hat{x} + e^{kz}\hat{y}) \tag{2.5}$$

At $kz = 0$ the phases of the beams will add in such a way as to produce a linearly polarized field of amplitude $\sqrt{2}E_o$ in direction $\hat{v} = \frac{1}{\sqrt{2}}(\hat{x}+\hat{y})$. At $kz = \frac{\pi}{4}$ the field will be circularly polarized with polarization vector $\hat{\sigma_-} = \frac{1}{\sqrt{2}}(\hat{x} - i\hat{y})$, and the polarization will be σ_+ $\frac{\pi}{2}$ later at $kz = \frac{3\pi}{4}$. A linearly polarized field will couple the ground state with excited states of the same magnetic quantum number, while circularly polarized light will couple the ground state most strongly with the $\Delta m_f = \pm 1$ state that matches the angular momentum of the beam. Since the polarization of the light varies spatially and the direction of the circular polarization switches every $\frac{\lambda}{4}$, the field alternates coupling the ground state to one of the two $\Delta m_f = \pm 1$ levels. Additionally, the polarization of the light splits the degeneracy of the ground state magnetic sublevels due to the AC Stark shift.

The dynamics that follow from this light-atom interaction and the choice of the lasers results in the cooling of the atoms. The polarization gradient cooling used in the experiment is designed to work between the magnetic sublevels of the $|F = 4, l = 0\rangle$ and $|F = 5, l = 1\rangle$ states. For an atom that starts at a location where the light is σ_+ polarized, the atoms are pumped into the highest m level of the ground state. The result of a spatially dependent AC stark shift is that as the atom moves through the field the energy of positive Fmagnetic sublevel states will alternate between being above and below the negative magnetic sublevel states. When the atom has travelled a quarter wavelength to a position where the light is σ_- polarized, the effect of the AC Stark shift on the energies of the magnetic sublevels will be maximized. The σ_- component of the light will pump the atom to the lowest magnetic sublevel state. The net result of this process is that the atom has released photons of higher energy than it has absorbed. The net energy must have come from the kinetic energy of the atom, and the atom will consequently slow down. As the atom moves a quarter wavelength further through the field into a region of σ_+ polarization the process occurs in the opposite direction, with the atom pumped into the highest m_f state as in Fig. (2-3). In this process, the kinetic energy of the atom is transformed into potential energy and then lost to the field. This procedure will repeat until the atom does not have sufficient kinetic energy to move an entire quarter wavelength. Thus, the temperature limit of this cooling process is the equivalent kinetic energy that would be necessary to come to the top of the barrier. This energy is the maximum potential energy difference between the magnetic sublevels. Treating the two magnetic levels as a two level system, time dependent perturbation theory will show the splitting between the levels to be $\frac{\hbar\Omega^2}{2\Delta}$ when $\Delta \gg \Omega$. Considering the angular momentum coupling, for $\Delta \gg \Gamma$ the complete expression for the energy difference is

$$\Delta U = \frac{\hbar\Omega^2 C_{ge}^2}{\Delta} \tag{2.6}$$

C_{ge} is the Clebcsh-Gordon coefficient for the coupling between the two angular momentum states. In comparing two possible transitions with different Clebsch-Gordon coefficients, even if one coefficient is only sligtly larger than the other, the cooling rate will be significantly larger because the process will occur an enormous number of times, rapidly multiplying the effect of the larger coupling coefficient. This expression for the difference in energy can be used to define a minimum temperature by setting ΔU to $k_B T$. Naïvely considering ΔU, we could conclude that decreasing the energy difference would result in a lower steady state temperature. However, lowering this barrier will radically decrease the cooling rate. Additionally, if the barrier were dropped below the recoil energy $\frac{\hbar^2 k^2}{2m}$, the cooling process would stop functioning altogether. A potential problem with PGC is that the atoms may fall into the $|F=3, l=0\rangle$ ground state, which is essentially not coupled to $|F=5, l=1\rangle$. If this system is left alone, all the atoms will eventually fall into this state, and the cooling will be completely ineffective. To remedy this problem, a repumper beam is always on to couple $|F=3, l=0\rangle$ to $|F=4, l=1\rangle$, from which it can fall into the $|F=4, l=0\rangle$ ground state and rejoin the PGC process.

2.1.4 Dipole Trap

While the MOT is effective in cooling the atoms to T_D and confining them spatially, PGC is necessary to reduce the atoms to even colder temperatures. Since PGC will not operate in the presence of a magnetic field, it is necessary to deactivate the MOT before cooling further with this scheme. Once the atoms have been further cooled, it will again be necessary to confine them spatially. In general, the confinement of the atom is parametrized by kx_0 where where x_0 is a measure of how far the atom moves from equilibrium. The dipole trap has the capability of confining the atoms to a small volume and reaching the Lamb-Dicke regime where the atom is confined to the limit that $kx_0 \ll 1$. In this limit the atom-field interaction meets all the assumptions of the dipole approximation, and there is little to no Doppler broadening. The dipole force can be understood classically as the polarization of the atom caused by the light field, and the interaction of the resultant dipole with the electric field. Quantum mechanically, a dipole force exists whenever there is spatial gradient of the light field.

The dipole trap will consist of a beam focused through a lens such that it has

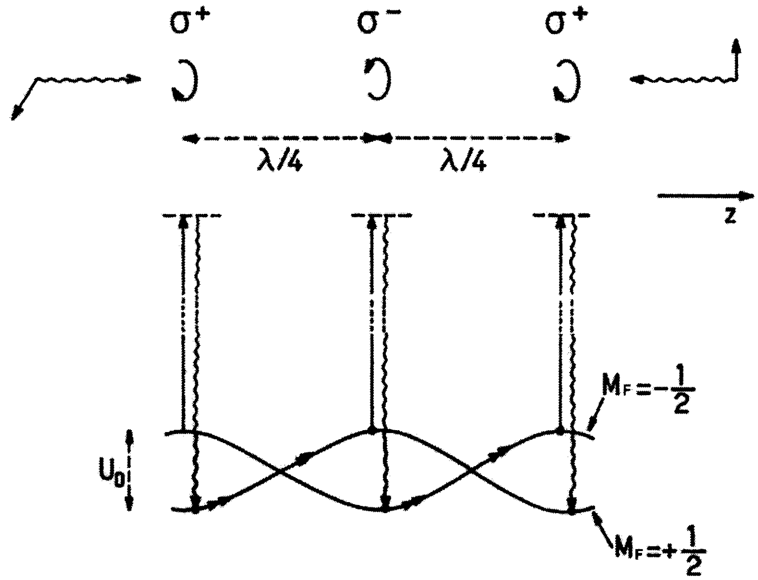

Figure 2-3: An illustration of the spatial dependence of the polarization. The helicity of the circularly polarized light switches every quarter wavelength. The lower diagram illustrates the coupling of the field to the $\Delta m_f = \pm 1$ magnetic sublevels. The strength of the coupling to each magnetic sublevel, and the energy of that sublevel, is dependent on the polarization of the light. U_0 is the energy difference between the magnetic sublevels, and sets the minimum temperature to which PGC can bring the atoms [7].

an extremely narrow waist. The field of a gaussian beam has a radial dependence $E(r) \propto \frac{1}{w(z)} e^{\frac{-r^2}{w(z)^2}}$, where $w(z)$ is the width over which the field drops to $1/e$ of its value at $r = 0$. It is this radial dependence of the field that will create a position dependent potential gradient to confine the atom radially. Additionally, by focusing the beam and sending it through a 2μm waist, the z-dependence of the field provides a weaker longitudinal intensity gradient strong enough to confine the atom within one Rayleigh range $z_R = \frac{\pi w_0^2}{\lambda}$.

Circularly polarized light will create an effective Zeeman shift and decohere the atoms. Therefore, the dipole trap uses linearly polarized light to couple the ground and excited states. Second order perturbation theory gives the result that for a two level system the eigenstates of the coupled system should lower and rise in energy in comparison to the original ground and excited state respectively. The degree to which the energy levels are shifted is dependent on the strength of the coupling, and is given by $\Delta U = \frac{\hbar\Omega(\vec{r})^2}{4\Delta}$. From this expression we see that if the beam were detuned to the red, the energy of the atomic ground state would lower with increased beam intensity, and the opposite would be true if the beam were detuned to the red. With regard to a gaussian beam, the laser should be detuned to the red of the atomic transition. In this case, the stronger the field, the better mixed the states will be, and the lower the energy of the first eigenstate. As the atom moves across the beam, the energy of the first eigenstate will be lowest in the center, and increase radially outwards. The potential energy of the atom will increase away from the center of the field, and this gradient will create a force $-\nabla(\Delta U)$, where ΔU is the energy difference of the state as a function of its position in space. However, since the energy of the higher eigenstate is also varying in the field, the detuning of the transition will effectively change, leading to a broadening of the transition. This will decrease the efficiency of the trap and invalidate the scheme. This issue would be negated if the transition energy between the eigenstates were independent of the strength of the coupling.

Almost miraculously, there is a solution. Performing second order perturbation on the entire collection of energy eigenstates, the ground state will be lowered in energy by the contribution from each term. The excited state, however, will be raised by the contribution from the first term, and will be lowered by the contribution from the rest. If the coupling between the ground and excited state is strong, the perturbed system will look like a two-level system, and the contribution from the other states can effectively be ignored. However, if the field is detuned from the resonance the coupling will be weak and the other states will contribute to the energy shift. In this regime, both eigenstates are lower in energy, and perhaps for a particular detuning, the energy difference between the two eigenstates will be constant regardless of the intensity. It so happens that this 'magic wavelength' does exist for certain atomic species, including cesium, for which the wavelength is 935 nm. Operating at a frequency so far detuned from resonance greatly suppresses the rate of spontaneous emission, and the atom exchanges photons with the field almost entirely through stimulated emission.

Chapter 3

Light-Matter Interaction

The interactions between light and matter in the context of atomic physics are clean enough that simple theoretical calculations can very accurately describe the experiments taking place. The theoretical study of cavity QED brings together many familiar topics from quantum mechanics. Phenomena described by cavity QED often have interpretations in classical physics. The quotidian language of atomic physics interchanges classical and quantum descriptions of the world in order to gain a more complete understanding of natural phenomena. This facet of the study makes it very interesting because the same phenomena can be understood many different ways, and different pictures of the same phenomena lend themselves to different analyses. Having such a broad range of descriptions of the phenomena is necessary to build effective systems, as it provides flexibility to choose the most convenient picture for understanding the physics.

3.1 Interaction of Light with Matter

Elementary quantum mechanics teaches us that the ground and excited states of an atom are eigenstates of a hydrogenic Hamiltonian, and are therefore orthogonal. Since the time evolution of an energy eigenstate is simply the accumulation of phase, it would seem impossible for an atom starting in its ground state to leap into its excited state, or for an atom in the excited state to decay to the ground state. However, the presence of an external field couples the eigenstates, mixing the levels. Thus, the time evolution of an atomic system in the presence of an external field can result in transitions between the eigenstates. Colloquially, we have learned to describe this phenomenon as the 'absorption' and 'emission' of radiation, and comfortably describe this process by the exchange of photons with the light field. Fundamentally, these transitions are allowed to occur because the atomic states are coupled by a time dependent electromagnetic field, causing the state of the atom to coherently oscillate between the levels. This coupling allows atoms to effectively store light that is incident upon them. In the spontaneous emission of a photon, the atomic states are coupled by fluctuations of the vacuum modes.

Many experiments in atomic physics requires the use of lasers to cool, trap, and read/write information from the atoms. How well a laser is capable of performing these tasks depends on the matrix element of the Hamiltonian coupling the states under consideration. In the simplest picture an atom can be considered a dipole, and therefore has an energy described by the dipole operator $\hat{d} = q\vec{r}$. The coupling Hamiltonian is the classical dipole energy $\hat{d}\cdot\vec{E}(\vec{r},t)$ for a varying electric field $\vec{E}(\vec{r},t)$. The matrix element of this operator $\langle i|\,\hat{d}\cdot\vec{E}(\vec{r},t)\,|f\rangle$ gives a measure of the strength of the dipole transition between the states $|i\rangle$ and $|f\rangle$. The value of this matrix depends only on f and m_f through matrix elements of representations of SU(2), and is closely related to the Clebsch-Gordon coefficients by the Wigner-Eckart theorem.

The quantization axis is used to define a reference frame in which to consider the polarization of the light with respect to the atom. The actual choice for the quantization axis is completely arbitrary, but some choices may make calculations much easier. Any polarization of light can be described as linear combinations of three polarizations with respect to the quantization axis. π polarized light is linear polarized in the direction of the quantization axis, σ_+ is circularly polarized light spinning in a positive sense with respect to the quantization axis, and σ_- spins in the opposite direction. Moving between different choices for the quantization axis can be accomplished by a change of basis of the atomic states. The strength of the coupling is an extremely important parameter in an atomic physics experiment, as the ability for an atom to hold a photon is the basis for all experiments in atomic physics. The presence of the cavity enhances the coupling between the atom and the light field, and is parametrized by the cooperativity η.

3.2 Two Level Atom in a Cavity

There are two formalisms when considering the interaction of light with individual atoms. The semi-classical interpretation views the atom as a quantum mechanical object in a classical field, and the full quantized picture considers the quantum nature of light. Time dependent perturbation theory is used to derive the coupling between the atom and the field, essentially by providing a description of which atomic state is occupied. Consideration of two atomic levels and a single mode of the light field can give important insight into many physical phenomena, eg Rabi flopping and quantum beats. In the case that the atom is treated like a two level system, the problem is reduced to solving the following set of coupled differential equations, presented in matrix form as $i\hbar\frac{d\psi}{dt} = \hat{H}\psi$ where $|\psi\rangle$ is a spinor whose components give the probability of the atom being in either the excited state or the ground state.

$$i\hbar\frac{d}{dt}\begin{pmatrix} C_e \\ C_g \end{pmatrix} = \begin{pmatrix} 0 & \Omega e^{i\Delta t} \\ \Omega e^{-i\Delta t} & 0 \end{pmatrix}\begin{pmatrix} C_e \\ C_g \end{pmatrix} \tag{3.1}$$

Ω is the magnitude of the coupling between the levels and is given by $|\mathscr{D} \cdot E|$ where $\mathscr{D}$ is the non-zero matrix element of the dipole operator $q\vec{r}$, and Δ is the detuning of the laser from the atomic resonance frequency. The resulting set of coupled differential equations can be solved exactly through straightforward means. The solution yields Rabi flopping and the AC Stark shift. Additionally, the density matrix can be introduced to account for the incoherent coupling of the two state system to the continuum of vacuum modes. The resulting first order dynamics can be solved to give the scattering rate of the atom into free space, which is a vital component in understanding and implementing laser cooling schemes.

There are phenomena that cannot be explained by a semi-classical theory can require a quantum mechanical description of light to be understood. Quantum electrodynamics (QED) gives a quantized description of electromagnetic radiation, giving rise to the notion of photons. In performing calculations in QED it is necessary to specify a quantization volume in which to describe the available modes of the electromagnetic field. Cavity QED provides an experimentally verifiable way of observing the quantized electromagnetic field.

In cavity QED the behavior of few photons contained in optical cavities is described and predicted. The spatial wavefunctions of the photons are given by linear combinations of the classical cavity modes. In this way, the spatial occupation of the quantum mechanical field is given by the Laguerre-Gaussian modes from classical optics. Since the cavity is not all-encompassing, the atom still couples to the freespace modes. This allows for an excited atom to emit a photon into free space where it will be lost, or into the cavity where it can be recorded by a photodiode with some efficiency. As desired from any quantum theory, QED will give the correct classical description of the atom-light interaction in the limit of high field intensity. In this regime, the signal to noise ratio of the light field increases to the point that individual photons cannot be seen. As desired from any quantum theory, QED will give the correct classical description of the atom-light interaction in the limit of high field intensity. In this regime, the signal to noise ratio of the light field increases to the point that individual photons cannot be seen.

The atom-photon interaction in the cavity is modeled by considering a two state system under the Jaynes-Cummings Hamiltonian. These states take into account the ability for the cavity to hold a number of photons, and for one of those photons to be absorbed by the atom. Including a third state, the ground state, allows for spontaneous emission of the atom, or the possibility of the photon leaking out of the cavity. This formalism does not allow for two photon interactions or other higher order processes. In many systems of interest to us those occurrences are so well suppressed as to warrant their being ignored. In general, the Jaynes-Cummings Hamiltonian has the form $\hat{H} = \hat{H}_{field} + \hat{H}_{atom} + \hat{H}_{int}$. The three terms in this Hamiltonian take into account the energy of the photons in the cavity, the energy of the atoms, and the energy in the interaction between the atom and the field. For a two level atom where the levels are separated by an energy $\hbar\omega$ interacting with a single cavity mode of frequency ν the Hamiltonian is

$$\hat{H}_{JC} = \hbar\nu a^\dagger a + \frac{\hbar\omega}{2}\sigma_z + \hbar g(a\sigma_+ + a^\dagger\sigma_-) \tag{3.2}$$

where the rotating wave approximation has been used to ignore 'energy non-conserving' terms. Quite intuitively, the eigenstates of the Hamiltonian without the interaction term are $|n, e\rangle$ and $|n+1, g\rangle$ which correspond to the atom in the ground state with $n+1$ photons in the cavity, and the atom in the excited state having absorbed a photon with n photons in the cavity. The energies of these states are $(n+1)\hbar\nu - \frac{1}{2}\hbar\omega$ and $n\hbar\nu + \frac{1}{2}\hbar\omega$, and thus are separated by an energy $\hbar(\omega - \nu) = \hbar\Delta$. g is the coupling between these states and is given by $\mathscr{D}\sqrt{\frac{\omega}{\hbar V\epsilon_0}}$ for a cavity volume V. This expression is equivalent to the coupling in the semi-classical model for a single photon in the cavity. It seems natural to give the matrix elements of the Hamiltonian in the uncoupled basis, and the Hamiltonian can be represented as

$$\hat{H}_{JC} = \begin{pmatrix} n\hbar\nu + \frac{1}{2}\hbar\omega & g\sqrt{n+1} \\ g\sqrt{n+1} & (n+1)\hbar\nu - \frac{1}{2}\hbar\omega \end{pmatrix} \tag{3.3}$$

The eigenvalues of this Hamiltonian are $(n+\frac{1}{2})\hbar\nu \pm \frac{1}{2}\hbar\Omega_n$, and correspond to the so called 'dressed states'. $\Omega_n = \sqrt{\Delta^2 + 4g^2(n+1)}$ is the Rabi frequency of the system and gives the rate of oscillation between the two uncoupled states. These states are the eigenstates of the system that are linear combinations of the original uncoupled states, and are lowered and raised in energy. The energy difference between the two states is $\hbar\Omega_n$.

Being a two level system, the Jaynes-Cummings model gives results exactly analogous to a spin $\frac{1}{2}$ particle in a magnetic field. Just like NMR, in which pulses of magnetic field can be used to precisely orient the spin of the hydrogen nucleus, laser pulses which couple the two levels for precise time periods [9] can move the atom from the excited state to the ground state, or one ground state to another, and back in a controllable manner. Under this premise a quantum memory scheme where information being carried by a photon needs to be coherently stored can be constructed. This scheme can gain additional complexity by considering the polarziation of the photon which will only couple states in such a way as to conserve angular momentum. This allows for the storage of a polarization state, which is another means of storing information. It turns out that the process of storing and retrieving a polarization state can be made much more efficient if the atomic excitation is written onto an ensemble of atoms. This distributed excitation is called a magnon and is an important advancement towards the realization of a usable quantum memory.

3.3 Cooperativity

The previous discussion on two level systems implies that the atom-cavity coupling is perfect. This would be true if the cavity were made of perfectly reflective walls and occupied all 4π steradians of free space. However, the photon has the ability to leak out of the cavity through the mirrors, or spontaneously emit from the atom into free space. Within the context of the two level system, these processes are non-conservative and cannot be described by a hermitian matrix. However, they can be included phenomenologically in the optical Bloch equations. The coupling between the atom and the cavity is parametrized by the single atom cooperativity, η, and is perhaps the most important parameter in engineering an atomic system. Effectively, the cooperativity is a measure of the probability of an atom scattering light into the cavity mode rather than into free space. In practice, the cooperativity measures the efficiency of the quantum process and determines how the system can be used. In essence, this is like setting up a large numerical aperture lens to focus the light at the center of the cavity and then collect much of the light emitted from that point. A precise expression for this probability is

$$P_{cavitymode} = \frac{\eta}{\eta + 1} \tag{3.4}$$

The cooperativity determines the collection efficiency and gives the probability that the photon will exit the cavity and thus be detected. Classically, η is the proportion of 4π steradians the cavity mode covers, multiplied by the finesse. The finesse is a measure of the number of roundtrips the photons in the cavity mode travel before leaking out. For an atom in the antinode of the light field in a cavity, the cooperativity can be expressed as

$$\eta = \frac{24\mathcal{F}}{\pi(kw_0)^2} \tag{3.5}$$

where k is the wavenumber of the light in the cavity, and w_0 is the waist of the cavity mode. w_0 is determined by the geometry of the optical resonator and the wavelength of the light. For a gaussian beam whose far field profile is linear with a waist $w = \frac{w_0}{z_R}z$, for $\frac{w_0}{z_R} \ll 1$, the fraction of solid angle the beam occupies is $\frac{1}{(kw_0)^2}$. It is clear to see that η is really a measure of the solid angle the cavity mode occupies. It would seem natural to desire a large value for η, in order to build a highly efficient system. However, the ways in which η can be adjusted are surprisingly limited. k cannot be changed as it is set to an atomic transition. w_0 can be decreased by changing the cavity geometry, for example by moving the mirrors closer together. However, the mirror separation affects the stability of the cavity, and the distance must be chosen such that coupling to a single cavity mode is much easier. The only parameter left is the finesse $\mathcal{F}$, which can be increased by using mirrors of greater reflectivity. As with all things, there is an exchange, and improving η by increasing the finesse also increases the sensitivity of the stable cavity mode to perturbations in the alignment. Operating in such a regime requires very careful control of the experimental parameters. Not surprisingly, there is a quantum mechanical description of the cooperativity that gives the same form for the probability of scattering into cavity mode. Consider a three level system composed of the empty cavity and the atom in the ground state $|g,0\rangle$, the empty cavity and the atom in the excited state $|e,0\rangle$, and the cavity filled with one photon with the atom in the ground state $|g,1\rangle$. The two excited states are coupled to the ground state by rates Γ and $\frac{4g^2}{\kappa}$ respectively, as in Fig. (3-1). These rates correspond physically to the processes of the atom spontaneously emitting the photon into free space, and the photon coupling to the cavity and the photon subsequently leaking out of the cavity. These rates correspond to a total probability of leaking out of the cavity

$$P_{cavitymode} = \frac{\frac{4g^2}{\kappa}}{\frac{4g^2}{\kappa} + \Gamma} = \frac{\frac{4g^2}{\kappa\Gamma}}{\frac{4g^2}{\kappa\Gamma} + 1} \tag{3.6}$$

By comparing eq.(3.4) with eq.(3.6) we can make the association $\eta = \frac{4g^2}{\kappa\Gamma}$. This is for the case of a cavity tuned to the atomic resonance. Should the cavity be detuned, the cooperativity will be suppressed by the square of the detuning. Controlling the cavity length is very important in this regard. Despite this quantum mechanical picture of the cooperativity, replacing g, κ, and Γ with their known values, we find that the expression reduces to

$$\eta = \frac{4g^2}{\kappa\Gamma} = \frac{\mathscr{D}^2(\frac{2\omega}{\hbar V \epsilon_0})}{\kappa \frac{1}{4\pi\epsilon_0}\frac{4\omega^3\mathscr{D}^2}{3\hbar c^3}} = \frac{6\pi c^3}{\kappa V \omega^2} = \frac{12\pi L\mathscr{F}}{Vk^2} = \frac{12\mathscr{F}}{(kw_0)^2} \tag{3.7}$$

Where we have used the expression for the free spectral range, and that the linewidth κ is the ratio of the free spectral range to the finesse. The most important thing to note about this expression is the absence of any quantity pertaining to the atom, and η is identical to its classical description up to a numerical factor. The naïve expression fo η seems to be dependent on the nature of the atom, in that g couples the atom to the cavity and Γ is the natural linewidth of the atom. However, the atom dependent terms cancel out leaving an expression dependent only on the geometry and composition of the cavity, and the frequency of the laser. Given a system with linewidths κ and Γ, it would seem as though the coherence of a quantum information process were limited by the larger of the two, as the loss rate through the cavity or to free space is $\frac{g}{\kappa}$ or $\frac{g}{\Gamma}$ respectively. A large linewidth corresponds to a small lifetime in that state, and will result in a rapid decay of the system to the ground state. The larger linewidth will determine the rate at which the system decays since the two decay processes essentially occur independently. The cooperativity can be increased by either improving the finesse of cavity with better mirrors, or decreasing the waist of the cavity mode. It is very important to recognize that the cooperativity is fundamentally

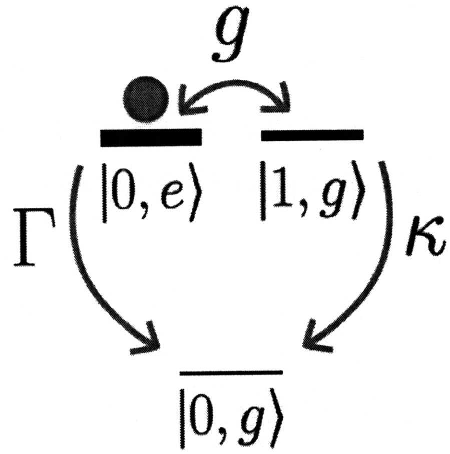

Figure 3-1: This level diagram of the atom in the cavity shows η as the ratio of the scattering rate out of the cavity to the total scattering rate. The excited states have linewidths Γ and κ which determines how quickly they fall to the ground state. The are coupled by g, and for an excited atom, the rate at which photons will be scattered into the cavity is $\frac{\eta}{1+\eta}$.

a classical quantity. It would seem possible to increase the cooperativity by increasing the length of the cavity, thereby increasing the linewidth of the cavity, but this effect will cancel since both κ and g^2 scale like $\frac{1}{L}$. Similarly, it would seem that decreasing the atomic linewidth would also increase the cooperativity, but again, this effect cancels since both g^2 and Γ scale like $\mathscr{D}^2$. interactions between light and atoms in the cavity. In essence, η is a measure of how many 'opportunities' the light has to interact with the atom trapped in the cavity in the time before it leaks out. The strong coupling between the atom and the cavity allows for easier observation of a few atoms, or the effect of the quantum nature of light. Increasing the value of η is especially important in trying to observe single atoms.For the case of an ensemble of atoms, the cooperativity is enhanced by the number of atoms in the collection. For a large ensemble of N atoms under certain circumstances the cooperativity will be enhanced by a factor of N. Thus, a system can have a high optical depth $N\eta$ even for relatively poor single atom cooperativity. This collective effect allows for the implementation of a high cooperativity system without the difficulty of developing the extreme precision and control necessary for a cavity with a high single atom cooperativity [32]. The number of cases where the process really scales like $N\eta$ are relatively few, and ultimately, the single atom coupling proves to be the most important quantity. In general, quantum information processing schemes scale like η and in this experiment we seek to explore the phenomena that are observable in the strong coupling regime.

3.4 Three Level System

Consider a two level atomic system as described before. In the dressed atom picture, the degenerate states $|e,0\rangle$ and $|g,0\rangle$ in a light field are split into linear combinations of the two undressed states, $|e,0\rangle+|g,0\rangle$ and $|e,0\rangle-|g,0\rangle$, up to a phase. These states, which have an equal probability of being in either of the two undressed states, each have an effective linewidth $\frac{\Gamma+\kappa}{2}$. Thus, it was long believed that an effective quantum system, one in which phenomena related to the cavity coupling were observable, would have to be designed such that $\frac{g}{\Gamma}$ and $\frac{g}{\kappa}$ were independently much larger than 1. However, it has been shown that many schemes limited in this a way can be redesigned such that the only parameter of import is $\eta = \frac{4g^2}{\kappa\Gamma}$.

In this experiment we aim to show that the relevant coupling really scales with η. In this setup, the three relevant states are two hyperfine ground states, one with a photon in the cavity, and one excited state. Three quantum states in either a 'ladder', a 'Λ' or a 'V' configuration can be used to explore some of the most important properties of coherent time evolution in three level systems. Most often in experiments the latter two are used. Coupling the levels of a two level system will cause the system to oscillate in time between the two states. This Rabi flopping transfers the atomic population between the ground and excited states with a frequency Ω. An important way to understand this is to consider that each atomic state accrues phase differently over time. The resulting interference changes the probability that the system is in either state over time. Similarly, coupling the three states correctly can allow for the accumulation of phase in such a way that the states destructively interfere as to eliminate atomic population entirely in some states. Such states are known as 'dark states', and are eigenstates of the Hamiltonian that have no contribution from one of the three original states. Not that this is only true in the large η limit.

The ability to create dark states gives rise to a number of quantum mechanical properties in materials including electromagnetically induced transparency, where the atoms are optically pumped to a dark state, which will not respond to in time Such configurations are often used to demonstrate adiabatic state transfer of a coherent atomic population. The ability to reversibly move a single atom or collection of atoms from one atomic state to another is an important facet in the design of quantum information processing or in testing the relative coherence of atomic populations. The simplest three level system is the atomic two level system with the cavity mode. Laser operations on this system can adiabatically transfer the state of the system between $|1, g\rangle$, $|0, g\rangle$, and $|0, e\rangle$. A photon can be reversibly written into the atom, and then read out at a later time. This implies the construction of a very simple quantum memory. The fidelity of this process depends on the coherence time of the atom and the cooperativity.

Chapter 4

Optical Resonators

4.1 A Standing Wave Cavity

In our experiment, a symmetric cavity, where the mirrors have the same radii of curvature, is used to enhance the coupling between the electromagnetic field and the trapped atoms. The simplest optical cavity consists of two coaxial parabolic mirrors of very high reflectivity. Analogously to a mechanical resonator that must be driven through by the periodic application of force, an optical resonator must be driven by a laser field. The laser couples into the cavity through one of the mirrors and creates a steady state field inside the cavity. Also, like a mechanical oscillator, the cavity will possess a resonant frequency such that a half integer number of wavelengths of light fits between the two mirrors. This is to match the boundary conditions at the highly reflective surfaces. The longitudinal modes of the cavity are separated by the free spectral range $\mathcal{FSR} = \frac{c}{2L}$ where c is the speed of light in the cavity and L is the length of the cavity. The cavity is designed such that one of its mirrors is affixed to a piezo electric actuator to alter the length of the cavity. The cavity does not have to be designed with micron precision because the actuator makes up for any additional length. Solving the paraxial wave equation shows that the light field inside the cavity can also have transverse modes in additon to the longitudinal modes. These modes each have a characteristic transverse profile and can be well described by the familar Laguerre polynomials. Additionally, each transverse mode has a phase shift associated with it. The (gouy) phase shift separates the frequencies of otherwise degenerate transverse modes. A confocal cavity is composed of two mirrors with the same radius of curvature, and has the property that all the transverse modes will overlap at the same frequency. This is a very difficult regime in which to conduct experiments, as it is impossible to know what the transverse field strength of the beam inside the cavity is, and thus we can no longer treat the cavity mode as a single spatial mode. Rather, cavities are built shorter or longer than the confocal limit such that the transverse modes are individually resolvable in frequency. As long as the beam coupling into the cavity has the correct profile, it is possible to make the beam couple only to the lowest order transverse mode in the cavity, which is the TEM_{00} mode. Finding the waist of the cavity is simply a matter of finding the eigenvalues of the ABCD matrix describing the roundtrip propagation of the beam from the waist. The matrices represent the free space propagation of the beam and its reflection from the mirrors. The overall ABCD matrix for the case of two coaxial mirrors with the same radius of curvature is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \mathbf{FS}\left(\frac{d}{2}\right) \cdot \mathbf{S}(R) \cdot \mathbf{FS}(d) \cdot \mathbf{S}(R) \cdot \mathbf{FS}\left(\frac{d}{2}\right) \tag{4.1}$$

where d is the separation between the faces of the two mirrors. The waist of the cavity as a function of the mirror separation distance gives a measure of the stability of the cavity. If the mirrors are farther than the concentric limit where the mirrors are separated by two radii of curvature, the cavity will be incapable of supporting a stable mode, and all the light that enters the cavity will leak out. An expression for the waist is

$$w = \sqrt{B\frac{\lambda}{\pi}\sqrt{\frac{1}{1-m^2}}} \tag{4.2}$$

where m is half the trace of the matrix. Fig. (4-1) shows the waist of a cavity built from two 1 cm radius of curvature mirrors as a function of the mirror separation distance d. The transmission of light through a two mirror cavity is a function of the wavelength of the light sent into the cavity. A laser frequency resonant with

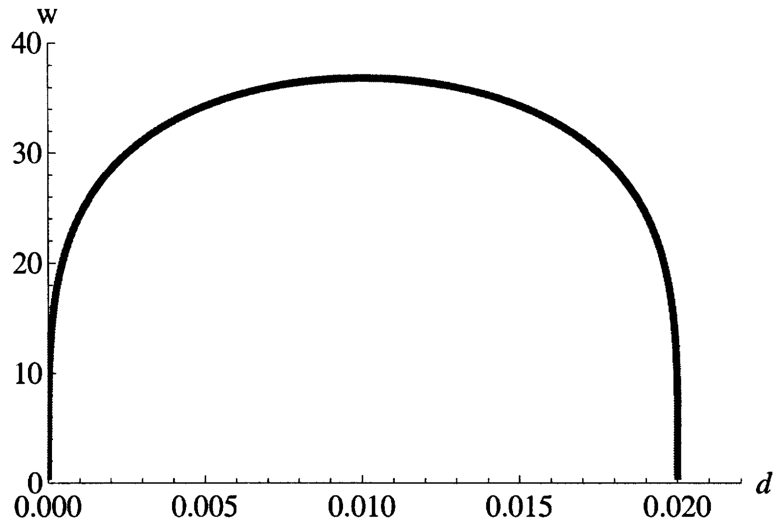

Figure 4-1: The waist of the beam in microns as a function of the mirror separation distance in meters. There is a peak in the waist size at the confocal limit(d = .01 m), and stability is lost entirely at the concentric limit (d = .02m). The experimental cavity follows this curve and was chosen to be separated 14mm, resulting in a waist size of 35.5μm.

the cavity is necessary to set up a strong steady state field in the cavity, which is the frequency of light at which the transmission through the cavity is maximized. The field transmission through a two mirror optical resonator as a function of the wavenumber of the light k and the roundtrip path L is given by

$$F = \frac{t^2 e^{ik\frac{L}{2}}}{1 - r^2 e^{ikL}} \tag{4.3}$$

where r and t are the reflection and transmission coefficients for the two mirrors respectively. Assuming the cavity is lossless, near resonance the expression for the transmission will have a Lorentzian lineshape. This curve has a maximum at the cavity resonance.

$$|F|^2 \approx \frac{1}{1 + \left(\frac{\nu_0 - \nu}{\kappa}\right)^2} \tag{4.4}$$

Where we have used the free spectral range $\mathcal{FSR} = \frac{c}{2L}$, and the finesse is $\frac{\pi r^2}{t^2}$ where r and t are the amplitude reflection and transmission coefficients for the mirrors. The Lorentzian has a linewidth κ related to the other parameters as the finesse divided by the free spectral range. Formally, the finesse is 2π divided by the round trip losses, and is a measure of the number of reflections a beam of light will make before it leaks out of the cavity. As an analogy to mechanical oscillators, the finnesse is like the quality factor, measuring how well coupled the oscillator is to the environment, and thus how quickly energy is lost from the resonant mode.

4.2 Running Wave Cavity

A running wave cavity is constructed by using three or more mirrors to project the beam along a non-overlapping closed path. The running wave cavity is different than a standing wave cavity in that the atoms only see travelling waves in one direction, and because there is no standing wave there is no spatial variation of intensity. This is important in the design of the cavity because it is not necessary to control the location of atoms as well as in a standing wave cavity. Normally, confining atoms to within half a cavity wavelength is a difficult task. The running wave cavity would obviate this condition, leading to a more robust experiment. Also, increasing the overall length of the cavity decreases the waist of the cavity mode, increasing the cooperativity. The analysis of such a cavity is very similar to that of a two mirror resonator, and simply requires computing the proper ABCD matrix. In the particular as shown in Fig. (4-2), the running wave cavity is composed of four mirrors in a bowtie configuration. Two of the mirrors are curved to focus the light into a tight waist, and the other two are flat. The flat mirrors could be replaced by convex mirrors to create an even smaller waist. The ABCD matrix for this arrangement is

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \mathbf{FS}\left(\frac{d}{2}\right) \cdot \mathbf{S}(R) \cdot \mathbf{FS}(L_2) \cdot \mathbf{I} \cdot \mathbf{FS}(L_1) \cdot \mathbf{I} \cdot \mathbf{FS}(L_2) \cdot \mathbf{S}(R) \cdot \mathbf{FS}\left(\frac{d}{2}\right) \quad (4.5)$$

where d is the distance between the two curved mirrors, and L_1 and L_2 are the lengths of the cavity arms.

The four mirror resonator has an added complication, however. When a beam reflects off of a curved mirror, the sagittal and tangential components of the beam reflect as though they are each reflecting off of mirrors with a slightly different radii of curvature. As a result, two different stability curves must be considered, and the only acceptable values of d are those that give rise to a mode in both polarizations of the light, as in Fig. (4-3). Ultimately, the standing wave cavity was chosen over the

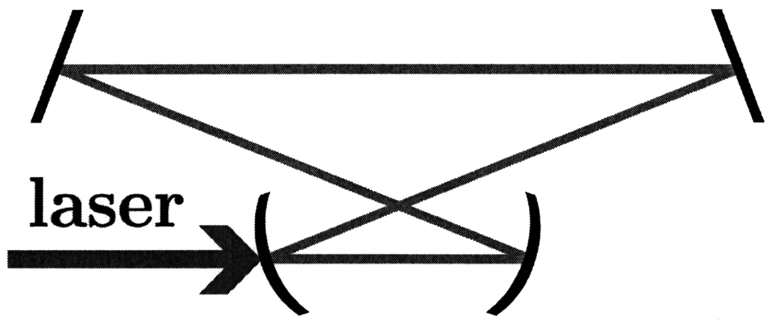

Figure 4-2: A simple schematic of a bowtie cavity. The laser couples into the cavity through one of the curved mirrors and completes a closed path travelling in one direction.

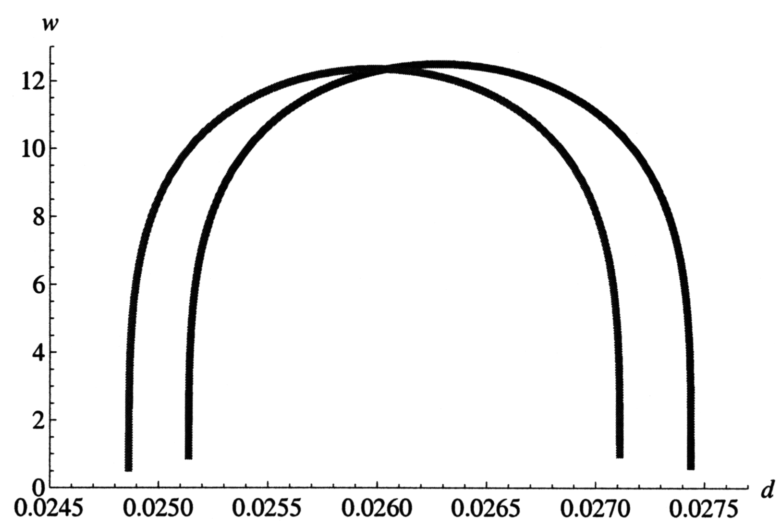

Figure 4-3: The waist of the beam in microns as a function of the mirror separation distance in meters. The fact that sagittal and tangential components of the beam reflect differently from the curved mirrors results in the existence of two separate stability curves that must both be satisfied for the existence of a stable mode.

running wave cavity in order to attain better experimental statistics and running the experiment quicker. A longer cavity has a smaller linewidth κ and thus photons leak out of the cavity at a slower rate. This decreased rate of photon collection leads to a slower rate of data collection overall, meaning that the experiment statistics will suffer. Even though the use of a running wave cavity will allow for easier trapping of the atoms, decoherence occurs faster than the photon leakage rate, making experiments that require coherence unfeasible.

Chapter 5

Trapping and Detection of a Single Atom

From our understanding of the theoretical and practical considerations of cooling and trapping atoms, we will consider some of the technical considerations important to build and operate a system where light will interact with one or two trapped atoms. For the case of a single atom, special consideration of the loading and loss rates of atoms into the trap must be made to reach the collisional blockade regime, where the probability of trapping one atom is much higher than the probability of trapping a greater number of atoms. Once atoms have been trapped in the cavity they will be detected either by seeing a scattered signal through the trap lenses, or through the cavity mirrors. The detection signal will provide information that can be used to determine the number of atoms in the cavity, as well as simply whether or not they are there.

If the center of the cavity overlaps with the center of the trap, it is possible to detect the atom by probing the cavity and observing the vacuum Rabi splitting. Ordinarily, the transmission through the cavity follows a Lorentzian distribution, but the presence of the atoms in the cavity splits the peak, and two separate peaks can be seen on either side of the resonance frequency. This effect was originally thought to be an important signifier of the validiy of cavity QED. As a quantum mechanical effect, the coupling of the atom to the cavity breaks the degeneracy of the two level system. Diagonalizing the Hamiltonian gives eigenstates, called dressed states, that are linear combinations of the photon in a cavity mode and the atom in the ground state, and the atom having absorbed the photon removing it from the cavity. The dressed states are raised and lowered in energy by $\sqrt{N}g$ for the case of a cavity tuned to the atomic resonance and N atoms trapped in the cavity. However, the Rabi splitting can also be understood as a classical effect in linear dispersion theory [35]. In this case the atoms are treated as classical oscillators, and on resonance scatter the light out of the cavity. The collection of atoms adds a phase shift to the light, such that the laser is resonant with the cavity at two frequencies outside of the original resonance, creating the vacuum rabi splitting. The appearance of these peaks in the transmission lineshape of the cavity marks the trapping of an atom, and their separation by $2\sqrt{N}\Omega_n$ indicates the number of atoms N.

The presence of the atoms can also be detected by the the high numerical aperture lens used for the dipole trap. If a probe beam resonant with the atomic transition is applied to the atoms, they will scatter at a rate $\frac{\Gamma}{2}$ in the strong field limit, and this scattered light can be collected through the trap lens, and detected by a photodiode. Assuming the atoms scatter light with a monopole radiation pattern $\frac{e^{ikR}}{R}$, the proportion of light captured by the trap is the overlap between the radiation pattern and the mode through the lens. Assuming the trap mode is axisymmetric around the z-axis, and the opening angle is very small such that $\frac{w_0}{z_R} \ll 1$, then $w \approx \frac{w_0}{z_R} z$ and $R = \sqrt{z^2 + r^2} \approx z$, and the overlap integral is

$$\frac{\sqrt{2}e^{ikz}}{wz} \int re^{-\frac{r^2}{w^2}} dr \approx \frac{2}{(kw_0)^2} \tag{5.1}$$

However, the atom will have a dipole radiation pattern $\propto \cos(\theta)$, and for a dipole oscillator oriented along $\hat{x}$ or $\hat{y}$, the energy will be concentrated along $\hat{z}$ such that the rate of emission into the trap will be $\frac{3}{2}$ as high. Since the radiation is randomly oriented, only 2 of the 3 modes will radiate along $\hat{z}$, so the above expression is correct. Including the quantum efficiency of the photodiode, the loss from interference filters, the loss from fiber coupling efficiency, the effect of beam splitter, the proportion of free space the detection mode occupies, and the scattering rate $\frac{\Gamma}{2}$, respectively, the detection rate for a single atom is

$$R = .4 \times .8 \times .8 \times \frac{1}{2} \times \frac{2}{(kw_0)^2} \times \frac{2\pi \times 5.2 \times 10^6 s^{-1}}{2} \approx 16\text{kHz} \tag{5.2}$$

Assuming a more realistic scattering rate of $\frac{\Gamma}{10}$, we would expect a 4 kHz scattering rate. The photodiode will effectively average the signal over a time T, resulting in a signal that is Poisson distributed with mean $N\langle RT\rangle$ for N atoms in the cavity. If T is such that $\langle RT\rangle \simeq \mathcal{O}(1)$, the photodiode will be unable to distinguish between any number of atoms, as the Poisson distributions will overlap. T must be large enough to separate the distributions, so that the number of atoms is clearly observable. If the scattering rate is 4 kHz, 10ms bins would give $\approx$ 40 counts per atom, more than enough to distinguish 1 atom from two. By contrast, using a cavity would provide approximately 500kHz of counts per atom, and so a much shorter time bin would be allowed to detect a single atom. This calculation is done by adjusting the cooperativity based on the S, P, and I. For a cavity constructed to have a maximum cooperativity of 10, the effective cooperativity is $\eta \times \frac{1}{2} \times \frac{2}{3} \times \frac{1}{3} \times .41 \approx .45$, from averaging over the cavity standing wave, averaging over polarizations, imperfect locking (rectifiable), and Doppler broadening from atoms at 100μK, respectively. As before, the total scattering rate must modify $\frac{\Gamma}{2}$ by considering, in addition to the cooperativity, the quantum efficiency of the photodiode, the fiber coupling efficiency, the fact that light is only collected from one of the two mirrors, and the effect of the interference filter, and finite losses on the cavity mirrors, respectively

$$R = .4 \times .8 \times \frac{1}{2} \times .7 \times .8 \frac{\eta}{1+\eta} \times \frac{2\pi \times 5.2 \times 10^6 s^{-1}}{2} \approx 450\text{kHz} \tag{5.3}$$

It is interesting to note that even though η has been reduced by a factor or 20, the atom cavity coupling is really $\frac{\eta}{1+\eta}$, which has only been reduced by a factor of 3.

However, atomic motion will worsen the cooperativity by introducing inhomogenous broadening. Doppler broadening creates an effective detuning Δ in coupling to the cavity, and the effective cooperativity is $\eta' = \frac{\eta}{1+(\frac{2\Delta}{\kappa})^2}$. For atoms cooled to 100μK the rms detuning will be 90 kHz. In a cavity with linewidth 150kHz the cooperativity will be depressed by a factor of 2.5. This leads to the factor of .41 in the above expression for the effective cooperativity.

5.1 Trap Loading and Collisional Blockade

In trapping atoms, it is necessary to first use a MOT and PGC in order to localize and cool them sufficiently to be trapped by the dipole trap. The dipole trap is intentionally designed to be extremely small and deep to encourage the atoms to collide, allowing the system to operate with the collisional blockade. The chances of capturing an atom from the vacuum chamber solely with the dipole trap are virtually non-existant considering its miniscule cross section and shallow potential as compared to room temperature. Therefore, other preliminary cooling and trapping methods must be used to prepare the atoms to be loaded into the dipole trap. It takes about 5s for the MOT to fill to its maximum capacity of $10^{10}/\text{cm}^3$ atoms. Initiating the dipole trap will load atoms from the MOT, where it can theoretically be maintained indefinately. However, experiments face a practical limitation which restricts the time the atom is in the trap to 100ms.

The study of loading the atoms into the dipole trap is very important in determining the parameters of the MOT, the PGC, and the dipole trap. Atoms are loaded into the dipole trap from the MOT simply by having spatial overlap of the two modes. Atoms from the MOT will fall into the trap at a rate determined by the density and tempearture of the MOT. The loading process competes with several other processes that scatter atoms out of the trap. Such processes include two body losses from atoms in the trap or one body losses from collisions with atoms in the surrounding gas, or free space scattering from the dipole trap beam. A steady state trap number is reached when the loading and scattering rates are equal. An expression providing the dynamics of the number of atoms in the trap N is

$$\frac{dN}{dt} = R - \gamma N - \beta N(N-1) \tag{5.4}$$

where R is the loading rate into the trap, γ is the one body loss rate rate at which atoms are scattered out of the trap by collisions with the background gas or free space scattering from the trap beam, and β is the two body loss rate at which atoms are scattered from the trap through collisions with other trapped atoms [27]. This description naturally lends itself to two different steady state limits. In the weak loading regime the loading rate R is very small, and collisions between atoms in the trap can be ignored. This leads to a steady state trap number of $N_{ss} = R/\gamma$. In the case that the loading rate is very high, collisions between atoms are the dominant limiting factor, and the steady state trap size is $N_{ss} = \sqrt{r/\beta}$. This continuous description for the loading dynamics is not applicable when the average number of atoms is fewer than 10. In this case, a set of first order differential equations must be solved for the dynamics. For a particularly small trap volume, the system can exhibit properties of a collisional blockade. This regime is characterized by the trap only being able to support one atom. This effect is due to β scaling inversely with the trap volume. For a trap mode below a certain volume, as soon as a second atom is loaded into the trap, it will collide with the already present atom and both will scatter out of the trap. Increasing the loading rate into such a trap, the average number of atoms in the trap will plateau at 1/2 over a range of loading rates, as the trap will either contain 1 or 0 atoms at any given time. A diagram of a range of loading regimes is given in Fig. (5-1). As long as the MOT is available to replenish the trap, the number of atoms will continue to cycle between 0 and 1 until the MOT is eliminated and there is no longer a source of atoms to load the trap. At this point we can expect that the trap has aquired one atom. For this experiment, the MOT will have to be turned off in order to make use of the PGC. Once the MOT has reached its maximum density and collected $\simeq 10^6$ atoms, the MOT will be turned off and PGC will cool the atoms for $\simeq 30ms$. Despite the PGC not providing a restoring force, the cloud will still be confined spatially, as the diffusion through the optical molasses is very slow. Atoms can now be loaded into the dipole trap, and the PGC can be turned off so the remaining cloud falls away. Removing the cloud from the proximity of the trap beam is necessary to prevent cycling of the trapped atom so it can be addressed by the read/write beams.

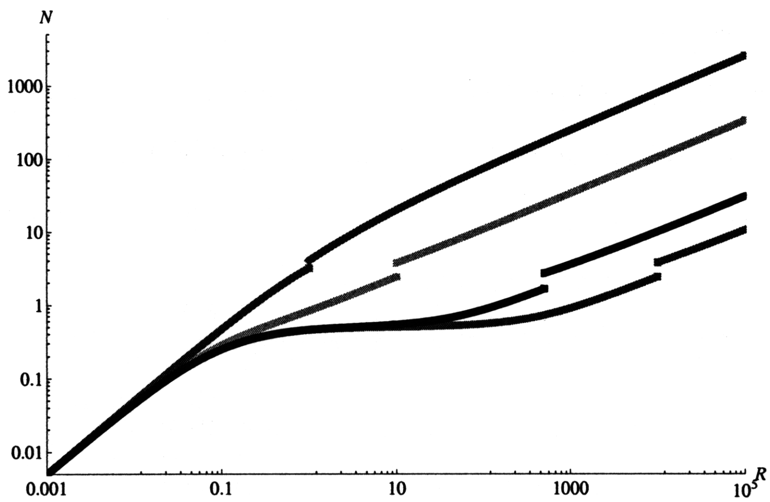

Figure 5-1: The steady state atom number versus loading rate in s^{-1} into the trap, with curves corresponding to w_0 = .7μm, 4μm, and 11μm going up. For w_0 = .7μm the collisional blockade becomes apparent, and the trap maintains an average atom number of 1/2 over a wide range of loading rates. This data assumes γ = .2s^{-1}, and β = 1000s^{-1} for w_0 = .7μm, where β scales like $\frac{1}{wo^4}$.

5.2 Reaching the Unitarity Limit

Building an effective dipole trap requires consideration of the loading dynamics, and capturing a single atom in an efficient manner requires designing a system that can operate in the collisional blockade regime. The system must be tuned correctly in R, γ, and β to produce a collisional blockade. In the absence of the blockade the probability of having any number of atoms in the trap is Poissonian, and under the correct conditions it is possible to significantly suppress the probability of having two or more atoms in the trap. A significant single atom loss γ prevents effective suppression of capturing two or more atoms in the trap. Thus, β must be very large compared to γ, and the system must be operating at a relatively small loading rate. The larger the value of $\frac{\beta}{\gamma}$, the greater the range of loading rates over which the collisional blockade will extend, making the process of trapping a single atom easier. Once any number of atoms is trapped, they will scatter light into the cavity, signalling their presence. Having a very strong collisional blockade assures that any signal will correspond to only a single atom in the trap. Reaching this level of certainty will require constructing a system with a very large β. The rate of two atom collisions between two atoms with identical cross sections σ trapped in a volume V is given by the following expression

$$\beta = \frac{2\sigma v}{V} \tag{5.5}$$

where v is the velocity of the atoms, and is related to the kinetic energy $\frac{3}{2}k_B T$. The volume the atom occupies will depend on its temperature, and will correspond to a contour of equipotential that is the classical turning point of the trapped atom. The shape of these equipotentials is shown in Fig. (5-2) An overestimate of the volume can

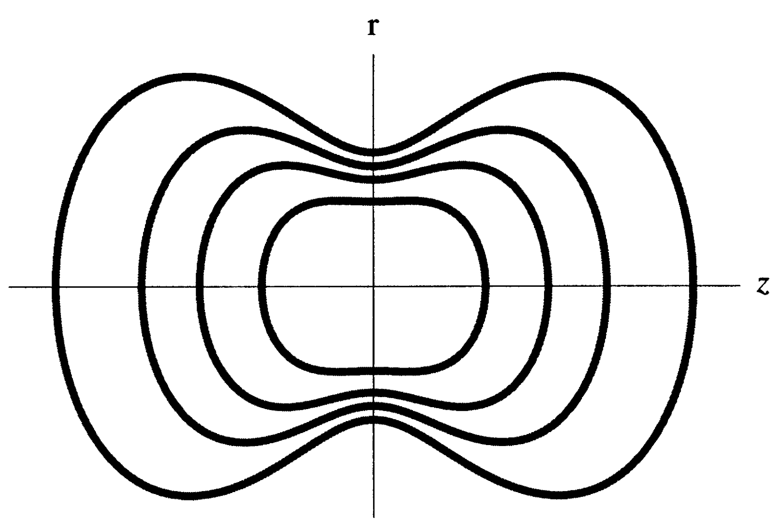

Figure 5-2: Lines of equipotential for a gaussian beam, axisymmetric around the z axis. The atoms in the dipole trap will see contours of this shape, and will be confined by the contour corresponding to their temperature.

be found by integrating over the hyperboloid defined by rotating $w = w_0\sqrt{1 + (\frac{z}{z_R})^2}$ around the z-axis. We begin by guessing incorrectly that the atom will be trapped within $r = \frac{w_0}{\sqrt{2}}$, by which point the field intensity will have fallen off by $1/e$. The atom is confined within one Rayleigh range, hence a rather severe overestimate of the trap volume volume V_L

$$V_L = \frac{8\pi^2}{3}\frac{w_0^4}{\lambda} = \frac{4\pi}{3} w_0^4 k \tag{5.6}$$

From scattering theory we know that the scattering cross section σ has an upper limit $\frac{4\pi}{k^2}$ for each partial wave. This (unitarity) limit enforces conservation of probability for the scattering event. In the low enengy limit $ka \ll 1$ all the scattering will be S-wave scattering, and the unitary limit represents the maximum scattering cross section observable. Using this value for σ, a gas of atoms with kinetic energy $\frac{1}{2}mv^2 = \frac{3}{2}k_B T$ has a two atom collision rate

$$\beta = \frac{6}{\pi}\frac{\lambda v}{k^2 w_0^4} = \frac{6\lambda}{\pi w_0^4 \hbar^2}\sqrt{\frac{1}{3k_B T m^3}} \tag{5.7}$$

For cesium in a dipole trap with a 2μm waist at 10μK, $\beta = .53s^{-1}$ given the previous computation for the volume. A much more accurate expression for the trap volume can be found by computing the volume found within the bounds of the classical turning points, by integrating one of the curves of equipotential corresponding to the trapped atom. Assuming the beam is an ideal gaussian beam and is small enough that the atom is effectively confined to one dimension, the lines of equipotential are given by

$$r_\alpha = \sqrt{\frac{w_0^2}{2}\left(1+\left(\frac{z}{z_R}\right)^2\right)\ln\frac{\alpha}{1+\left(\frac{z}{z_R}\right)^2}} \tag{5.8}$$

where $\alpha = \left(1 - \frac{T}{2U_0}\right)^{-1}$ is a measure how well the atom is confined. α compares the atom's energy, parametrized in its temperatrue T, to the trap depth U_0. The closer α is to 1, the better confined the atom will be spatially. Fig. (5-3) shows the operating conditions for our trap. The volume of the trap is found by evaluating $\int \pi r_\alpha^2 dr$ between the two classical turning points. For a trap depth of 3000μK and an atom at a temperature of 100μK, the two atom collision rate is 111s^{-1}. The trap presented in ref. [27] exhibits a value of $\beta = 10$s^{-1} for the same waist size. Thus, the two atom collision rate can be greatly enhanced by increasing the scattering cross section to the unitarity limit. Reaching the unitarity limit can be accomplished by photoassociation

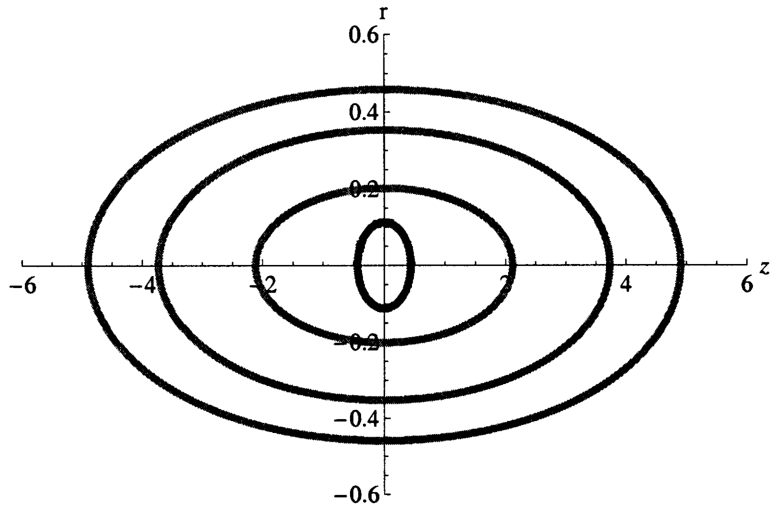

Figure 5-3: Lines of equipotential for a dipole trap used for capturing single atoms. The axes give distances in microns. The trap depth is 3000μK, and the green contours are equipotentials at 500μK, 300μK, and 100μK. Note that the axis scales are not the same, and the particle is effectively in a one dimensional space. The red contour in the center corresponds to the trap used in ref. [27].

resonances [34].

5.3 Laser Stability

We want to design a process in which the lasers will interact with the trapped atoms in a manner that is efficient and reproducible. In studying an application to quauntum information processing, the experiment must try to accurately represent the fully engineered realization of the process in order to demonstrate the validity of the scheme as more than a proof of concept. Similarly, in probing the fundamental physics of the atom-light interaction, it is necessary to operate with extreme precision. To this end, the frequency of the lasers must be very well controlled in order to interact with the atom in a predictable and understandable way. The atom has a natural linewidth that is determined by its coupling to the vacuum and is given theoretically by $\Gamma = \frac{1}{4\pi\epsilon_0}\frac{4\omega^3\mathcal{D}^2}{3\hbar c^3}$. This linewidth is a measure of the range of photon frequencies over which the atom will respond. The laser must have a linewidth smaller than the atom's in order to resolve the atom's resonance frequency. If the linewidth of the laser is too large, it will be impossible to get precise coupling between the atom and the field. Rather, the interaction will be smeared over a broad range of frequencies and there will be very little control over the process.

To avoid this situation, a collection of devices, techniques, and methodologies has been developed to maintain a very high level of control. Stabilizing the temperature of the lasers gives course control of their frequency and is the first step to minimizing fluctuations. A simple temperature control scheme can be implemented with a PI loop and a thermoelectric module. However, much more sensitive control is necessary to keep the fluctuations in an acceptable range. This control is achieved through more sophisticated equipment and intelligently designed control schemes. After all the systems are in place, the linewidths of each laser is 1 MHz, except the experimental laser which should have a linewidth smaller than the linewidth of the transfer cavity. At present the linewidth of the experimental cavity is 150kHz relative to the cavity linewidth due to the low finesse of the experimental and transfer cavities at the locking frequency 780nm.

5.3.1 Electro-optic Modulators

The Pockels effect is an example of an electro-optical effect, that is, a proprety of a material that changes its response to light based on an applied electric field. The Pockels effect changes the refractive index of a material linearly with the strength of the applied field. Materials that exhibit this effect strongly include lithium niobate and many organic polymers and are often used for electro-optic modulators used in experiments. The electro-optic modulator is a device that uses the Pockels effect to phase modulate a laser. The device is driven with a sinusoidal voltage applied across the crystal through which a laser is directed. For a complex electric field oscillating at frequency ω, if it is phase modulated with an amplitude $\phi_{max} = \alpha$ at a modulation frequency β, the resulting field will be given by

$$E \propto e^{i\omega t + i\alpha \sin \beta t} = e^{i\omega t} \sum_{n=-\infty}^{\infty} i^n J_n(\alpha) e^{in\beta t} \tag{5.9}$$

In the case that the phase modulation amplitude α is very small, E can be expanded to give

$$E \approx e^{i\omega t}(1 + \alpha e^{i\beta t} - \alpha e^{-i\beta t}) \tag{5.10}$$

where J_m is the Bessel function of order m. We see that the overall effect of the phase modulation for small α has been to add sidebands onto the signal. Thus the laser will be composed of sinusoidal components at three frequencies; the carrier frequency ω, and the sideband frequencies $\omega \pm \beta$. It is very important to note that sidebands can be added through amplitude modulation as well. However, since the stability of the laser is the most important consideration in this scheme, it is necessary that the amplitude of the beam does not change. The result from amplitude modulation differs from the result from phase modulation by the notable absence of a factor of i from the two complex exponential terms. Without this i the field amplitude is time-dependent and the beam cannot be used in the experiment. Therefore, phase modulation must be used to add sidebands. The purpose of these sidebands is to make use of the Pound-Drever-Hall scheme for locking the frequency of the lasers.

5.3.2 Acousto-optic Modulator with Respect to a Cavity

The acousto optic modulator is a device that is used to change the direction and frequency of a beam of light. The device functions by creating a running wave of phonons of a specific frequency. The presence of the phonons creates regions of high and low density in the material as the wave travels. Since the refractive index of the material is dependent on its density, the phonons moving through the material create a sinusoidally varying refractive index. The spatially varying refractive index causes the material to behave like a diffraction grating. The incident beam will be deflected by the grating into collimated modes, corresping to paths where the deflected beams interfere constructievly. Changing the driving frequency of the acousto optic modulator will change the diffraction spacing, which is the wavelength of the phonons, and will consequently change the angle at which the beam deflects. This effect can be used to rapidly change ($< 1\mu$s) the direction in which a laser is traveling. The use of the acousto optic modulator allows for the sudden introduction of a laser into a beam path. Such a rapid implementation or removal of a beam will be necessary for the precisely timed read/write processes used to probe the atom. This procedure cannot be accomplished by simply turning the laser on at the time it is needed, as it will not be in lock immediately, and there is not enough time in the experimental sequence to wait for the laser to stabilize. Rather, the beam is kept on at all times and introuced into the appropriate beam path when necessary.

An important consequence of using phonons to create the diffraction grating is that the phonons will interact with the incident photons and transfer their energy in a four wave mixing process. The frequency of the light increases by the equivalent frequency of the phonon. In this way, the acousto optical modulator can be used to sensitively adjust the frequency of the laser beam. Altering the frequency by this method will be used to feedback to stabilize the frequency of the laser. Also, by the simultaneous conservation of energy and momentum, the effective absorption of a phonon by a photon will alter the beam path of the laser slightly therefore the acousto-optic modulator is often used by passing the laser through twice. In what is known as a double pass configuration, the first mode diffracted beam of the incident laser will be retroreflected onto the acousto optic modulator and into its original path.

Setting up an acousto optic modulator requires a careful consideration of phase matching conditions to bring as much of the incident laser power into the first diffraction mode. This can be understood classically as either the conservation of energy and linear momentum, or as the interference of waves scattering off the grating, which must interfere constructively to produce a beam.

5.3.3 Pound-Drever-Hall Lock

The Pound-Drever-Hall scheme is a methodology for locking a laser to a cavity. This is accomplished by deriving an error signal from the beam reflected off the cavity and using that signal to adjust either the frequency of the laser or the length of the cavity until the reflected signal is minimized. The minimization of the reflected signal is an indication that the transmission through the cavity is maximised and the cavity is on resonance with the laser. The reflection coefficient for a lossless two mirror optical cavity is

$$\frac{r(e^{ikl} - 1)}{1 - r^2 e^{ikl}} \tag{5.11}$$

Where k is the wavenumber of the laser, and l is the single pass length of the cavity. This signal cannot be used as an error signal in a control scheme because it is symmetric in kl, as seen in Fig. (5-4). It would be impossible to know on which side of stability the laser/cavity system is situated, and it would therefore be impossible to feedback in the correct direction. While it is possible to attain control by

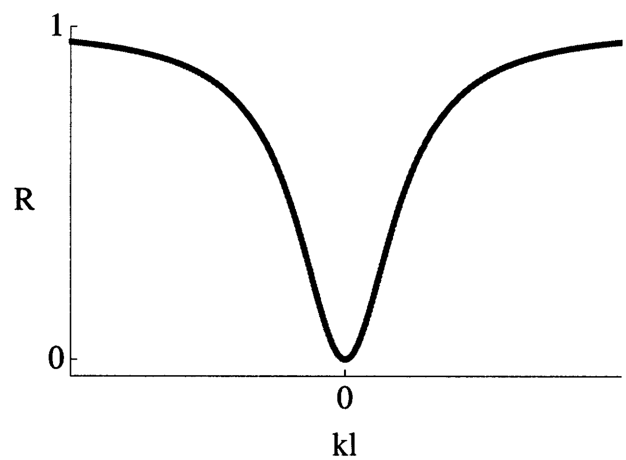

Figure 5-4: The reflection coefficient of a two mirror optical resonator as a function of the phase kl. Since the signal is symmetric it cannot be used as an error signal to control the laser.

also observing the derivative of the transmission signal to determine which side of the cavity transmission line the system is on, ultimately any scheme that uses the transmission intensity as the error signal is not robust. The detection of any stray light will change the error signal and prevent proper control. The Pound-Drever-Hall error signal eliminates this problem by creating a signal that is linear and asymmetric across resonance. The Pound-Drever-Hall lock is an example of a lock-in amplifier that modulates the probe signal such that the error signal is at a high frequency. This makes the system very robust to environmental noise, which tends to obey a $\frac{1}{f}$ intensity. For the case of a laser, the beam is phase modulated, giving it side bands. The reflected side bands interfere with the reflected carrier and result in an error signal that can be used to stabilize the cavity. The reflected beam is measured by a photodiode and the resulting signal is mixed with $\sin \beta t$ to extract a usable error signal. This signal has the property that it is odd, is linear around the lock point, and thus gives a value of equal magnitude and opposite sign for a displacement in either direction. Fig. (5-5) gives an example of a PDH error signal. A basic scheme utilizing

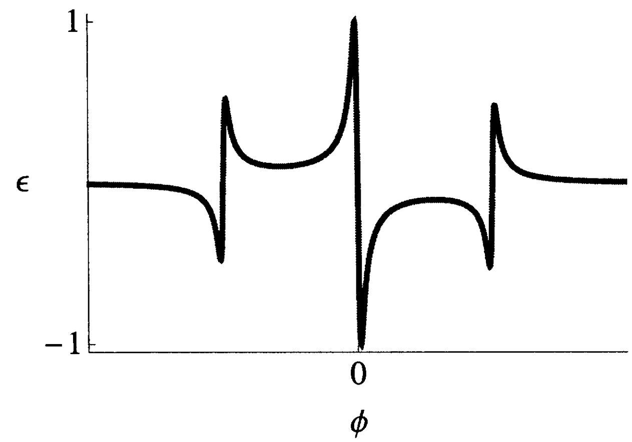

Figure 5-5: A normalized Pound-Drever-Hall error signal as a function of the phase kl. At very small deviations from the stable center, the signal is linear and steep. It has the important property that the sign is different on either side of the lock point.

the PDH lock will operate as follows. The laser will be phase modulated by an electro optic modulator driven at a frequency β, altering the laser so it has sidebands at its primary frequency $\pm\beta$. The laser is then sent through a beam splitter and into the the cavity. The reflected beam is sent back through the beam splitter and is recorded by a photodiode. The ensuing electronic signal is mixed with the same signal that was given to the electro optic modulator to produce the phase modulation. The resulting signal will consist of the PDH error signal and a modulated version of it. Since the modulation is at high frequency, a low pass filter will effectively select the PDH error signal. This signal can then be used to feedback on either the length of the cavity by a piezo electric actuator, or on the frequency of the laser by adjusting the current to the laser. A simple example is given in Fig. (5-6). The particular control scheme by which the error signal is used to control the cavity is dependent on the specific properties of the system. Generally, a simple PI loop properly tuned will be sufficent for good control. In the case that lag introduced by the PI control results in a narrow bandwidth, it is possible to improve the response of the control loop by feeding the PHD error signal directly, without applying the PI gain. As a note, derivative control is in general ignored because it increases the order of the transfer function, potentially decreasing the stability of the system by introducing poles and phase shift.

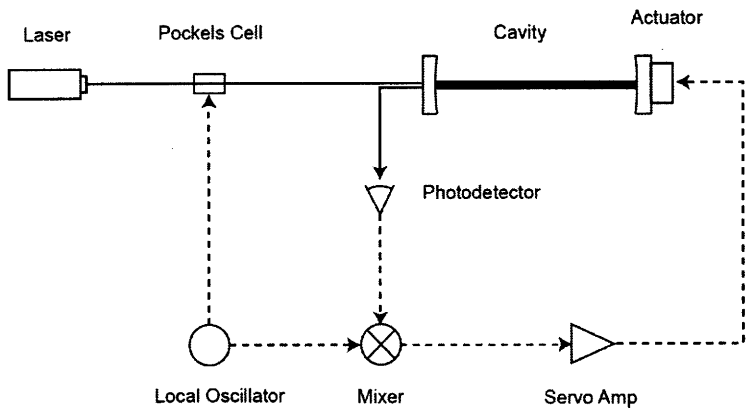

Figure 5-6: A basic implementation of a PDH locking scheme. In this case, the feedback is on the length of the cavity, and the PI gain on the error signal is given by the servo amp.

5.3.4 Resonant Photodiode

Creating good control systems involving detection of light requires specialty electronics sensitive enough to resolve small changes in the amplitude of the light field, and fast enough to follow vacillations in the light intensity. A typical photodiode functions on the principle of the photoelectric effect, where light with an energy higher than the work function of bound electrons excites them into conducting bands, creating a measurable current, put though a resistor to derive a measurable signal. Additional design elements may include the ability to bias the photodiode so it operates with greater sensitivity. Our photodiode circuits are composed of two gain stages. The first is usually an op amp designed to run as an impedance gain stage, whose purpose is to transform the signal from the current source into a voltage, while the second stage is given a gain of perhaps 10 to amplify the photodiode signal. The resistor in the impedance gain stage must be high in order to increase the SNR from the johnson noise. For a current source I, the signal will be IR, with a corresponding Johnson noise $\sqrt{4k_BTR}$, so the SNR goes like $\sqrt{R}$. The signal should be limited by the shot noise on the current source $\sqrt{2Ie}$. Increasing the resistance has the unfortunate side effect of decreasing the stage's bandwidth, decreasing the gain of the system. In any feedback system with numerous elements in series, stability is a serious concern. Phase shifts from subsequent elements can add in a non-trivial manner to create a closed loop gain whose transfer function has poles. The presence of these singularities is an enormous hindrance to running a successful control scheme.

The intrinsic gain of an ideal op amp is $\frac{GBWP}{i\omega}$, and any stray capacitance in the op amp's closed loop gain will allow the possibility for an instability to arise. Stability issues can be controlled by adding capacitance into the closed loop gain to roll it off. This standard technique will prevent the system from oscillating, but will also lower the bandwidth. For the purposes of measuring and controlling an optical signal, where the linewidth of the beam needs to be less than 1 MHz, having a photodiode with a smaller linewidth is necessary to control the laser. Additionally, electrical noise in the circuits can severely limit the ability of the photodiode to respond correctly. Noise will be an increasingly high problem at higher frequencies where the gain is smaller, and thus the signal to noise ratio decreases. It is possible to compensate for a lower SNR by using a laser with a higher power. However, in the case that we would want to feedback on the length of the cavity with a PDH error signal, the experimental beam that enters the experimental cavity will have to be very weak so as not to scatter or decohere the atoms. Thus, the photodiode will need to operate with a high SNR at a large frequency, and with a large bandwidth. This is possible in principle by building a resonant circuit into the normal photodiode circuit. A simple RLC circuit will be effective in producing a large signal only at the resonant frequency, allowing for the use of a weak beam through the cavity.

The RLC circuit can be installed in parallel with the photodiode. The op amp reading the signal must have an incredibly large GBWP in order to respond at the high frequencies, $\mathcal{O}(10\text{ MHz})$ of the signal it will receive, minimizing the noise. Additionally, the inherent resistance in the inductor and capacitor of the RLC circuit are sufficient to fulfill the purpose of the impedance gain stage, so the first op amp can be used as a voltage follower, whose only purpose is to isolate the photodiode circuit from other electrical elements. The RLC circuit is coupled to the op amp by a 100pF capacitor.

5.3.5 Transfer Cavity

Often, the reflection of the principal beam off the experimental cavity is used to create the PDH error signal. However, since this beam corresponds to the atomic transition of the atom, it has the potential to scatter or decohere the atoms in the trap. In cooling or a controlled read/write operation, this effect is exactly what is desired. Using the reflection of a beam tuned to the atomic resonance to derive the PDH error signalr would require that the beam be in constant operation, which would affect the atoms in an uncontrolled and undesirable fashion. However, under the condition that we want to trap and perform operations on a single atom, it would certainly be detrimental to the experiment to constantly shine a beam into the cavity. Instead, a more elaborate locking scheme will be necessary to stabilize the laser.

We implement a second cavity, called a transfer cavity, for this purpose. The construction and implementation of a stable transfer cavity will provide the ability to lock the laser to the cavity without ever shining it into the experimental cavity. This control scheme will require the use of an additional laser. This second laser will be of a frequency far detuned from the atomic resonance such that it can be locked to the experimantel cavity without disrupting the atom. The experimental laser and the transfer laser will both be locked to the transfer cavity, and the experimental cavity will be locked to the frequency of the experimental laser via the transer cavity.

The transfer cavity is an important piece of equipment in this experiment as it provides a function necessary to the realization of a single atom dipole trap. The transfer cavity was designed to match the geometry of the experimental cavity. The cavity was machined out of a single piece of cylindrical aluminum and possesses $D3h$ symmetry. The symmetry of the cavity is very important in minimizing acoustical coupling, and in particular to vibrational modes that change the length of the cavity. The cavity is held vertically for this reason, and is coupled into optically from below. It is supported by teflon rods and screws, which are screwed into an aluminum base, which is screwed into an aluminum breadboard. This construction is successful in decoupling the cavity from acoustical noise and consequently the system is extremely stable. However, to use the transfer cavity effectively, a control scheme will need to be implemented. This design is an important factor in implementing effective control schemes. The system responds quicker if the stability of the transfder cavity matches the stability of the experimental cavity.

The actual scheme to lock the experimental laser to the cavity is not too complicated, but very delicate. First, the experimental laser at 852 nm is locked to a cesium reference cell to stabilize the laser frequency. This is done by using the beat note from the overlap of the laser and the reference cell to feed back on the frequency of the experimental laser via current control. To lock the cavity to this laser, the experimantal laser is modulated by an electro optic modulator at a frequency of $\approx$ 30MHz and is projected into the transfer cavity. The PDH error signal derived from the reflection of the experimantal beam off the transfer cavity is fed back onto the piezo electric actuator that changes the length of the transfer cavity, thus locking it to the frequency of the laser. Similarly, the transfer laser at 780nm is modulated with an electro optic modulator at a frequency of $\approx$ 30MHz and projected into the transfer cavity. The resulting error signal is fed back onto the laser current, locking the transfer laser to the cavity.

The transfer laser is used to control the length of the experimental cavity. Control is achieved through two separate loops. The transfer laser is projected into the experimental cavity a PDH error signal is aquired which is used to feed back on the length experimanteal cavity. In principle, this control scheme is complete. Since the experimental cavity is locked to the transfer beam, and the transfer beam is locked to the transfer cavity, which is locked to the experimental beam, the original objective has been achieved. The experimental laser will be on resonance with the experimental cavity without having ever probed it. However, the piezo response is typically quite slow, and so this feedback loop will not have enough bandwidth to suppress high frequency noise. Increasing the gain could introduce phase shifts such that the unity gain response is unstable. That is why a second, faster loop must be included to deal with high frequency noise in the system. Before entering the experimental cavity, the beam is sent through a double pass acousto optic modulator, changing the frequency. The driving frequency of the acousto optic modulator is fed back on by the PDH error signal from the cavity reflection. This loop will prevent the generation of instability in the feedback, and will allow the first loop to follow at much higher gain. Essentially, this works by altering the laser frequency to follow the cavity resonance very quickly such that no large phase shifts are introduced. In the case that the feedback to the AOM broadens the transfer laser substantially, the experimental laser will be unable to properly track the experimental cavity. In this case the laser will be wider than the cavity and the cooperativity will be greatly reduced. A complete picture of control scheme is given in Fig. (5-7). It may be necessary to feed the error signal from the experimental cavity back on the experimental laser as well. Although this may require putting the experimental laser into the cavity, it should be at a sufficiently low power that the experiment is unaffected. Also, note that the linewidth of the laser is much wider than the bandwidth of the piezo feedback. For the feedback to function properly, the laser must already be well stabilized before the PDH error signal can be used to feed back on the cavity length. The order in which the control loops are initiated is very important to being able to maintain stability.

The experimental cavity has a linewidth of 150 kHz, such that the laser's linewidth will actually need to be narrowed by the control loop. The relative finesses of the lasers with respect to the experimental cavity is very important in determining the level of control that can be achieved. The finesse of the 780nm in the experimental cavity is 5000, whereas the finesse of the 852nm laser is 80000. The difference in finesse is due to the specially designed mirrors, which reflect in a highly wavelength dependent manner. This ratio tells us that the control loop will lock the 852nm laser such that it is 16 times as broad as the locked 780nm laser. The use of a longer wavelength laser as the transfer laser will circumvent this problem as this wavelength will have a higher finesse in the cavity, and thus the ratio of the finesses between the experimental and transfer lasers will be lower, resulting in superior control on the 852nm laser.

In order to operate the beam at a lower power, a polarizing beam splitter is used such that all polarization of one linear polarization is transmitted through the beam splitter, and the other linear polarization is reflected. A quarter wave plate situated between the beam splitter and the cavity is used to rotate the linear polarization 90° so no power is ever reflected away by the beam splitter.

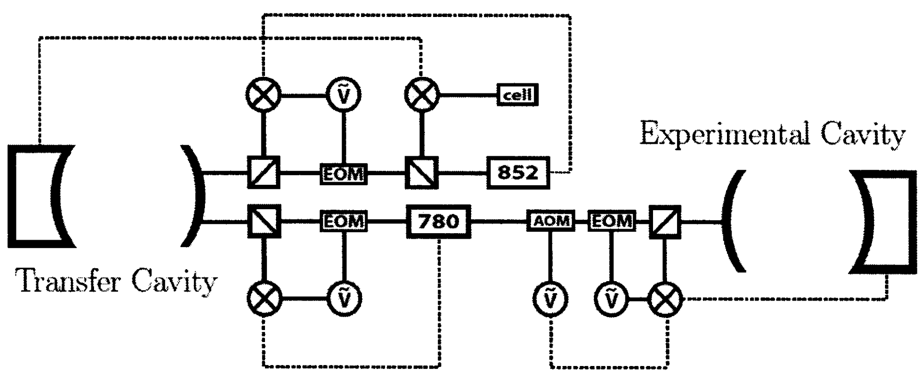

Figure 5-7: The overall control scheme coupling the transfer cavity and the experimental cavity. Dotted lines represent control signals.

5.4 Experimental Setup

In our experiment, we will utilize all three of the aforementioned cooling and trapping schemes to the effect of cooling a single atom (hopefully) to 10μK and holding it for 1s [27]. There are three MOT beams that together create a three dimensional trap. Each beam is 1cm wide, and in this configuration the MOT can hold 10^6 atoms. Additionally, a constant magnetic field can be added in any direction to bias the field from the anti-helmholtz coils. This bias allows for the MOT to be moved and precisely placed in the center of the cavity. At the beginning of the experiment the MOT coils must be turned off so as not to interfere with the PGC. A precise timing system ensures that the sequence of beams activates in the right order. At this point the dipole trap will have trapped a single atom. The experiment has the capacity to work with two dipole traps simultaneously. Since the MOT beams are much larger than the dipole trap beams, each trap can take an atom from the same MOT. This system will allow for the interactions of the two atoms via van der Waals forces. The atoms will remain coherent for s, in which time. In the experiment, when the atoms have been brought to T_D and confined to some extent, the MOT will be turned off and PGC will be implemented. This cooling scheme will bring the atoms to a temperture below T_D, on the order of 10μK.

Anti Helmholtz coils inside of the vacuum chamber are used to create a constant gradient magnetic field centered at the center of the cavity.

By sending a σ_+ polarized beam through the chamber, and then reflecting it through a quarter waveplate, it will return as σ_- polarized light. This setup will create two counterpropogating beams of opposite helicity, as desired.

For the cesium MOT in our experiment, we couple the ground state of cesium $\left|6^2S_{\frac{1}{2}}, F = 4\right\rangle$ state, to the $\left|6^2S_{\frac{3}{2}}, F = 5\right\rangle$ and $\left|6^2S_{\frac{3}{2}}, F = 5\right\rangle$ states . The linear magnetic field gradient is $\approx$ 10 G/cm, and the laser frequency is tuned 4 - 5 MHz below the atomic resonance.

Chapter 6

Conclusion

In this thesis I have considered the theoretical background, as well as an analysis, for an experiment in atomic physics. The purpose experiment involves developing a system to reliably capture a single atom, at which point experiments on the non-linear nature of the atom-photon interaction can be performed. These type of experiments require extreme precision in the lasers used to trap the atoms and probe the atomic transitions. Setting up a successful collisional blockade is accomplished by constructing a trap and loading scheme with the correct physical parameters. The process of capturing the atom requires extreme precision, and is accomplished in part through careful consideration of technical issues. High laser stability can be achieved through the use of an intelligently designed and carefully controlled feedback scheme. Part of developing good control is implementing hardware that is inherently stable. To this end I designed and built an ultrastable transfer cavity. Thus, the construction of the electronics and cavity are paramount to successful experimentation. Once the system is constructed, the single atom can be trapped in the strong coupling regime and the nature of the photon-atom interaction probed.

Appendix A

Optics and Lasers

A.1 Lasers

A laser projects a coherent beam of light formed by the stimulated emission of photons. A laser is constructed by placing a gain medium between two highly reflective mirrors. The gain medium is a material wherein the atoms can be brought to an excited state by the application of energy. Often, the energy is applied in the form of acoustic, thermal, electrical, and photonic, and effectively primes the atoms to emit photons by stimulation from passing light. The atom is initiated by powering the gain medium and sending light into the cavity. Light that enters the optical cavity passes through the gain medium and causes the coherent release of more photons by stimulated emission. Upon reaching the next mirror, as long as a greater proportion of the light reflects off the mirror rather than exits the cavity, the light will continue gathering power from subsequent passes through the gain medium. Eventually, a steady state will be established, and as much power will be exiting the cavity as is put into the light field inside of it. The light is coherent, and can be made highly directional by the geometry of the cavity.

Microscopically, laser operation can be understood by considering an atomic three level system. For a two level system, time reversal symmetry requires that the rate of spontaneuos emission be the same as the rate of spontaneous absorption. As a consequence, a two level system is incapable of lasing, since the absorption and emission of radiation by the medium will be equal. Including a third level in the transition scheme will allow for the creation of a state that is not in thermodynamic equilibrium, which will lead to the ability of the atom to lase. For a gain medium composed of atoms with a well understood three level structure, energy is put into the gain medium such that the atoms are pmuped to the higher excited state. In this state of population inversion, the system is not in thermal equilibrium, and the atoms will tend to fall to the ground state. But because the atoms can fall to a state above the actual ground state of the system, equilibrium will never be reached. Of the two possible transitions, the atom will undergo one transition by spontaneous emission, but the other transition will occur by stimulated emission, creating the laser. The optical cavity is designed such that the atomic frequency corresponding to the stimulated emission corresponds to an integer multiple of the free spectral range. Designing the cavity in this manner will allow the cavity to support the frequency of the atom that will produce the laser.

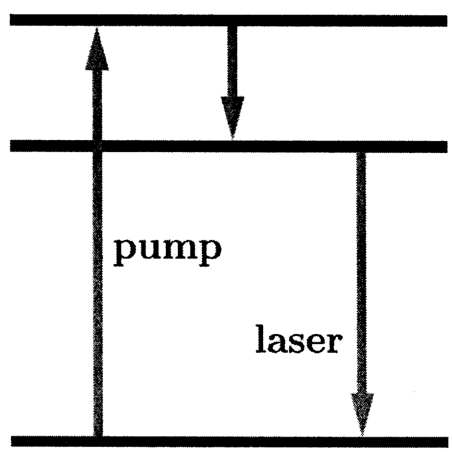

Figure A-1: The three level system is the basis for laser operation. The atoms in the gain medium are pumped to the upper level from which they fall by spontaneous emission into a lower state. The cavity is designed to be on resonance with the transition from the second level to the ground state, and photons supported by the cavity mode build up by many stimulated emission events, leading to steady state operation of the laser.

A.2 Gaussion Beams

A gaussian beam is a mathematical formulation that describes the propogation of light in the limit where it its expansion in the radial is very small as it travels forward. The field strength of a gaussian beam is $\propto e^{-r^2/w^2}$ and is a solution to the paraxial wave equation $(\nabla_T^2 - 2ik\frac{\partial}{\partial z})u = 0$. For the gaussian beam equations to be valid, the conditions that define the paraxial wave equation must be met. The gaussian beam is an effective description for a laser beam, and can be used for classical calculations detailing the behavior of the laser when it is projected through a variety of optical elements. The field amplitude of a gaussian beam is parametrized by two properties for any location in its path of propagation. The width w is a measure of how quickly the field of the beam falls to $1/e$ of its maximum value in the radial direction, and the radius of curvature R gives a locus for the contours of constant phase in the light field. A useful formulation is to consider the complex gaussian beam, where the complex q parameter holds all the information of the light field.

$$\frac{1}{q} = \frac{1}{R} - i\frac{\lambda}{\pi w^2} \tag{A.1}$$

Where λ is the wavelength of the light. In terms of q, the field strength for a radially symmetric beam can be expressed as

$$u = \frac{1}{q} e^{-ik\frac{r^2}{2q}} \tag{A.2}$$

This formalism is incredibly useful because it allows use of the ABCD matrices from classical optics. It can be shown that propagating a gaussian beam through a classical optical element that can be described by an ABCD matrix is the same as solving for the complex q_2 parameter in the equation $\begin{pmatrix} q_2 \\ 1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} q_1 \\ 1 \end{pmatrix}$ given a starting q_1. Note that the ABCD matrix must be unitary to conserve the power in the beam. For an optical resonator at steady state, propagating the beam through any closed loop should yield an identical value for q as the starting value. This formalism transforms an integral equation into a matrix eigenvalue problem whose solution will give the width of the beam w and the radius of curvature R everywhere in the cavity. In designing an optical cavity or dipole trap, the width of the beam at the center, called the waist, is very important in deciding what kind of experiment the trap can be used for, as it determines the volume, cross section, and beam intensity of the trap. For a diffracting beam, the dependence of the beam width w on its propagated distance z is

$$w = w_0\sqrt{1 + \left(\frac{z}{z_R}\right)^2} \tag{A.3}$$

where w_0 is the waist of the beam, and $z_R = \frac{\pi w_0^2}{\lambda}$ is the Rayleigh range.

ABCD Matrices		
Free space	$\mathcal{FS}$	$\begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix}$
Reflection from a flat mirror	$\mathcal{I}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
Reflection from a curved mirror	$\mathcal{S}$	$\begin{pmatrix} 1 & 0 \\ \frac{-2}{R} & 1 \end{pmatrix}$
Propagation through a thin lens	$\mathcal{L}$	$\begin{pmatrix} 1 & 0 \\ \frac{-1}{f} & 1 \end{pmatrix}$

Cavity Data for Experimental and Transfer Cavity		
Cavity Length	13.739mm	13.316mm
Radii of Curvature of Mirrors	1cm	2.5cm
Free Spectral Range of Cavity	10909 ± 1MHz	11257 ± 1MHz

Appendix B

Design Schematics

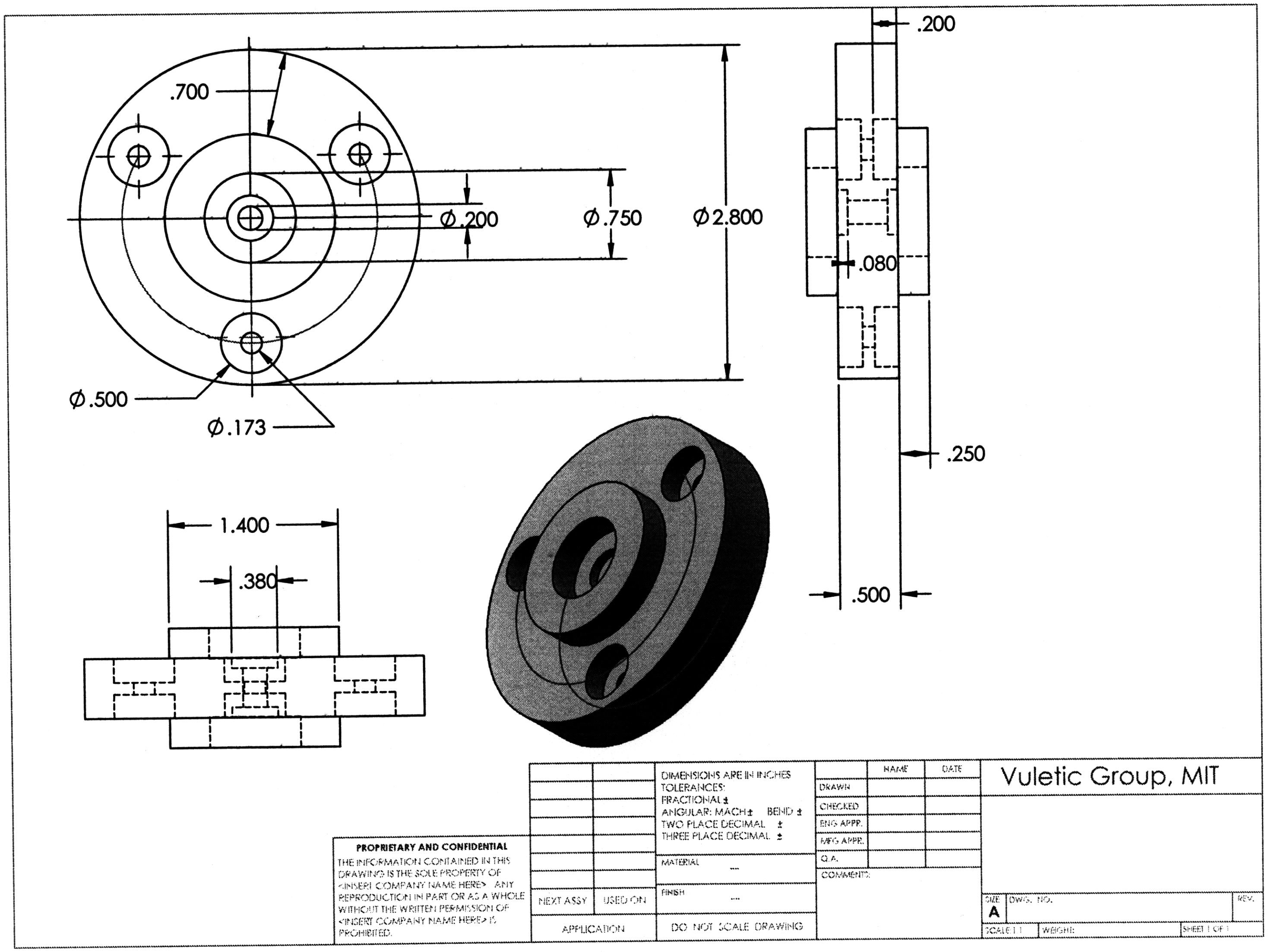

Figure B-1: Schematic of the Transfer Cavity. Machined out of aluminum.

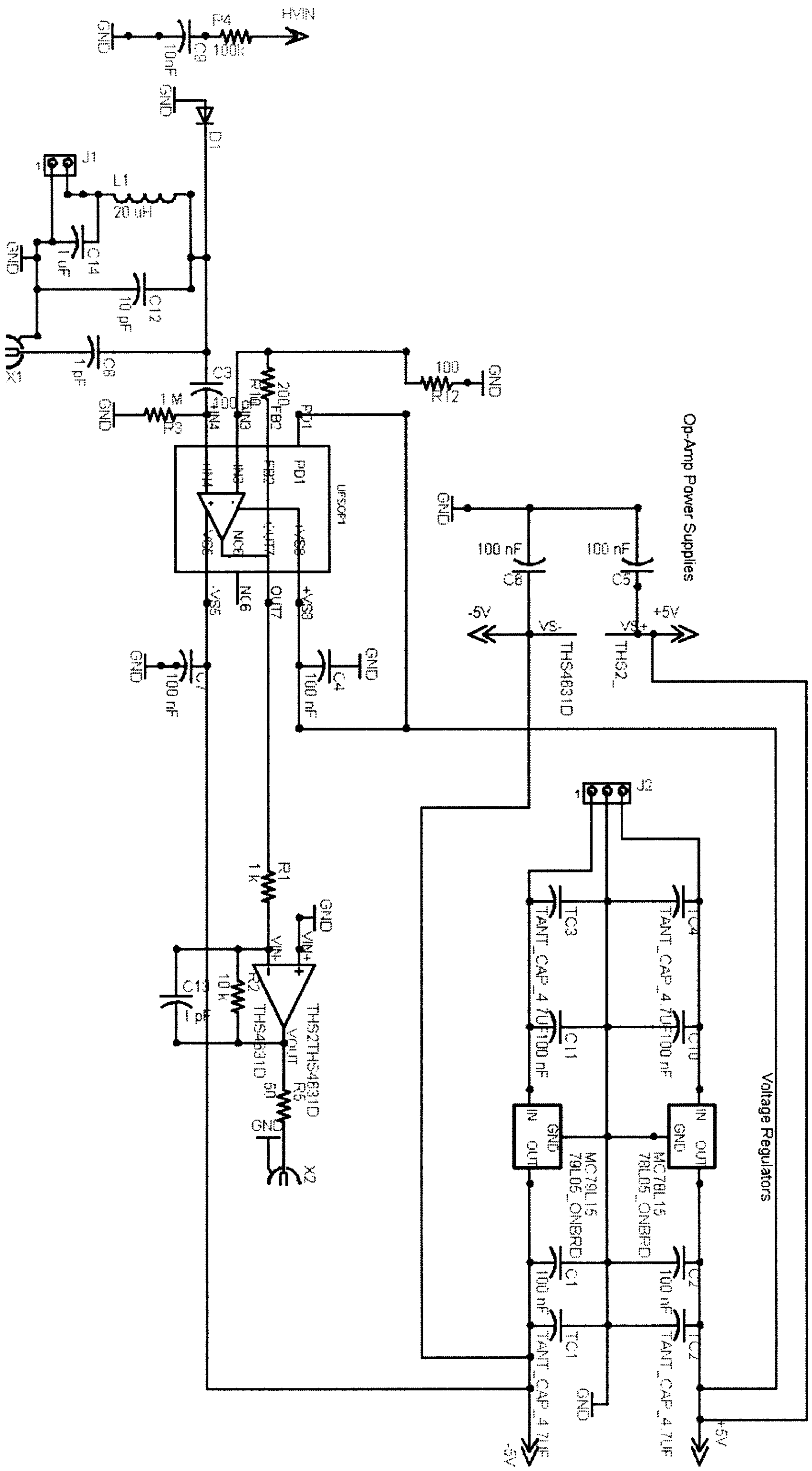

Figure B-2: Schematic of the resonant photodiode.

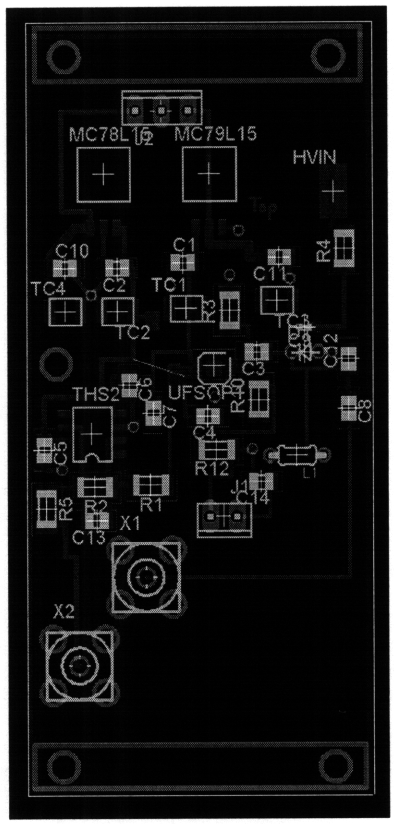

Figure B-3: PCB layout of the resonant photodiode.

Appendix C

Collisional Blockade Simulation

```
γ = .2;
β = .016 (11/11)^4;
Vec = {P0, P1, P2, P3, P4, P5, P6}
H = {{-R, γ, 2 β, 0, 0, 0, 0}.Vec == 0, {R, -R - γ, 2 γ, 6 β, 0, 0, 0}.Vec == 0,
   {0, R, -2 γ - 2 β - R, 3 γ, 12 β, 0, 0}.Vec == 0, {0, 0, R, -3 γ - 6 β - R, 4 γ, 20 β, 0}.Vec == 0,
   {0, 0, 0, R, -4 γ - 12 β - R, 5 γ, 30 β}.Vec == 0, {0, 0, 0, 0, R, -5 γ - 20 β - R, 6 γ}.Vec == 0,
   {0, 0, 0, 0, 0, R, -6 γ - 30 β}.Vec == 0, {1, 1, 1, 1, 1, 1, 1}.Vec == 1};
Plot1 = Show[LogLogPlot[n /. Solve[0 == R - γ n - β n (n - 1), n][[2]],
    {R, 1, 10^5}, PlotRange → {{10^-3, 10^5}, {.005, 5000}}, Ticks → {True, True},
    AxesLabel → {R, N}, BaseStyle → {FontSize → 16}, PlotStyle → Thickness[.01]],
   LogLogPlot[(Vec /. (Solve[H, Vec]) // N)[[1]].{0, 1, 2, 3, 4, 5, 6},
    {R, 10^-3, 1}, PlotRange → {{10^-3, 10^5}, {.005, 5000}}, Ticks → {True, True},
    AxesLabel → {R, N}, BaseStyle → {FontSize → 16}, PlotStyle → Thickness[.01]]];
```

Figure C-1: Mathematica code connecting the steady state of a first order linear dynamics simulation of atom loading to a continuous approximation. The connection occurs around $n = 1$, in which the continuous approximation is not valid, leading to the observed discontinuities. Each simulation was run with a different value for β, and in the figure β increases going down. Blue, green, and red correspond to experimennts done in [27], and black corresponds to our experiment.

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