

XVIII. NETWORK SYNTHESIS

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A. TIME-DOMAIN SYNTHESIS BY FOURIER METHODS

The present note supplements the discussion given in the Quarterly Progress Report of January 15, 1952, page 66. Its purpose is to call attention to properties of the result given there that became apparent when the relevant notes were recently rewritten for inclusion in the book "Network Synthesis," which will be published by John Wiley and Sons, Incorporated, in the current year. A detailed discussion of this and related methods of time-domain synthesis will be found in Chapter XV of this volume. It will be recalled that, in the aforementioned discussion, the resultant system function was represented (for purposes of physical interpretation) as the net admittance of a series connection of boxes N_1 and N_2 whose impulse responses are, respectively, even-order and odd-order periodic repetitions of the desired time-domain pulse shape. We can, however, alternately define the box N_1 as having the same odd-order periodic response as box N_2 , for an excitation voltage that is the derivative of a unit square wave (for $t > 0$). In fact, this alternate attitude was taken in the original conception of the method, but it was then revised to the equivalent definition given in the discussion in the Quarterly Progress Report of January 15, 1952, because the latter seemed less awkward and more consistent with the definition of the properties of box N_2 .

It turns out, however, that if we proceed from the more awkward (but theoretically equivalent) definition of box N_1 , a resultant system function is obtained that is not only simpler because it does not contain redundant critical frequencies, but its right half-plane analyticity is automatically assured. The conclusion that the redundancy (in the system function given in the aforementioned discussion) disappears as the number of terms in the pertinent Fourier representation becomes larger and larger, so that ultimately the resultant system function obtained by either process is the same, is an important one. The revised approach is preferable in any practical situation in which a moderate number of terms is involved.

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B. A SADDLEPOINT THEOREM

In transient theory, second-order saddlepoints moving with time in the complex frequency plane are often used for approximate integration. Using the same notations as those of reference 1, we have

$$f(t) = \frac{1}{2\pi i} \int_{\gamma_s} F(s) e^{W(s,t)} ds$$

$$W(s, t) = st + \phi(s)$$

$$\frac{dW(s, t)}{ds} = t + \frac{d\phi(s)}{ds}$$

$$\frac{d^2W(s, t)}{ds^2} = \frac{d^2\phi(s)}{ds^2}$$

In the vicinity of a second-order saddlepoint, s_s ,

$$W(s, t) - W(s, t)_{s=s_s} \approx \frac{(s-s_s)^2}{2!} \left(\frac{d^2W(s, t)}{ds^2} \right)_{s=s_s}$$

We also put

$$\frac{1}{2} \left(\frac{d^2W(s, t)}{ds^2} \right)_{s=s_s} = \frac{1}{2} \left(\frac{d^2\phi(s)}{ds^2} \right)_{s=s_s} = A e^{j\alpha_2}$$

$$s - s_s = re^{j\theta}$$

$$\left(\frac{ds}{dt} \right)_{s=s_s} = v_s = |v_s| e^{j\psi_s}$$

In the saddlepoint method of integration, the contour of integration is deformed along the lines of steepest descent through the saddlepoint. It is shown in reference 1 that the directions of the lines of steepest descent are given by

$$\theta_1 = \frac{\pi}{2} - \frac{\alpha_2}{2}; \quad \left(\theta_3 = \frac{3\pi}{2} - \frac{\alpha_2}{2} \right) \quad (1)$$

At the saddlepoint, we have

$$\left(\frac{dW(s, t)}{ds} \right)_{s=s_s} = t + \left(\frac{d\phi(s)}{ds} \right)_{s=s_s} = 0$$

so that along the saddlepoint trajectory

$$\left(\frac{ds}{dt} \right)_{s=s_s} = v_s = - \frac{1}{\left(\frac{d^2\phi(s)}{ds^2} \right)_{s=s_s}}$$

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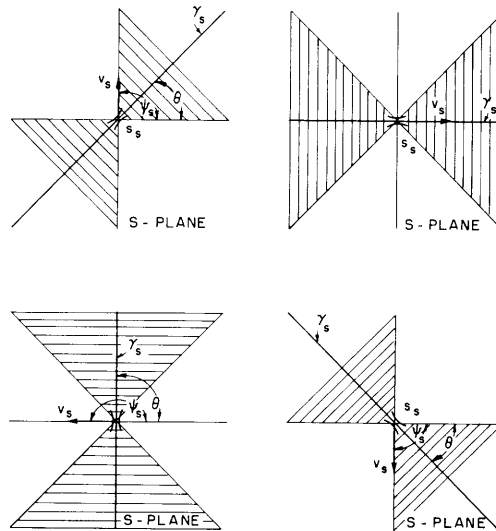


Fig. XVIII-1. Second-order saddlepoints in the complex frequency plane.

Thus

$$\psi_s = \pi - \alpha_2 \quad (2)$$

Equations 1 and 2 yield

$$\theta_1 = \frac{\psi_s}{2}; \quad \left(\theta_3 = \pi + \frac{\psi_s}{2} \right)$$

Theorem. At a second-order saddlepoint in the (upper Riemann sheet of the) complex frequency plane, the angle of the contour of integration along a line of steepest descent is half the angle of the saddlepoint velocity vector.

Figure XVIII-1 gives some simple illustrative examples of the connection between the movement of the second-order saddlepoint and the contour of integration.

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References

1. M. V. Cerrillo, Technical Report 55:2a, Research Laboratory of Electronics, M.I.T., May 3, 1950.