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## Electrothermal feedback in superconducting nanowire single-photon detectors

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We investigate the role of electrothermal feedback in the operation of superconducting nanowire single-photon detectors (SNSPDs). It is found that the desired mode of operation for SNSPDs is only achieved if this feedback is unstable, which happens naturally through the slow electrical response associated with their relatively large kinetic inductance. If this response is sped up in an effort to increase the device count rate, the electrothermal feedback becomes stable and results in an effect known as latching, where the device is locked in a resistive state and can no longer detect photons. We present a set of experiments which elucidate this effect and a simple model which quantitatively explains the results.

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Superconducting nanowire single-photon detectors (SNSPDs) combine high speed, high detection efficiency (DE) over a wide range of wavelengths, and low dark counts.<sup>1-4</sup> Of particular importance is their high *single-photon* timing resolution of  $\sim 30$  ps,<sup>4</sup> which permits extremely high data rates in photon-counting communications applications.<sup>5,6</sup> Full use of this electrical bandwidth is limited, however, by the fact that the maximum count rates of these devices are much smaller (a few hundred MHz for  $10 \mu\text{m}^2$  active area and decreasing as the area is increased<sup>2</sup>), limited by their large kinetic inductance and the input impedance of the readout circuit.<sup>2,7</sup> To increase the count rate, therefore, one must either reduce the kinetic inductance (by using a smaller active area or different materials or substrates) or increase the load impedance.<sup>7</sup> However, either of these approaches causes the wire to “latch” into a stable resistive state where it no longer detects photons.<sup>8</sup> This effect arises when negative electrothermal feedback, which in normal operation allows the device to reset itself, is made fast enough that it becomes stable. We present experiments which probe the stability of this feedback, and we develop a model which quantitatively explains our observations.

The operation of an SNSPD is illustrated in Fig. 1(a). A nanowire (typically  $\sim 100$  nm wide and 5 nm thick) is biased with a dc current  $I_0$  near its critical current  $I_c$ . The nanowire has kinetic inductance  $L$  and is read out using a load impedance  $R_L$  (typically a  $50\Omega$  transmission line). When a photon is absorbed, a short ( $< 100$  nm long) normal domain is nucleated, giving the wire a resistance  $R_n(t)$ . This results in Joule heating which causes the normal domain (and consequently,  $R_n$ ) to expand in time exponentially. The expansion is counteracted by negative electrothermal feedback from the load  $R_L$ , which forms a current divider with  $R_n$ , and diverts a current  $I_L$  into the load (so that the current in the nanowire is reduced to  $I_d \equiv I_0 - I_L$ ), reducing the heating. However, in a correctly functioning device, this feedback is unstable: the inductive time constant is long enough so that before  $I_L$  becomes appreciable, Joule heating has already increased  $R_n$ , so that  $R_n \gg R_L$ . The current  $I_d$  then drops nearly to zero, turning off the heating and allowing the nanowire to quickly cool down and return to the superconducting state, after which  $I_d$  recovers with a time constant  $\tau_e \equiv L/R_L$ .<sup>2</sup> If one attempts to shorten  $\tau_e$  too much, the negative feedback be-

comes fast enough to counterbalance the Joule heating before it runs away, resulting in a stable resistive domain, known as a self-heating hotspot.<sup>9,10</sup>

In a standard treatment of these hotspots,<sup>9</sup> solutions to a one-dimensional heat equation are found in which a normal-superconducting (NS) boundary propagates at constant velocity  $v_{\text{NS}}$  for fixed device current  $I_d$ .<sup>9,11</sup> This results in a solution of the form

$$v_{\text{NS}} = v_0 \frac{\alpha(I_d/I_c)^2 - 2}{\sqrt{\alpha(I_d/I_c)^2 - 1}} \approx \frac{1}{\gamma} (I_d^2 - I_{\text{ss}}^2), \quad (1)$$

where  $v_0 \equiv \sqrt{A_{\text{cs}} \kappa h / c}$  is a characteristic velocity ( $A_{\text{cs}}$  is the wire's cross-sectional area,  $\kappa$  is its thermal conductivity, and  $c$  and  $h$  are the heat capacity and heat transfer coefficient to the substrate, per unit length, respectively),  $I_c$  is the critical current, and  $\alpha \equiv \rho_n I_c^2 / h(T_c - T_0)$  is known as the Stekly parameter, which characterizes the ratio of Joule heating to

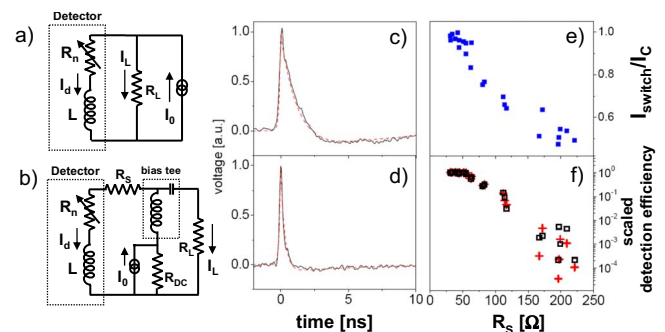


FIG. 1. (Color online) Speedup and latching of nanowire detectors with increased load impedance. (a) electrical model of detector operation. A hotspot is nucleated by absorption of a photon, producing a resistance  $R_n$  in series with the wire's kinetic inductance  $L$ . (b) Experimental circuit, including series resistor  $R_S$ , bias tee, and impedance of current source  $R_{\text{dc}}$ . (c) and (d) Averaged pulse shapes for  $R_S = 0, 250\Omega$  ( $L \sim 50$  nH); dashed lines are predictions with no free parameters. (e)  $I_{\text{switch}}$  vs  $R_S$ . As  $R_S$  is increased,  $I_{\text{switch}}$  decreases, becoming less than  $I_c$ . (f) DE at  $I_0 = 0.975 I_{\text{switch}}$  vs  $R_S$  (open squares). Also shown (crosses) are the expected DEs assuming that latching affects DE simply by limiting  $I_0$  (obtained from DE vs  $I_0$  at  $R_S = 0$ ).

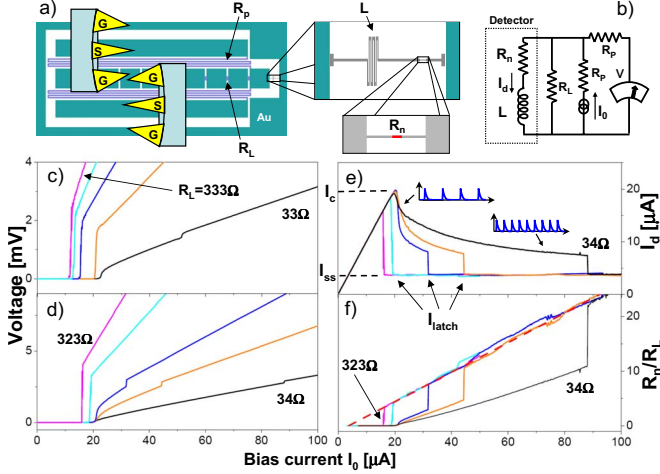


FIG. 2. (Color online) Hotspot stability measurements. (a) Device schematic; two ground-signal-ground probes perform a high-impedance three-point measurement of  $R_n$ , with  $R_L$  determined by probe position. (b) Equivalent electrical circuit. (c) and (d) Example  $V$ - $I$  curves, with  $L=60$  nH and  $L=605$  nH, respectively. (e) and (f) Inferred  $I_d$  and  $R_n/R_L$  for the data shown in (d). In (e) the relaxation oscillations in the region  $I_c < I_0 < I_{\text{latch}}$  are shown schematically. Dashed lines show (e)  $I_d = I_c$  and  $I_d = I_{\text{ss}}$ . (f)  $R_{\text{ss}} = R_L(I_0/I_{\text{ss}} - 1)$ .

conduction cooling in the normal state<sup>9</sup> ( $\rho_n$  is the normal resistance of the wire per unit length and  $T_0$  and  $T_c$  are the substrate and critical temperatures). Equation (1) is valid when  $T_0 \ll T_c$ , and the approximate equality holds when  $|I_d - I_{\text{ss}}| \ll I_{\text{ss}}$  with  $\gamma \equiv (T_c - T_0)(c/\rho_n)\sqrt{h/\kappa A_{\text{cs}}}$  and  $I_{\text{ss}}^2 \equiv 2h(T_c - T_0)/\rho_n$ . The physical meaning of Eq. (1) is clear: the NS boundary is stationary only if the local power density ( $\propto I_d^2$ ) is equal to a fixed value; if it is greater, the hotspot will expand ( $v_{\text{NS}} > 0$ ), if less it will contract ( $v_{\text{NS}} < 0$ ).

We can use Eq. (1) to describe the electrothermal circuit in Fig. 1(a), by combining it with the circuit equation  $I_d R_n + L dI_d/dt = R_L(I_0 - I_d)$  (where  $dR_n/dt = 2\rho_n v_{\text{NS}}$ ). To determine when the device will latch, we analyze the stability of the resulting second-order nonlinear system for small deviations from its steady-state solution [ $I_d \rightarrow I_{\text{ss}}, R_n \rightarrow R_L(I_0/I_{\text{ss}} - 1) \equiv R_{\text{ss}}$ ] to obtain a damping coefficient  $\zeta = \frac{I_0}{4I_{\text{ss}}} \sqrt{\tau_{\text{th}}/\tau_e}$ , where  $\tau_{\text{th}} \equiv R_L/2\rho_n v_0$  is a thermal time constant. This can be re-expressed in terms of  $R_{\text{tot}} \equiv R_L + R_{\text{ss}}$  thus:  $\zeta = \frac{1}{4} \sqrt{\tau_{\text{th,tot}}/\tau_{e,\text{tot}}}$  ( $\tau_{e,\text{tot}} \equiv L/R_{\text{tot}}$  and  $\tau_{\text{th,tot}} \equiv R_{\text{tot}}/2\rho_n v_0$ ), which clearly shows that the stability is determined by a ratio of electrical and thermal time constants.

In normal device operation, where the damping  $\zeta$  is small, the feedback cannot stabilize the hotspot during the initial photoresponse, as described above. However, as  $I_0$  is increased,  $\zeta$  increases, making the hotspot more stable (this occurs because  $R_{\text{ss}} \propto I_0$  and larger  $R_{\text{ss}}$  gives a shorter inductive time constant  $\tau_{e,\text{tot}}$ ). Eventually, at a bias current  $I_0 = I_{\text{latch}}$  the device latches. For a correctly functioning device,  $I_{\text{latch}} > I_c$ , so that latching does not affect its operation. However, if  $\tau_e$  is decreased,  $I_{\text{latch}}$  decreases, and eventually it becomes less than  $I_c$ . This prevents the device from being biased near  $I_c$ , resulting in a drastic reduction in performance.<sup>12</sup>

Devices used in this work were fabricated from

$\sim 5$ -nm-thick NbN films, deposited on  $R$ -plane sapphire substrates in a UHV dc magnetron sputtering system (base pressure  $< 10^{-10}$  mbar). Film deposition was performed at a wafer temperature of  $\sim 800$  °C and a pressure of  $\sim 10^{-8}$  mbar.<sup>13</sup> Aligned photolithography and liftoff were used to pattern  $\sim 100$ -nm-thick Ti films for on-chip resistors<sup>8</sup> and Ti:Au contact pads. Patterning of the NbN was then performed with  $e$ -beam lithography.<sup>3</sup> Devices were tested in a cryogenic probing station at 2 K as described in Refs. 2 and 3.

Figures 1(c)–1(f) show data for a set of ( $3 \mu\text{m} \times 3.3 \mu\text{m}$  area) devices having various resistors  $R_S$  in series with the  $50\Omega$  readout line<sup>8</sup> [Fig. 1(b)], so that  $R_L = 50\Omega + R_S$ . For  $R_S = 0$ , these devices had similar performance to those in Ref. 3. Panels (c) and (d) show averaged pulse shapes for devices with  $R_S = 0, 250\Omega$ , respectively. Clearly, the reset time can be reduced; however, this comes at a price. Panels (e) and (f) show, for devices with different  $R_S$ , the current  $I_{\text{switch}} \equiv \min(I_c, I_{\text{latch}})$  above which each device no longer detects photons and the measured DE at  $I_0 = 0.975 I_{\text{switch}}$ . The data show that as  $R_S$  is increased,  $I_{\text{switch}}$  decreases far below  $I_c$  (due to reduction in  $I_{\text{latch}}$ ), resulting in a significantly reduced DE.<sup>14</sup>

To investigate the latched state, we fabricated devices designed to probe the stability of self-heating hotspots as a function of  $I_0$ ,  $L$ , and  $R_L$ . Each device consisted of three sections in series, as shown in Fig. 2(a): a  $3\text{-}\mu\text{m}$ -long  $100\text{-nm}$ -wide nanowire where the hotspot was nucleated, a wider ( $200$  nm) meandered section acting as an inductance, and a series of nine contact pads interspersed with Ti-film resistors. Also shown are the two electrical probes, which result in the circuit of Fig. 2(b): a high-impedance ( $R_p = 20$  k $\Omega$ ) three-point measurement of  $R_{\text{ss}}$ . We varied  $R_L$  by moving the probes along the line of contact pads and  $L$  by testing different devices (with different  $L$ ). We tested 66 devices on three chips and selected from these only unconstricted<sup>12</sup> nanowires with nearly identical linewidths ( $I_c \approx 22\text{--}24 \mu\text{A}$ ) and with  $R_L = 20\Omega\text{--}1000\Omega$  and  $L = 6\text{--}600$  nH.

For each  $L$  and  $R_L$ , we acquired a dc  $V$ - $I$  curve like those shown in Figs. 2(c) and 2(d), sweeping  $I_0$  downward starting from high values where the hotspot was stable.<sup>15</sup> These data can be converted to  $I_d$  and  $R_n$ , as shown in Figs. 2(e) and 2(f) [for the data of Fig. 2(d)]. From data of this kind,  $I_{\text{latch}}$  can be identified by the sudden jumps in  $I_d$ : for  $I_0 > I_{\text{latch}}$ ,  $I_d$  is fixed (at  $I_{\text{ss}}$ ), independent of  $I_0$  and  $R_L$ , as predicted by Eq. (1).<sup>16</sup> For the largest values of  $R_L$ ,  $I_d$  never reaches  $I_c$  [shown by a horizontal dashed line in Fig. 2(e)] because once it latches  $I_d \rightarrow I_{\text{ss}} < I_c$ . As  $R_L$  is decreased,  $I_{\text{latch}}$  increases as expected, until another feature appears when  $I_{\text{latch}} > I_c$ . In this region ( $I_c < I_0 < I_{\text{latch}}$ ) the nanowire can neither superconduct nor latch and instead undergoes relaxation oscillations,<sup>9,17</sup> as indicated in the figure, producing a periodic pulse train with a frequency that increases as  $I_0$  is increased.<sup>14</sup> The average resistance (from the dc  $V$ - $I$  curve) increases with this frequency, producing the observed continuous decrease in  $I_d$  until  $I_{\text{latch}}$  is reached.

The data in Fig. 3 show the measured  $I_{\text{latch}}$  as  $R_L$  and  $L$  are varied, plotted in dimensionless form as  $2\tau_e/\tau_{\text{th}} (\propto L/R_L^2)$  vs  $(I_{\text{latch}}/I_{\text{ss}})^2$ , which can be thought of as defining the boundary between stable and unstable hotspots. Our simple model de-

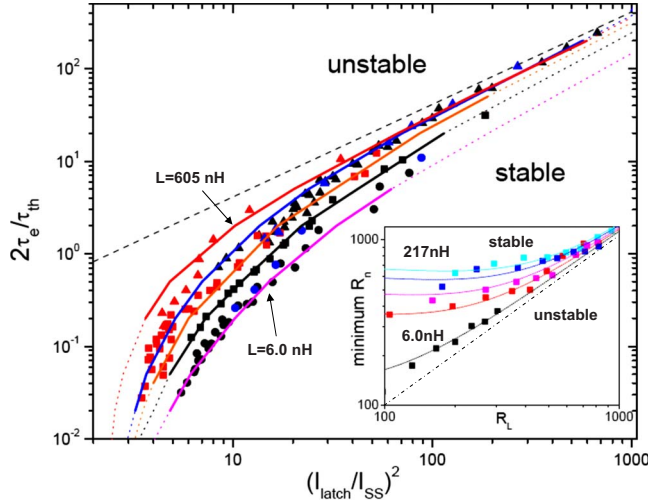


FIG. 3. (Color online) Summary of hotspot stability results. Data are shown from three different chips (indicated by different colors). Circles, squares, and triangles are data for  $L=6$ – $12$ ,  $15$ – $60$ , and  $120$ – $600$  nH, respectively. In the  $\tau_e \gg \tau_a$  limit (where NS domain-wall motion dominates the feedback), the data approach the dashed line, which is the prediction based on Eq. (1). The solid curves are obtained from Eq. (8) with a phase margin of  $30^\circ$ ; each curve corresponds to a fixed  $L$  in the set  $(6, 15, 30, 60, 600)$  nH and spans the range of  $R_L$  in the data. The dotted lines extend these predictions over a wider range of  $R_L$ . For  $\tau_e \ll \tau_{th}$  the NS domain walls are effectively fixed and the temperature feedback dominates. In this regime the feedback is always unstable when  $R_n \leq R_L$  (or equivalently,  $I_0 \lesssim 2I_{SS}$ ), as shown in the inset; the minimum stable  $R_n$  are all above the dashed-dotted line ( $R_n = R_L$ ).

scribed above predicts a line of slope 1 (indicated by the dashed line). The data do approach this line, although only in the  $\tau_e \gg \tau_{th}$  limit. This is consistent with the assumption of constant (or slowly varying)  $I_d$  under which Eq. (1) was derived. As  $\tau_e/\tau_{th}$  is decreased, the data trend downward, away from this line, and  $I_{latch}/I_{SS}$  becomes *almost independent* of  $\tau_e/\tau_{th}$  (all data approach the same vertical asymptote); this implies a minimum  $I_{latch}/I_{SS}$ , or equivalently, a minimum  $R_n/R_L$ , below which the hotspot is *always* unstable. This is shown in the inset: the measured minimum stable  $R_n$  is always greater than  $\sim R_L$ , over a range of  $L$  values from 6 to 217 nH (shown by solid symbols—solid lines are guides for the eyes).

This behavior can be explained in terms of a time scale  $\tau_a$  over which the temperature profile of the hotspot stabilizes into the quasisteady-state form which yields Eq. (1). For power-density variations faster than this, the NS boundaries do not have time to start moving, resulting instead in a temperature deviation  $\Delta T$ . Since the NS boundary occurs at  $T \approx T_c$ , where  $\rho_n$  is strongly temperature dependent (defined by  $d\rho/dT \equiv \beta > 0$ ), this changes  $R_n$ , giving a second parallel electrothermal feedback path which dominates for frequencies  $\omega \gg \tau_a^{-1}$ . We can describe this by replacing Eq. (1) with

$$\frac{\gamma \rho_n}{2} \left( \tau_a \frac{d^2 l}{dt^2} + \frac{dl}{dt} \right) = I_d^2 \rho(\Delta T) - I_{SS}^2 \rho_n, \quad (2)$$

$$c \frac{d\Delta T}{dt} = \frac{\gamma \rho_n \tau_a}{2} \frac{d^2 l}{dt^2} - h \Delta T. \quad (3)$$

Here,  $l$  is the hotspot length,  $\rho(\Delta T)$  is the resistance per unit length [with  $\rho(0) \equiv \rho_n$ ], and  $R_n = \rho(\Delta T)l$ . In Eq. (2),  $\tau_a$  is the characteristic time over which  $2v_{NS} = dl/dt$  adapts to changes in power density: for slow time scales  $dt \gg \tau_a$ , we have  $\tau_a d^2 l/dt^2 \ll dl/dt$  and Eq. (2) reduces to Eq. (1) (with  $\Delta T = 0$ ). For faster time scales,  $\tau_a d^2 l/dt^2$  becomes appreciable and acts as a source term for temperature deviations in Eq. (3). When  $dt \ll \tau_a$ ,  $\tau_a d^2 l/dt^2 \gg dl/dt$  and Eqs. (2) and (3) can be combined to give  $c \cdot d\Delta T/dt \approx I_d^2 \rho(\Delta T) - I_{SS}^2 \rho_n - h \Delta T$ .<sup>18</sup> In this limit, if  $R_L \leq R_n$  the bias circuit including  $R_L$  begins to look like a current source, which then results in positive feedback: a current change produces a temperature and resistance change of the same sign. Therefore, the hotspot is always unstable when  $R_n \leq R_L$ .

Expressing Eqs. (2) and (3) in dimensionless units ( $i \equiv I_d/I_0$ ,  $r \equiv R_n/R_L$ ,  $\lambda \equiv l\beta T_c/R_L$ , and  $\theta \equiv T/T_c$ ) and expanding to first order in small deviations ( $\delta i$ ,  $\delta r$ ,  $\delta \lambda$ ,  $\delta \theta$ ) from steady state, we obtain

$$\delta i' = -(i_0 \delta i + i_0^{-1} \delta r), \quad (4)$$

$$\delta r = \eta(i_0 - 1) \delta \theta + \eta^{-1} \delta \lambda, \quad (5)$$

$$\frac{\tau_a}{\tau_e} \delta \lambda'' + \delta \lambda' = 2\eta^2 \frac{\tau_e}{\tau_{th}} (\delta \theta + 2i_0 \eta^{-1} \delta i), \quad (6)$$

$$\delta \theta' = \frac{\Theta \tau_{th} \tau_a}{\eta \tau_e \tau_c} \delta \lambda'' - \frac{\tau_e}{\tau_c} \delta \theta. \quad (7)$$

Here, the prime denotes differentiation with respect to  $t/\tau_e$ ,  $i_0 \equiv I_0/I_{SS}$ ,  $\Theta \equiv (T_c - T_0)/T_c$ ,  $\eta \equiv \beta T_c/\rho_n$  characterizes the resistive transition slope, and  $\tau_c \equiv c/h$  is a cooling time constant. When  $\tau_e \gg \tau_{th}, \tau_a$ , the system reduces to  $\delta i'' + i_0 \delta i' - 4\tau_e/\tau_{th} \approx 0$ , which has damping coefficient  $\zeta = i_0(4\sqrt{\tau_e/\tau_{th}})^{-1}$ , as above. In the opposite limit, where  $\tau_e \ll \tau_{th}, \tau_a$ , we obtain  $\delta i'' + i_0 \delta i' + (2\eta\Theta\tau_e/\tau_c)(i_0 - 2) \approx 0$ . In agreement with our argument above, the oscillation frequency becomes negative for  $R_n < R_L$  ( $I_0 < 2I_{SS}$ ).

We characterize the stability of the system of Eqs. (4)–(7) using its “open loop” gain  $A_{ol}$ : we assume a small oscillatory perturbation by replacing  $\delta r$  in Eq. (4) with  $\Delta r e^{j\omega t}$  and responses  $(\delta i, \delta \theta, \delta \lambda, \delta r) e^{j\omega t}$ . Solving for  $A_{ol} \equiv \delta r/\Delta r$ , we obtain

$$A_{ol} = \frac{4 \frac{\tau_e}{\tau_{th}} \left( 1 + j\omega \frac{\tau_e}{\tau_c} \right) - 4\eta\Theta\omega^2 (i_0 - 1) \frac{\tau_a}{\tau_c}}{j\omega i_0 \left( 1 + \frac{j\omega}{i_0} \right) \left[ 2j\omega\eta\Theta \frac{\tau_a}{\tau_c} - \left( 1 + j\omega \frac{\tau_e}{\tau_c} \right) \left( 1 + j\omega \frac{\tau_a}{\tau_c} \right) \right]}. \quad (8)$$

The stability of the system can then be quantified by the phase margin  $\pi + \arg[A_{ol}(\omega_0)]$ , where  $\omega_0$  is the unity gain ( $|A_{ol}| = 1$ ) frequency. In the extreme case, when the phase margin is zero ( $\arg[A_{ol}(\omega_0)] = -\pi$ ), the feedback is positive. The solid lines in Fig. 3 show our best fit to the data. Note that although the stability is determined only by  $\tau_e/\tau_{th}$  and  $i_0$  in the two extreme limits (not visible in the figure), in the



intermediate region of interest here this is not the case, so several curves are shown. Each solid curve segment corresponds to a single  $L$ , over the range of  $R_L$  tested; the dotted lines continue these curves for a wider range of  $R_L$ . The data are grouped into three inductance ranges: 6–12, 15–60, and 120–600 nH, indicated by circles, squares, and triangles, respectively. We used fixed values  $\Theta=0.8$  and  $\eta=6.5$ , which are based on independent measurements, and fitted  $\tau_a=1.9$  ns and  $\tau_c=0.47$  ns to all data.<sup>19</sup> Separate values of  $\rho_n v_0$  were fitted to data from each of the three chips, differing at most by a factor of  $\sim 2$ . These fitted values were  $\rho_n v_0 \sim 1 \times 10^{11}$   $\Omega/s$ ; since  $\rho_n \sim 3 \times 10^9$   $\Omega/m$ , this gives  $v_0 \sim 30$  m/s, which is a reasonable value.

A natural question to ask in light of this analysis is whether it suggests a method for speeding up these devices. The most obvious way would be to increase the heat transfer

coefficient  $h$ , which increases both  $I_{ss}$  and  $v_0$ , moving the wire further into the unstable region and allowing its speed to be increased further without latching. However, at present it is unknown how much  $h$  can be increased before the DE begins to suffer. At some point, the photon-generated hotspot will disappear too quickly for the wire to respond in the desired fashion. In any case, experiments like those described here will be a useful measurement tool in future work for understanding the impact of changes in the material and/or substrate on the thermal coupling and electrothermal feedback.

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- <sup>10</sup>Since the negative feedback opposes the fast joule heating that produces the rising edge of the output pulse, it may also affect the timing jitter of this edge.
- <sup>11</sup>This description is further simplified by the fact that near the NS boundary all material properties can be approximated by their values at  $T_c$ .
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- <sup>14</sup>For the circuit in Fig. 1(b), if  $I_{latch} > I_c$ , the detector oscillates [see Fig. 2(e) and associated discussion] for the time constant of the bias tee, after which it senses the larger  $R_{dc}=5$  k $\Omega$  and latches. The absence of this burst of pulses is a signature for  $I_{latch} < I_c$ .
- <sup>15</sup>The results were almost identical when sweeping  $I_0$  upward, since the dark counts of the device allow it to lock into the latched state if it is stable.
- <sup>16</sup>From the observed  $I_{ss} \approx 5$   $\mu A$  and  $\rho_n \sim 3 \times 10^9$   $\Omega m^{-1}$ , Eq. (1) gives  $h \sim 5 \times 10^{-3}$   $W m^{-1} K^{-1}$ ; this gives  $\alpha \sim 30$  ( $T_c \approx 10$  K,  $T_0=2$  K, and  $I_c=22$   $\mu A$ ).
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