

XIX. NETWORK SYNTHESIS

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A. SYNTHESIS OF SAMPLED SIGNAL NETWORKS

A sampled signal network is defined as a network that contains a finite number of linear, passive circuit elements and which, when excited by a signal consisting of a series of impulses occurring at fixed, equally spaced time intervals, gives a response that consists only of a series of impulses occurring at these same time intervals. Such networks might find use in sampled-data systems or in radar systems.

A general synthesis procedure for such networks, which includes the basic realizability conditions and general realization methods, was developed. These networks are composed of resistors and open- and short-circuited transmission lines all of which have the same time delay. Synthesis with such transmission lines has also been studied by Richards (1).

The approximation problem for these networks was also studied. A general approximation procedure was developed, as well as an approximate lowpass-to-sampled-signal transformation. A paper is being prepared for publication which will give the main results of this research. These results will be summarized here.

1. Realizability Conditions

It is not difficult to show that a system function of this type must be a rational function of $e^{-\tau s}$:

$$H(s) = \frac{a_0 + a_1 e^{-\tau s} + a_2 e^{-2\tau s}, \dots, a_n e^{-n\tau s}}{b_0 + b_1 e^{-\tau s} + b_2 e^{-2\tau s}, \dots, b_n e^{-n\tau s}}$$

The realizability conditions become apparent immediately when the behavior of this function is considered in the complex s -plane. These conditions are exactly the same as those for lumped networks; this becomes clear when we remember that the conditions were derived from the standpoint of energy considerations and do not depend on the network being lumped. These conditions are well known and concern the analyticity of the function in the right-half plane and certain constraints on the j -axis real part. In the case of rational functions of $e^{-\tau s}$, these conditions imply the additional constraint that (a) $b_0 \neq 0$, and (b) $a_0 \neq 0$ for driving-point functions. These conditions can be immediately transformed to the z -plane, in which $z = e^{\tau s}$.

It is convenient to introduce another transformation

$$e^{-\tau s} = \frac{1 - w}{1 + w}, \quad w = \frac{1 - e^{-\tau s}}{1 + e^{-\tau s}} = \tanh \tau s/2$$

(Note that this is the negative of the usual w -transformation.) This transformation has the property that it transforms the j -axis of the s -plane into the j -axis of the w -plane with the left- and right-half planes transforming into the left- and right-half planes, respectively. Thus the system function, when it is expressed in terms of w , represents a realizable lumped network, and all of the well-known realizability tests for these networks can be used.

2. Realization Methods

The system function, expressed in terms of w , represents a realizable lumped network and can be realized by using the ordinary methods of lumped-network synthesis. Then we have only to perform some operation on this network so as to "undo" the change of variable. This operation is easily performed by using the transformation in the form $w = \tanh \tau s/2$.

The function $\tanh \tau s/2$ represents the input impedance of an ideal shorted transmission line with a characteristic impedance of one ohm and a one-way time delay of $\tau/2$ sec (or the input admittance of an open-circuited line). Therefore, the inverse transformation can be accomplished by substituting a short-circuited line with $Z_o = L$ ohms for each inductor of value L henrys, and an open-circuited line with $Y_o = C$ mhos for each capacitor — all of these lines having a one-way time delay of $\tau/2$ seconds. This completes the synthesis.

3. The Approximation Problem

Two approximation methods will be discussed; one is an adaptation of a well-known method (2), and the other is an approximate lowpass-to-sampled-signal transformation.

a. Guillemin's method

This method can be used to approximate any arbitrarily given magnitude characteristic plotted against $j\omega$. The given magnitude function will be an even function of ω and will be periodic, with a period $\omega_o = 2\pi/\tau$. It can therefore be approximated by the first n terms of a Fourier series of cosines

$$|F(\omega)|^2 = \sum_0^n a_k \cos k \tau \omega$$

The Tchebycheff multiple-angle formulas can then be used to transform this expression into a polynomial in $\cos k \tau \omega$:

$$|F(\omega)|^2 \sum_0^n b_k \cos k \tau \omega$$

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Finally, making use of the fact that the transformation

$$e^{-\tau s} = \frac{1+w}{1-w}$$

implies the existence of the relation

$$\cos \tau \omega = \frac{1 - \bar{\omega}^2}{1 + \bar{\omega}^2}$$

where $s = \sigma + j\omega$ and $w = \bar{\sigma} + j\bar{\omega}$, and we can write, for the magnitude function,

$$|G(\bar{\omega})|^2 = \sum_0^n b_k \left(\frac{1 - \bar{\omega}^2}{1 + \bar{\omega}^2} \right)^k$$

This sum can be combined into a rational function of $\bar{\omega}^2$ which is the approximation to the required function. Then standard methods can be used to find the realizable function of w that has this magnitude.

b. Lowpass-to-sampled-signal transformation

Another method would be to construct first, by any of the standard methods, a lumped lowpass network, which approximates the required characteristic and is all "contained" in the interval $-\pi/\tau < \bar{\omega} < \pi/\tau$, and then to make a suitable change of variable similar to the lowpass-to-bandpass transformation which will transform the network into an equivalent sampled-signal filter. Such a transformation is given by $\bar{\omega} \longleftrightarrow 2/\tau \tan \omega\tau/2$. The factor $2/\tau$ is included so that the two variables will have the same slope at the origin. The transformation can be interpreted physically as substituting a short-circuited ideal transmission line with characteristic impedance $2/\tau L$ ohms and one-way time delay of $\tau/2$ sec for each inductor of value L henrys in the lowpass network; and an open-circuited line with characteristic admittance of $2/\tau C$ mhos for each capacitor of value C farads.

This is not an exact transformation (neither is the usual lowpass-to-bandpass transformation), but it is quite good for frequency values near the origin. Moreover, since the transformation affects only the abscissa (ω -axis) and not the ordinate, the absolute tolerances on the error are not affected; for example, an equiripple filter would still be equiripple.

As far as time response is concerned, it can be shown (by computing derivatives at the origin) that, for any sample time τ , the average value and the first two moments of the impulse response of the lumped and sampled filters are identical, and if these two moments should be zero, then the first two nonzero moments are equal. In general, the smaller the sample time, the closer the higher-order moments agree.

Because the average values of the time response of the lumped and sampled networks

are equal, the envelope of the impulses produced by the sampled network will be a constant multiplied by the time response of the lumped network – the constant depending upon the sample time. If it is desirable to have the envelopes of the lumped and sampled signals equal, then an impedance-level change can be made on one or the other of the networks.

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References

1. P. I. Richards, Resistor transmission line circuits, Proc. IRE 36, 217-220 (1948).
2. E. A. Guillemin, Synthesis of RC networks, J. Math. Phys. 28, 22-42 (1949).