

### XIII. MECHANICAL TRANSLATION\*

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#### A. ON THE LIMITS OF FINITE-STATE DESCRIPTION

In the Quarterly Progress Report of April 15, 1956, two types of grammars were described formally: finite-state grammars with no independent memory that produce sentences word by word, and  $[\Sigma, F]$  grammars which can be represented as slightly less elementary finite-state processes and which impose phrase structure on the generated sentences rather than produce them from "left-to-right." A theorem was stated to the effect that every language describable in terms of a finite-state grammar (every finite-state language) is describable in terms of a system of phrase structure (is a terminal language) but not conversely. The natural question to raise is whether or not there are existent languages that fall outside the range of finite-state description, but within the range of phrase-structure grammars. Further investigation has shown that certain syntactic properties of English exclude it from the set of finite-state languages, but not from the set of terminal languages.

Suppose that  $A$  represents the alphabet of language  $L$ , and that  $S = a_1 \wedge a_2 \wedge \dots \wedge a_n$  ( $a_i \in A$ ) is a sentence of  $L$ .

Definition 1.  $S$  has an  $(i, j)$ -dependency with respect to  $L$  if and only if

- (i)  $1 \leq i < j \leq n$
- (ii) there are  $b_i, b_j \in A$  so chosen that  $S_1$  is not a sentence of  $L$  and  $S_2$  is a sentence of  $L$ , where  $S_1$  is formed by replacing the  $i^{\text{th}}$  symbol ( $a_i$ ) of  $S$  by  $b_i$ , and  $S_2$  is formed by replacing the  $j^{\text{th}}$  symbol ( $a_j$ ) of  $S_1$  by  $b_j$ .

Definition 2.  $D = \{(a_1, \beta_1), \dots, (a_m, \beta_m)\}$  is a dependency set for  $S$  in  $L$  if and only if

- (i) for  $1 \leq i \leq m$ ,  $S$  has an  $(a_i, \beta_i)$ -dependency with respect to  $L$
- (ii) for each  $i, j$ ,  $a_i < \beta_j$
- (iii) for  $i \neq j$ ,  $a_i \neq a_j$  and  $\beta_i \neq \beta_j$ .

If  $S$  contains an  $m$ -termed dependency set, then at least  $2^m$  states are necessary in the finite-state grammar that generates the language  $L$  that contains  $S$ . Hence, a necessary condition on finite-state languages is that there must be a finite upper limit to the size of their dependency sets. With this condition in mind, we can easily construct many nonfinite-state languages. For example, let  $L_1$  be the language containing the "sentences"  $aa, bb, abba, baab, aabbaa \dots$ , and, in general, all "mirror image" sentences consisting of a string  $X$  of  $a$ 's and  $b$ 's followed by  $X$  read from back to front, and only these. Then, for any  $m$ , we can find a dependency set  $D_m = \{(1, 2m), (2, 2m-1), \dots, (m, m+1)\}$ , so that  $L_1$  is not a finite-state language.

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### (XIII. MECHANICAL TRANSLATION)

Turning now to the English language, we find that there are infinite sets of sentences with just the mirror-image properties of  $L_1$ . For example, let  $S_1, S_2, S_3, \dots$  be declarative sentences. Then the following are all English sentences:

- (1) (i) If  $S_1$ , then  $S_2$ .
- (ii) Either  $S_3$ , or  $S_4$ .
- (iii) The man who said that  $S_5$ , is arriving today.

These sentences have dependencies between "if" and "then," "either" and "or," "man" and "is." But we can choose  $S_1, S_3$ , and  $S_5$  in (1) as (li), (lii), or (liii) themselves. Proceeding to construct sentences in this way, we arrive at sentences with dependency sets of more than any fixed number of terms, just as in the case of  $L_1$ . English is therefore not a finite-state language.

Note that  $L_1$  is a terminal language. It has the  $[\Sigma, F]$  grammar with  $\Sigma = \{Z\}$  and  $F = \{Z \rightarrow aZa, Z \rightarrow bZb, Z \rightarrow aa, Z \rightarrow bb\}$ . Hence, the argument that we have just given does not show that English is not a terminal language, since the sentences we have discussed could be given a  $[\Sigma, F]$  grammar in the same way as  $L_1$ . The question of the literal possibility or impossibility of a phrase-structure description of English therefore remains open, even though there is considerable evidence that more powerful methods are required if English is to be described effectively.

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