INVESTIGATION OF ANOMALIES OF MINIMUM AND OF MAXIMUM TEMPERATURE FOR WINTER AND SUMMER AT OMAHA, NEBRASKA

by

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ABSTRACT

Normal minimum and maximum temperature are computed for each day of the winter and summer seasons from the 100-year Omaha temperature record. After removal of trend from the raw data, statistical significance levels are calculated and a count of significant values of the daily means has been made for the entire period, for each of the phases and for the minor and the major halves of the double sunspot cycle. It has been found that the phase of the minor maximum for the summer season and the phase preceding the minor maximum (NN-M) for the winter season show significantly more singularities than any other period.

The standard deviations of both maximum and minimum temperatures for the eight phases exhibit cycles with double phase that correlate highly with the double sunspot cycle. The results of statistical tests indicate that the cycle is real and as expected more significant for the summer than for the winter season, and for Omaha than for Boston. It has not been possible to prove any significance of daily singularities.

Thesis Supervisor: Hurd C. Willett
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To my children.

Table of Contents

List of Tables

List of Tables (Cont.)

List of Tables (Cont.)

List of Tables (Cont.)

List of Figures

List of Figures (Cont.)

Chapter I

Introduction

In a paper of December 1965 [1], E. Newman made a statistical investigation of anomalies in the winter temperature record of Boston, Massachusetts. He showed that the probability of the chance occurrence of certain anomalous points in the temperature is very small-- less than 10%-- only during some phases of the 20-22 year double sunspot cycle. He concluded: "It is impossible to establish the reality of the 'January Thaw', or any other singularity but the variation in temperature variability is quite cyclic in character and relates to the 20-24 year double sunspot cycle".

It has been thought interesting to do the same study for another place in the United States and especially for one with a completely different climatological character. That is one reason why Omaha -- Epply Airfield, Nebraska, 41° 18'N, 95° 54'W, elevation 977 ft. -- has been chosen; a second reason is because a very long record of data is available there. These consist of 100 years (1871-1970) daily values of temperature for the winter and summer seasons, defined respectively as December 1 to February 28 and June 1 to August 31.

It was decided to make a statistical analysis of both the minimum and maximum daily temperatures to try to show whether the results obtained for Boston, Massachusetts must be expected to hold for other places and other seasons, and if, in the affirmative, to what degree.

Following the terminology generally used, we adopt the following definitions for the terms "singularity" and "anomaly". Anomaly will denote a peak or a trough in the time profile of a parameter, and singularity an anomaly which can be shown statistically significant at above the 5% level. Although it is almost impossible to prove that a singularity is physically meaningful, it is possible to show that the probability of the chance occurrence of these anomalous points in the temperature profile is very small.

We hope this kind of study will be another step towards forming a physical cause and effect relationship between the singularities and some variables such as the sunspots cycle, variable geomagnetic activity or total atmospheric ozone. With this idea as a starting point, we decided to analyze both the winter and summer season, on the assumption that if any solar-climatic relationship exists, it must be more significantly shown for a continental station in the summer; the influence of the sun, stronger during this season than during the winter time, not being interfered with by any maritime thermal influence.

Special emphasis has been placed on the computer problem. It was made as general and as complete as possible in such a way that the final results -- including some persistence parameters -- can be obtained easily for any station and any period of time, provided the data are available in a computer form. Already we plan to make the same statistical investigation for European climatological stations where records are available for at least 150 to 200 years. If the results justify it, Professor H.C. Willett hopes to extend these studies to other American stations. Hopefully such a complete set of results for specific locations with different climatological and geographical character will enable us to draw some geophysical conclusions.

Chapter II

The Statistical Problem

Section 1. The daily normal temperature

Since minimum and maximum for winter and summer are treated separately, the same notation used for the temperature may be applied without confusion to either. (In the following theoretical explanation, the term 'temperature' will refer to either minimum or maximum for either the summer or the winter season.)

Working first with the complete set of data and then after with each of the phases of the double sunspot cycle, we will take care to specify the size of the data population we use to define the mean and other statistical parameters.

Let N equal the number of years

M equal the number of days (M=90 for winter; M=92 for summer)

X_{ij} the temperature at day j of the year i

X_{-j} the normal temperature of the day j computed over N years

$$X_{-j} = \frac{1}{N} \sum_{i=1}^{N} X_{ij}$$

X_{i-} the mean temperature for the season of the year i computed for M days

$$X_{i-} = \frac{1}{M} \sum_{j=1}^{M} X_{ij}$$

Since the data contain the annual trend, it is necessary for a statistical analysis to subtract it out from the actual data to make them more independent. The usual method to determine this seasonal trend is the harmonic analysis [2,3,4] if the sample covers a period showing a periodic fluctuation. Since we have only the winter and summer season daily temperatures it is not possible to use this method. To approximate the daily normal temperature profile by a smooth curve, the fitting method of least squares has been applied to a polynomial of the second degree, This analytical expression for the trend has been used also by H.A. Panofsky [5] and suggested by C.E.P. Brooks [2] and many others. To find the coefficient C_i of the different powers of d in the equation of the parabola (1), the orthogonal Gram polynomials [6] have been used to considerably simplify the computations.

If Y_{-j} is the trend value of the day j we can write

$$Y_{-j} = C_0 + C_1 d_j + C_2 d_j^{\,2} \qquad (1)$$

where $Y_{-j} = \hat{X}_{-j} = X_{-j} - e_{-j}$

e_{-j} is the departure of the daily normal temperature from the trend for the day j

d_j is the date: December 1 = 1, ---- February 28 = 90

June 1 = 1, -------- August 31 = 92

Since C_i's are such that $\frac{\partial \overline{e^2_{-j}}}{\partial C_i} = 0$ for $i = 1, 2, 3$; $j = 1, \dots M$

they are given by

$$C_0 = E_0 + E_1 + E_2$$

$$C_1 = -2\left(\frac{E_1}{M+1} + \frac{3E_2}{M+2}\right)$$

$$C_2 = \frac{6E_2}{(M+1)(M+2)}$$

with

$$E_0 = \frac{1}{M}\sum_{j=1}^{M} p_{0j} X_{-j}$$

$$E_1 = \frac{3}{M}\sum_{j=1}^{M} p_{1j} X_{-j} \qquad p_{ij} = \sum_{k=0}^{i} \binom{i}{k}\binom{i+k}{k}\frac{\binom{j-1}{k}}{\binom{M-1}{k}}$$

$$E_2 = \frac{5}{M}\sum_{j=1}^{M} p_{2j} X_{-j} \qquad i = 0, 1, 2$$

We obtain respectively for

minimum summer temperature $Y_{-j} = 57.49 + 0.3866\, d_j - 0.0037\, dj^2$

maximum summer temperature $Y_{-j} = 77.08 + 0.4024\, dj - 0.0038\, dj^2$

minimum winter temperature $Y_{-j} = 24.75 - 0.4854\, dj + 0.0050\, dj^2$

maximum winter temperature $Y_{-j} = 41.29 - 0.4547\, dj + 0.0050\, dj^2$

The trend value is then subtracted from each of the daily temperatures and these departures are called e_{ij} (the meaning of the indices in e_{ij} being coherent with those used for X_{ij}). For each of the different sets of data, the trend curve was computed only from the daily normal temperature distribution computed for the entire period, on the assumption that in mean it corresponds to the seasonal trend for each year. (Then the seasonal means e_{i-} are not zero $i = 1,$ --- N). Since we wish to test the tendency for each of the sunspot cycle periods to produce singularities and to do that relatively to the whole period, this same 'normal' trend was used. As a consequence we have to point out that the interpretation of the results with regard to the significance level must be carefully made (see section 3.3).

Section 2. Standard Deviation *

The standard deviation of the e_{-j}'s has been computed according to $\overline{e'^2_{-j}} = \overline{e^2_{-j}} - \overline{e}^2_{-j}$ and compared to the standard error $\left[\frac{\overline{\overline{e'^2_{ij}}}}{N}\right]^{\frac{1}{2}}$ *. Because working with a distribution of mean values e_{-j} of samples we know that the effect of averaging random data is to reduce the standard deviation of these data by a factor equal to the sqare root of the number of data appearing in the processes of averaging. The comparison of these two values can be interpreted to mean that grouping the data into date means is not random, but the persistence being present there is more than random tendency towards formation of singularities [16].

Section 3. Significance of the extremes

3.1 Significance levels

A significance level represents the probability of a random number's taking a certain value different from the mean. Generally significance levels are computed for random data from the relation (2)

$$t = \pm \frac{x - \bar{x}}{(\overline{x'^2})^{1/2}}$$

where t is the normalized (mean = 0, standard deviation = 1) value of (mean = $\bar{x}$, standard deviation = $\overline{x'^2}^{1/2}$),and the normal distribution function.

Although we know that our data are not independent [16] this test must be applied with a very good confidence as shown by

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* the results of the two next sections appear essentially in tables II, III and IV. ** (see next page)

McIntosh [7]. Some selected levels of significance l and their respective values of t given by $l = \sqrt{\frac{2}{\pi}} \int_t^{\infty} e^{-\frac{x^2}{2}} dx$ are the following.[8]

Levels of significance l in %	t
1.0	2.57582
2.5	2.24
5.0	1.95996
10.0	1.64485
20.0	1.28155
50.0	0.67449

On each of the figures (1,2,3,4) showing the daily mean temperature profile and the trend curve, the curves corresponding respectively to the 5% and 1% levels of significance have been drawn assuming that the normal seasonal mean of the temperature departure from the trend is zero for each of the periods analyzed. The significance level's curves are parallel to the trend curve at a distance given in tables III.b. by $|\Delta x| = \overline{x'^2}^{1/2}\, t$ (3)

Although the trend curve is kept the same for all the different sub-periods we considered, we can see that these significance levels are a function of the variance and thus will vary from one period to another, reflecting the variability or the steadiness around the assumed mean state.

** (see previous page) -- We will denote by $\bar{\bar{A}}$ the mean of A computed over the population data for the analyzed period and by $\bar{A}$ the mean of A over any sample. We recall that $\bar{\bar{e}}_{y} = 0$ for the whole period but not for the periods of the sunspot cycle.

To illustrate the physical meaning of these significance levels, let's consider a point falling outside an 1% level. This means that the ratio of its deviation from the mean value $\bar{x}$ to the standard deviation $\overline{x'^2}^{1/2}$ is greater than the value of t corresponding to the 1% level given by the normal distribution. Thus we can conclude that there is more than (100-1)% probability that this value is significantly different from the mean and occurs not merely by chance. It is evident that the larger the value of t, the greater is the significance of this point. We compute the level of significance for all extremes significant at less than the 10% level. The results are shown in tables IV.

3.2 Significance counts

Before deciding whether a peak must be definitely considered as significant or not, we have to check whether the number k' of peaks falling outside the 1% significance bands has a small probability or not to occur only by chance. A significance count k has been made only for dates significant at any level less than 1% and in such a way that 2 consecutive peaks are at least a number of days apart corresponding to the length of persistence [16] (the values of which are given in tables II).

The significance of the counts is now determined by normalizing k and assuming that it follows the binomial distribution. We have:

$$K = \frac{k - Mp}{\sqrt{Mpq}}$$

where p is the probability of the event occuring, $p = \frac{1}{100}$

$q = 1-p$

Mp is the expected number of counts

The significance level for K is then obtained from the normal distribution by

$$\sqrt{\frac{2}{\pi}} \int_{|K|}^{\infty} e^{-\frac{u^2}{2}} du$$

and its sign determines whether the number of peaks is larger $(K>0)$ or smaller $(K<0)$ than expected by chance. The number of peaks k', the significance count k, its normalized value K and its significance level for the 5,2.5 and 1% bands are reported in tables III.

3.3 Significance levels if the mean is not zero

From (2) we see immediately that the non-normalized significance levels x are symmetric around the mean. Hence if the mean is equal to zero, they are given by (3) and will appear symmetric to the trend curve as shown on figures 1 and following.

But if the mean is no longer zero, the significance levels for the positive and negative values of the temperature departure from the trend will no longer have the same amplitude when referring to the trend. This is a direct consequence of the fact that for any sub-period we subtracted from the daily temperature the trend computed for the whole period instead of recomputing a specific trend each time. As shown in tables II by the values of the seasonal means of the daily temperatures, some sub-periods do appear warmer or colder than the whole period. When the significance levels are computed taking into account that the subperiod has a mean temperature different from the trend, the significance of the counts is a measure of the "ability" of this subperiod to produce more or less singularities than the entire period. However, this analysis does not enable us to decide whether a specific maximum or minimum of the temperature departure curve is more or less significant with respect to the whole period.

Referring to the values given in Tables III.a. we can note the periods of the double sunspot cycle that show more significant singularities than the whole period or any other sub-period chosen randomly.

Minimum	Winter	m-Half
Maximum	Winter	N-MM, NN-M, M
Minimum	Summer	N-MM
Maximum	Summer	M, N

The criteria used are based essentially on the total number of peaks at the 3 levels and upon the level of significance of the counts at each of the 3 levels.

Tables V.a. give the calendar dates for each phase of the double sunspot cycle showing a singularity significant at less than 2% level which appears to be insignificant for the whole period (i.e. with a level>10%).

On the other hand, we can also consider that the trend for the whole period is the expected value of the mean daily temperature variation even for each of the sub-periods. In doing this we assume that the grouping of data has been made randomly. The results have then to be interpreted as a test of this null hypothesis and we will have to conclude that because of the specific tendency of the temperature during one sub-period to average lower (higher) than during the whole period the minima (maxima) of the temperature departure from the trend must be expected to be more significant with respect to the whole period but not necessarily with respect to this sub-period. The results are shown in tables III.b. for a mean length of persistence equal to 3. Tables V.b. give the significance level of the most important peaks appearing in each phase but not significant in the whole period (level $>$ 10%).

Chapter III

The Sunspot Cycle

Section 1. Physical meaning and solar climatic relationship

Since the atmosphere can be considered as a thermally driven system, the pattern of temperature variation is related to the variation of the general circulation. Both are significantly related to the 20-24 year double sunspot cycle -- Willett, H.C. [1,9,10,18] -- therefore it was decided to analyze the data for different phases associated with the double sunspot cycle relatively to the entire period. We can recall briefly the main features of the sunspot cycle. (White, O.R. [13] and Waldmeir, M. [14]). The current data are given as the daily sunspot number which are averaged for the whole year. The Wolf-number, also called the Zurich relative sunspot number and designed by RSS, is the common sunspot index. It measures both the daily number of visible spots and their tendency to occur in groups. Its basic cycle is the so-called eleven-year cycle but the length between two successive maxima ranges from 7 to 17 years. The average length from a maximum (minimum) to the following minimum (maximum) is 6.6 (4.3) years and fluctuates from 3 to 11 years (3 to 7 years). The maximum (minimum) values of RSS fluctuate from 46 in 1816 to 190 in 1957 (0 in 1810 to 11 in 1843) with an average of 104 (6). Figure 6 shows the year-to-year variation of the annual sunspot number from 1871 to 1970.

In addition to the count variation, sunspots show characteristic changes of latitude and of orientation of magnetic field during a single cycle. The zone of sunspot activity in both solar hemispheres moves toward the solar equator as the cycle advances in time. During a single cycle the magnetic polarity of sunspots pairs is opposite in the northern and southern hemisphere. In the next cycle, the two hemispheres retain their opposite magnetic character but the spot fields reverse direction from the earlier cycle. Therefore in addition to the 11-year cycle the sun shows a 22 year magnetic cycle with a tendency for the sunspot number to be alternately lower and higher with successive maxima.

As is evident on the graph, and shown schematically in [1] and defined in [9], the double sunspot cycle may be divided into the following 8 three-year phases:

- N-MM : the 3 years of most rapid increase in average value of RSS following the minimum after a minor maximum M
- MM : the 3 successive years of maximum annual values of RSS
- MM-NN : the 3 years of most rapid decrease of annual value of RSS following the major maximum MM
- NN : the 3 successive years of minimum annual values of RSS following MM
- NN-M : the 3 years of most rapid increase of annual value of RSS following MM
- M : the 3 successive years of maximum annual value of RSS at the minor maximum M (usually but not always lower than MM)

M-N : the 3 years of most rapid decrease of annual value of RSS following M

N : the 3 successive years of minimum annual value of RSS following M

In addition to these 8 phases we can also consider two others: the major and the minor halves of the 22 year sunspot cycle.

These are respectively the samples containing the years beginning at the year following N (NN) and ending with the year of NN (N).

Whereas the shortening and lengthening of the 11-year cycle may occasionally cause one year's overlap or less frequently one year missing between the 3-year phases, the major and minor half periods contain all the years and in such a way that no overlapping occurs. Table VI gives us the years corresponding to these 8 phases of the double for the period 1871-1971.

Section 2. Solar-Climatic relationship

Following H.C. Willett[11], the correlation of long-term indices of solar activity with those of atmospheric circulation and the double sunspot cycle phase relationships of these indices indicate clearly that significant solar-climatic relationships exist on the northern hemisphere. They are highly variable with season, with geographical location and with phase of the double sunspot cycle as indicated by all past synoptic and statistical analysis.

According to [15], three basic circulation patterns determining rather completely the primary character of the northern hemisphere weather pattern, can be defined as follows:

High latitude zonal pattern (HLZ) is characterized by strong zonal westerlies centered in the higher middle latitudes, poleward of their normal latitude, and by lower than average meridional circulation. It is the high zonal index pattern.

Low latitudinal zonal pattern (LLZ), stable intermediate stage between the high-index and low-index meridional types, is characterized by an equatorward shift of the zonal westerlies and zonal climatic belts.

Blocking cellular (BC) or weak-zonal westerlies, characterized by large amplitude and short wave length of the upper level wave pattern. It is the low zonal index pattern.

It is interesting to note [12] that the index cycle -- BC → LLZ → HLZ -- is closely related to the double sunspot cycle: "Climatically the double sunspot cycle is manifested primarily by an opposite trend of change of the pattern of the general circulation in passing from sunspot minimum to alternate maximum". We summarize in Table VII the general character of this solar-climatic relationship. We can see that going from N to MM there is a strong tendency towards increasing prevalence of the climatic stress circulation patterns, measuring a predominance of polar continental anticyclones in high latitudes in winter, and warm dry summers in the interior of the continents. It is the time both of maximum maritime-continental and of maximum summer-winter contrasts. In passing from NN to M, the trend is towards increasing prevalence of LLZ, meaning a more southerly course of the prevailing storm tracks in middle latitudes, generally wetter conditions and wetter cooler summers over continents in lower middle latitudes. It is a period of increasing glaciation. After the minor sunspot maximum M, there is a pronounced tendency towards a shift of the zonal circulation pattern from lower to higher latitudes. This implies a poleward displacement of the storm tracks with a return to warm and dry conditions in lower and middle latitudes. Because of the prevailing dryness conditions, it is essentially an "interglacial" period.

Section 3. Singularities and double sunspot cycle

The same computations and statistical analysis as for the entire period have been made for each of these phases of the double sunspot cycle to determine whether there might be a predominance of singularities during certain of these phases. The numerical and graphical results are shown in Tables II, III, IV and figures 2 to 5.

Table VI. Classification of years 1871-1971 by Phases of Double Sunspot-Cycle.

N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
1890	1871	1872	1877	1879	1882	1885	1888
1891	1872	1873	1878	1880	1883	1886	1889
1892	1892	1874	1879	1881	1884	1887	1890
1915	1893	1895	1900	1903	1905	1908	1911
1916	1894	1896	1901	1904	1906	1909	1912
1917	1917	1897	1902	1905	1907	1910	1913
1935	1918	1919	1922	1924	1927	1930	1932
1936	1919	1920	1923	1925	1928	1931	1933
1937	1937	1921	1924	1926	1929	1932	1934
1955	1938	1940	1942	1945	1947	1950	1952
1956	1939	1941	1943	1946	1948	1951	1953
1957	1957	1942	1944	1947	1949	1952	1954
	1958	1960	1963	1966	1968	1971	
	1959	1961	1964	1967	1969		
		1962	1965	1968	1970		

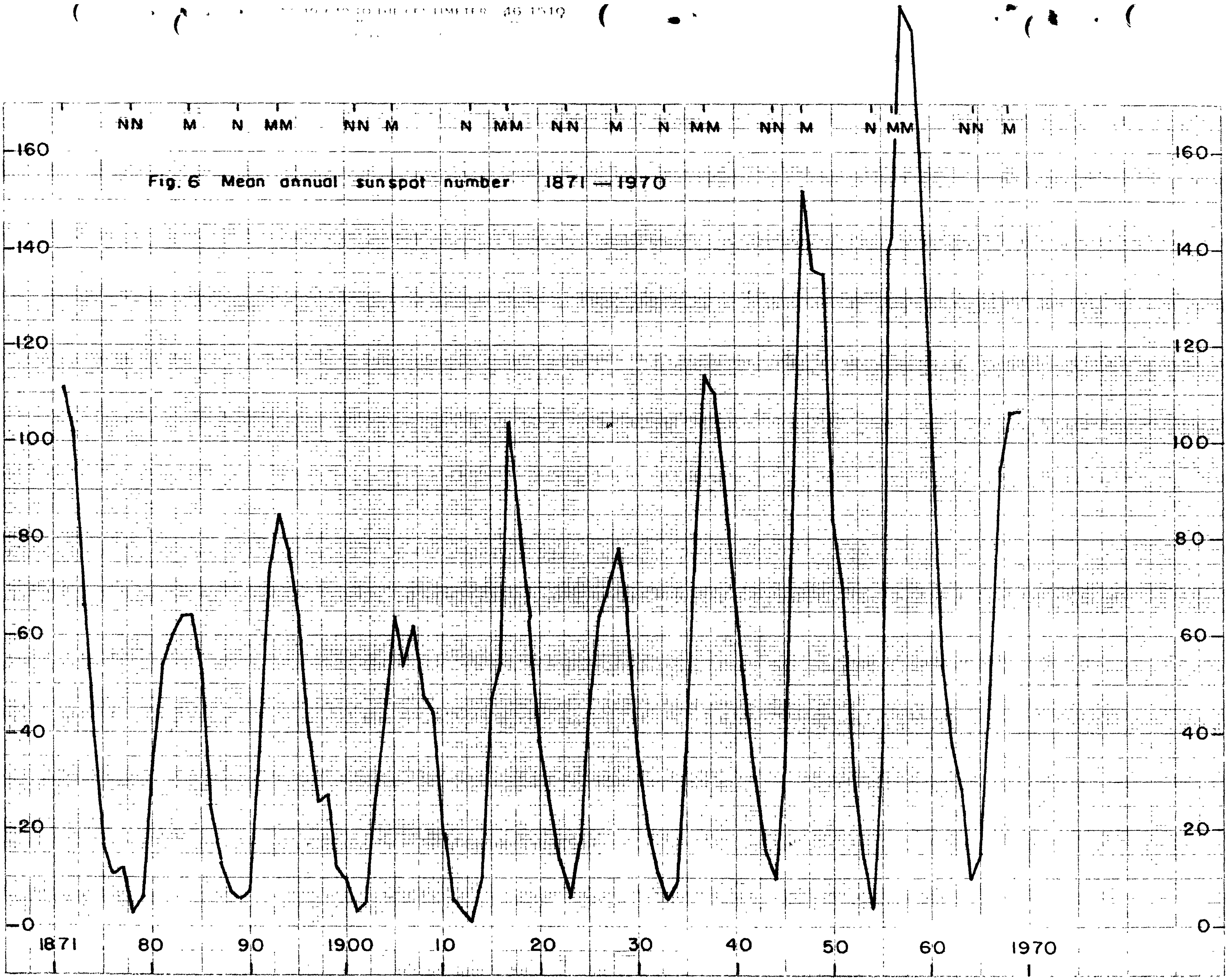


Fig. 6 Mean annual sunspot number 1871 — 1970
NN M N MM NN M N MM NN M N MM NN M N MM NN M
160
140
120
100
80
60
40
20
0
1871
80
90
1900
10
20
30
40
50
60
1970

Table VII. Solar Climatic Relationships

Index Cycle		Climatic Stress	LLZ	HLZ
22-year sunspot cycle		N-MM	NN	M-N
		MM	NN-M	N
		MM-NN	M	
	RSS	highest	low to moderate	moderate to low
SUN	U.V.	strong	steady	low
	SCR	burst	little	strong M-region
Atmospheric Ozone		variable (high at times)	low and increasing	high and decreasing
Geomagnetism				high activity

RSS Wolf number

U.V. Ultra-violet radiation

SCR Solar corpuscular radiation

Chapter IV

Conclusion

Section 1. Significance of the double sunspot cycle

1.1. Standard deviation cycle

From tables II showing the standard error of maximum and minimum temperature for the 8 phases of double sunspot cycle and table VI giving the number of years contained in each phase, it is easy to compute the standard deviations of the daily temperature departures from the trend. Table VIII shows these computed values for the minimum and the maximum temperature during the winter and summer seasons at Omaha and the values found by Newman for Boston [1] for minimum and maximum winter temperature.

The cycle present in these standard deviations (fig. 7) appears very interesting and rather significant. If one considers a large population of random data and divides them randomly into 8 equal subgroups, one would not expect to find the standard deviations for each group of values varying in a systematic way. By means of the F. test of the variance ratio we can compute the level of significance of our double sunspot phase grouping for the minimum and for the maximum temperature data during both the winter and the summer seasons.

As indicated in the last column of table VIII by the significance level expressed in % , the variance of the temperature between the eight phases of the double sunspot cycle is significantly larger than the variance within each phase. Thus the cycle is definitely significant and undoubtedly real.

Moreover as expected if there is any solar-climatic relationship the significance of the cycle appears to be stronger with the direct influence of the sun. It is more significant:in summer than in winter; for the maximum temperature (occuring during the day) than for the minimum (occuring during the night) -- at least for Boston ; and, for Omaha -- a completely continental station -- than for Boston with some maritime influence. These results confirm those obtained by Newman for the winter temperature at Boston [1]. The only differences are that the significance levels are higher for Omaha than for Boston and that, for the winter season at Omaha, the phase relationship of the standard deviation cycle to the sunspot cycle is less significant for the maximum temperature than for the minimum. Probably the most important difference is that for both the minimum and maximum temperature data in winter and in summer at Omaha, unlike Boston, the cycle of the standard deviation has a double rather than single period during the double sunspot cycle and the phase relationships are different during summer and winter. At Omaha, a higher value is found for the winter (summer) during the M-N (N-MM) phase and with a secondary maximum respectively 3 and 4 phases later, whereas at Boston the cycle has only one maximum located during the N phase of the double sunspot cycle. It will be interesting to see if this double-cycle for Omaha is still present for other variables related to the sun activity.

Table VIII Significance of the double sunspot cycle

		N-MM	MM	MM-NN	NN	NN-M	M	M-N	N	(3)
M W Omaha	1	12.96	13.25	12.97	12.96	12.53	12.79	13.89	12.91	10^{-5}
	2	-0.42	-0.73	1.02	0.90	-0.71	-0.53	-0.55	1.80	
M W Boston	1	10.28	10.15	9.93	9.80	9.98	10.24	10.53	10.58	0.06
M W Omaha	1	13.94	14.18	13.43	13.54	13.77	13.23	14.49	14.23	10^{-3}
	2	-0.93	-0.58	-0.08	1.43	0.01	-0.42	-0.36	1.43	
M W Boston	1	10.09	9.90	9.51	9.43	9.97	10.28	10.62	10.97	10^{-6}
M S Omaha	1	6.42	5.80	5.80	6.00	6.32	5.84	6.12	6.37	10^{-13}
	2	-0.01	0.63	0.15	0.06	-0.56	-0.8	-0.02	1.25	
M S Omaha	1	8.80	7.35	7.12	7.46	7.60	7.37	7.86	8.50	10^{-23}
	2	0.66	0.83	-0.09	-0.34	-0.72	-0.96	-0.44	2.15	

1 Standard deviation computed from all data points in the phase (not from average phase values)

2 Mean temperature departure from trend

3 F-level of significance in %

_______ Main maximum; _ _ _ _ _ _ Secondary maximum

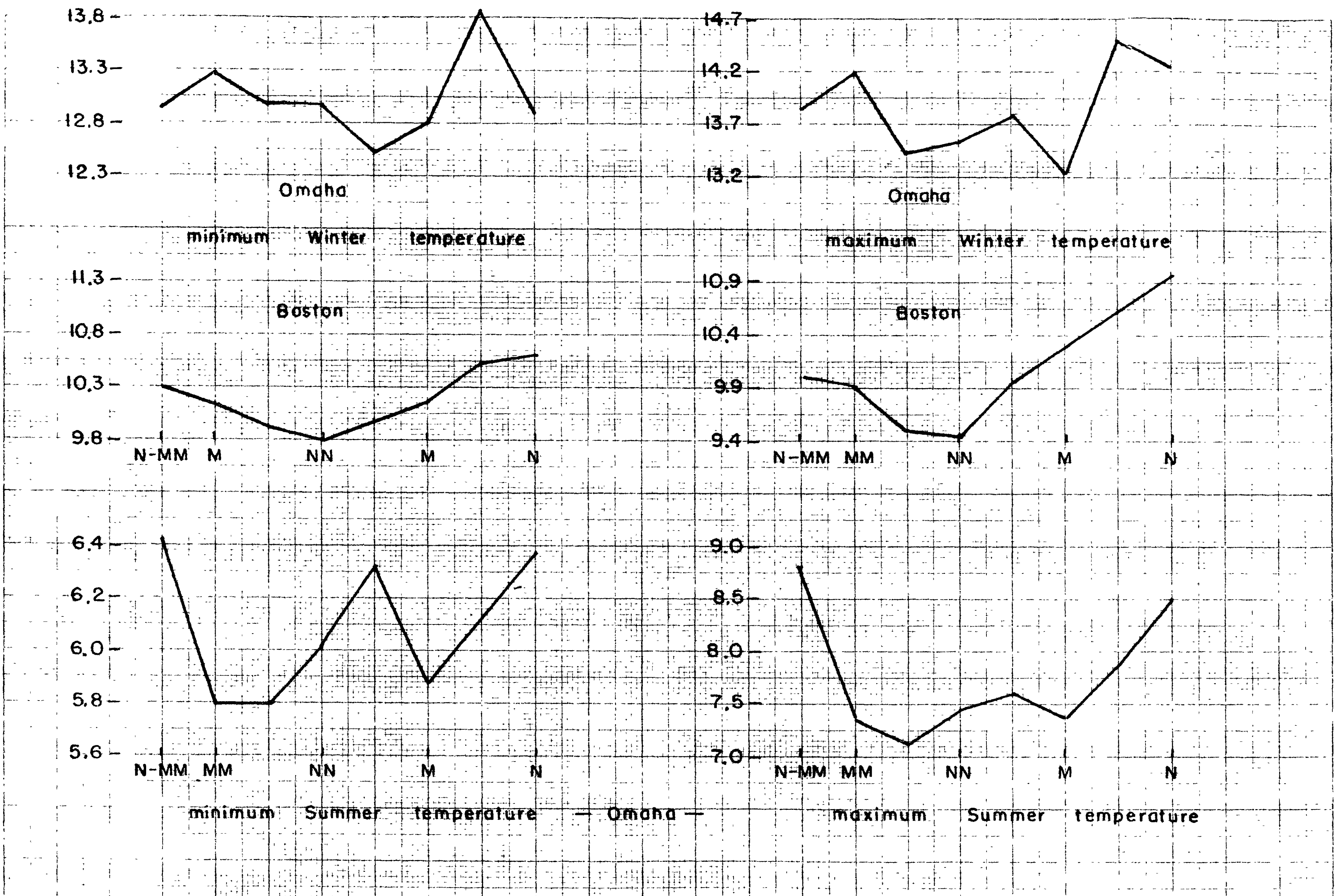


Fig. 7. Significance of the double Sunspot cycle.

This seems to be in agreement with the result found by F. Baur [17] who obtained an excessive frequency of very cold winters and very dry and wet summers in particular parts of the solar activity cycle with a probability of chance occurrence less than 0.027.

1.2 Temperature for the double sunspot cycle phases

The variation of the mean of the daily temperature departures from the normal trend with the phase of the double sunspot cycle shows a probability less than 1% of occuring only by chance. For the minimum temperature in winter, the big jump between the major maximum phase and the MM-NN phase, from the coldest period ($\bar{T} = 15.6$) to one of the warmest ($\bar{T} = 17.3$) probably is a consequence of an eastward displacement of the cellular blocking pattern characterizing this period, thus shifting Omaha toward the warm side of the continental high pressure system.

But for each set of data the maximum contrast between the mean temperature of all the phases of the double sunspot cycle occurs between the M-N and the N phases -- the last one being each time the warmest. The maximum differences appear to be respectively for:

Minimum	Winter	2.3°F
Maximum	Winter	1.8°F
Minimum	Summer	1.3°F
Maximum	Summer	2.6°F

which in every case are significant at less than 10^{-3}% level.

Section 2. Significance of the counts

Tables III.b. contain a count by significance levels of singularities and the significance level of these counts for each of the ten phases and for the entire period. The results are not impressive. Looking through all the periods we have respectively:

	Winter	Summer
for 5% category	3 minimum singularities	4 min.
	2 maximum	2 max.
for 2.5% "	1 minimum	3 min.
	1 maximum	2 max.
for 1% "	0 minimum	2 min.
	1 maximum	1 max.

If the categories are summed individually for maximum and minimum values only the 1% maximum summer category shows a real significance.

In fact, looking through the level of significance of the counts, only their relative values have any meaning. As mentioned in Section 3 it is difficult -- perhaps impossible -- to interpret the absolute values since the effect of the persistence upon singularities is not known.

Section 3. Significance of the singularities

3.1 The entire period

As shown graphically and computed automatically, significance bands were placed first upon the curves for the entire period to determine whether there are any significant singularities. Upon inspection it is found that anomalous values occur on certain dates and their levels of significance are listed in tables IV.a.

3.1.1 Winter season

There are 8 and 5 values respectively for the minimum and maximum temperature which exhibit the largest significance Unlike Newman's results for Boston [1], the minimum temperatures at Omaha show more significant singularities than the maximum temperatures (tables III.b. 1 and 2). Investigating the minimum temperature curve we see also a contrast in the levels of significance that appears in the first half of the winter as compared to the second half. From 1 December to 10 January there are 6 days with a level of 4% or less whereas from mid-January to the end of February only one day (February 6) exceeds this level at 2% level. [At Boston the latter half of the winter shows more significant peaks than the early half.] The most outstanding singularity occurs on 22 December at the 0.1% level. In fact, 21-22-23 December seems to be a highly significant anomalous period but the levels for these 3 days cannot be thought of as separate entities due to the persistence.

The chance probability of obtaining only one value at the 1% level with a total of 90 data points being high (0.92), it is difficult to accept with confidence the future recurrence of a singularity during 21-22-23 December. But before drawing any definitive conclusion about the significance of the counts, it is necessary to analyze carefully the assumptions made in the computation of their levels of significance. In fact applying (2) implies that the data are independant. However tables II.a and 2 give a coefficient of persistence not at all negligible. It was thus decided to estimate again (2) but with M equal this time to the equivalent number of independent data defined by

$$M_e = M \frac{1 - r_1}{1 + r_1}$$

assuming now that the Markov chain model of order 1 is applicable. In fact this model gives a good approximation to the interdependance of our data [16] but we have considered it to be too strict a criterion. Indeed we don't have necessarily reoccurrence of a peak within the period of persistence and this test does not take into account the conditional probability of having a singularity if the preceding day has had one. It was therefore decided to compute (2) with M equal this time to the number of persistent periods:

$$M_\ell = \frac{M}{\ell_a} = M (1 - r_1)$$

The results are labelled L′ and shown in tables III. The probability at the 1% level is still high (0.27) in such a way that definitively we cannot draw conclusions as to the reoccurrence of this singularity.

3.1.2 Summer season

As expected we find for this season considerably lower variance than for the winter season both for the minimum and maximum temperature. This result applies to all the phases of the double sunspot cycle and is a direct consequence of the fundamental difference between the patterns of behavior of the general circulation during the two seasons. In summer both for the minimum and maximum temperature the only period which seems to show any noteworthy singularities is August. For the minimum temperature, the most significant singularity is a minimum occuring 12 August, while for the maximum it is a maximum occurring 30 August. These singularities are still present in the opposite data set but with a lesser degree of significance. However even with the strongest criterion the chance of having one peak at the 1% level of significance is still high and the physical reality of these singularities seems doubtful.

3.2 Phases of the double sunspot cycle

If these singularities have a physical cause which operates only in certain years, this type of averaging overall years can not clearly bring out significant singularities. It shows only a residual effect. Some rationally objective sub-grouping of the data should be tested. Upon inspection it is found that singularities occur on certain dates during some phases of the double sunspot cycle but not during others (tables V.b).

3.2.1 Singularities in winter

The preferred date of 22 December appears significant only during the major half of the double sunspot cycle in both minimum (0.2%) and maximum (0.8%) temperature data. During the phase of the minor maximum this significant peak is found 2 days earlier on

20 December (0.3%) only for the minimum temperature and during the NN-M phase, later on 26 December for the minimum temperature (-0.1%) and 27 December for the maximum (-0.7%) (but with a reversed sign). On the other hand, 2-3 January, significant at about 5% level for the entire period, appears to be a highly significant minimum in the minimum temperature for only the minor half of the double sunspot cycle (-0.3%). For all these dates the only significant count at 1% level occurs during the NN-M phase for the maximum temperature and seems to confirm the result found earlier that this phase has a relatively higher probability of producing singularities than any other period.

Returning to the idea of an eventual division in the winter season, average values of significance were computed for each phase by using the absolute value of the individual levels. In addition an average was computed for each of the two apparent halves of the winter season. Another method of testing this discrepancy in winter involves use of a weighted mean. The reciprocals of the values in tables IV.b and their averages are computed in the same manner as before. The occurrence of a value at the 1% level is in this scheme weighted 10 times more than an occurrence at 10% level and the larger values indicate higher significance. The results are shown in table IX and seem to show that higher anomalous values appear for the period from 1 December to 10 January during all the period but principally during the major half of the double sunspot cycle, while for the remainder

<u>Table IX</u> Significance of the singularities for winter season

	Minimum Temperature						Maximum Temperature					
	Mean			Weighted Mean			Mean			Weighted Mean		
	All days	D14-J3	F1-F6	All days	D14-J3	F1-F6	All days	D4-D22	F1-F17	All days	D4-D22	F1-F17
Normal	2.9	2.5	3.7	1.47	2.04	0.32	5.3	6.5	3.0	0.29	0.18	0.50
Major-Half	30.6	33.4	25.0	0.70	1.03	0.05	26.8	28.0	24.3	0.19	0.26	0.05
Minor-Half	34.1	39.1	24.0	0.78	0.14	0.08	36.6	42.4	25.0	0.23	0.30	0.09
N-MM	45.3	30.5	59.0	0.05	0.05	0.03	53.8	59.8	41.7	0.15	0.05	0.34
MM	36.1	31.5	45.3	0.16	0.06	0.34	32.1	30.7	35.0	0.10	0.06	0.18
MM-NN	39.8	46.2	27.0	0.05	0.05	0.04	65.3	61.5	73.0	0.02	0.02	0.01
NN	27.3	27.3	27.3	0.13	0.12	0.15	39.8	41.5	36.3	0.08	0.05	0.14
NN-M	35.2	27.8	50.0	0.16	0.23	0.02	29.1	36.0	15.3	0.07	0.04	0.12
M	36.3	25.7	57.7	0.06	0.08	0.02	62.0	62.3	61.3	0.02	0.02	0.02
M-N	42.0	54.5	17.0	0.04	0.02	0.07	53.3	60.2	39.7	0.03	0.02	0.04
N	38.6	31.2	53.3	0.04	0.04	0.02	42.3	50.8	25.3	0.04	0.03	0.07

of the winter they occur only during the major maximum phase of the double sunspot cycle.

3.2.2 Singularities in summer

This season seems to show less consistency in the singularities than the winter. In fact, referring to the 2 significant singularities found for the entire period, neither of them has any significance for the different phases of the double sunspot cycle.

The only interesting feature seems to be the appearance of singularities specific to certain phases only and especially to the N-MM, M and N phases for both the minimum and maximum temperature.

During the N-MM phase, the period 5-6-7-8 June shows a very significant minimum in both minimum (J7 - 0.07%) and maximum (J6 - 0.2%) temperature data. The probability of the chance occurrence of the counts at 1% level remains however relatively high, respectively 7% and 15%; hence it does not enable us to count on any recurrence of this singularity in the future. During the M phase, 12 June and 3 August (13 June and 3 August) are 2 minima in the maximum (minimum) temperature data, significant respectively at 0.4 and 0.2% (0.6 and 3%). In this case the chance probability of the number of peaks significant at less than 1% level is very small, but for the maxima only (0.0007%). As this probability is also obtained when we analyze this phase independently of the entire period it is difficult to reject it's significance.

For the N phase in the minimum temperature, 28-29-30 June is a period with a daily singularity significant at less than 0.6% level. Moreover the N phase shows a count of the significant peaks at 5, 2.5 and 1% level which is significant at respectively 0.02, 0.1 and 0.02. Since this does not hold as we analyze this phase independently of the entire period, it is doubtful that the singularity is real. For the maximum temperature, although the chance probability of the significant counts at 5% level is 0.005, the significance of the counts at 1% level is not strong enough to show any positive conclusion.

As a conclusion, it is a matter of fact that the entire period shows singularities in the latter half of the summer season, whereas singularities appear essentially during June when we analyze each of the phases of the double sunspot cycle relatively or not to the whole period. Over 16 (16) singularities significant at less than 2% level and particular at the different phases of the double sunspot cycle, 13 (9) are located in June for the minimum (maximum) temperature and it is possible to conclude that there is a tendency for singularities to occur in the first half of the summer during the M and N phases of the double sunspot cycle.

List of Symbols

Quantity	Symbol
Normal seasonal temperature in °F	x
Mean of the daily temperature departure from the trend in °F	e
Standard deviation of the normal daily temperature departure from the trend in (°F)	$\left[\overline{e'^{2}_{-j}}\right]^{\frac{1}{2}}$
Standard error in(°F)	$\left[\overline{e''^{2}_{ij}}/N\right]^{\frac{1}{2}}$
Coefficient of autocorrelation at lag 1	r_1
Normalized Besson coefficient of persistence	r_a
Length of persistence in days	l_a
Amplitude of the 1% significance level for the temperature departure from the trend in °F	$\lvert\Delta x\rvert$
Number of anomalies significant at 1% level	k'
Number of anomalies significant at 1% level and at least l_a days apart	k
Normalized significant counts	K
Significance level of the normalized counts in %	L

Table I.1 Normal daily minimum temperature for winter

Days	Period	Trend	M half	m half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
Dec 1	25.0	24.3	25.2	24.8	27.8	25.5	23.9	24.3	22.5	25.5	23.7	30.5
2	24.0	23.8	25.1	23.0	26.9	28.2	21.3	23.5	21.1	23.3	22.3	30.2
3	23.5	23.3	23.7	23.3	25.3	24.1	20.4	25.3	21.9	22.6	22.2	30.9
4	22.9	22.9	23.7	22.0	24.5	22.5	24.9	21.5	22.1	25.1	19.4	26.2
5	22.3	22.4	22.9	21.8	22.6	21.9	22.5	21.9	21.7	23.1	19.5	27.7
6	21.8	22.0	23.0	20.7	21.8	17.6	27.1	22.9	22.2	21.6	18.2	24.1
7	20.6	21.6	21.9	19.3	19.8	16.9	25.9	24.5	22.3	18.3	16.5	24.1
8	20.0	21.2	19.6	20.4	21.8	15.9	21.4	22.3	22.8	19.9	20.5	23.8
9	19.5	20.8	18.4	20.6	21.1	14.9	19.1	20.5	20.9	20.3	22.9	23.0
10	20.4	20.4	19.8	21.0	21.5	15.3	22.8	20.4	19.0	17.5	25.9	25.4
11	20.7	20.0	20.0	21.4	20.8	15.4	25.6	18.9	21.2	24.3	22.2	19.8
12	18.9	19.6	17.6	20.2	18.4	14.8	19.7	18.2	19.7	21.9	21.9	16.7
13	18.2	19.3	17.1	19.3	17.3	12.9	16.2	16.7	18.6	18.1	20.4	21.9
14	16.9	18.9	17.5	16.4	19.6	18.2	17.8	12.1	15.9	15.2	17.8	19.9
15	18.4	18.6	19.8	17.0	22.1	20.7	21.9	15.3	18.2	15.6	16.8	20.6
16	18.2	18.3	20.1	16.2	23.3	22.2	23.6	14.9	15.6	17.9	16.5	20.4
17	16.9	17.9	18.2	15.5	18.8	20.0	20.8	14.6	13.3	18.7	15.2	19.8
18	17.4	17.6	19.8	15.0	19.8	23.1	21.3	14.3	13.9	18.9	11.9	21.6
19	18.2	17.3	19.4	16.9	17.8	21.1	19.1	17.5	17.1	20.9	12.7	22.8
20	18.1	17.0	18.3	18.0	18.5	18.9	17.7	17.2	19.5	23.8	14.0	20.2
21	18.8	16.7	19.9	17.6	19.9	22.1	18.3	21.4	16.1	21.9	14.5	20.0
22	19.5	16.5	21.9	17.0	23.1	22.3	20.2	20.9	13.0	20.6	15.3	21.0
23	18.5	16.2	20.7	16.3	23.0	21.6	18.0	17.3	13.1	16.9	15.0	21.3
24	17.4	16.0	18.5	16.3	21.3	17.2	17.8	17.5	12.2	19.0	16.2	19.0
25	16.6	15.7	16.8	15.7	14.8	14.6	18.2	17.8	9.9	19.7	16.1	18.9
26	15.0	15.5	15.7	14.2	14.8	13.7	17.5	17.1	6.4	16.9	16.9	17.1
27	15.4	15.3	16.1	14.6	18.5	15.5	13.4	14.9	7.1	17.9	17.2	14.3
28	16.1	15.1	16.9	15.2	21.4	17.6	16.9	15.6	8.1	16.1	19.1	16.2
29	15.4	14.9	15.7	15.1	15.7	14.5	18.8	16.9	11.5	14.2	18.2	14.6
30	15.1	14.7	16.4	13.9	16.3	15.4	17.9	13.9	10.7	11.7	17.7	18.1
31	15.2	14.5	17.0	13.3	22.4	17.2	16.6	14.5	11.4	7.8	17.0	15.1

Days	Period	Trend	M half	m half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
Jan 1	14.3	14.3	16.7	12.0	19.3	14.4	15.0	14.3	10.7	9.3	13.4	12.3
2	12.2	14.1	14.7	9.7	16.9	11.9	10.7	11.3	8.3	8.0	10.0	9.8
3	11.5	14.0	13.5	9.5	17.5	10.2	6.7	11.3	7.0	8.9	10.2	9.0
4	13.2	13.8	13.9	12.4	19.6	15.5	6.9	9.7	11.5	8.9	12.2	17.1
5	12.7	13.7	13.9	11.4	18.8	14.9	6.5	11.1	12.5	10.9	7.5	13.3
6	12.8	13.6	15.0	10.6	14.7	11.9	15.1	13.9	12.3	12.5	2.5	9.8
7	13.6	13.5	15.0	12.3	13.9	10.9	16.2	17.6	14.0	13.7	3.9	12.1
8	13.7	13.4	15.1	12.2	18.1	11.0	12.6	16.3	13.0	12.4	9.4	13.2
9	13.7	13.3	16.3	11.2	16.7	12.5	15.0	18.0	12.7	12.2	7.5	12.0
10	14.0	13.2	15.6	12.3	14.5	10.2	17.8	14.9	13.2	15.1	9.4	11.7
11	13.8	13.1	16.3	11.3	13.5	14.2	17.3	15.7	12.5	15.5	10.2	7.8
12	13.7	13.1	14.6	12.9	8.4	12.9	20.7	12.8	11.7	17.2	12.6	9.8
13	21.7	13.0	14.0	11.4	5.8	13.6	20.8	11.6	11.9	14.0	10.2	8.2
14	13.9	13.0	15.8	12.0	7.4	10.2	18.6	18.7	13.9	12.3	10.1	9.5
15	13.2	12.9	14.3	12.2	5.8	9.8	17.4	16.3	14.5	12.5	9.7	9.2
16	12.3	12.9	13.2	11.4	4.1	12.1	15.5	12.5	11.2	11.7	11.8	9.9
17	13.5	12.9	14.7	12.3	8.8	14.8	15.1	16.4	14.0	13.7	10.7	12.3
18	12.8	12.9	12.7	12.9	6.5	12.2	12.7	14.9	16.8	14.1	10.0	12.3
19	12.7	12.9	13.5	11.8	3.7	14.7	16.3	13.7	16.8	7.9	11.0	14.2
20	14.4	12.9	16.5	12.3	8.3	18.6	17.3	18.5	17.5	9.7	12.6	8.5
21	13.0	12.9	15.1	11.0	6.5	12.5	15.9	19.7	13.7	7.1	12.5	11.2
22	12.4	13.0	12.2	12.5	3.6	8.8	13.3	19.1	16.3	7.7	11.4	14.0
23	12.0	13.0	10.0	13.9	5.1	10.8	7.9	14.9	17.1	8.6	13.5	16.8
24	12.4	13.1	10.8	14.0	11.0	14.9	6.1	15.1	17.4	9.4	13.7	14.4
25	13.3	13.1	11.8	14.7	11.3	10.9	9.3	17.1	14.0	10.6	15.8	21.8
26	12.7	13.2	12.5	12.9	8.5	10.7	11.3	16.7	13.6	12.5	14.7	10.7
27	11.9	13.3	10.9	12.8	9.3	10.1	7.1	15.9	13.9	16.2	12.2	16.5
28	12.5	13.4	12.0	12.9	12.8	15.5	9.1	16.1	13.7	14.7	12.3	14.4
29	13.2	13.5	13.4	13.0	11.3	11.5	15.6	21.7	14.9	13.9	5.3	17.3
30	13.0	13.6	13.0	12.9	12.0	9.7	15.3	16.3	14.9	11.4	8.8	18.6
31	12.4	13.7	12.2	12.7	12.0	6.5	15.1	16.0	14.8	13.2	7.3	19.3

Days	Period	Trend	M half	m half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
Feb 1	11.9	13.9	12.3	11.4	8.0	5.7	18.8	15.0	10.7	10.5	9.2	17.9
2	13.3	14.0	13.3	13.2	8.7	8.9	18.9	16.3	11.0	11.1	15.2	21.6
3	14.2	14.2	13.6	14.7	11.2	8.7	19.5	16.4	15.0	10.9	17.6	30.3
4	13.9	14.4	14.3	13.5	10.8	9.9	18.3	16.9	15.5	10.5	15.5	15.6
5	16.4	14.5	17.3	15.5	12.8	15.1	19.1	20.7	15.3	13.7	18.9	16.2
6	17.0	14.7	16.7	17.3	14.8	17.1	17.7	19.7	16.8	16.3	20.5	16.0
7	15.1	14.9	15.6	14.6	16.0	15.8	15.0	19.5	15.5	10.9	18.5	13.7
8	14.9	15.1	15.1	14.8	10.3	14.5	16.2	19.7	18.8	10.1	18.1	13.9
9	14.3	15.3	14.5	14.3	10.3	11.5	20.1	17.5	17.6	9.5	15.8	16.3
10	15.7	15.6	14.6	16.8	13.8	12.9	20.1	14.9	18.3	13.1	16.9	20.8
11	17.1	15.8	15.2	19.0	15.1	13.6	21.8	14.3	19.0	17.7	20.7	20.1
12	16.6	16.0	17.1	16.1	16.4	20.4	20.0	12.7	16.5	14.5	15.6	20.7
13	16.6	16.3	16.5	16.7	10.1	20.1	16.8	12.4	15.5	12.7	16.7	25.5
14	16.2	16.6	15.7	16.7	17.1	11.8	16.9	14.1	15.3	15.1	15.9	24.9
15	16.5	16.8	17.4	15.6	19.5	12.5	23.5	15.3	13.8	18.0	11.0	24.4
16	17.3	17.1	17.5	17.1	18.3	14.5	21.1	14.7	13.8	20.3	15.1	24.2
17	18.4	17.4	19.2	17.6	20.3	14.8	22.7	18.9	13.9	14.3	20.8	24.8
18	18.2	17.7	18.5	17.9	17.8	15.9	19.5	20.1	12.7	12.3	22.3	23.9
19	17.3	18.0	17.4	17.2	18.9	12.6	17.4	20.6	15.1	14.9	17.9	22.3
20	17.6	18.3	17.7	17.4	18.9	12.9	18.9	17.7	16.3	14.4	19.3	17.3
21	18.4	18.7	18.1	18.8	19.7	15.5	18.2	20.5	17.4	16.9	22.6	16.8
22	19.5	19.0	17.6	21.4	18.8	18.8	16.7	22.33	20.8	21.5	22.9	19.7
23	20.4	19.4	18.2	21.9	23.2	23.2	17.7	19.7	21.4	22.8	21.9	21.4
24	20.5	19.7	17.6	23.4	20.1	20.9	16.3	18.1	25.0	25.5	21.3	22.9
25	19.5	20.1	16.3	22.6	16.8	21.9	11.9	15.3	19.9	26.1	24.4	19.3
26	20.2	20.5	17.6	22.8	14.4	25.0	17.5	16.3	20.1	25.9	24.2	18.3
27	20.4	20.9	18.2	22.6	15.1	25.7	15.0	21.7	20.1	24.9	25.1	17.3
28	21.5	21.3	18.8	24.2	16.8	21.2	16.3	23.0	21.6	23.9	28.3	20.7

Table II.1 Statistical parameters for minimum temperature in winter

Parameters	Period	Trend	M half	m half	N–MM	MM	MM–NN	NN	NN–M	M	M–N	N
X	16.3	16.3	16.7	15.9	15.9	15.6	17.3	17.2	15.6	15.8	15.8	18.1
e	0		0.35	-0.35	-0.42	-0.73	1.02	0.90	-0.71	-0.53	-0.55	1.8
$\left[\overline{e'^2_{ij}}\right]^{1/2}$	0.970		1.767	1.512	3.929	3.343	3.686	2.885	2.841	3.201	3.526	3.541
$\left[\overline{\overline{e''^2_{ij}}}/N\right]^{1/2}$	1.302		1.841	1.841	3.742	3.674	3.349	3.347	3.235	3.302	3.854	3.725
r_1	0.614		0.745	0.679	0.782	0.654	0.702	0.672	0.779	0.724	0.744	0.711
r_a	0.611		0.749	0.693	0.785	0.647	0.704	0.666	0.772	0.721	0.758	0.718
l_a	2.6		3.9	3.1	4.6	2.9	3.4	3.1	4.5	3.6	3.9	3.5

Table III.a.1 Significance of singularities for minimum temperature in winter

Parameters		Period	Total	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
$\lvert\Delta x\rvert$	20%	1.24		2.62	1.58	4.61	3.55	5.74	4.60	2.93	3.58	3.97	6.30
		-1.24		-1.91	-2.29	-5.46	-5.02	-3.70	-2.80	-4.35	-4.63	-5.06	-2.78
	10%	1.60		3.26	2.13	6.04	4.77	7.08	5.64	3.96	4.74	5.25	7.58
		-1.60		-2.56	-2.84	-6.88	-6.23	-5.04	-3.85	-5.38	-5.79	-6.35	-4.06
	5%	1.90		3.82	2.61	7.28	5.82	8.24	6.55	4.86	5.75	6.35	8.70
		-1.90		-3.11	-3.32	-8.12	-7.28	-6.20	-4.80	-6.28	-6.80	-7.46	-5.18
	2.5%	2.17		4.31	3.03	8.38	6.76	9.28	7.36	5.65	6.65	7.35	9.69
		-2.17		-3.60	-3.74	-9.22	-8.22	-7.24	-5.56	-7.08	-7.70	-8.44	-6.17
	1%	2.50		4.90	3.54	9.70	7.88	10.51	8.33	6.61	7.72	8.54	10.88
		-2.50		-4.20	-4.25	-10.54	-9.34	-8.47	-6.53	-8.03	-8.77	-9.63	-7.36
k'	5%	8	45	3	5	4	2	6	4	5	2	4	2
	2.5%	4	23	3	4	2	0	3	2	2	1	2	0
	1%	1	9	1	3	0	0	0	1	2	0	1	0
k	5%	5	34	2	4	3	2	4	4	3	2	3	2
	2.5%	3	16	2	3	1	0	2	2	1	1	1	0
	1%	1	7	1	2	0	0	0	1	1	0	1	0
L	5%	81		-23	-81	-47	-23	-81	-81	-47	-23	-47	-23
	2.5%	61		-87	61	-40		-87	-87	-40	-40	-40	
	1%	92	-35	92	24				92	92		92	
L'	5%	1		41	3	4	72	2	3	4	48	8	53
	2.5%	2		6	1	46	-37	10	14	47	63	57	-41
	1%	27	1	11	0.2				19	7		11	

Table III. b. 1 Significance of singularities for minimum winter temperature, assuming e = 0

Parameters		Period	Total	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
|Δx|	20%	1.24		2.26	1.94	5.04	4.28	4.72	3.70	3.64	4.10	4.52	4.54
	10%	1.60		2.91	2.49	6.46	5.50	6.06	4.75	4.67	5.27	5.80	5.82
	5%	1.90		3.46	2.96	7.70	6.55	7.22	5.65	5.57	6.27	6.91	6.94
	2.5%	2.17		3.96	3.39	8.80	7.49	8.26	6.46	6.36	7.17	7.90	7.93
	1%	2.50		4.55	3.90	10.12	8.61	9.49	7.43	7.32	8.25	9.08	9.12
k'	5%	8	55	4	5	5	2	5	6	6	2	4	8
	2.5%	4	27	2	3	3	1	0	3	4	0	3	3
	1%	1	10	1	2	0	0	0	1	2	0	2	1
k	5%	5	36	3	4	4	1	3	5	3	2	3	5
	2.5%	3	19	1	2	3	1	0	3	2	0	2	2
	1%	1	7	1	1	0	0	0	1	1	0	1	1
L	5%	81		-47	-81	-81	-9	-47	81	-47	-23	-47	81
	2.5%	61		-40	-87	61	-40		61	-87		-87	-87
	1%	92	-35	92	92				92	92		92	92
L'	5%	1		8	3	0.2	-65	14	0.3	4	49	7	0.1
	2.5%	2		57	13	0.03	80	-41	1	3	-42	6	9
	1%	27	1	11	18					7		11	14

Table IV.a.1 Level of significance for selected calendar dates
Minimum winter temperature

Dates	Period	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
Dec. 14	-4	-30	-16	78	-99	-56	-0.7	-42	-32	-86	-83
21	4	11	41	36	7	88	19	97	7	-64	67
22	0.1	0.3	55	7	5	46	23	-33	15	-87	43
23	2	2	79	7	7	83	94	-41	70	-85	35
Jan. 2	-4	90	-0.6	42	-66	-22	-20	-7	-8	-31	-9
	-1	-62	-0.7	32	-37	-2	-21	-3	-15	-36	-6
Feb. 1	-4	-27	-16	-16	-3	29	94	38	-37	-24	52
5	5	16	39	-73	70	33	7	59	-93	17	-97
6	2	35	5	91	35	60	16	32	51	7	-90

Table IV.b.1 Level of significance for selected calendar dates, assuming e=0. Minimum winter temperature.

Dates	Period	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
Dec. 14	-4	-41	-10	87	-82	-76	-2	-29	-24	-74	78
21	4	7	55	42	11	67	11	-83	11	-53	36
22	0.1	0.2	72	9	8	31	13	-22	20	-74	20
23	2	1	97	8	11	63	70	-28	82	-73	15
Jan. 2	-4	74	-0.3	48	-51	-35	-33	-4	-6	-24	-22
3	-1	-77	-0.3	37	-26	-5	-35	-1	-11	-29	-16
Feb. 1	-4	-38	-10	-13	-1	18	70	-26	-29	-19	25
5	5	11	53	-65	87	21	3	78	-81	22	64
6	2	26	9	99	48	42	9	46	63	10	71

Table V.1 Level of significance for highly significant peaks
Minimum winter temperature

Periods	a	b
Period	D22 0.1 , F3 -1	D22 0.1 , F3 -1
M-Half		
m-Half	F24 0.7	F24 1
N-MM		
MM		
MM-NN	F25 -1	
NN	J29 1	J29 0.5
NN-M	D26 -0.3 , D27 -0.9	D26 -0.1 , D27 -0.4 ,D28 -1
M	D20 0.2	D20 0.3
M-N	F6 -0.3 , F7 -1	F6 -0.2 , F7 -0.7
N		F13 0.9

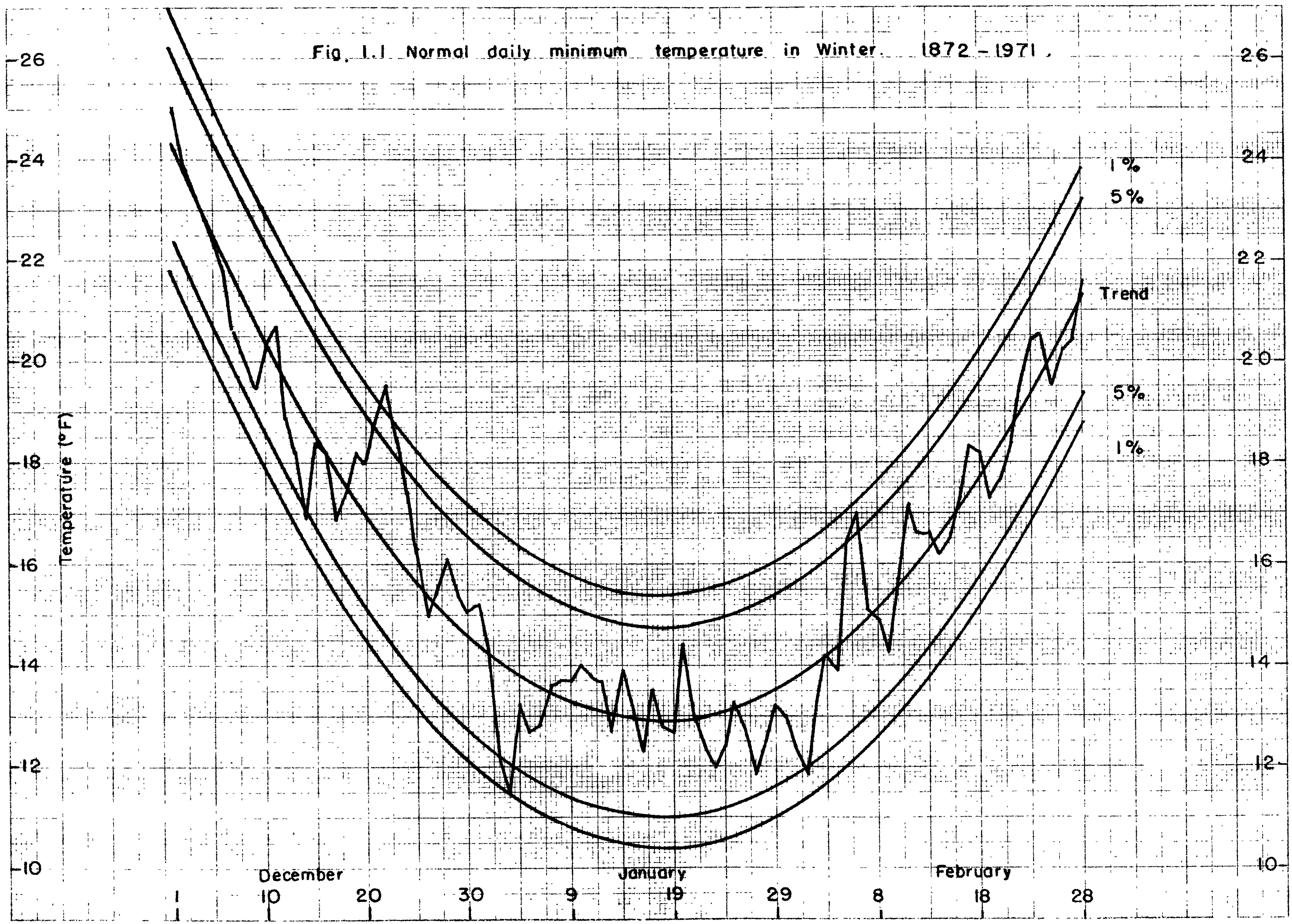


Fig. 1.1 Normal daily minimum temperature in Winter. 1872 - 1971.
Temperature (°F)
26
24
22
20
18
16
14
12
10
1%
5%
Trend
5%
1%
December
January
February
1
10
20
30
9
19
29
8
18
28

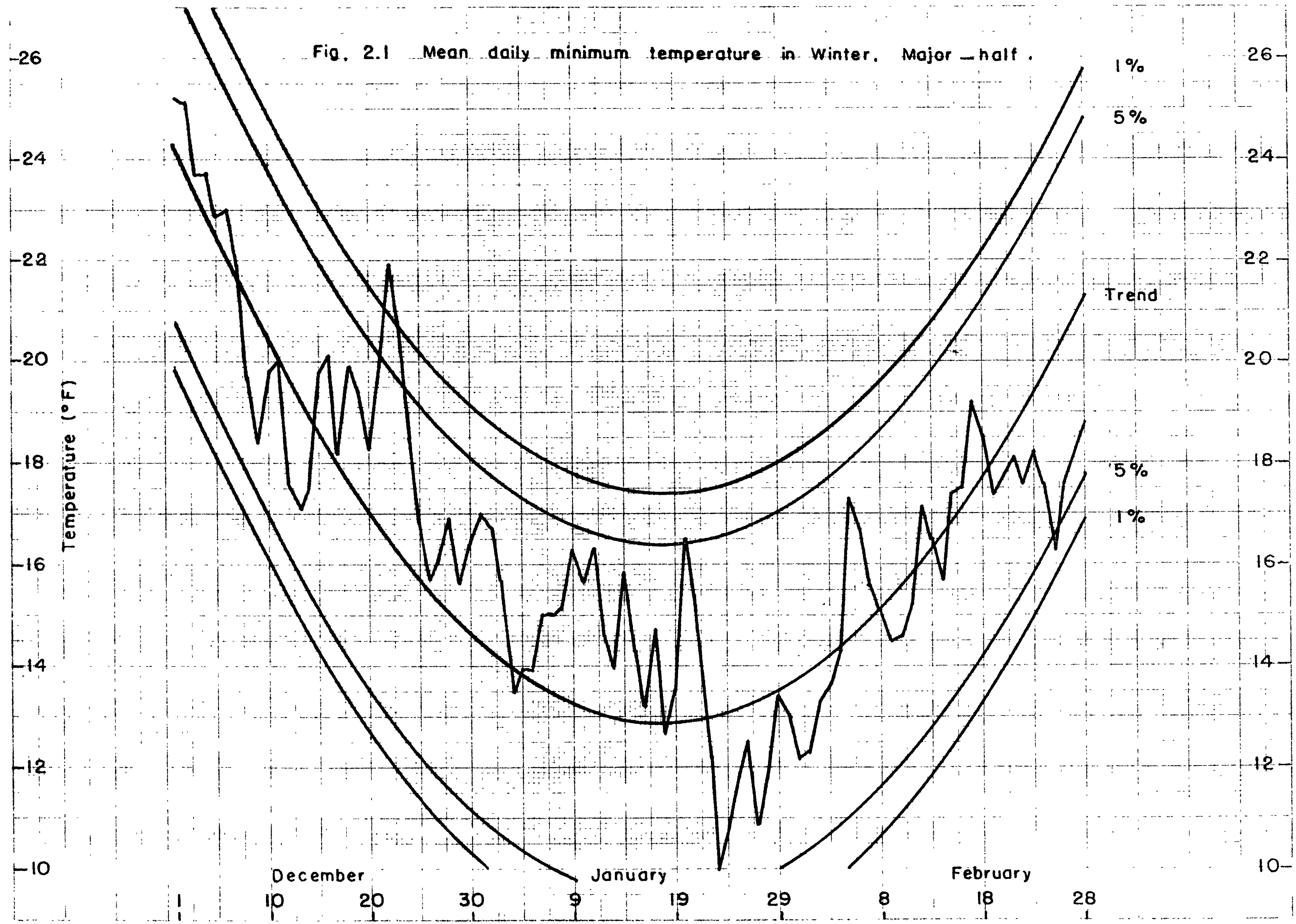


Fig. 2.1 Mean daily minimum temperature in Winter. Major — half.
Temperature (°F)
26
24
22
20
18
16
14
12
10
1%
5%
Trend
5%
1%
December
January
February
1
10
20
30
9
19
29
8
18
28

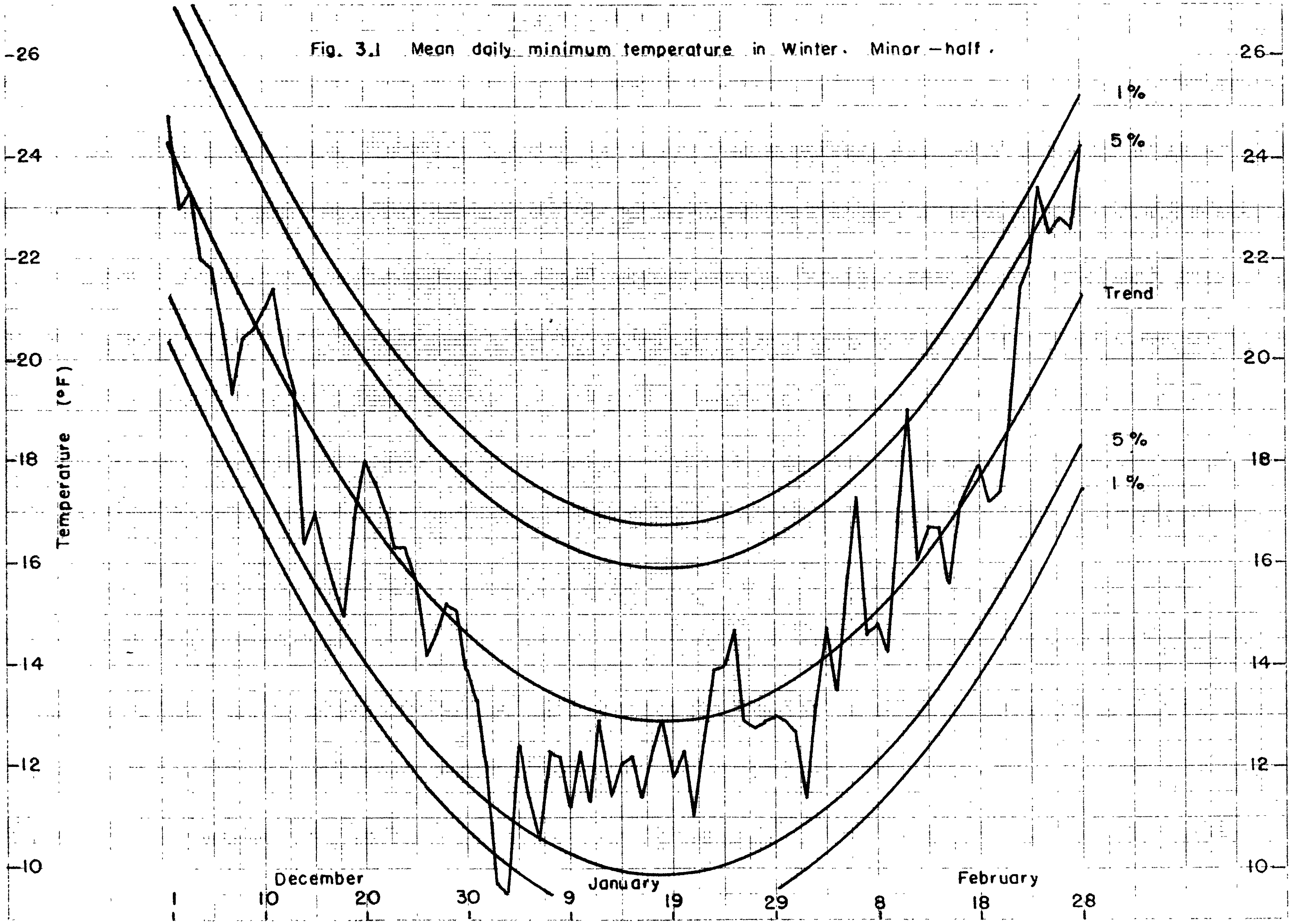
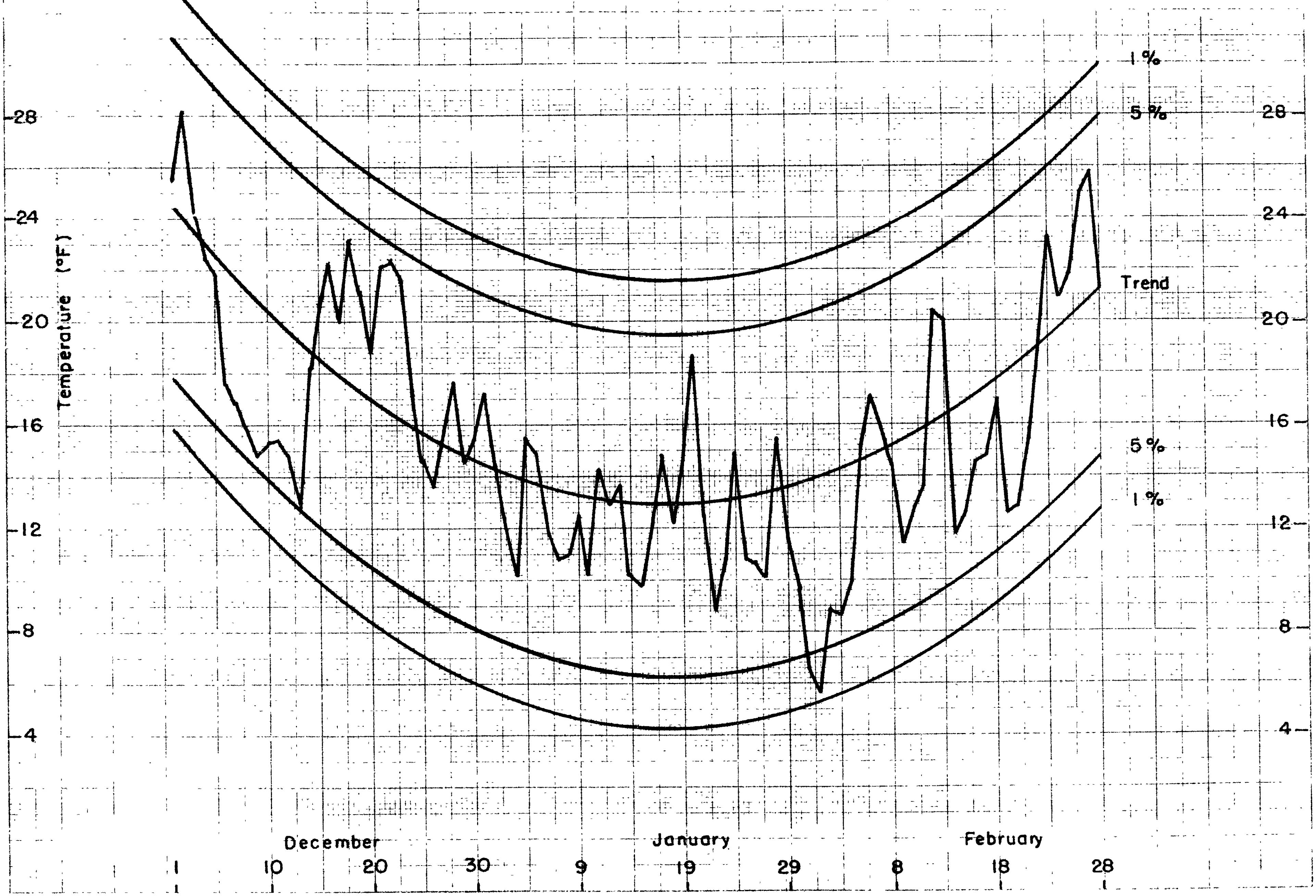


Fig. 3.1 Mean daily minimum temperature in Winter. Minor-half.
Temperature (°F)
-26
-24
-22
-20
-18
-16
-14
-12
-10
26
24
22
20
18
16
14
12
10
1%
5%
Trend
5%
1%
December
January
February
1
10
20
30
9
19
29
8
18
28

Fig. 4.1. Mean daily minimum temperature in Winter. Major maximum.



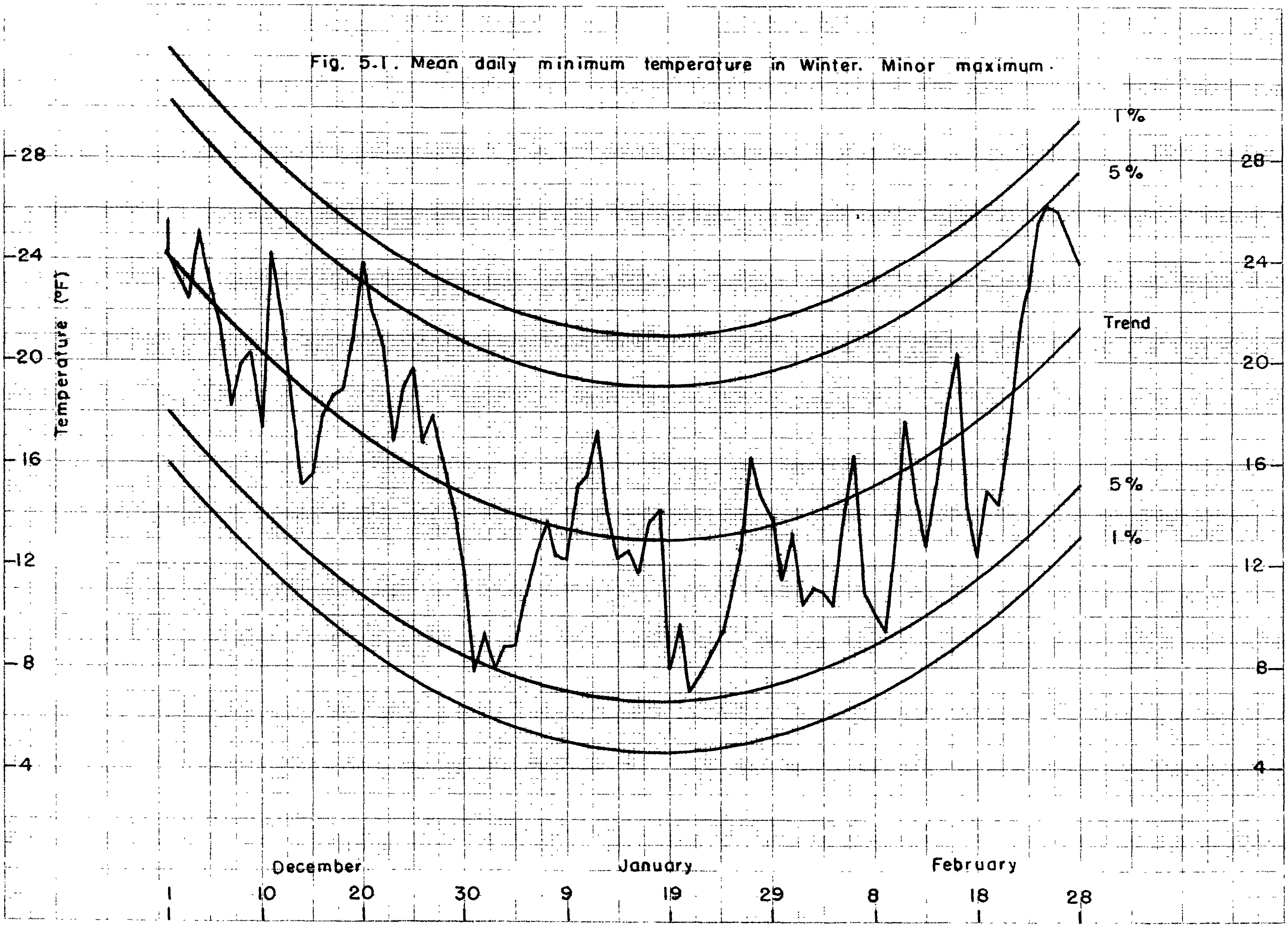


Fig. 5.1. Mean daily minimum temperature in Winter. Minor maximum.
Temperature (°F)
28
24
20
16
12
8
4
1 %
5 %
Trend
5 %
1 %
December
January
February
1
10
20
30
9
19
29
8
18
28

Table I.2 Normal daily maximum temperature for winter

Days	Period	Trend	M half	m half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
Dec 1	42.3	40.8	42.1	42.6	44.1	44.5	38.9	44.1	41.1	43.3	41.5	45.8
2	41.2	40.4	41.3	41.1	43.7	45.6	37.7	41.6	40.5	40.9	40.5	46.8
3	40.9	40.0	41.3	40.5	44.0	44.1	39.3	41.3	41.3	41.1	40.5	45.7
4	41.2	40.0	42.9	39.5	40.9	43.4	42.8	42.8	37.8	42.5	38.6	44.9
5	40.3	39.1	40.6	39.9	37.9	37.5	44.7	40.9	38.9	43.5	35.9	43.4
6	38.6	38.7	38.3	38.8	33.9	32.5	42.9	40.8	40.3	41.8	35.5	41.8
7	37.5	38.4	38.9	36.0	38.1	35.1	43.4	40.9	42.3	35.0	33.4	38.9
8	36.2	38.0	36.1	36.3	36.9	31.3	38.6	37.5	40.1	36.9	36.1	38.1
9	36.7	37.6	36.6	36.8	40.6	33.8	39.0	36.7	37.3	36.3	37.4	40.1
10	36.8	37.2	36.7	36.9	37.9	33.8	38.1	36.3	35.2	36.9	39.3	41.3
11	35.6	36.9	34.8	36.4	35.7	31.9	37.1	35.6	37.3	40.9	36.5	33.3
12	36.0	36.5	35.6	37.5	35.8	29.4	35.1	35.5	35.9	43.7	33.8	35.6
13	34.2	36.2	33.6	34.8	33.3	30.3	34.8	33.3	33.5	35.9	33.2	34.4
14	39.3	35.9	34.7	33.9	34.2	32.6	38.2	30.4	33.2	31.7	34.9	39.0
15	35.4	35.6	38.1	32.7	36.3	39.2	39.8	36.4	33.5	31.0	30.9	37.7
16	35.8	35.3	38.2	33.3	36.0	40.9	41.8	35.5	33.6	36.3	31.9	34.7
17	33.3	35.0	35.8	30.7	34.0	36.8	37.8	34.0	29.9	35.7	30.9	32.7
18	34.4	34.7	37.2	31.6	37.5	39.0	35.7	35.2	33.8	34.9	28.0	36.9
19	34.2	34.4	35.2	33.1	35.4	36.2	32.7	35.1	34.0	37.0	30.2	38.5
20	34.7	34.2	35.5	34.0	34.3	36.2	32.8	37.0	36.0	37.7	32.2	35.0
21	34.5	33.9	36.7	32.3	34.8	38.7	34.4	39.1	31.0	35.4	29.3	34.7
22	35.7	33.7	38.4	33.0	42.2	37.2	33.0	38.9	30.3	34.8	32.9	39.3
23	34.8	33.4	35.1	34.5	40.0	36.2	28.3	35.6	33.3	33.5	33.1	41.0
24	33.9	33.2	34.7	33.2	37.6	37.2	29.9	32.5	29.4	35.3	34.2	37.4
25	33.3	33.0	33.6	33.1	34.4	33.5	33.0	33.5	26.9	39.5	34.0	35.0
26	33.5	32.8	33.5	33.5	37.1	36.1	32.1	32.9	28.5	36.9	35.0	32.9
27	31.2	32.6	30.7	31.8	35.6	33.9	25.3	29.7	24.5	34.4	35.8	33.5
28	33.4	32.4	34.2	32.6	37.3	33.2	33.4	33.0	25.8	32.9	37.2	35.2
29	32.5	32.2	33.0	32.1	39.3	34.2	32.7	32.5	28.4	31.0	36.2	32.0
30	33.1	32.1	33.7	32.5	35.9	34.1	34.4	32.5	30.4	27.2	38.2	35.0
31	31.9	31.9	33.2	30.6	39.8	33.0	30.8	28.8	30.9	22.1	34.5	36.3

Days	Period	Trend	M half	m half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
Jan 1	31.2	31.8	33.7	28.6	36.4	34.9	31.1	29.9	29.5	23.0	29.2	28.8
2	30.3	31.6	32.7	28.0	36.8	32.6	26.4	29.3	26.3	26.8	29.2	25.2
3	30.2	31.5	31.7	28.8	38.9	28.2	25.9	28.7	26.8	23.9	30.9	31.5
4	31.5	31.4	32.5	30.5	39.3	35.9	25.8	28.9	31.4	27.5	29.1	33.2
5	29.9	31.3	30.0	29.9	35.6	30.9	24.3	26.9	29.5	30.3	25.9	29.7
6	31.3	31.2	32.9	29.6	31.9	32.2	32.9	33.8	32.2	31.3	23.0	24.5
7	30.4	31.1	31.7	29.1	29.6	26.4	31.1	35.5	31.6	28.5	24.2	27.1
8	31.9	31.0	33.7	30.2	31.2	32.9	31.9	36.3	31.9	31.7	25.9	28.2
9	31.8	30.9	33.3	30.3	34.4	28.9	32.3	34.9	31.5	33.5	26.5	29.9
10	32.0	30.9	32.9	31.1	31.4	30.5	35.8	33.6	31.3	34.8	25.7	30.4
11	31.2	30.8	32.9	29.4	28.9	29.5	36.7	32.8	29.9	32.3	29.4	24.8
12	30.6	30.8	30.4	30.7	25.7	30.2	36.7	38.1	30.3	34.0	29.5	25.2
13	30.5	30.8	32.2	28.8	26.3	29.5	33.7	33.4	31.5	29.9	28.0	24.2
14	31.5	30.8	33.2	29.8	28.1	28.3	33.0	36.3	31.7	29.9	28.0	27.8
15	30.4	30.7	30.7	30.1	18.4	26.1	33.7	35.6	30.9	28.7	29.7	27.1
16	30.2	30.7	31.4	29.0	24.4	32.1	32.7	32.5	31.6	27.7	29.5	28.5
17	29.7	30.8	29.8	29.7	23.5	28.5	29.1	35.8	33.2	32.1	26.9	27.5
18	30.0	30.8	29.4	30.5	24.0	30.1	29.9	32.5	37.0	29.9	27.4	29.6
19	32.0	30.8	32.0	32.0	27.0	31.6	34.5	3.37	38.7	27.9	31.0	31.7
20	31.6	30.8	33.4	29.8	25.6	33.5	33.5	37.6	33.5	27.3	32.5	27.2
21	31.3	30.9	32.5	30.2	23.8	31.2	33.2	41.3	31.5	26.3	31.0	29.8
22	31.3	31.0	31.0	31.6	24.7	30.1	29.9	38.7	32.9	29.1	30.0	32.9
23	30.2	31.0	28.3	32.1	23.3	31.2	25.2	33.7	35.3	29.1	31.5	33.5
24	32.1	31.1	30.2	33.9	31.3	34.6	23.9	36.4	37.1	28.3	36.0	37.3
25	31.2	31.2	29.9	32.5	31.3	30.8	26.2	33.2	32.5	28.5	34.2	38.3
26	31.0	31.3	30.0	31.9	29.0	27.0	27.1	34.1	32.5	33.2	32.8	37.2
27	30.2	31.4	29.5	31.0	29.2	29.5	24.4	35.5	31.6	31.1	32.9	35.1
28	31.9	31.5	32.4	31.5	30.1	32.3	29.0	34.8	30.4	32.8	31.9	36.5
29	32.1	31.6	31.5	32.7	28.4	30.3	34.4	38.4	32.4	31.2	28.5	39.2
30	31.0	31.8	31.8	30.3	28.8	26.1	34.9	34.0	33.1	29.5	24.2	38.7
31	31.5	31.9	30.4	32.5	27.5	24.0	34.3	33.3	33.4	32.6	25.1	40.8

Days	Period	Trend	M half	m half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
Feb 1	30.2	32.1	29.7	30.6	21.3	23.1	33.4	36.6	27.5	31.0	29.4	38.7
2	32.1	32.3	31.9	32.2	26.4	26.7	34.9	37.1	31.9	29.2	34.6	38.3
3	31.6	32.4	31.3	31.8	28.1	28.4	35.2	36.3	32.9	31.1	32.4	34.7
4	33.4	32.6	34.6	32.2	32.8	31.2	31.3	38.6	33.5	27.7	34.5	37.3
5	34.8	32.8	35.3	34.3	30.3	36.9	31.5	39.6	39.1	31.9	34.6	30.4
6	33.9	33.0	32.8	35.0	30.0	33.6	30.5	38.5	37.0	32.5	37.2	34.2
7	33.5	33.2	33.9	35.1	31.4	36.7	29.9	39.7	34.3	29.0	37.1	33.9
8	32.4	33.5	32.4	32.5	29.5	31.7	34.1	35.3	35.5	28.3	35.3	32.0
9	32.7	33.7	30.6	34.7	37.4	28.5	34.7	33.1	37.3	29.7	38.4	35.3
10	34.9	33.9	33.2	36.6	34.6	30.2	36.7	34.1	34.7	32.7	40.3	42.3
11	34.5	34.2	32.3	36.7	29.2	32.3	39.7	31.6	35.1	35.9	41.5	36.5
12	35.6	34.5	35.3	35.9	34.5	41.5	37.8	32.0	36.5	33.7	35.5	39.4
13	35.3	34.7	36.3	34.3	38.0	39.2	35.9	30.3	35.7	31.1	31.1	46.1
14	34.3	35.0	33.9	34.5	33.7	29.7	37.8	31.3	36.5	30.5	32.2	42.6
15	35.8	35.3	34.6	37.1	37.5	28.9	39.5	31.6	36.8	39.4	30.9	44.7
16	36.8	35.6	37.5	36.0	35.3	33.5	43.7	33.9	30.3	40.9	33.6	42.0
17	38.3	35.9	37.5	39.0	37.6	37.1	36.9	36.3	32.7	39.0	41.1	43.0
18	36.9	36.3	36.3	37.5	33.4	34.2	34.7	40.1	30.6	37.6	39.5	41.9
19	35.4	36.6	35.6	35.2	33.3	30.5	35.7	40.1	34.2	34.7	36.5	34.7
20	35.4	36.9	34.5	36.2	33.3	27.9	35.9	36.9	34.5	34.1	39.6	33.0
21	37.1	37.3	35.5	38.7	34.3	32.6	36.2	39.3	40.1	36.5	40.2	35.4
22	38.0	37.7	36.0	39.9	36.3	35.9	33.5	40.5	42.1	38.9	38.7	36.5
23	37.9	38.0	35.4	40.3	39.8	39.3	34.5	34.9	40.9	39.5	40.2	38.6
24	38.3	38.4	35.2	41.3	36.7	39.2	32.9	36.4	42.9	39.9	41.2	39.4
25	39.1	38.8	37.2	40.9	35.9	43.9	34.3	38.1	4.15	41.1	42.2	33.4
26	38.5	39.2	36.0	41.0	38.2	42.4	33.9	37.2	38.4	42.1	42.9	36.0
27	38.9	39.6	36.4	41.4	28.9	39.3	35.9	42.1	39.7	41.4	46.2	35.3
28	39.7	40.0	37.2	42.2	34.3	36.1	34.3	43.5	44.9	39.1	44.9	38.1

Table II.2 Statistical parameters for maximum temperature in winter

Parameters	Period	Trend	M-half	m half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
X	34.1	34.1	34.2	33.9	33.1	33.5	34.0	35.5	34.1	33.6	33.7	35.5
e	0		0.12	-0.12	-0.93	-0.58	-0.08	1.45	0.01	-0.42	-0.38	1.41
$\left[\overline{e'^2_j}\right]^{1/2}$	0.957		1.791	1.553	4.225	3.710	3.530	3.180	3.021	3.156	3.522	4.158
$\left[\overline{\overline{e''^2_{ij}}}/N\right]^{1/2}$	1.378		1.948	1.951	4.025	3.933	3.468	3.497	3.557	3.415	4.018	4.107
r_1	0.405		0.576	0.635	0.719	0.576	0.686	0.688	0.654	0.637	0.764	0.710
r_a	0.415		0.587	0.646	0.724	0.581	0.695	0.690	0.661	0.633	0.766	0.710
ℓ_a	1.7		2.4	2.7	3.6	2.4	3.2	3.2	2.9	2.8	4.2	3.5

Table III a.2 Significance of singularities for maximum temperature in winter

Parameters		Period	Total	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
$\lvert\Delta z\rvert$	20%	1.23		2.42	1.86	4.49	4.17	4.44	5.52	3.88	3.63	4.16	6.74
				-2.17	-2.11	-6.34	-5.34	-4.60	-2.63	-3.87	-4.46	-4.87	-3.92
	10%	1.57		3.07	2.43	6.02	5.52	5.73	6.68	4.98	4.77	5.44	8.25
				-2.82	-2.68	-7.86	-6.68	-5.89	-3.78	-4.96	-5.61	-6.15	-5.43
	5%	1.88		3.63	2.92	7.35	6.69	6.84	7.68	5.93	5.77	6.55	9.56
				-3.39	-3.17	-9.21	-7.85	-7.00	-4.79	-5.92	-6.60	-7.26	-6.74
	2.5%	2.14		4.14	3.36	8.54	7.73	7.83	8.57	6.77	6.65	7.53	10.73
				-3.89	-3.60	-10.39	-8.89	-7.99	-5.68	-6.76	-7.49	-8.25	-7.90
	1%	2.46		4.74	3.88	9.96	8.98	9.01	9.64	7.79	7.71	8.71	12.12
				-4.49	-4.13	-11.81	-10.14	-9.17	-6.75	-7.78	-8.55	-9.43	-9.30
k'	5%	5	44	1	3	7	4	4	3	7	5	4	1
	2.5%	1	18	1	2	3	2	1	1	2	4	0	1
	1%	0	8	1	1	1	0	0	1	2	2	0	0
k	5%	5	38	1	3	6	3	4	2	5	4	4	1
	2.5%	1	17	1	2	3	2	1	1	2	3	0	1
	1%	0	7	1	1	1	0	0	1	2	1	0	0
L	5%	81		-9	-47	47	-47	-81	-23	81	-81	-81	-9
	2.5%	-40		-40	-87	61	-87	-40	-40	87	61		-40
	1%		-35	92	92	92				24	92		
L'	5%	14		-50	28	0.02	42	3	61	0.5	6	0.3	-78
	2.5%	-77		96	19	0.3	28	73	72	16	1	-46	66
	1%		7	31	24	14			17	0.3	24		

Table III.b.2 Significance of singularities for maximum winter temeprature assuming e=0

Parameters		Period	Total	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
$\|\Delta z\|$	20%	1.23		2.29	1.99	5.41	4.75	4.52	4.08	3.87	4.05	4.51	5.33
	10%	1.57		2.94	2.55	6.95	6.10	5.81	5.23	4.97	5.19	5.79	6.84
	5%	1.88		3.51	3.04	8.28	7.27	6.92	6.23	5.92	6.19	6.90	8.15
	2.5%	2.14		4.01	3.48	9.46	8.31	7.91	7.12	6.77	7.07	7.89	9.31
	1%	2.46		4.61	4.0	10.88	9.56	9.09	8.19	7.78	8.13	9.07	10.71
k'	5%	5	49	1	5	4	3	5	6	7	5	4	4
	2.5%	1	21	1	2	3	2	1	2	2	4	1	2
	1%	0	9	1	1	1	0	0	1	2	2	0	1
k	5%	5	38	1	3	4	2	5	3	5	4	3	3
	2.5%	1	18	1	2	3	2	1	1	2	3	1	1
	1%	0	8	1	1	1	0	0	1	2	1	0	1
L	5%	81		-9	-47	-81	-23	81	-47	81	-81	-47	-47
	2.5%	-40		-40	-87	61	-87	-40	-40	-87	61	-40	-40
	1%		-54	92	92	92			92	24	92		92
L'	5%	15		-50	28	1	95	0.2	17	0.5	6	5	13
	2.5%	-77		96	19	0.3	28	72	72	16	1	51	66
	1%		2	31	24	14	17			0.2	24		15

Table IV.a.2 Level of significance for selected calendar dates
Maximum winter temperature

Dates	Period	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
Dec. 4	8	7	94	59	23	35	57	-56	29	-87	34
8	-7	-28	-33	-98	-10	84	-54	49	-83	-66	-76
13	-4	-13	-42	-65	-15	-71	-17	-37	98	-46	-44
14	-10	-46	-24	-85	-47	50	3	-37	-24	-85	68
17	-7	70	-0.8	-99	52	41	-45	-9	73	-28	-37
22	3	1	-73	3	27	-87	24	-27	62	-90	31
Feb. 1	-4	-16	-39	-2	-2	69	33	-13	-83	-51	21
5	4	19	31	-70	21	-72	9	4	-88	54	-36
17	1	42	4	54	64	76	-74	-29	27	12	17

Table IV.b.2 Level of significance for selected calendar dates assuming e=0
Maximum winter temperature

Dates	Period	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
Dec. 4	8	6	-99	75	30	36	31	-56	36	-79	20
8	-7	-31	-29	-80	-7	86	-87	49	-73	-59	98
13	-4	-15	-38	-50	-11	-69	-37	-38	-91	-40	-67
14	-10	-50	-21	-68	-38	51	-8	-37	-19	-77	45
17	-7	65	-0.6	-82	63	42	-76	-9	83	-24	-58
22	3	0.8	-67	4	35	-85	10	-27	72	-82	17
Feb. 1	-4	-18	-35	-1	-2	71	16	-13	-73	-44	11
5	4	17	35	-54	27	-70	3	4	-78	61	-56
7	1	38	5	70	76	78	90	-29	33	14	9

Table V. 2 Level of significance for highly significant peaks.
Maximum winter temperature

Periods	a			b		
Period						
M-Half						
m-Half	J2 -2			J2 -2		
N-MM	J15 -0.7			J15 -0.3	F27 -1	
MM						
MM-NN						
NN	J21 0.5			J21 0.1		
NN-M	D27 -0.7	J19 0.9		D27 -0.7	J19 0.9	
M	D12 2	D31 -0.3	F1 -0.8	D12 2	D31 -0.2	F1 -0.5
M-N						
N	F13 2			F13 0.6		

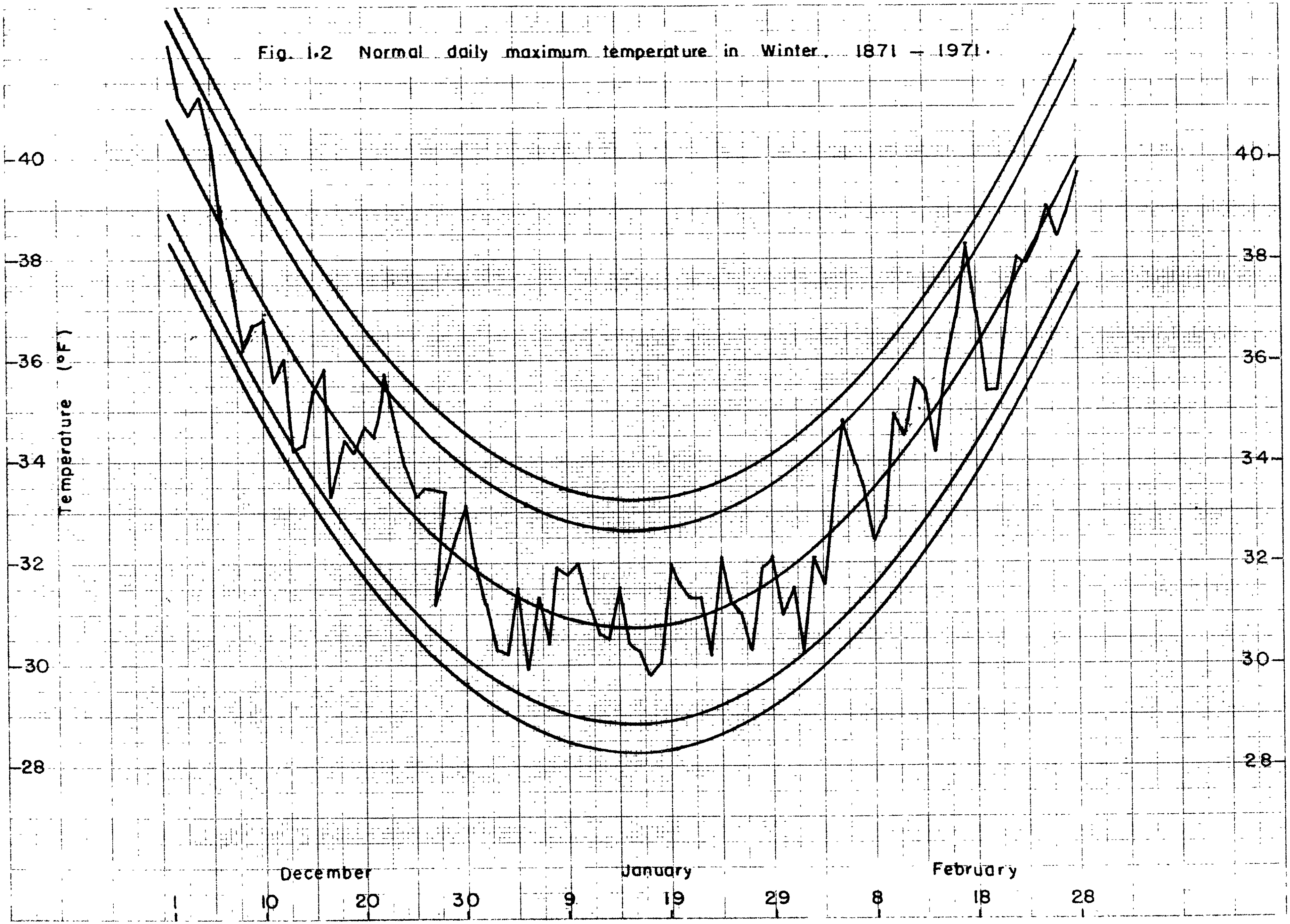


Fig. 1.2 Normal daily maximum temperature in Winter, 1871 – 1971.

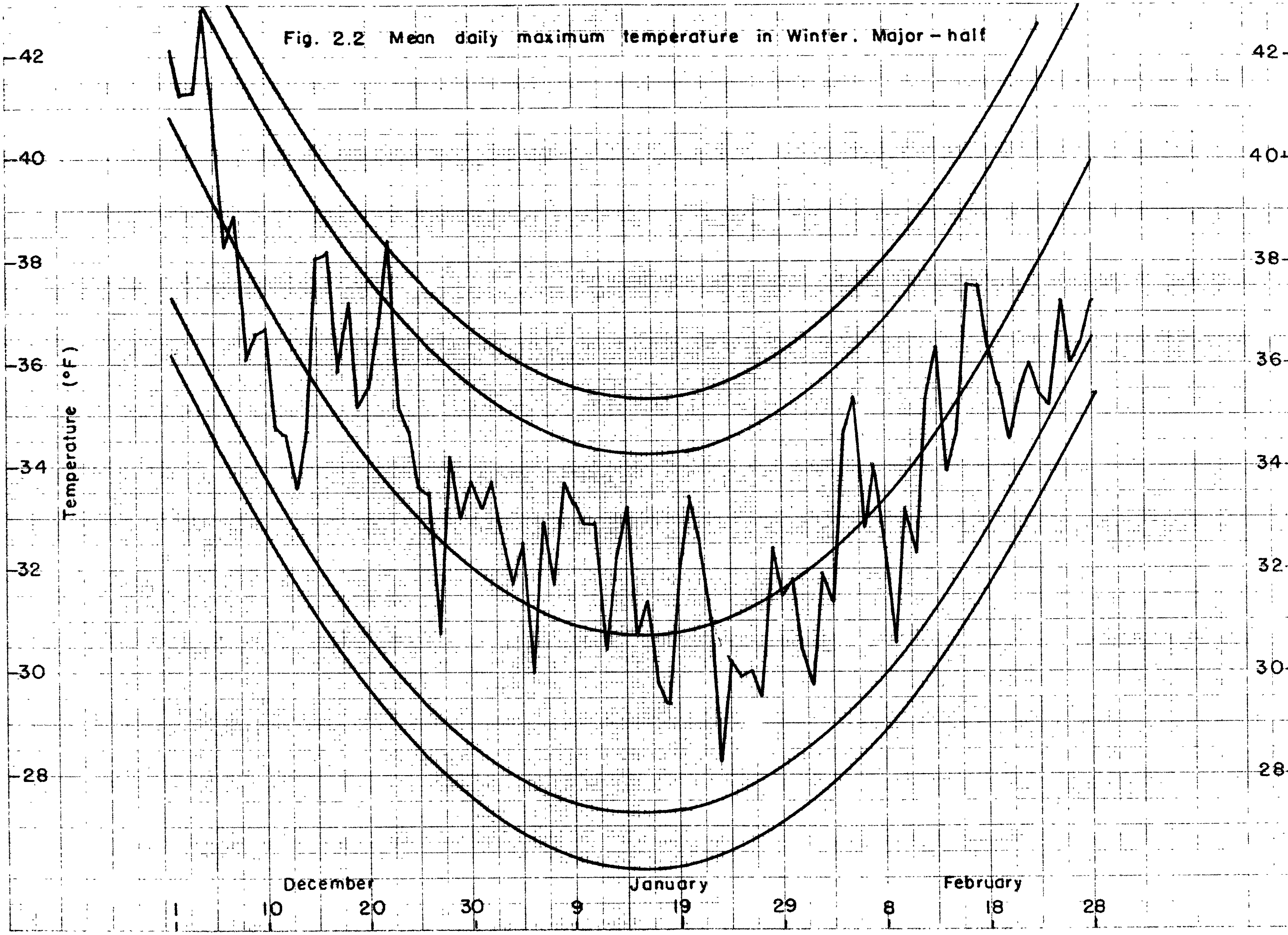


Fig. 2.2 Mean daily maximum temperature in Winter. Major-half

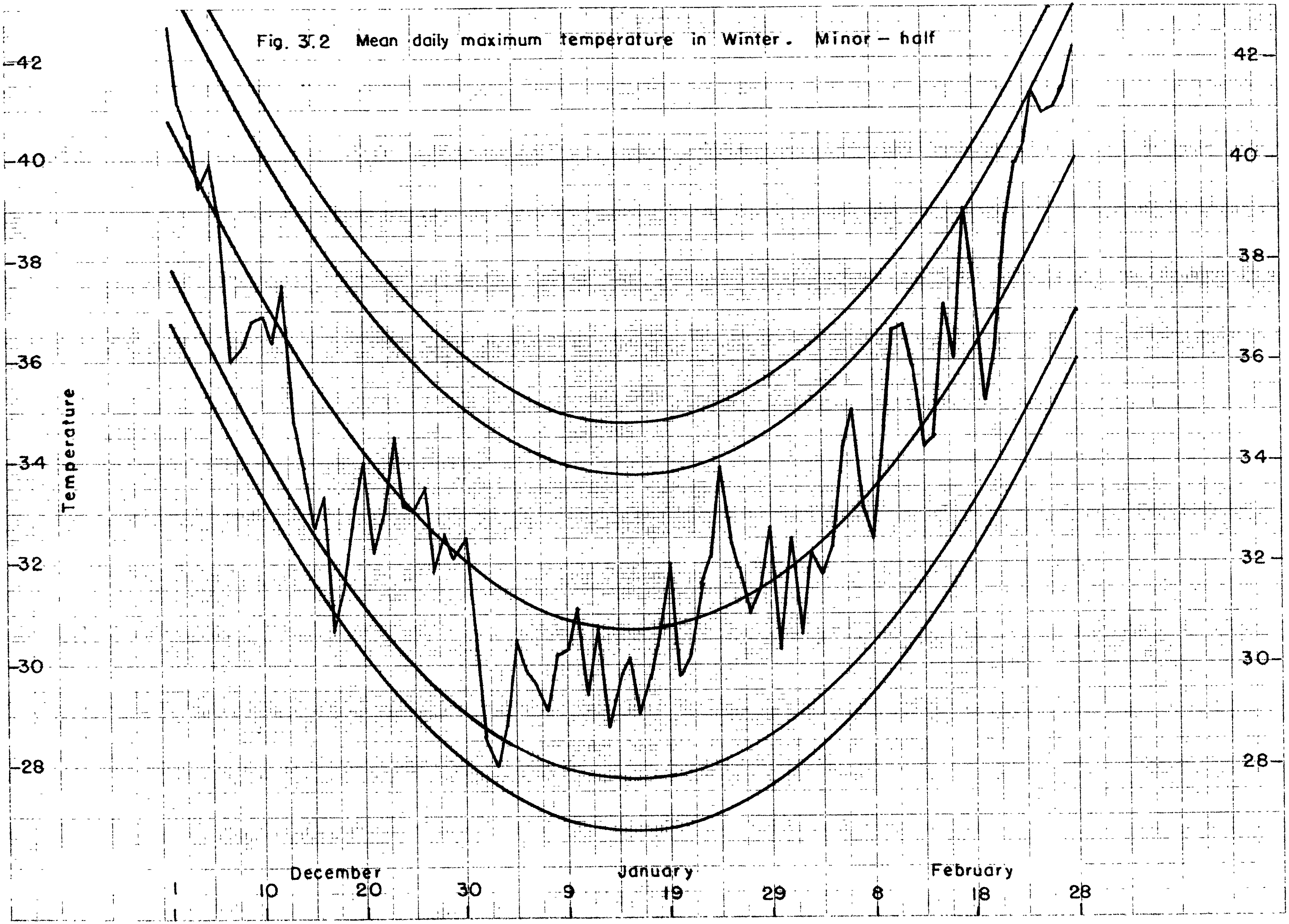


Fig. 3.2 Mean daily maximum temperature in Winter. Minor – half

Fig. 4.2. Mean daily maximum temperature in Winter. Major Maximum.

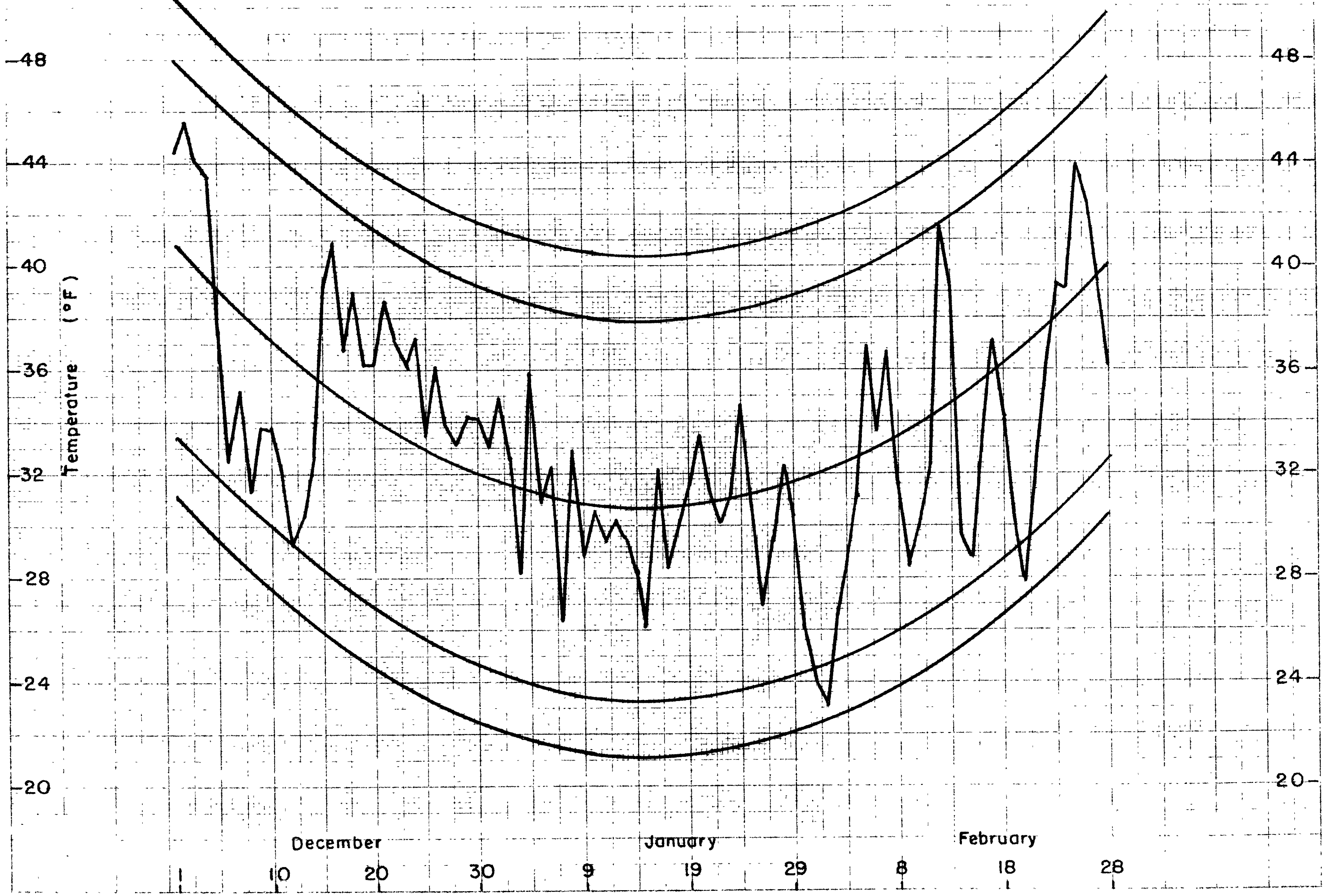


Fig. 5.2 Mean daily maximum temperature in Winter. Minor maximum.

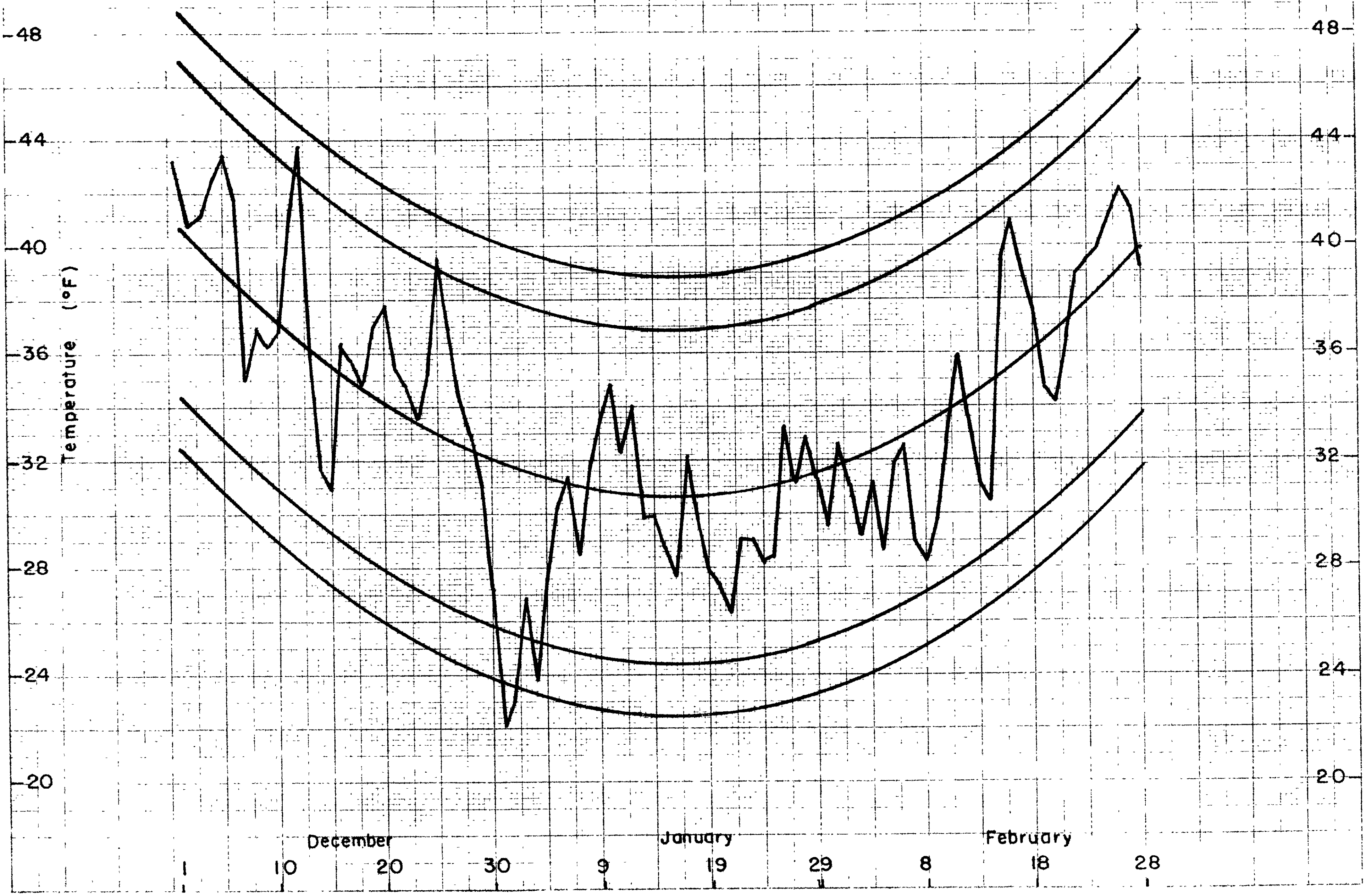


Table I.3 Normal daily minimum temperature for summer

Days	Period	Trend	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
June 1	57.7	57.9	59.4	55.9	57.1	58.1	59.9	59.1	53.7	56.0	57.3	57.6
2	57.8	58.3	59.4	56.0	56.9	57.1	59.9	60.7	54.7	54.4	57.7	59.9
3	58.2	58.6	59.2	57.1	57.4	59.3	58.0	60.4	56.5	54.3	58.4	61.8
4	59.3	59.0	59.6	59.0	57.3	60.7	57.7	60.8	60.5	58.7	58.1	61.2
5	59.0	59.3	59.0	59.0	58.5	59.9	58.3	60.3	61.7	57.3	58.4	62.2
6	59.9	59.7	59.2	60.6	54.7	58.1	61.8	59.5	60.6	60.5	62.2	61.3
7	59.7	60.0	58.8	60.6	52.8	57.9	61.9	58.5	60.1	61.2	61.1	62.8
8	59.6	60.4	59.7	60.0	54.9	59.4	62.5	58.9	59.4	61.1	58.8	60.9
9	59.9	60.7	59.8	60.1	55.8	61.6	61.3	58.9	61.3	61.4	58.6	60.4
10	60.6	61.0	60.7	60.5	58.4	62.2	62.1	59.5	59.7	59.3	60.2	62.5
11	61.4	61.3	61.0	61.8	60.8	63.1	61.5	61.5	62.0	59.0	61.8	64.3
12	61.5	61.6	61.4	61.6	62.2	63.4	61.7	61.7	61.3	57.7	63.3	63.3
13	62.0	61.9	62.8	61.2	62.3	63.7	61.6	62.4	60.7	57.1	64.8	65.2
14	62.4	62.2	62.9	61.9	61.3	62.2	62.1	62.7	60.3	60.2	63.4	64.8
15	63.2	62.5	63.7	62.7	62.3	64.1	62.1	63.6	62.5	60.4	64.0	65.3
16	63.3	62.7	63.4	63.2	62.8	65.9	62.4	61.5	61.5	63.5	63.8	64.3
17	63.3	63.0	63.4	63.2	61.0	65.8	63.7	62.4	61.1	64.7	62.7	64.8
18	63.9	63.3	64.4	63.4	61.8	67.0	66.5	62.5	62.1	62.3	64.2	67.3
19	63.9	63.5	64.7	63.1	63.6	66.4	65.6	63.8	61.8	60.3	63.8	66.7
20	63.5	63.8	64.0	62.9	64.3	66.2	62.9	64.1	63.2	61.8	63.3	65.5
21	63.6	64.0	64.5	62.7	63.9	65.1	64.5	62.3	61.9	62.3	65.3	64.4
22	64.1	64.2	64.9	63.3	63.4	64.1	64.8	64.9	61.5	62.5	66.1	64.3
23	64.9	64.4	65.2	64.5	63.1	65.0	64.1	65.5	65.2	62.3	66.2	67.3
24	65.3	64.7	66.0	64.5	64.5	66.7	64.6	65.3	64.0	64.0	65.7	68.3
25	65.0	64.9	66.4	63.5	65.0	65.5	65.2	66.5	61.2	62.0	66.8	67.7
26	65.6	65.1	66.9	64.1	65.1	66.1	66.3	66.1	61.9	62.4	66.8	68.8
27	65.0	65.3	65.6	64.3	64.5	63.8	67.3	66.1	62.5	61.8	65.6	68.5
28	65.1	65.4	64.9	65.2	65.0	63.8	66.6	63.9	63.7	65.0	65.2	70.5
29	65.8	65.6	65.2	66.3	66.3	64.1	66.2	62.9	64.6	68.3	64.3	71.2
30	66.3	65.8	65.6	67.1	64.8	64.6	67.3	63.7	65.3	66.9	66.6	71.3

Days	Period	Trend	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
July 1	66.1	66.0	65.6	66.6	65.3	64.5	66.7	64.7	63.9	66.8	64.7	70.5
2	66.0	66.1	65.7	66.4	66.0	65.0	66.2	64.0	63.7	66.6	65.0	69.9
3	66.1	66.3	66.3	65.9	66.8	66.6	65.8	65.7	63.5	65.7	65.3	68.3
4	66.3	66.4	66.8	65.8	65.2	66.1	66.4	67.8	63.9	66.5	63.3	68.5
5	66.2	66.5	66.6	65.9	66.8	65.2	65.1	67.7	65.0	65.3	65.3	67.9
6	66.9	66.7	67.6	66.2	69.7	66.3	66.0	68.3	66.0	65.9	64.9	68.6
7	67.1	66.8	67.5	66.8	68.9	68.2	67.3	67.3	67.5	65.5	64.4	68.2
8	67.0	66.9	66.2	67.8	66.8	66.2	66.6	67.7	68.1	66.8	66.3	67.2
9	66.5	67.0	66.3	66.7	66.2	66.5	65.9	67.9	67.6	64.3	66.7	66.4
10	66.9	67.1	67.4	66.3	68.9	67.2	65.8	66.9	66.5	64.5	66.4	68.6
11	67.4	67.2	67.8	67.0	70.7	68.4	65.5	67.2	67.1	66.3	67.3	68.3
12	68.4	67.3	68.9	68.0	71.0	71.3	66.5	68.5	68.9	66.1	66.3	69.8
13	68.1	67.3	68.9	67.4	70.3	70.6	67.3	68.3	66.9	67.4	65.6	72.2
14	67.8	67.4	68.7	66.8	68.6	69.6	68.1	68.7	67.3	67.3	65.8	70.0
15	67.8	67.5	68.5	67.0	68.1	68.1	67.5	69.7	68.0	66.8	67.4	66.8
16	68.1	67.5	68.4	67.7	68.3	67.6	66.1	70.5	68.8	67.5	69.3	67.8
17	67.5	67.6	67.8	67.2	67.8	65.7	68.7	67.9	68.6	65.5	69.3	68.8
18	67.8	67.6	67.8	67.8	68.8	66.1	67.3	67.9	68.1	66.6	70.1	69.4
19	67.2	67.6	67.2	67.2	67.8	66.6	67.1	67.3	66.7	65.5	70.3	68.3
20	67.2	67.7	66.8	67.7	67.3	66.9	67.0	67.7	67.6	65.7	70.6	70.2
21	67.3	67.7	67.2	67.4	67.8	66.3	67.1	67.7	66.8	66.3	68.7	70.1
22	66.9	67.7	68.1	65.7	68.9	66.8	67.1	68.0	65.2	64.1	66.6	68.7
23	67.7	67.7	69.2	66.0	71.2	68.9	68.1	69.5	66.1	65.2	66.1	67.0
24	67.4	67.7	69.0	65.8	68.5	68.2	68.7	69.4	65.2	66.6	65.7	65.8
25	68.3	67.7	68.8	67.7	69.0	68.9	69.0	68.1	66.4	68.3	68.4	68.8
26	68.2	67.7	68.9	67.5	70.3	70.3	67.6	66.9	66.1	68.2	69.3	68.1
27	68.9	67.6	69.3	68.4	72.4	71.1	67.7	67.5	66.9	67.3	71.3	70.1
28	68.2	67.6	68.5	68.0	70.7	70.1	68.3	66.1	67.3	66.7	70.6	68.6
29	67.7	67.6	68.0	67.3	69.8	68.8	70.1	65.7	66.4	65.0	70.3	67.8
30	67.6	67.5	68.1	67.1	70.8	69.1	69.6	66.0	66.6	64.9	68.6	69.4
31	66.6	67.5	67.6	65.6	70.2	68.2	66.7	67.6	65.5	63.7	65.9	67.8

Days	Period	Trend	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
Aug. 1	66.9	67.4	68.0	65.8	68.8	69.0	67.1	68.9	66.5	63.9	66.3	67.0
2	67.0	67.3	67.9	66.0	68.1	67.9	65.8	70.1	66.9	64.6	67.4	67.9
3	66.6	67.2	67.6	65.5	69.5	67.3	66.9	68.2	66.4	62.9	67.9	66.6
4	66.6	67.2	67.4	65.7	66.9	68.1	67.7	68.1	67.7	63.5	67.3	66.1
5	67.2	67.1	68.3	66.0	67.5	68.5	68.7	67.4	68.1	65.1	66.7	65.9
6	66.9	67.0	68.0	65.7	68.5	68.5	66.9	68.0	66.5	67.5	64.7	65.1
7	66.7	66.9	67.4	65.9	67.6	68.5	66.3	67.1	64.3	67.4	66.3	67.0
8	66.6	66.7	68.0	65.1	68.8	68.6	67.9	65.0	64.3	66.3	67.3	65.9
9	66.5	66.6	67.8	65.1	67.7	67.8	67.5	64.9	64.7	66.0	66.6	66.0
10	65.8	66.5	67.0	64.6	66.6	67.1	66.9	65.1	63.1	66.5	65.8	65.5
11	65.0	66.3	65.8	64.2	65.8	65.8	66.3	65.0	64.1	65.5	65.6	63.3
12	64.7	66.2	64.8	64.6	65.1	66.4	64.3	64.0	63.9	65.9	64.8	63.7
13	65.8	66.1	65.4	66.2	64.7	66.2	65.9	64.6	65.1	66.9	66.3	66.4
14	65.7	65.9	65.3	66.2	65.3	66.8	65.5	64.4	64.6	66.0	67.0	67.3
15	66.2	65.7	65.7	66.7	68.2	68.4	63.8	63.5	66.0	66.6	67.3	67.7
16	65.8	65.6	65.1	66.4	67.8	66.8	62.6	62.1	65.0	66.1	67.3	67.3
17	65.7	65.4	64.9	66.6	68.2	65.1	62.7	62.4	65.7	67.1	65.8	65.9
18	65.3	65.2	65.1	65.5	67.7	65.9	62.3	63.9	66.1	66.9	62.9	65.6
19	65.3	65.0	65.5	65.1	65.3	67.3	64.1	65.1	65.9	67.0	61.8	64.6
20	65.0	64.8	64.9	65.0	63.8	66.2	63.8	67.4	66.8	65.7	62.8	65.0
21	63.7	64.6	63.3	64.0	61.7	64.0	63.0	65.6	66.1	64.9	62.5	63.1
22	63.8	64.4	64.3	63.3	61.5	66.1	63.7	63.3	64.3	65.4	62.3	61.5
23	63.2	64.1	63.3	63.2	61.3	65.1	62.7	63.2	65.6	63.8	62.7	61.3
24	63.2	63.9	63.0	63.5	61.3	64.4	63.8	63.1	64.7	63.3	62.8	63.1
25	63.3	63.7	62.9	63.7	63.1	62.9	65.2	62.9	63.9	62.9	62.1	63.9
26	63.8	63.4	63.2	64.4	64.3	63.6	64.0	63.3	64.7	64.9	62.3	65.3
27	64.1	63.2	63.6	64.7	61.8	64.9	64.4	64.5	65.3	66.0	63.3	65.4
28	63.8	62.9	63.3	64.4	62.3	62.4	65.1	63.4	66.1	65.4	63.2	64.2
29	64.0	62.6	63.1	64.9	59.1	62.4	65.7	65.4	66.2	65.0	63.6	65.1
30	63.3	62.4	62.6	64.1	57.4	61.2	64.0	64.9	65.1	63.7	63.1	64.8
31	62.1	62.1	62.3	62.0	59.8	60.9	62.5	63.9	62.8	63.0	59.7	62.7

Table II.3 Statistical parameters for minimum temperature in summer

Parameters	Period	Trend	M half	m half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
X	65.0	65.0	65.0	64.7	65.0	65.6	65.1	65.0	64.4	64.2	64.9	66.2
e	0		0.31	-0.32	-0.01	0.63	0.15	0.06	-0.56	-0.8	-0.02	1.25
$\left[\overline{e'^2_j}\right]^{1/2}$	0.356		0.764	0.942	2.128	1.416	1.259	1.437	1.599	1.774	1.533	1.835
$\left[\overline{\overline{e''^2}}/N\right]^{1/2}$	0.608		0.847	0.871	1.853	1.549	1.497	1.548	1.632	1.507	1.765	1.838
r_1	0.641		0.700	0.711	0.782	0.665	0.590	0.706	0.691	0.707	0.602	0.749
r_a	0.634		0.715	0.726	0.780	0.660	0.597	0.711	0.720	0.705	0.669	0.756
l_a	2.8		3.4	3.5	4.6	3.0	2.4	3.5	3.2	3.4	3.0	4.0

Table III. a.3 Significance of singularities for minumum temperature in summer

Parameters		Period	Total	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
|Δx|	20%	0.69		1.29	0.89	2.71	2.44	1.76	1.91	1.49	1.49	1.94	3.60
				-0.67	-1.53	-2.74	-1.19	-1.47	-1.78	-2.61	-3.06	-1.99	-1.10
	10%	0.88		1.56	1.23	3.49	2.96	2.22	2.43	2.07	2.13	2.50	4.27
				-0.95	-1.87	-3.52	-1.70	-1.93	-2.30	-3.19	-3.70	-2.54	-1.77
	5%	1.05		1.80	1.53	4.16	3.40	2.61	2.88	2.57	2.69	2.98	4.85
				-1.19	-2.17	-4.19	-2.15	-2.32	-2.75	-3.69	-4.26	-3.03	-2.35
	2.5%	1.20		2.02	1.79	4.75	3.80	2.96	3.28	3.02	3.19	3.41	5.36
				-1.40	-2.43	-4.78	-2.54	-2.67	-3.16	-4.14	-4.76	-3.46	-2.86
	1%	1.38		2.27	2.11	5.47	4.28	3.39	3.77	3.56	3.78	3.93	5.98
				-1.66	-2.75	-5.50	-3.02	-3.10	-3.64	-4.68	-5.36	-3.97	-3.48
k'	5%	5	50	3	4	6	4	5	4	4	4	4	7
	2.5%	4	25	1	1	6	1	4	1	2	1	1	3
	1%	1	4	0	1	1	0	0	0	1	0	0	0
k	5%	4	35	3	2	3	4	4	3	2	4	3	3
	2.5%	3	19	1	1	3	1	4	1	1	1	1	2
	1%	1	4	0	1	1	0	0	0	1	0	0	0
L	5%	-77		-44	-21	-44	-77	-77	-49	-21	-77	-44	-44
	2.5%	64		-39	-39	64	-39	26	-39	-39	-39	-39	-84
	1%	93	-5		93	93				93			
L'	5%	6		15	55	4	4	11	15	62	2	23	8
	2.5%	2		70	68	0.04	79	0.1	69	73	69	80	6
	1%	24	62		15	7				18			

Table III.b.3 Significance of singularities of minimum summer temperature assuming e = 0

Parameters		Period	Total	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
$\lvert\Delta x\rvert$	20%	0.69		0.98	1.21	2.78	1.81	1.61	1.84	2.05	2.27	1.96	2.35
	10%	0.88		1.26	1.55	3.50	2.33	2.07	2.36	2.63	2.90	2.52	3.02
	5%	1.05		1.50	1.85	4.17	2.77	2.47	2.82	3.13	3.48	3.00	3.60
	2.5%	1.20		1.71	2.11	4.77	3.17	2.82	3.22	3.58	3.97	3.43	4.11
	1%	1.38		1.97	2.43	5.48	3.65	3.24	3.70	4.12	4.57	3.95	4.73
k'	5%	5	64	5	8	6	7	6	3	6	8	3	9
	2.5%	4	33	1	2	6	4	4	1	2	3	1	5
	1%	1	10	0	0	1	2	0	0	1	1	0	4
k	5%	4	44	4	5	4	4	4	2	4	5	3	5
	2.5%	3	26	1	2	4	3	3	1	2	3	1	3
	1%	1	8	0	0	1	2	0	0	1	1	0	2
L	5%	-77		-77	85	-77	-77	-77	-21	-77	85	-44	85
	2.5%	64		-39	-84	26	64	64	-39	-84	64	-39	64
	1%	93	-50			93	26			93	93		26
L'	5%	6		2	0.1	0.2	4	11	57	3	0.1	23	0.02
	2.5%	2		70	10	0.006	1	3	69	12	0.4	80	0.1
	1%	24	0.6			7	0.2			18	16		0.02

Table IV.a.3 Level of significance for selected calendar dates
Minimum summer temperature

Dates	Period	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
July 12	3	9	26	8	2	-49	40	18	37	-52	47
27	2	7	26	2	4	-97	-91	-93	81	2	51
Aug. 11	-1	-24	-6	-79	-40	-90	-33	-28	-99	-63	-2
12	-0.5	-2	-18	-61	-78	-10	-12	-29	79	-38	-4
21	-9	-4	-82	-17	-40	-17	50	20	54	-18	-14
23	-9	-12	-49	-19	78	-20	-49	20	80	-35	-3
27	7	91	4	-54	42	38	39	9	4	90	58
28	9	92	6	-77	-44	10	76	2	6	85	99
29	1	87	0.6	-10	-53	2	6	1	7	52	51
30	6	93	3	-2	-21	23	9	4	22	62	53

Table IV.b.3 Level of significance for selected calendar dates assuming e=0
Minimum summer temperature

Dates	Period	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
July 12	3	4	44	8	0.5	-56	38	32	65	-51	16
27	2	3	43	2	1	93	-95	-66	-84	2	18
Aug. 11	-1	-43	-3	-78	-69	-99	-35	-15	-65	-62	-9
12	-0.5	-7	-9	-60	87	-12	-13	-16	-85	-37	-17
21	-9	-9	-57	-17	-69	-21	47	35	87	-18	-42
23	-9	-25	-31	-19	47	-25	-52	36	-85	-34	-13
27	7	61	9	-53	21	32	36	17	11	91	22
28	9	62	12	-76	-74	8	72	4	16	86	49
29	1	57	2	-10	-85	2	5	3	18	53	18
30	6	75	6	-2	-42	19	8	8	44	63	19

Table V.3 Level of significance of highly significant peaks.
Minimum summer temperature

Periods	a	b
Period	A11 -1 , A12 -0.5 , A29 1	A11 -1 , A12 -0.5 , A29 1
M-Half		J26 1
m-Half		J2 -2
N-MM	J6 -2 , J7 -0.08 , J8 -1	J6 -2 , J7 -0.07 , J8 -1
MM		J18 0.8
MM-NN	J18 2 , A16 -1	J18 1
NN	A16 -1	A16 -2
NN-M	J1 -3	J1 -1
M	J13 -2	J3 -2 , J13 -0.6 , A3 -2
M-N		
N	J29 2 , J30 2	J28 0.6 , J29 0.3 , J30 0.3

J = June   Jl = July   A = August

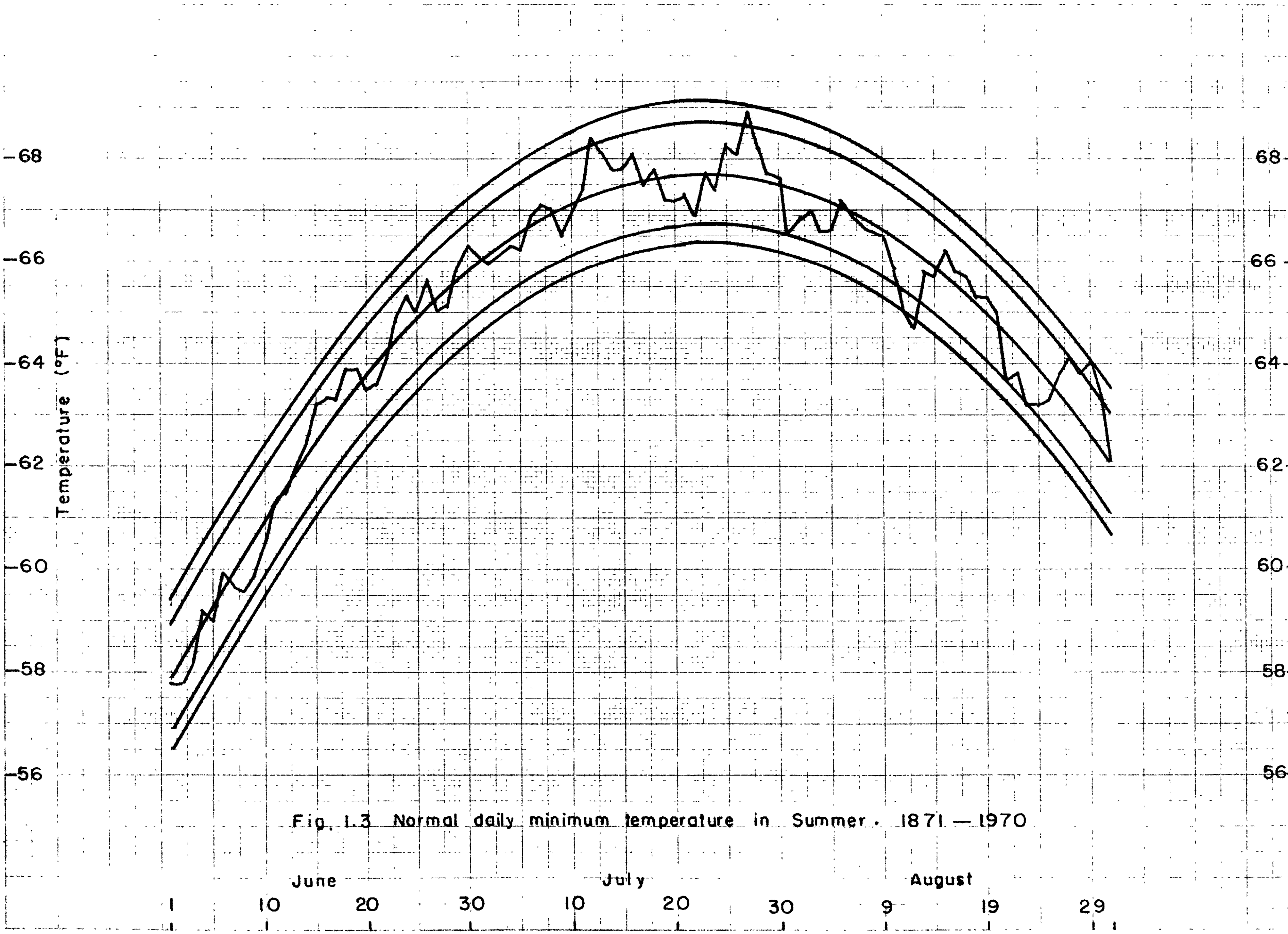


Fig. 1.3 Normal daily minimum temperature in Summer. 1871 — 1970

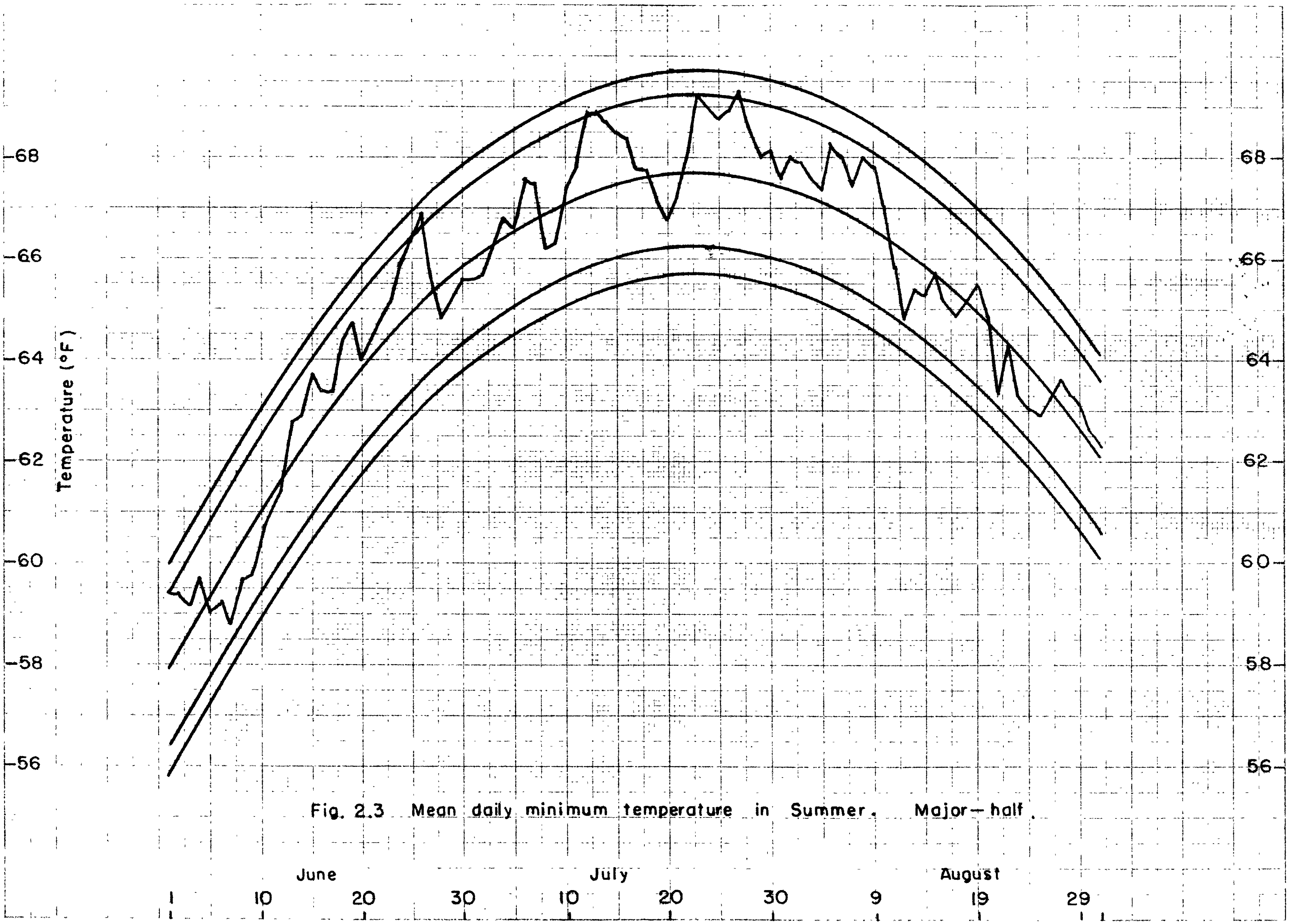


Fig. 2.3 Mean daily minimum temperature in Summer. Major—half.

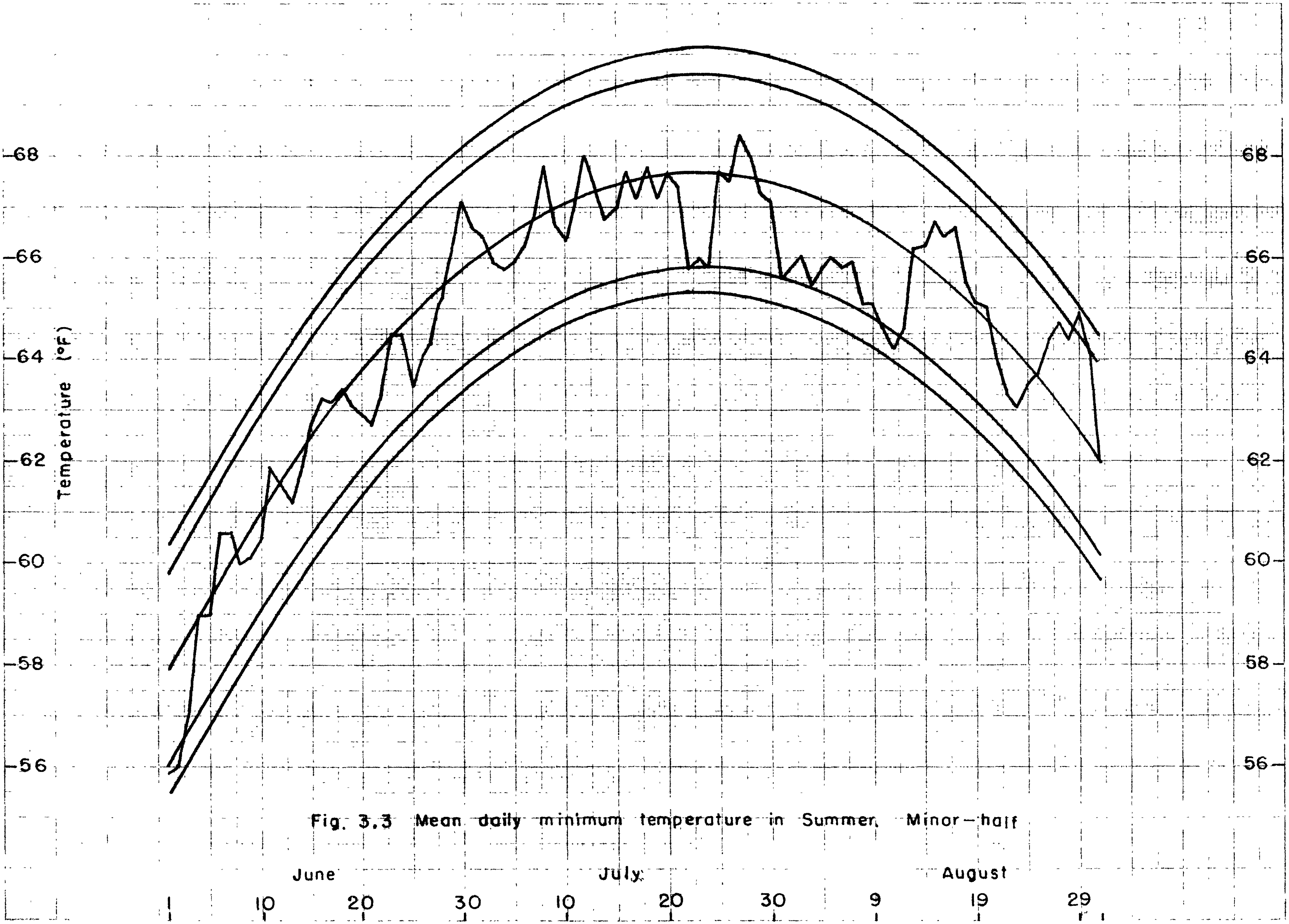


Fig. 3.3 Mean daily minimum temperature in Summer, Minor-half

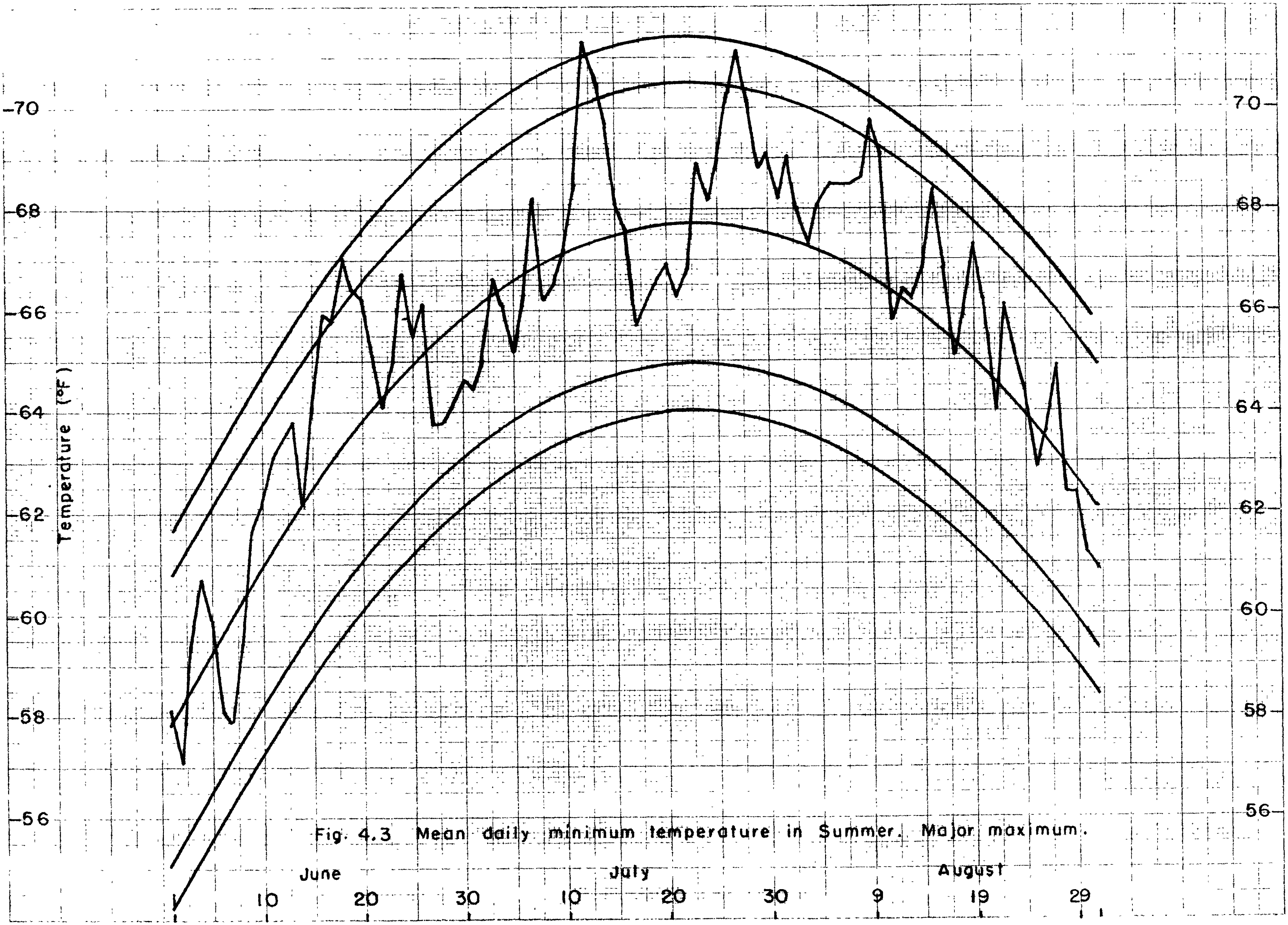


Fig. 4.3 Mean daily minimum temperature in Summer. Major maximum.

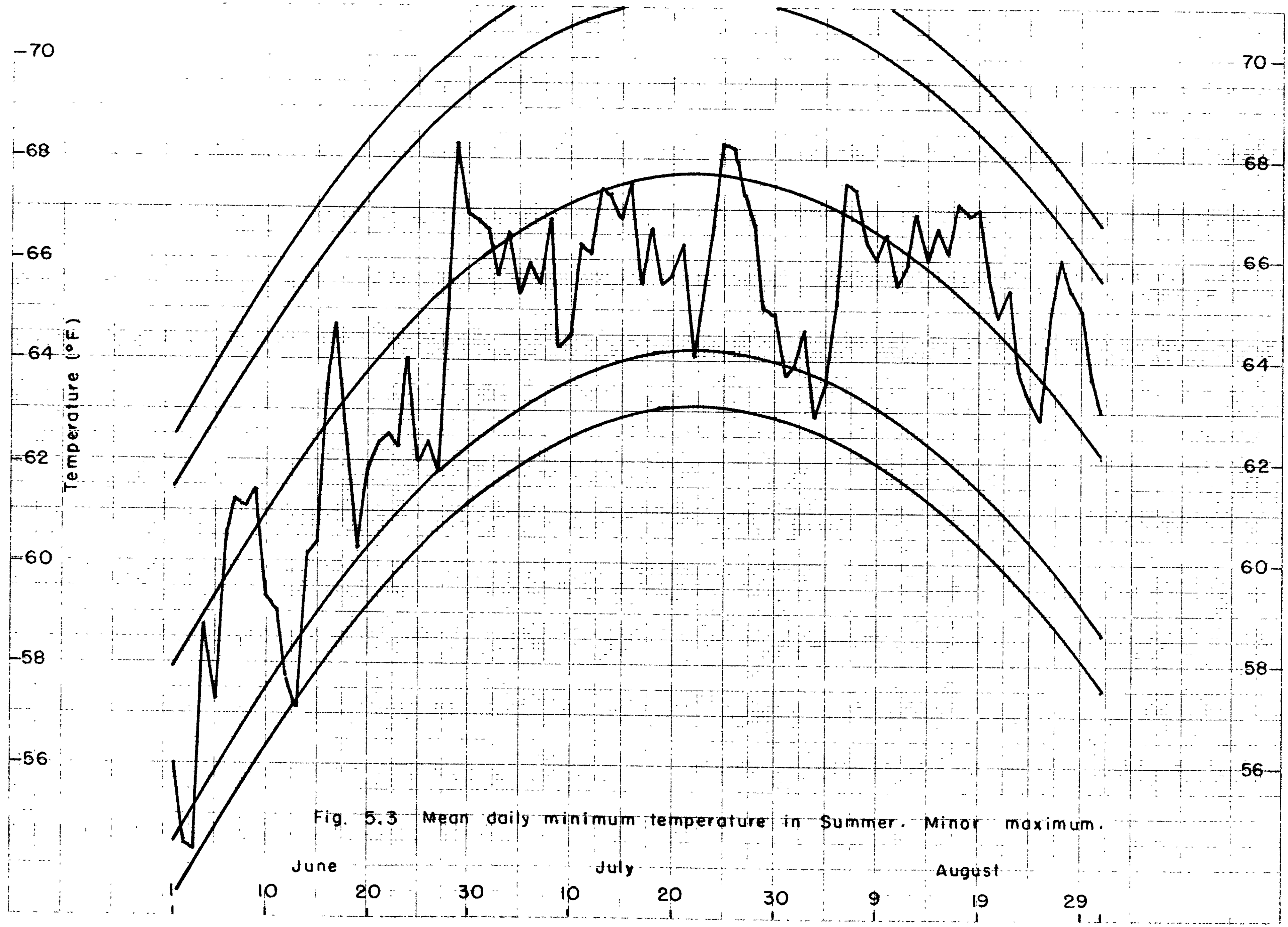


Fig. 5.3 Mean daily minimum temperature in Summer. Minor maximum.

Table I.4 Normal daily maximum temperature for summer

Days	Period	Trend	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
June 1	76.9	77.5	78.9	74.8	74.3	75.5	79.6	78.7	73.0	74.2	77.3	78.4
2	77.5	77.9	78.8	76.2	77.1	77.1	77.2	80.9	74.5	74.9	76.6	83.5
3	77.8	78.3	78.9	76.6	77.4	79.8	75.3	82.9	77.5	74.5	75.5	81.2
4	79.1	78.6	79.3	78.9	76.0	79.1	78.3	81.5	81.4	76.7	78.3	82.3
5	79.3	79.0	78.1	80.6	72.5	77.7	78.6	79.1	81.4	80.9	80.2	83.8
6	79.3	79.4	77.9	80.7	70.9	78.9	80.5	78.6	79.6	82.5	81.4	81.8
7	79.0	79.7	78.0	79.9	73.6	79.5	80.5	76.7	79.3	81.8	78.3	83.7
8	78.6	80.1	77.9	79.2	74.3	79.6	78.1	77.1	80.3	80.9	76.9	82.2
9	79.1	80.4	78.1	80.1	76.7	80.5	78.3	76.5	81.7	82.6	78.1	79.9
10	80.4	80.7	80.4	80.5	79.2	83.5	81.3	79.9	80.1	79.3	78.5	82.3
11	80.9	81.1	79.8	82.1	80.5	81.7	79.7	81.9	82.8	79.1	79.6	85.8
12	81.4	81.4	81.9	80.9	83.7	84.6	80.4	80.7	80.3	74.8	85.5	85.2
13	82.0	81.7	82.1	81.9	83.4	83.4	79.1	80.2	82.5	78.1	85.3	86.6
14	82.3	82.0	82.2	82.4	81.3	82.0	81.3	82.7	81.3	81.3	82.3	85.8
15	82.5	82.3	82.3	82.7	83.1	83.3	82.3	78.9	79.8	83.1	83.4	86.3
16	82.4	82.6	82.5	82.2	82.3	85.3	83.4	79.3	80.6	82.8	80.4	85.8
17	83.2	82.8	83.6	82.9	80.3	86.5	85.1	81.3	80.8	84.1	82.3	86.1
18	83.7	83.1	85.1	82.1	80.9	87.4	88.1	82.7	82.0	79.7	82.7	87.9
19	83.9	83.4	85.2	82.7	84.0	88.6	85.9	82.9	80.7	80.7	82.9	86.3
20	83.0	83.6	83.9	82.1	85.5	86.1	81.4	82.3	82.3	79.3	83.8	84.8
21	83.1	83.9	84.3	81.8	81.9	84.5	84.2	83.5	80.5	80.5	83.2	85.3
22	84.2	84.1	85.2	83.1	86.0	86.2	83.1	84.9	82.7	80.4	84.8	87.4
23	85.7	84.3	86.7	84.7	86.0	86.2	85.4	85.9	84.8	83.0	86.1	90.1
24	85.4	84.6	86.5	84.4	85.9	86.0	85.3	86.2	82.5	83.1	86.9	90.3
25	85.5	84.8	86.6	84.4	86.8	85.2	84.3	85.9	81.5	81.8	87.9	91.8
26	84.8	85.0	86.0	83.5	82.3	83.0	84.7	86.1	81.3	81.7	84.8	89.3
27	85.3	85.2	85.9	84.7	84.7	84.4	88.0	85.6	83.1	82.8	86.9	89.3
28	85.1	85.4	84.6	85.6	86.6	82.9	84.0	82.1	85.8	86.2	85.8	90.1
29	86.1	85.6	84.8	87.4	86.4	83.9	85.6	81.2	84.7	90.1	86.8	91.7
30	86.3	85.7	85.9	86.8	86.6	85.7	87.8	81.4	84.3	88.3	85.4	91.5

Days	Period	Trend	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
July 1	86.2	85.9	85.6	86.9	86.0	85.6	85.7	81.8	84.2	87.8	84.1	92.3
2	86.3	86.1	86.5	86.1	86.1	86.4	87.3	84.7	83.1	86.6	84.0	90.8
3	86.1	86.2	86.2	86.0	87.3	85.3	87.0	85.5	82.5	87.5	84.3	89.4
4	86.8	86.4	86.9	86.8	85.8	86.6	86.4	87.3	84.3	86.7	84.3	92.3
5	87.1	86.5	87.4	86.7	89.7	85.4	86.5	88.5	84.5	86.7	84.8	92.3
6	87.4	86.7	87.7	87.0	91.2	87.4	85.3	89.3	85.5	86.5	85.3	91.2
7	87.3	86.8	87.8	86.8	89.9	88.1	88.5	88.3	87.1	86.7	84.3	88.7
8	87.2	86.9	86.5	87.9	87.4	86.5	86.8	87.5	87.6	87.7	86.8	88.3
9	86.9	87.0	87.4	86.4	86.8	87.2	87.0	87.7	87.1	84.3	86.8	89.6
10	87.1	87.1	87.7	86.5	90.9	87.6	85.5	86.6	87.8	85.6	86.0	89.1
11	88.2	87.2	88.7	87.8	91.7	90.7	86.7	85.8	89.1	86.5	86.9	90.7
12	88.8	87.3	88.9	88.8	91.6	93.4	85.2	87.1	90.0	87.8	86.4	93.4
13	88.1	87.4	88.8	87.5	90.7	89.4	87.5	88.9	87.9	86.3	85.6	94.8
14	87.8	87.5	88.5	87.1	88.2	85.5	87.9	90.9	88.5	85.7	86.3	90.7
15	87.7	87.5	89.0	86.3	90.2	87.4	86.7	90.3	87.9	85.1	88.1	86.4
16	88.3	87.6	88.5	88.1	88.8	86.8	87.6	89.3	88.4	85.1	92.2	89.9
17	87.4	87.6	87.9	86.8	90.3	83.4	89.1	87.3	86.1	83.9	89.3	90.6
18	87.6	87.7	88.1	87.0	89.8	87.9	86.3	86.1	87.6	86.4	89.0	88.8
19	86.9	87.7	87.1	86.6	87.1	86.7	86.4	85.0	85.9	84.6	90.5	89.8
20	86.7	87.7	86.5	86.9	86.2	86.3	87.2	86.3	87.8	85.1	89.0	89.5
21	87.4	87.8	88.2	86.6	88.8	87.3	87.6	86.5	84.9	86.5	88.5	89.6
22	87.6	87.8	88.9	86.4	90.8	87.6	87.0	89.5	84.4	85.6	86.6	91.2
23	87.5	87.8	89.8	85.1	92.2	90.1	88.2	90.5	85.1	85.1	84.3	85.2
24	87.5	87.8	89.5	85.4	88.3	89.9	89.9	86.5	84.6	85.4	86.9	85.4
25	88.0	87.8	89.3	86.6	89.9	90.1	90.0	86.7	83.9	88.0	87.8	88.5
26	88.8	87.7	89.7	87.9	92.8	90.6	89.0	86.5	85.4	88.5	89.8	90.5
27	88.3	87.7	88.2	88.5	92.3	91.4	86.1	85.9	87.1	85.5	94.5	91.3
28	88.0	87.7	88.4	87.7	90.3	88.1	88.2	87.2	86.4	84.7	91.7	89.8
29	87.5	87.6	87.9	87.1	90.4	88.7	90.3	85.1	85.7	83.7	89.6	90.2
30	87.4	87.6	88.6	86.2	93.3	90.0	88.9	87.1	85.7	81.7	87.7	91.6
31	87.4	87.5	89.1	85.6	92.3	89.0	88.3	89.7	85.1	84.3	86.3	86.3

Days	Period	Trend	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
Aug. 1	87.1	87.5	87.8	86.4	89.6	88.0	85.8	90.4	85.9	84.2	88.7	88.0
2	86.5	87.4	87.5	85.5	88.8	87.3	85.5	89.7	86.4	82.3	87.1	87.5
3	87.0	87.3	88.5	85.4	89.3	90.6	89.4	86.5	87.8	80.1	88.9	87.8
4	86.7	87.2	87.9	85.5	88.6	90.6	88.0	85.2	88.5	82.1	86.4	87.4
5	86.5	87.2	87.7	85.3	88.9	89.9	85.9	85.0	87.7	85.9	85.3	85.6
6	85.8	87.0	87.4	84.2	88.8	89.8	85.9	86.4	84.1	85.7	85.3	84.1
7	85.8	86.9	87.3	84.2	88.5	88.9	86.3	84.9	83.7	86.5	85.9	84.0
8	87.0	86.8	88.7	85.2	91.9	89.4	88.0	84.9	85.3	85.3	87.3	86.8
9	86.3	86.7	88.1	84.4	88.0	86.9	87.1	84.7	83.7	85.5	85.9	85.3
10	86.5	86.6	88.0	85.0	88.4	86.1	88.0	87.1	82.9	86.5	86.2	87.2
11	84.8	86.4	85.7	83.8	86.1	86.5	85.3	85.7	83.8	86.9	82.8	81.3
12	84.6	86.3	85.5	83.6	85.7	87.4	83.0	85.7	82.3	86.1	81.4	83.4
13	85.5	86.1	86.3	84.6	88.5	88.4	84.9	83.5	82.9	85.9	82.9	86.3
14	85.7	86.0	85.6	85.7	87.2	87.4	84.6	84.7	84.1	85.8	85.7	88.6
15	85.9	85.8	86.0	85.9	90.9	87.0	84.8	82.3	83.9	84.4	87.3	88.5
16	85.8	85.6	85.2	86.5	89.4	86.4	83.7	80.5	85.2	87.5	87.2	86.8
17	85.7	85.4	85.1	86.3	89.4	86.6	80.9	83.7	88.7	85.9	82.9	86.4
18	84.9	85.2	85.1	84.8	86.4	86.1	82.3	83.4	85.7	86.9	80.6	86.3
19	84.7	85.0	85.4	84.0	84.8	86.4	83.7	87.9	84.9	85.6	80.8	83.8
20	84.0	84.8	83.4	84.6	81.5	83.6	82.3	86.7	87.5	85.7	82.9	80.7
21	84.4	84.6	85.2	83.6	84.3	87.0	83.6	85.6	86.6	85.4	82.2	82.6
22	83.2	84.4	83.2	83.1	81.3	82.9	83.9	82.0	85.3	86.3	80.9	80.3
23	83.9	84.2	83.8	84.0	81.2	84.4	81.7	84.7	86.0	85.5	83.5	82.1
24	84.1	83.9	83.8	84.4	84.0	83.1	83.7	84.9	86.3	83.5	83.2	83.7
25	83.7	83.7	83.9	83.6	87.3	83.8	84.8	82.0	83.5	83.4	78.8	86.0
26	84.1	83.4	83.6	84.7	85.6	85.5	83.5	83.5	86.5	85.0	79.9	86.8
27	84.2	83.2	83.7	84.8	84.5	82.8	85.4	82.9	86.9	85.2	82.6	85.7
28	83.7	82.9	83.3	84.0	80.9	82.5	84.3	85.5	85.7	85.2	80.9	84.8
29	83.8	82.6	83.5	84.1	78.3	82.9	84.5	84.9	83.6	85.1	82.7	85.0
30	84.4	82.4	84.1	84.7	80.5	83.4	85.3	83.1	84.7	84.3	83.5	88.4
31	82.9	82.1	83.5	82.3	80.3	84.1	83.3	82.3	83.8	81.7	78.8	86.8

Table II.4 Statistical parameters for maximum temperature in summer

Parameters	Period		M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
x	84.9	84.9	85.4	84.5	85.6	85.8	84.9	84.6	84.2	84.0	84.5	87.1
e	0.0		0.43	-0.45	0.66	0.83	-0.09	-0.34	-0.72	-0.96	-0.44	2.15
$[\overline{e'^2_{ij}}]^{1/2}$	0.689		0.974	1.182	2.755	1.742	1.628	2.036	2.007	2.284	2.147	2.734
$[\overline{\overline{e''^2_{ij}}}/N]^{1/2}$	0.778		1.086	1.110	2.540	1.965	1.837	1.926	1.963	1.903	2.269	2.454
r_1	0.649		0.625	0.705	0.713	0.522	0.388	0.643	0.676	0.717	0.577	0.703
r_a	0.654		0.640	0.724	0.714	0.529	0.396	0.638	0.699	0.719	0.583	0.706
l_a	2.85		2.67	3.40	3.48	2.09	1.6	2.8	3.1	3.5	2.4	3.4

Table III.a. 4 Significance of singularities for maximum temperature in summer

Parameters	Period	Total	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
$\|\Delta x\|$ 20%	0.88		1.68	1.06	4.19	3.06	2.0	2.27	1.86	1.97	2.31	5.65
			-0.81	-1.97	-2.87	-1.40	-2.18	-2.95	-3.29	-3.88	-3.19	-1.36
10%	1.13		2.04	1.49	5.19	3.69	2.59	3.01	2.59	2.80	3.09	6.64
			-1.17	-2.40	-3.87	-2.04	-2.77	-3.69	-4.02	-4.71	-3.97	-2.35
5%	1.35		2.34	1.86	6.06	4.24	3.10	3.65	3.22	3.52	3.77	7.50
			-1.47	-2.77	-4.74	-2.59	-3.28	-4.33	-4.65	-5.43	-4.65	-3.21
2.5%	1.54		2.62	2.19	6.83	4.73	3.56	4.22	3.78	4.16	4.37	8.27
			-1.75	-3.10	-5.51	-3.07	-3.74	-4.90	-5.21	-6.07	-5.25	-3.98
1%	1.77		2.94	2.59	7.76	5.31	4.10	4.90	4.45	4.93	5.09	9.19
			-2.07	-3.50	-6.44	-3.66	-4.28	-5.58	-5.89	-6.84	-5.97	-4.90
k' 5%	6	43	4	3	4	4	2	4	2	4	7	3
2.5%	3	25	2	1	4	3	2	2	0	3	2	3
1%	1	12	2	0	2	2	2	0	0	1	1	1
k 5%	5	34	2	3	2	3	2	3	2	4	6	2
2.5%	2	20	1	1	2	3	2	2	0	3	2	2
1%	1	10	1	0	1	2	2	0	0	1	1	1
L 5%	85		-21	-44	-21	-44	-21	-44	-0.21	-77	50	-21
2.5%	-84		-39	-39	-84	64	-84	-84		64	-84	-84
1%	93	-97	93		93	26	26			93	93	93
L' 5%	0.6		1	0.1	13	21	91	28	67	2	2	0.005
2.5%	18		21	10	67	7	61	19		0.3	4	0.5
1%	23	0.1	55		15	2	5			0.0007	33	16

Table III.b.4 Significance of singularities for maximum summer temperature assuming e=0

Parameters		Period	Total	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
$\lvert\Delta x\rvert$	20%	0.88		1.25	1.51	3.53	2.23	2.09	2.61	2.57	2.93	2.75	3.50
	10%	1.13		1.60	1.94	4.54	2.86	2.68	3.35	3.30	3.76	3.53	4.50
	5%	1.34		1.91	2.32	5.40	3.41	3.19	3.99	3.93	4.48	4.21	5.36
	2.5%	1.54		2.18	2.65	6.17	3.90	3.65	4.56	4.50	5.12	4.81	6.12
	1%	1.77		2.51	3.04	7.10	4.49	4.19	5.24	5.17	5.88	5.53	7.04
k'	5%	6	66	6	8	5	7	3	5	2	6	6	12
	2.5%	3	29	2	3	2	4	2	2	0	5	3	3
	1%	1	11	0	0	1	2	2	0	0	3	1	1
h	5%	5	45	5	5	3	4	3	3	2	4	5	6
	2.5%	2	23	2	2	1	3	2	2	0	3	3	3
	1%	1	11	0	0	1	2	2	0	0	3	1	1
L	5%	85		85	85	-44	-77	-44	-44	-21	-77	85	50
	2.5%	-84		-84	-84	-39	64	-84	-84		64	64	64
	1%	93	-78			93	26	26			3	93	93
L'	5%	0.6		1	0.1	13	21	91	28	67	2	2	0.005
	2.5%	18		21	10	67	7	61	19		0.3	4	0.5
	1%	23	0.02			15	2	5			0.0007	33	16

Table IV.a.4 Level of significance for selected calendar dates
Maximum summer temperature

Dates	Period	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
June 8	-3	-0.9	-74	-2	-45	-24	-19	62	42	-21	-99
9	-6	-0.5	88	-11	-68	-23	-8	31	17	-38	-34
23	5	5	50	71	54	48	36	56	-87	31	19
July 12	3	25	10	19	0.3	-22	93	9	52	-84	15
Aug. 6	-8	-94	-4	68	27	-53	-88	-27	-85	-53	-6
7	-10	-94	-6	74	50	-72	-41	-20	81	-79	-6
11	-2	-23	-7	-72	-67	-51	-84	-34	52	-14	-0.7
12	-1	-20	-6	-64	89	-5	-89	-10	74	-4	-7
22	-7	-9	-49	-18	-19	-79	-31	41	22	-16	-2
30	0.3	16	2	-36	92	6	60	12	21	46	15

Table IV.b.4 Level of significance for selected calendar dates assuming e=0
Maximum summer temperature

Dates	Period	M-Half	m-Half	N-MM	MM	MM-NN	NN	NN-M	M	M-N	N
June 8	-3	-3	-47	-4	-78	-22	-14	89	70	-14	44
9	-6	-2	-82	-18	95	-21	-5	50	33	-28	-86
23	5	2	77	54	28	51	45	82	-56	41	4
July 12	3	11	21	12	0.05	-20	-93	18	83	-68	3
Aug. 6	-8	71	-2	52	12	-49	-75	-15	-55	-40	-28
7	-10	71	-2	57	25	-68	-33	-10	-86	-63	-28
11	-2	-45	-3	-90	97	-48	-71	-19	82	-9	-6
12	-1	-41	-3	-82	54	-4	-76	-5	-93	-2	-30
22	-7	-22	-28	-27	-40	-74	-24	64	41	-11	-14
30	0.3	7	5	-50	56	7	72	23	40	59	3

Table V.4 Level of significance of highly significant peaks. Maximum summer temperature

Periods	a	b
Period	A12 -1 , A30 0.3	A12 -1 , A30 0.3
M-Half		
m-Half		
N-MM	J5 -0.9 , J6 -0.1 , J7 -1	J5 -2 , J6 -0.2
MM	J1 1 , J1 17 -0.3	J18 1 , J19 0.3 , J1 17 -1
MM-NN	J18 0.2 , A17 -0.7	J18 0.2 , A17 -0.6
NN	J3 1	J3 2 , A16 -1
NN-M		
M	J12 -1 , A3 -0.6	J12 -0.4 , J1 30 -0.9 , A3 -0.2
M-N	J1 27 0.08	J1 27 0.2
N		J25 1 , J31 2 , J1 13 0.6

J = June J1 = July A = August

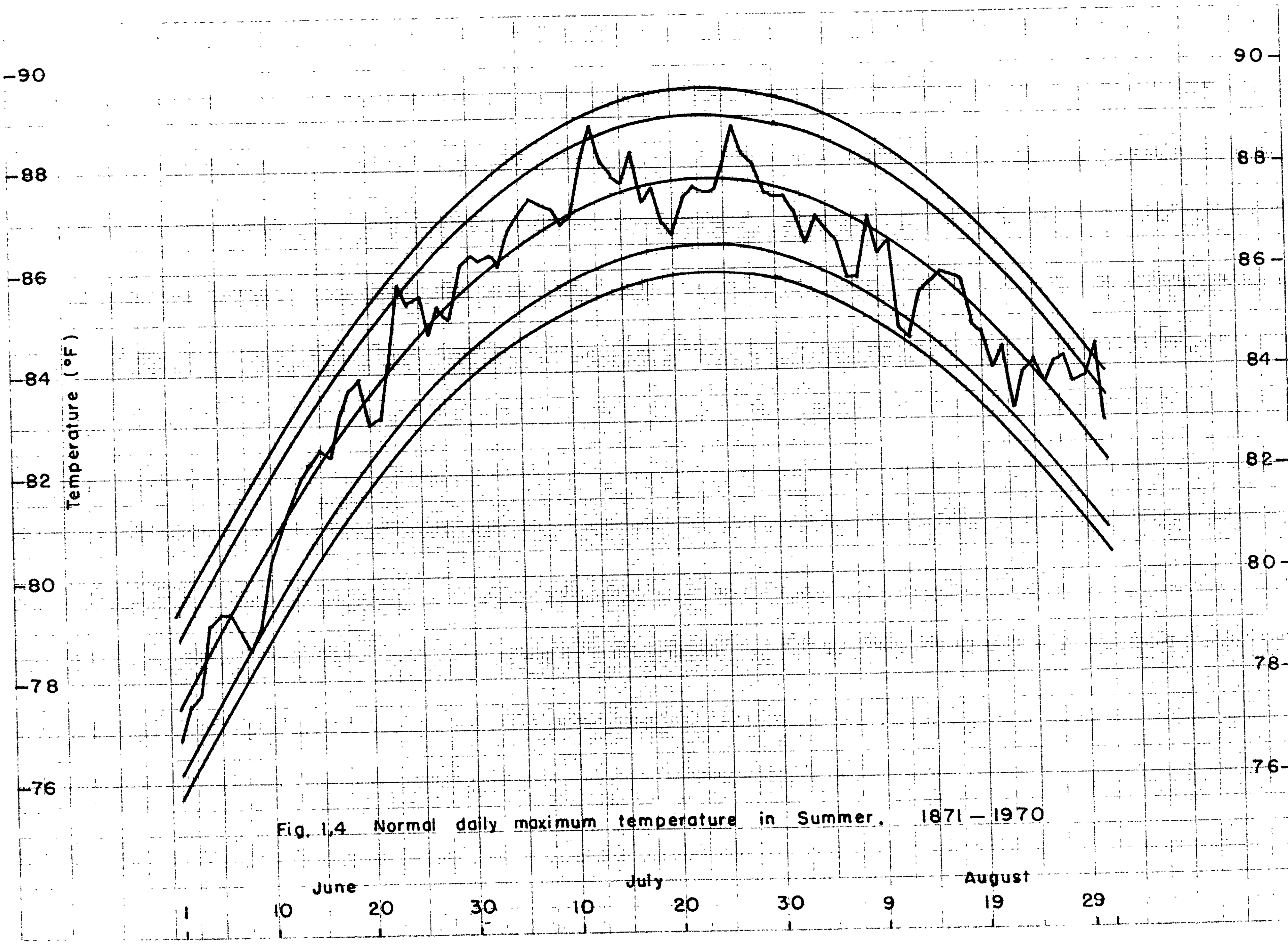


Fig. 1.4 Normal daily maximum temperature in Summer. 1871 — 1970

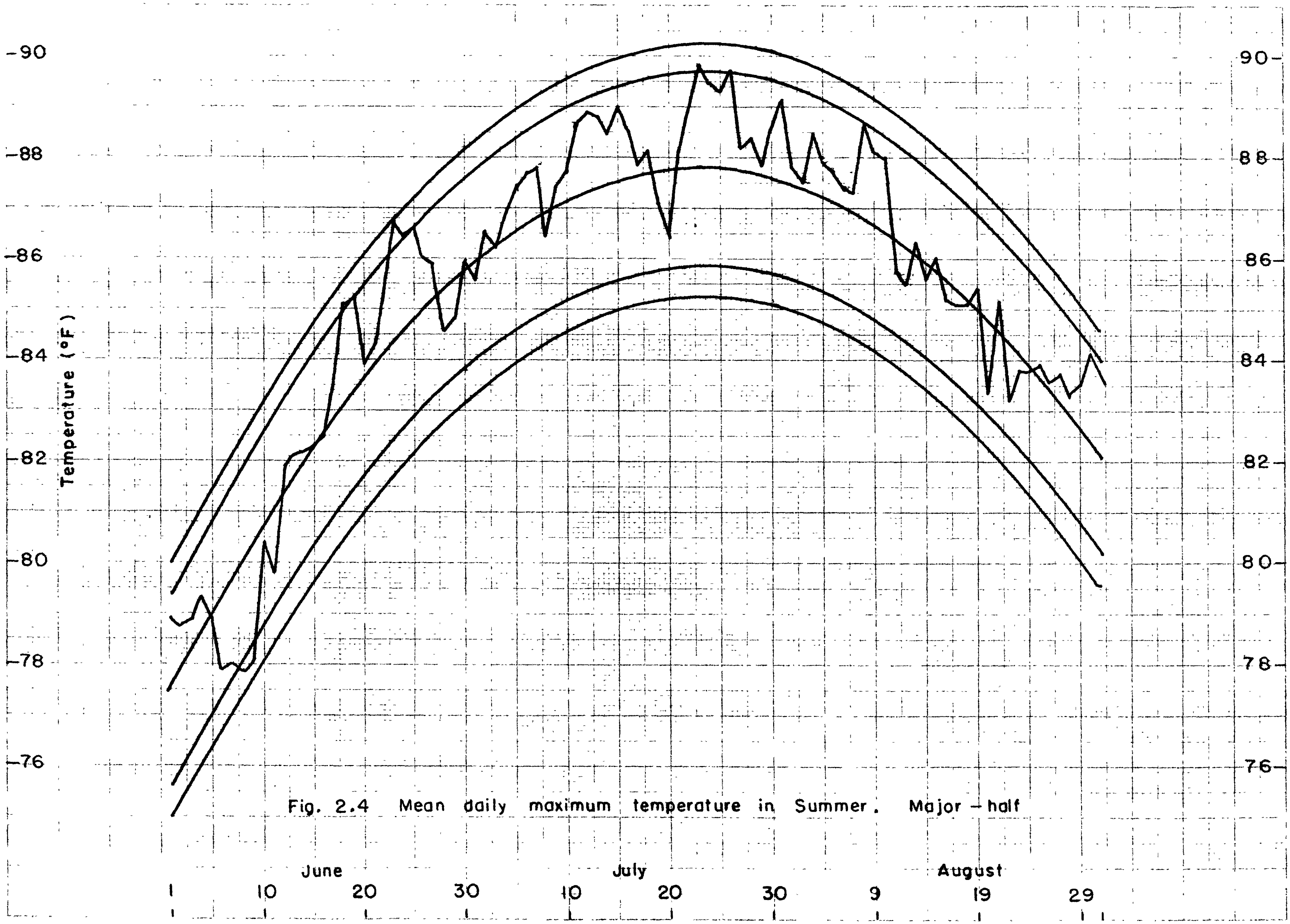


Fig. 2.4 Mean daily maximum temperature in Summer. Major – half

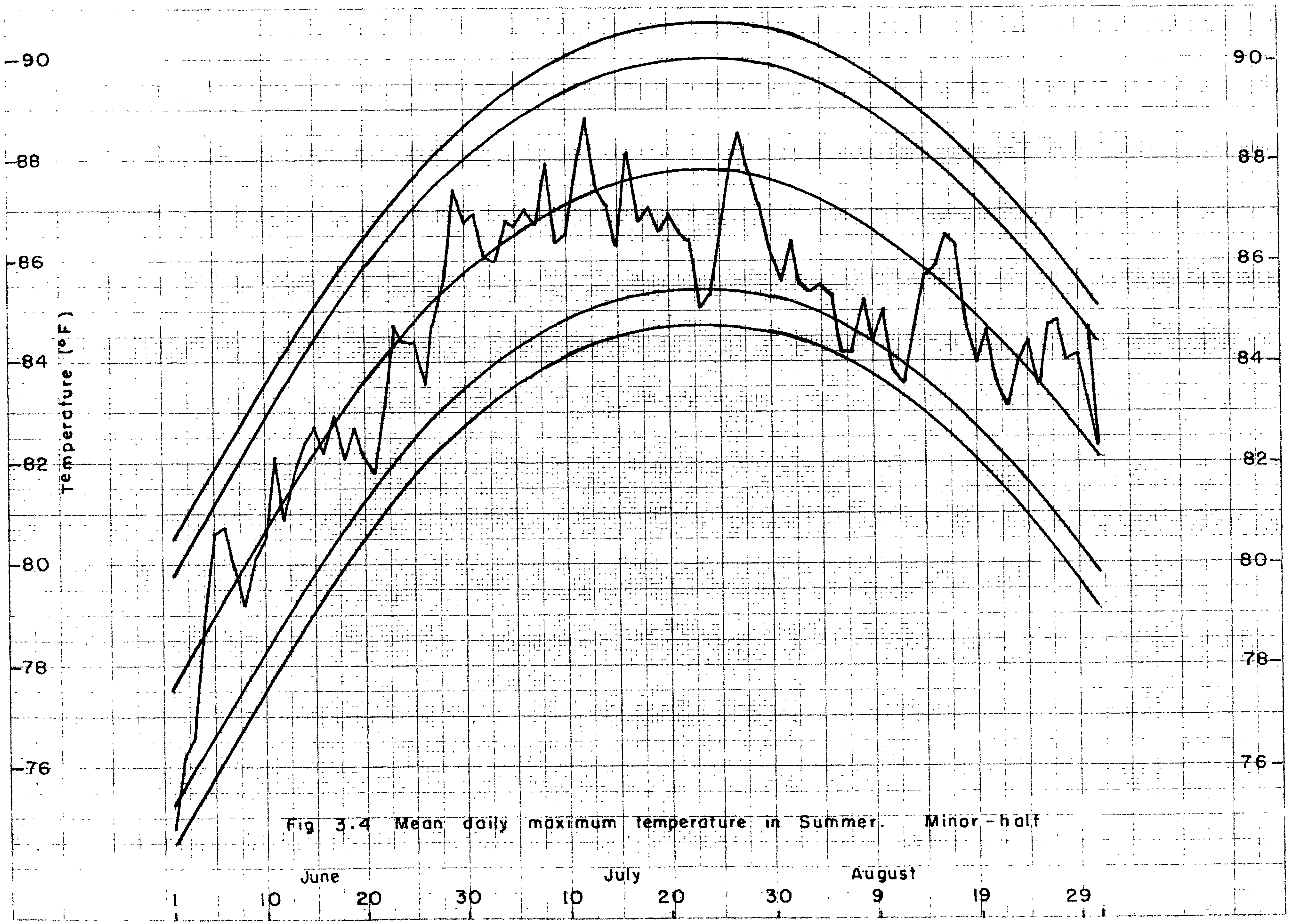


Fig 3.4 Mean daily maximum temperature in Summer. Minor-half

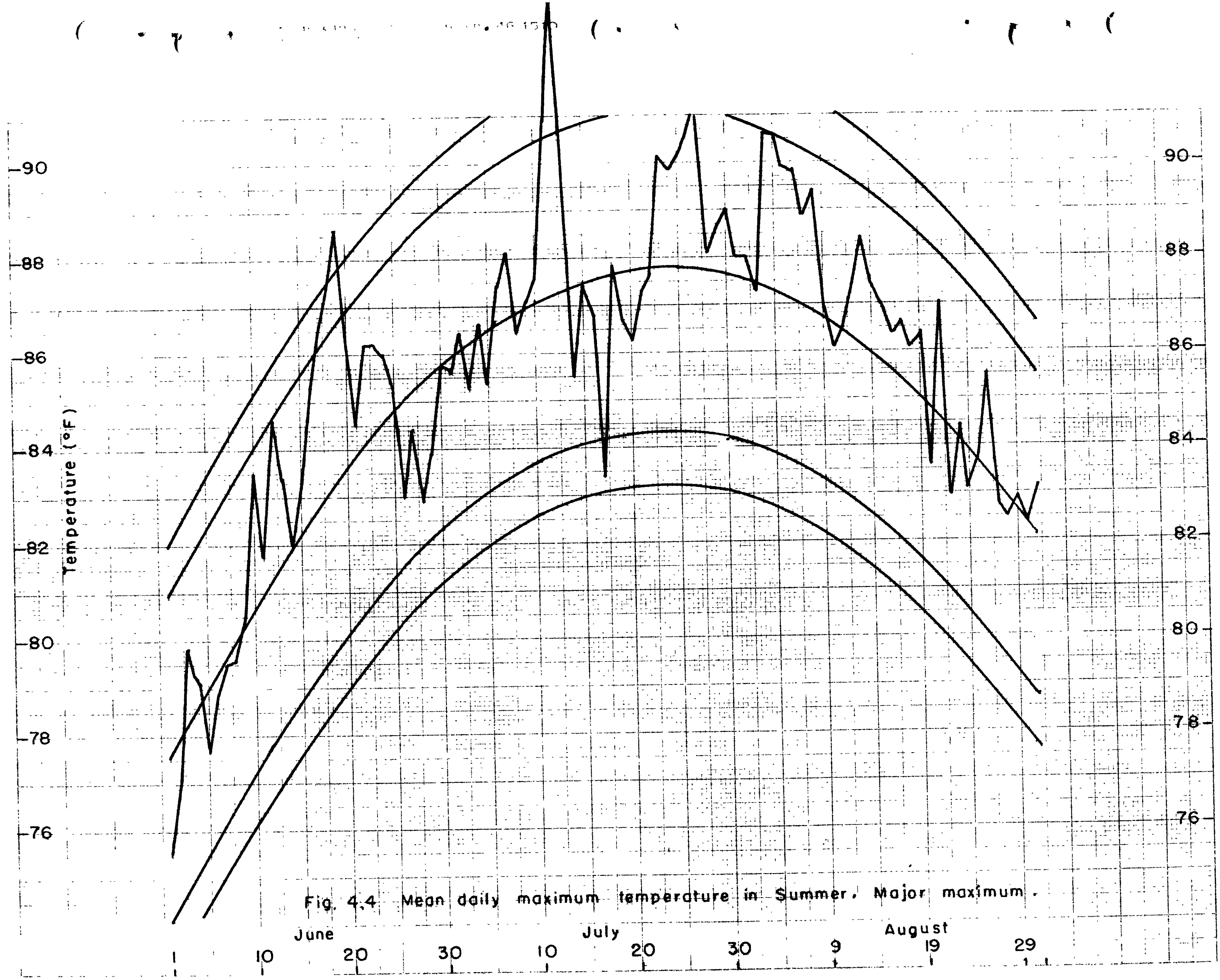


Fig. 4.4 Mean daily maximum temperature in Summer, Major maximum.

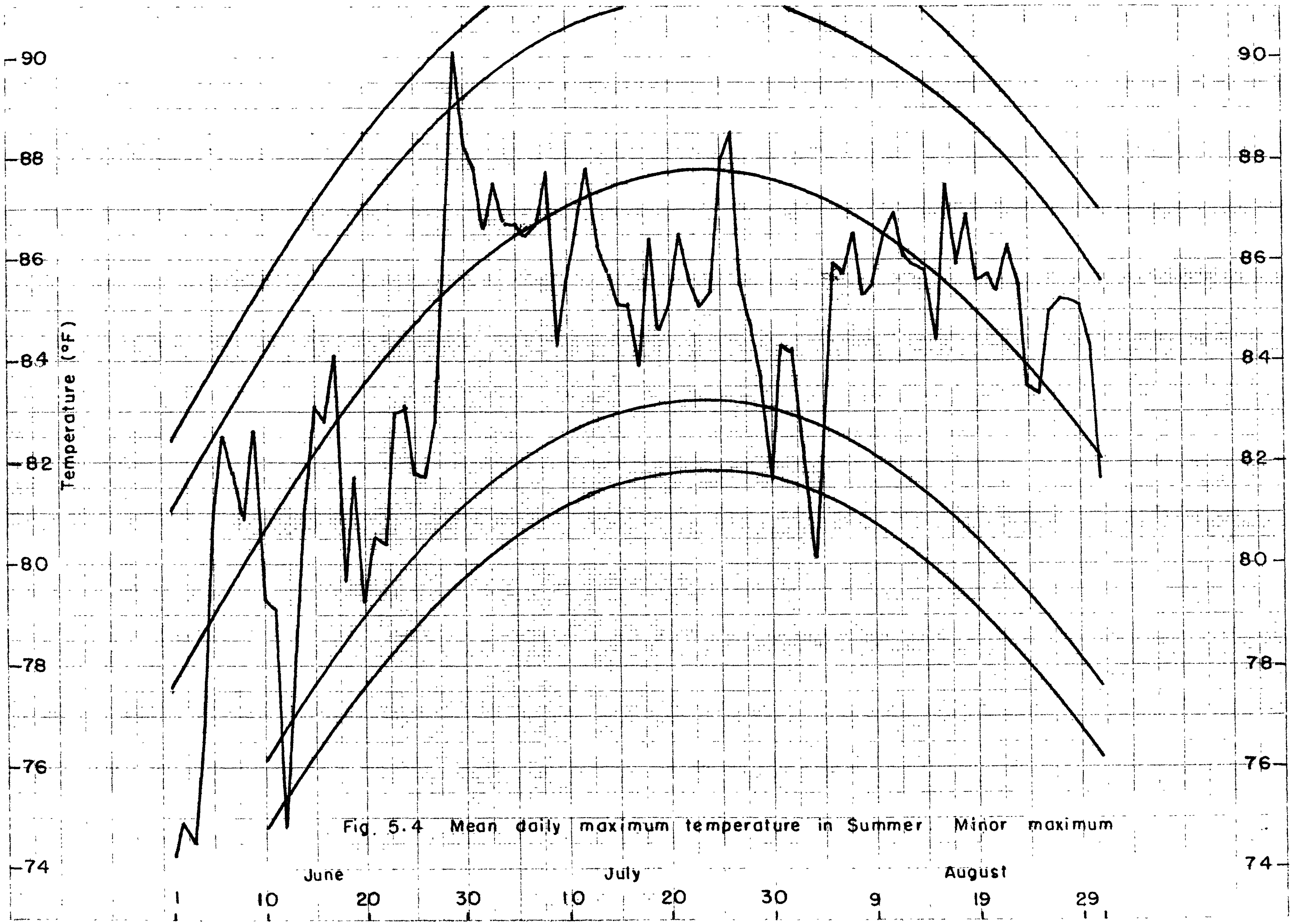


Fig. 5.4 Mean daily maximum temperature in Summer. Minor maximum

Appendix

Persistence

For the theory and explanation we make references to [2] and [16], but we want here to give the formula used to compute a measure of the day to day persistence.

The coefficient of autocorrelation at lag 1 of a variable x is defined by

$$r_1 = \frac{\overline{x_i' x_{i+1}'}}{\overline{x_i'^2}}$$

and the normalized Besson Coefficient of Persistence by

$$r_a = 1 - \frac{\sigma_d^2}{2\sigma^2}$$

where σ_d^2 is the variance of the differences from one observation to the next

σ^2 is the variance of the series of observations as a whole.

From these definitions it is easy to prove that in a Markov chain model of order 1 (i.e. the value of a variable x at time t is dependent only on the value of the same variable x at time t-1) a continuous variate which is strictly random apart from persistence has a length of persistence (the length of period over which, on the average, a single observation can be regarded as representative) given by

$$l_a = \frac{1}{1-r_1}$$

and if N is large enough $r_1 \sim r_a$

F-test

Testing the significance of a systematic variation is equivalent to testing whether the mean square which includes the variance due to this systematic variation, is significantly greater than that which is an estimate of the residual variance alone. The F-test is suitable for this purpose, provided that the mean squares are independent and that the deviations from which the residual is calculated may be assumed to be distributed normally. If x_1^2 and x_2^2 are independent random variables following chi-square distributions with v_1 and v_2 degrees of freedom respectively, then the distribution of F

$$F = \frac{x_1^2 / v_1}{x_2^2 / v_2}$$

is said to follow the variance ratio or F-distribution with v_1 and v_2 degrees of freedom. When v_2 is large, it's distribution function can be approximated by the distribution function of a χ^2 distribution with v_1 degrees of freedom

$$\lim_{v_2 \to \infty} Q(F \mid v_1, v_2) = Q(x^2 \mid v_1) \qquad \text{with } x^2 = v_1 F$$

which, when v_1 is odd, can be computed from

$$Q(x^2 \mid v) = 2\,Q(x) + 2\,Z(x) \sum_{r=1}^{\frac{v-1}{2}} \frac{x^{2r-1}}{1.3.5 \ldots (2r-1)}$$

where
$$Z(x) = \frac{1}{\sqrt{2\pi}}\, e^{-\frac{x^2}{2}}$$

$$Q(x) = \int_x^{\infty} Z(t)\, dt$$

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