

VI. MICROWAVE ELECTRONICS*

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A. VARIATIONAL PRINCIPLE FOR UNIFORM AND PERIODIC ELECTRON BEAM SYSTEMS

A variational principle has been derived for the propagation constant of a wave in a uniform lossless system or of the propagation constant of one of the spatial harmonics in a periodic lossless system containing an electron beam. The principle can be simplified somewhat when it is limited to longitudinal beams and Brillouin beams with zero curl of the generalized momentum. We shall present the background, write down the principle for uniform structures, and give a brief summary of the salient results. The details are presented in reference 1.

The variational principle is based on the small-signal power theorem (2, 3). It is a consequence of this theorem that the existence of a solution of the form $\exp(-\gamma z)$ in a lossless uniform system always implies the existence of a solution $\exp(+\gamma^* z)$. Denote the transverse dependence of the excitation quantities in the solution with the z -dependence $\exp(-\gamma z)$ by the subscript (+), those pertaining to $\exp(+\gamma^* z)$ by the subscript (-). The variational principle for the longitudinal beam then is

$$\begin{aligned} \gamma = \frac{1}{4} \left\{ \int \left[(\nabla_T \times \bar{E}_+ + j\omega u_o \bar{H}_+) \cdot \bar{H}_-^* - (\nabla_T \times \bar{H}_+ - j\omega \epsilon_o \bar{E}_+ - \bar{J}_+) \cdot \bar{E}_-^* \right. \right. \\ \left. \left. + (j\omega \frac{m}{e} \bar{u}_+ - \bar{E}_+) \cdot \bar{J}_-^* + \frac{m}{e} (j\omega \bar{J}_+ - j\omega \rho_o \bar{u}_+) \cdot \bar{u}_-^* \right] da \right. \\ \left. - \oint \bar{E}_+ \cdot \bar{K}_-^* ds \right\} \left\{ \frac{1}{4} \int \left[\bar{E}_+ \times \bar{H}_-^* + \bar{E}_-^* \times \bar{H}_+ + V_-^* \bar{J}_+ + V_+ \bar{J}_-^* \right] \cdot \bar{i}_z da \right\}^{-1} \end{aligned}$$

The integration is carried over one cross section of the system. The vectors \bar{E} and \bar{H} are three-dimensional vectors, but functions of x and y only; u is the z -directed small-signal Eulerian velocity; V is Chu's kinetic voltage, $V = (m/e) u_o u$; and ∇_T is the two-dimensional operator

*This research was supported in part by Purchase Order DDL-B222 with Lincoln Laboratory, which is supported by the Department of the Army, the Department of the Navy, and the Department of the Air Force under Contract AF 19(122)-458 with M. I. T.

†From Raytheon Manufacturing Company.

$$\nabla_T = \bar{i}_x \frac{\partial}{\partial x} + \bar{i}_y \frac{\partial}{\partial y}$$

The contour integral $\oint \bar{E}_+ \cdot \bar{K}_-^* ds$ involves the field of the (+) solution and the surface current \bar{K}_- of the (-) solution on the conductors on which the trial solutions may fail to satisfy the electric boundary conditions. If these boundary conditions are fulfilled, the contour integral is automatically equal to zero.

The corresponding principle for the Brillouin beam is

$$\begin{aligned} \gamma = \frac{1}{4} \left\{ \int \left[(\nabla_T \times \bar{E}_+ + j\omega \bar{u}_O \bar{H}_+) \cdot \bar{H}_-^* - (\nabla_T \times \bar{H}_+ - j\omega \epsilon_O \bar{E}_+ \right. \right. \\ \left. \left. - \bar{J}_+\right) \cdot \bar{E}_-^* + \left[j\omega \frac{m}{e} \bar{u}_+ + \nabla_T \frac{m}{e} (\bar{u}_O \cdot \bar{u}_+) - \bar{E}_+ \right] \cdot \bar{J}_-^* \right. \\ \left. + (j\omega \bar{J}_+ + \bar{u}_O \nabla_T \cdot \bar{J}_+ - j\omega \rho_O \bar{u}_+) \cdot \frac{m}{e} \bar{u}_-^* \right] da - \oint \bar{E}_+ \cdot \bar{K}_-^* ds \left. \right\} \\ \left\{ \frac{1}{4} \int \bar{i}_z \cdot \left[\bar{E}_+ \times \bar{H}_-^* + \bar{E}_-^* \times \bar{H}_+ + \frac{m}{e} (\bar{u}_-^* \cdot \bar{u}_O) \bar{J} + \frac{m}{e} (\bar{u}_+ \cdot \bar{u}_O) \bar{J}_-^* \right] da \right\}^{-1} \end{aligned}$$

Here \bar{u} and \bar{J} are vectors with, in general, three components; \bar{u} is the small-signal Eulerian velocity; \bar{J} is the ac volume current density supplement by the surface current $\bar{r}_1 \cdot \bar{n}_O \bar{u}_O$, where $\bar{r}_1 \cdot \bar{n}$ is the displacement normal to the beam boundary of an electron on the boundary. Note that the denominator in both expressions is equal to the small-signal self-power of the solution if γ is imaginary, or to the small-signal cross power between the growing and decaying waves when γ is complex.

In spite of their involved appearance, the variational principle leads to simple results when appropriate trial solutions are introduced. It is reasonable to use as a trial solution a linear superposition of the wave solutions in the beam, with the slow-wave structure removed, and the wave solutions in the slow-wave structure, with the beam removed. When these trial solutions are introduced and when the coefficients are optimized, we obtain, in essence, a set of lossless coupled-mode equations of the type derived from the mode expansion (4). There are advantages in using the variational principle. First, we obtain a good idea of the approximations involved in using a finite set of coupled-mode equations. Second, the use of the variational principle assures that we obtain the best value of the propagation constant (and thus the best value for the gain) for the approximation used. Third, the coupled-mode formalisms derived from the variational principle for the traveling-wave tube and magnetron amplifier can be shown to reduce to the existing formalisms for these devices. Thus the principle

(VI. MICROWAVE ELECTRONICS)

provides the basis for a unified treatment of all microwave beam tubes.

The variational principle for periodic structures is similar to that for uniform structures.

H. A. Haus

References

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B. DIODE NOISE AT LOW FREQUENCIES (1 mc-60 mc)

In order to determine the behavior of shot-noise suppression in the space charge in front of a space-charge-limited cathode, we have been conducting experiments on planar diodes at frequencies below 60 mc. Experiments by North (1) on space-charge suppression of shot noise at 1 mc showed that in a diode structure the noise was considerably higher than predicted. North attributed this difference to the re-entry of elastically reflected electrons from the anode to the region of the potential minimum, which contributes to a reduction of space-charge suppression of the full shot noise from the cathode. North's experiments were in excellent agreement with predicted noise output in a negative-grid triode in which he claimed the elastically reflected anode electrons were not able to return to the potential minimum. A private communication from W. Veith (2) gave the results of two experiments involving movable element structures. Both structures had provisions for preventing elastically reflected electrons from returning to the region of potential minimum. Veith's measurements made at 1 mc showed no variation of noise output with element spacing. This is in direct contrast to North's predicted value of noise output, which is given (see ref. 1) by

$$\overline{i_a^2} = \left[1.29 \frac{kT}{e} \frac{g}{I} \right] \overline{i_s^2} \quad (1)$$

where i_a is the anode noise current, i_s is the full shot noise for I , I is the anode current, and $g = \partial I / \partial E_b$.

Since Veith's experiments were made at constant anode current, the shot-noise output should have been directly proportional to g , the diode conductance. In his

experiments, g was varied by a ratio of 50 to 1, and yet the value of $\overline{i_a^2}$ remained constant.

To further investigate these phenomena, we constructed a planar diode structure with provision for preventing elastically reflected anode electrons from returning to the region of the potential minimum. This was accomplished by making the anode a long, tapered tube that is closed at the end farthest from the cathode and has a disc with an aperture at the end nearest the cathode. The aperture is approximately 1.5 times the diameter of the planar cathode surface. The cathode-to-anode spacing is approximately 6 mm; the cathode diameter is 2.5 mm.

The noise from this structure was measured with a low-noise (noise figure, 1.5 db) amplifier whose bandwidth is approximately 10 mc at 30-mc center frequency. The output from the amplifier was fed to a Rada-Node (Kay Electric Co.), which essentially detected and averaged the output from the low-noise amplifier. The anode of the test diode and the anode of a temperature-limited tungsten diode were simultaneously connected to the input of the amplifier. Stray capacities were resonated with a shunt inductor. The circuit, which is shown in Fig. VI-1, is the same as the circuit used by North (1) except for the operating frequency. Measurements were made by applying anode voltage to either the test diode or to the reference diode. Since the anode resistance of the reference diode was infinite, its conduction or nonconduction did not affect the input circuit impedance. The anode resistance of the test diode varied with the current drawn, but its minimum value was more than 50,000 ohms. Since the amplifier input impedance was only 400 ohms, conduction or nonconduction of the test diode did not affect the input circuit impedance. Adjustment of the reference diode current made the output of the amplifier the same for the test diode input and the reference diode input. The reference diode noise current is thus equal to $\overline{i_a^2}$, Eq. 1, for the test diode. The ratio of $\overline{i_s^2}/\overline{i_a^2}$ (usually called Γ^2) is seen to be the factor by which the action of the potential minimum reduces the anode current fluctuations from full shot noise.

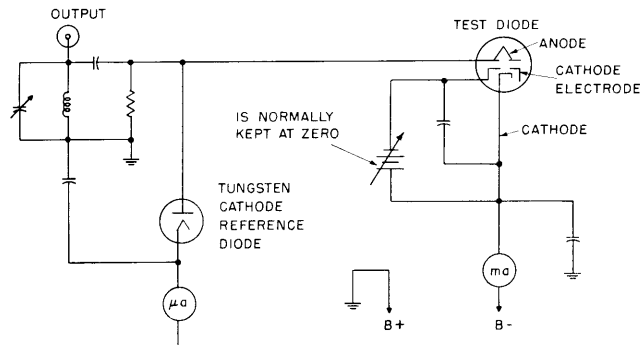


Fig. VI-1. Test circuit.

(VI. MICROWAVE ELECTRONICS)

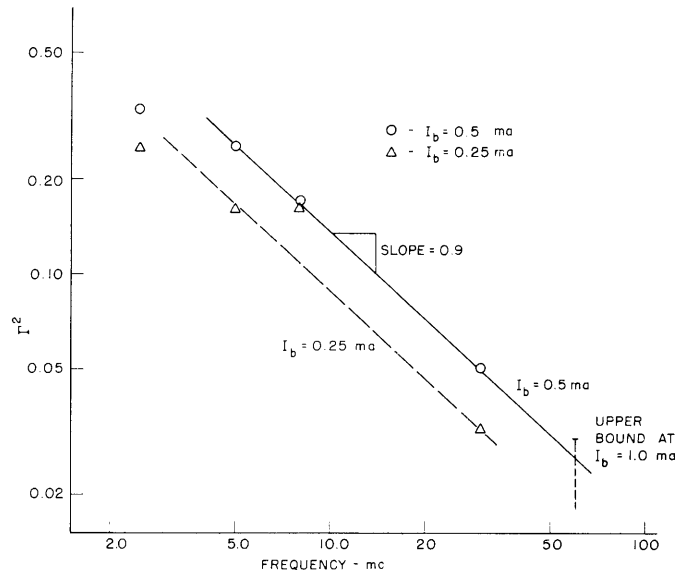


Fig. VI-2. Ratio of equivalent shot-noise current to anode current as a function of frequency.

Since both reference and test diodes have the same load impedance and the same bandwidth, the experimental value of Γ^2 is

$$\Gamma_{\text{exp}}^2 = \frac{I_n}{I_o} \quad (2)$$

where I_n is the reference diode current, and I_o is the test diode current. The predicted value of Γ^2 is the bracketed quantity in Eq. 1; that is,

$$\Gamma^2 = 1.29 \frac{kT}{e} \frac{g}{I} \quad (3)$$

Measurements at 30 mc showed Γ_{exp}^2 to be about 500 times the predicted value. By using a sensitive radio receiver in place of the 30-mc amplifier and Rada-Node, measurements were taken down to 2.5 mc. They are shown in Fig. VI-2. A 60-mc measurement was taken by the same technique except for the use of 60-mc components. As the plot shows, the output exhibits the $1/f$ characteristic of flicker noise. Lindemann and van der Ziel (3) investigated the same phenomenon and attributed it to the fact that ions were partially neutralizing the space charge in the potential minimum region. Since these ions have a varying lifetime in the space charge, they produce a fluctuation of anode current with a spectral density proportional to $1/f$. By inserting a negative electrode between cathode and anode, Lindemann and van der Ziel were able to eliminate this excess noise.

The diode structure used in our experiments is a parallel-flow Pierce gun, with an

insulated cathode electrode normally run at cathode potential. By biasing the electrode negative with respect to the cathode, we hoped to drain off positive ions. This was attempted, with the anode voltage increased to produce a constant anode current, and the noise output was measured. No change was observed for an auxiliary electrode voltage variation from zero to -15 volts. This result does not entirely rule out ions as the cause of the noise output; the variation of the cathode-anode field may not be sufficient to cause noticeable deflection of ions from the potential minimum region.

In future experiments we shall attempt to eliminate the ion source. It is believed to be the anode surface, which receives a deposit of cathode material during the conversion and activation process. By constructing the anode from titanium, this deposit will be bound more strongly to the anode surface, and the production of ions by electron bombardment may be reduced.

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