XVII. NETWORK SYNTHESIS

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A. EXACT SOLUTION OF THE APPROXIMATION PROBLEM FOR EQUIRIPPLE RC FILTERS

Theoretically exact results of this study, with their detailed derivation, are presented in the writer's thesis (1). In particular, exact expressions are given for the poles and zeros of the transfer admittance of a lowpass filter that has equiripple (often called "Tschebycheff") behavior in either the passband or the stop-band, as well as simple poles restricted to the negative real axis. These expressions were obtained with the aid of potential analogy and conformal mapping (2, 3, 4, 5, 6), and they involve elliptic functions (7, 8) for which twelve-place tables are available (9).

T. V. D. Pampoucas

References

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B. SYNTHESIS OF A CONSTANT REACTIVE TRANSFER FUNCTION

The problem of constructing a lossless network whose transfer function approximates a constant (imaginary) for a finite band of frequencies has been studied (1). When such a network is connected in series with a resistor, the transfer function of the resulting configuration can be made to approximate any arbitrary complex number for a finite

frequency band, provided that an ideal transformer is used to accomodate a possible negative sign. For the special case R = 0, a 90° phase-shift network is obtained.

The transfer function of a lossless network can be written as

$$T_{n+1}(s) = \frac{P_{2n+2}(s)}{s\left(s^2 - \omega_1^2\right) \dots \left(s^2 - \omega_n^2\right)}$$

where $P_{2n+2}(s)$ is an even function of degree $2n + 2$. We assume that the ω_i are ordered in such a way that

$$\omega_1 < \omega_2 < \dots < \omega_n$$

and that we want $T_{n+1}(s)$ to approximate a constant for $0 < \omega < \omega_1$. By a frequency transformation, we can always make $\omega_1 > 1$.

For any choice of ω_i, subject to these conditions, it is possible to find a $P_{2n+2}(s)$ that is such that the resulting $T_{n+1}(s)$ has the properties

$$T_{n+1}(j) = j$$

$$\left.\frac{d^k T_{n+1}(s)}{d\omega^k}\right|_{s=j} = \begin{cases} 0 & k \leq n + 1 \\ \neq 0 & k > n \end{cases}$$

The polynomial $P_{2n+2}(s)$ can be found by solving a set of $n + 2$ linear equations for its coefficients.

The properties of $T_{n+1}(s)$ make it a potential solution to our problem. The only question is, For a given n, is there an optimal choice of ω_i? The results of the present investigation indicate that the choice $\omega_1 = \omega_2 = \dots = \omega_n = \infty$ is optimal. This choice does not lead to a physically realizable network. However, it has been found that for reasonably large ω_i, say, $3 < \omega_i < 10$, the resulting functions are close to the optimal functions in the approximation band, $0 < \omega < \omega_1$.

<div align="right">J. D. R. Kramer, Jr.</div>

References

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