XV. PHYSICAL ACOUSTICS*

Prof. U. Ingard
Prof. R. D. Fay
Dr. W. W. Lang
L. W. Dean III
P. Gottlieb
G. C. Maling, Jr.
E. J. Martens, Jr.
M. D. Mintz
M. B. Moffett
H. L. Willke, Jr.

A. SPHERICAL WAVES OF FINITE AMPLITUDE

A somewhat unconventional method has been devised for the analysis of spherical waves of finite amplitude. Difficulties have been encountered in devising a satisfactory system of symbols whereby the concepts can be expressed in a manner that is convincing and free from ambiguity.

The problem and the progress that has been made can best be indicated by presenting the pertinent results that have been obtained.

In a sound field comprising oppositely directed plane waves of finite amplitude, it has been shown (1) that to a specified degree of approximation: (a) Linear superposition is valid for particle velocities, and (b) the speed of propagation is the same in both directions, the incremental speed being proportional to the difference in the particle velocities in the incident and reflected waves.

These findings are extended to spherical waves by assuming that the conical paths of transmission are composed of a series of cylinders of infinitesimal length with infinitesimal increments in area at the junctions.

The particle velocities of the incident and the reflected waves are designated v_i and v_r (the v's being assumed positive in the outward direction). The speed of propagation of each wave is

$$S = S_o + a(v_i - v_r)$$

where S_o is the speed of infinitesimal waves, and $\underline{a}$ is a constant of controversial magnitude. The classical value is $a = (\gamma+1)/2$, whereas the value obtained by the author (2) is $a = (\gamma+1)/3$.

An observer traveling outward at speed S_o would maintain a fixed distance from the axis crossings of v_i. Within the cylindrical section of length dx, he would observe a change in the magnitude of v_i, given by

$$dv_i = -(S - S_o)\frac{\partial v_i}{\partial x}\,dt$$

In crossing the area discontinuity from A to A + dA, there are additional increments arising from reflections.

*This research was supported in part by the U.S. Navy (Office of Naval Research) under Contract Nonr-1841(42). Reproduction in whole or in part is permitted for any purpose of the United States Government.

The total change from the beginning of cylinder A to the beginning of cylinder A + dA is, then

$$\frac{\partial v_i}{\partial t} = -(S - S_o)\frac{\partial v_i}{\partial x} - S_o \frac{dA}{2Adr}(v_i + v_r)$$

The corresponding change in the reflected wave as seen by the same observer (i.e., traveling at speed S_o in the direction opposite to that in which v_r is propagated) is, then

$$\frac{\partial v_r}{\partial t} = +(S + S_o)\frac{\partial v_r}{\partial x} - S_o \frac{dA}{2Adr}(v_i + v_r)$$

Now assume that v_i is separated into two components: v_l, which is equal in magnitude and direction to v_r (but propagated in the positive direction), and the residue $v_t = v_i - v_l$ $(= v_i - v_r)$. The subscript t indicates that this component represents the transmitted energy. The reflected portion of v_r is to be considered as an increment in v_l. We then have (neglecting viscous losses):

$$\frac{\partial v_t}{\partial t} = -av_t \frac{\partial v_t}{\partial x} - S_o \frac{dA}{2Adr} v_t$$

$$\frac{\partial v_l}{\partial t} = -av_t \frac{\partial v_l}{\partial x} - S_o \frac{dA}{2Adr}(v_l + v_r)$$

$$\frac{\partial v_r}{\partial t} = (2S_o + av_t)\frac{\partial v_r}{\partial x} - S_o \frac{dA}{2Adr}(v_t + v_l + v_r)$$

We next assume that when a steady state exists (in unbounded spherical space) the relations between velocity components are such that the apparent direction of propagation of v_r is reversed. This implies that $\partial v_r/\partial t = \partial v_l/\partial t$, or that

$$2(S_o + av_t)\frac{\partial v_r}{\partial x} = S_o \frac{dA}{2Adr} v_t$$

or that

$$\frac{\partial v_r}{\partial x} = \frac{dA}{4Adr}\left(\frac{v_t}{1 + \frac{a}{S_o} v_t}\right)$$

For spherical waves, the value of (dA)/(4Adr) is of course 1/2r, but the correct interpretation of the relation is, perhaps, more readily grasped in the form given. The slope $\partial v_r/\partial x$ is that which exists within a cylindrical section, that is, when the area A is constant. From this point of view,

$$v_r = \frac{dA}{4Adr}\int\left(\frac{v_t}{1 + \frac{a}{S_o} v_t}\right)dx$$

This interpretation is consistent with the familiar solution for unbounded spherical waves of infinitesimal amplitude, for which case we have

$$\frac{a}{S_o} v_t \ll 1$$

Thus, if

$$v_t = V e^{jkx}$$

$$v_r = \frac{-j}{2kr} v_t$$

$$v_i = v_t + v_r = v_t\left(1 - \frac{j}{2kr}\right)$$

the following familiar relation is obtained

$$v = v_i + v_r = v_t\left(1 - \frac{j}{kr}\right)$$

The indicated procedure from this point on is to find the waveform for v_t that is most stable and the corresponding value for v_r, and then from these to find the excess pressure from the relations previously given (1). Since the most stable waveform for a plane progressive wave (at least in closed form) has not yet been found, this procedure cannot be expected to yield completely satisfactory results for spherical waves.

The expression for v_t can, however, be put into a more workable form by making the substitution $v_t = U(r_o/r)$, where r_o/r represents $(A_o/A)^{1/2}$. Then

$$\frac{\partial v_t}{\partial t} = \frac{r_o}{r}\frac{\partial U}{\partial t} - U\frac{r_o}{r^2} S_o$$

because, to the traveling observer, the area is certainly a function of time. But

$$\frac{\partial v_t}{\partial x} = \frac{r_o}{r}\frac{\partial U}{\partial x}$$

since it is the gradient at constant area that is involved. Therefore

$$\frac{\partial U}{\partial t} = -\frac{ar_o}{r} U \frac{\partial U}{\partial x} - \text{losses}$$

This equation states the time rate of change in magnitude of a point on the wave which is at a fixed distance from an axis crossing (that is, progressing at speed S_o) in terms of the magnitude, the slope, and the radius at a given instant and place.

With this interpretation understood, it is possible to derive useful information concerning the behavior of the spherical wave. The derivation of this information is being carried out.

R. D. Fay

References

1. R. D. Fay, Oppositely directed plane finite waves, J. Acoust. Soc. Am. 29, 1200-1203 (1957).

2. R. D. Fay, Successful method of attack on plane progressive finite waves, J. Acoust. Soc. Am. 28, 910-914 (1956).

B. EXCITATION OF ACOUSTIC CAVITIES BY DC FLOW

There have been many experimental investigations concerning the effects of dc flow on acoustic resonators. In particular, it has been found that there is a relationship between the cavity Q and the dc velocity that is necessary to excite oscillations in the cavity. We shall attempt to explain this relationship from a theoretical point of view.

The most convenient example is a simple pipe resonator, as shown in Fig. XV-1. If the equations of motion are linearized in acoustic variables, they become

$$\frac{\partial \rho}{\partial t} + U\frac{\partial \rho}{\partial x} = -\rho_o\frac{\partial v}{\partial y} - \rho_o\frac{\partial u}{\partial x} \qquad \text{(continuity equation)} \tag{1}$$

$$\left.\begin{aligned} \frac{\partial v}{\partial t} + U\frac{\partial v}{\partial x} - \nu\nabla^2 v &= -\frac{c_o^2}{\rho_o}\frac{\partial \rho}{\partial y} \\ \frac{\partial u}{\partial t} + U\frac{\partial u}{\partial x} - \nu\nabla^2 u &= -\frac{c_o^2}{\rho_o}\frac{\partial \rho}{\partial x} \end{aligned}\right\} \qquad \text{(momentum equations)} \tag{2}$$

where u = velocity in x direction, v = velocity in y direction, and ν = kinematic viscosity. If u and v are eliminated, we obtain the wave equation for ρ:

$$\frac{\partial^2 \rho}{\partial t^2} - \nu\nabla^2\frac{\partial \rho}{\partial t} - c_o^2\frac{\partial^2 \rho}{\partial y^2} = c_o^2\frac{\partial^2 \rho}{\partial x^2} - 2U\frac{\partial^2 \rho}{\partial x \partial t} - U^2\frac{\partial^2 \rho}{\partial x^2} - \nu U\nabla^2\frac{\partial \rho}{\partial x} \tag{3}$$

If we consider just the inside of the pipe shown in Fig. XV-1, all the terms on the right-hand side of Eq. 3 will be zero, since there is no x variation for the lowest modes of the pipe resonator. If we confine our attention to the lowest mode, the lumped-circuit parameters can be determined from the boundary conditions. (In other words, the resonance frequency is found from the pipe length, and the Q is found from the total of all the damping mechanisms.) Thus the left-hand side of Eq. 3 becomes an ordinary resonance equation

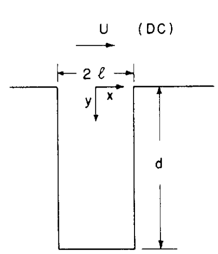

Fig. XV-1. Resonator and coordinate system.

$$\frac{\partial^2 \rho}{\partial t^2} + \frac{\omega_o}{Q} \frac{\partial \rho}{\partial t} + \omega_o^2 \rho = 0 \tag{4}$$

The solutions that satisfy Eq. 4 are of the form

$$\begin{aligned} \rho_1 &= \frac{\rho_o A_1}{c} \sin \frac{\pi y}{2d} \left(\sin \omega_o t + \frac{1}{Q} \cos \omega_o t \right) \\ v_1 &= A_1 \cos \frac{\pi y}{2d} \left(\cos \omega_o t - \frac{1}{Q} \sin \omega_o t \right) \end{aligned} \tag{5}$$

Turning to the right-hand side of Eq. 3, we see that all the terms contain derivatives with respect to x. This means that they will be finite only outside the mouth of the pipe. The first term on this side is not multiplied by U, so that it is present in the absence of U and will therefore not contribute to the resonance excitation caused by U. Examining the time dependence of the remaining terms, we note that $\partial^2 \rho / \partial x \partial t$ has the same time dependence as the damping term of Eq. 4. The term $U^2 (\partial^2 \rho / \partial x^2)$ has the same time dependence as the reactive terms of Eq. 4 for the $\sin \omega_o t$ term. (Thus this term merely shifts the frequency of the resonator, as has been amply demonstrated experimentally.) The smaller $(\cos \omega_o t)$ part of this term has the damping time dependence. The last term on the right-hand side of Eq. 3 is a viscosity term that has the reactive time dependence for the large $(\sin \omega_o t)$ term and resistive time dependence for the small $(\cos \omega_o t)$ term. If the viscous effects are small, this term can be neglected. Thus we are interested only in the second term and the small part of the third term on the right-hand side of Eq. 3. To put these terms in a lumped-parameter form such as Eq. 4 would require a knowledge of the exact wave field over the interaction volume. Therefore, it is most convenient to leave the parameters for these terms unspecified, except for sign. Figure XV-2 reveals that the interaction volume is greatest on the side of the tube where the dc flow actually enters the tube slightly. At this side $\partial \rho / \partial x \approx -(\rho / \ell)$, since we expect the sound field to be maximum near the center of the mouth. Thus the resistive terms of Eq. 3 give

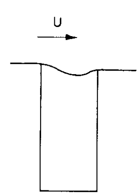

Fig. XV-2. Resonator with dc flow.

$$\frac{\omega_o}{Q} = \frac{2Ug}{\ell} - \frac{U^2 h}{\ell^2 Q \omega_o}$$

where g and h are the unspecified positive parameters.

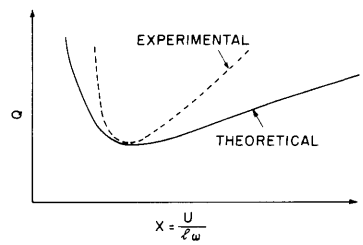

Fig. XV-3. Excitation regions.

In Fig. XV-3, Q is plotted as a function of U, and the resulting curve agrees qualitatively with the experimental results (1). These oscillations can be thought of as being excited initially by the noise in the dc flow, which is caused by the geometry of the mouth of the pipe. In other words, the mouth of the pipe will disturb the stream, whether the pipe resonates or not. The spectrum of this noise would be expected to have a peak near $\omega = U\pi/\ell$, which is also near the minimum of the curve of Fig. XV-3. The actual experimental curve then would lie inside the theoretical one, which is, qualitatively, the situation shown in Fig. XV-3.

P. Gottlieb

References

1. L. W. Dean III, Excitation of acoustic resonators by wind, S.M. Thesis, Department of Electrical Engineering, M.I.T., June 1957.

C. SOUND TRANSMISSION THROUGH A VELOCITY DISCONTINUITY

This report presents a discussion and comparison of methods and results for the problem of the reflection and refraction of a sound wave at the interface between two media in motion relative to each other, as shown in Fig. XV-4. When the problem was first reported (1), incorrect boundary conditions were assumed. The correct treatment (2) was given later, but remains unverified experimentally because such an interface is unstable. Indeed, the validity, or even the necessity, of any derivation under such conditions of instability might be questioned. In order to answer these questions, the problem of the propagation of a plane wave in a continuous flow, with velocity gradient perpendicular to the flow direction, has been solved by dividing the flow field into layers of constant velocity, as shown in Fig. XV-5. Then each interface between the layers

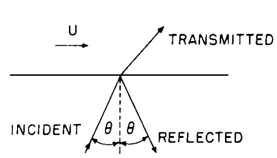

Fig. XV-4. Interface between moving media.

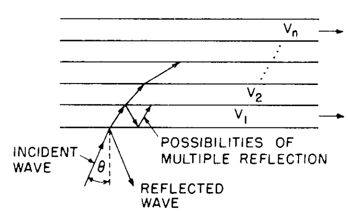

Fig. XV-5. Layers of constant velocity.

is similar to the situation shown in Fig. XV-4, and a continuous velocity distribution can be approximated by letting the thickness of all the layers approach zero. This solution is then compared with the analytic solution obtained from the wave equation for the continuous flow.

With the use of Ribner's solutions, the sound pressure, after passing into the first layer, can be written

$$\frac{p_1}{p_o} = T = \frac{2\delta_1}{\delta_1 + \dfrac{(\delta_1 - \sin^2\theta)^{1/2}}{\cos\theta}} \tag{1}$$

where $\delta_1 = (1 - M_1 \sin\theta)^2$, with $M_1 = V_1/c$. In order to obtain the exact expression for the sound transmission through layers, we must sum all of the multiple reflections of the form indicated in Fig. XV-5, in addition to taking the product of the transmission coefficients for each layer. However, if $M \sin\theta \ll 1$ throughout the given region, it is seen that the reflection coefficient at the n^{th} layer is of the order of magnitude $M_n \sin\theta$. Because any fraction of the wave that is reflected once must be reflected a second time before it can contribute to the transmitted fraction, the contribution to the transmitted wave caused by multiple reflections will be of the order of magnitude $(M \sin\theta)^2$. In the present case, the approximation will be carried only to the order $M \sin\theta$, and so the multiple reflections do not have to be considered. If the approximation is further restricted so that $\sin\theta \ll 1$, then Eq. 1 can be rewritten as

$$\frac{p_1}{p_o} \approx 1 - \frac{M_1 \sin\theta}{2} \tag{1a}$$

With the same approximation, it is found that

$$\frac{p_n}{p_{n-1}} \approx 1 - \frac{(M_n - M_{n-1}) \sin\theta}{2}$$

since the change in angle of incidence from layer to layer will give only a higher order correction. Thus, the total transmission coefficient is given by

$$\frac{p_n}{p_o} \approx 1 - \frac{M_n \sin\theta}{2} \tag{2}$$

For the analytic solution it is convenient to consider a two-dimensional coordinate system with a fluid velocity in the x-direction, given by $M = by$ (where b is a constant). If the sound pressure is assumed to be of the form

$$p = \exp\left(-i\omega t + \frac{i\omega x \sin\theta}{c}\right) F(y) \tag{3}$$

then the wave equation becomes

$$\frac{d^2F}{dy^2} + \frac{2b \sin\theta}{1 - by \sin\theta} \frac{dF}{dy} + \frac{\omega^2}{c^2} [(1-by \sin\theta)^2 - \sin^2\theta] F = 0 \tag{4}$$

If the approximation $M \sin\theta \ll 1$ again is used, the solution of Eq. 4 is

$$F(y) = Nu^{1/2} Z_{1/3}\left(\frac{2}{3} u^{3/2}\right) e^{-by \sin\theta}$$

where $N(\theta)$ is a normalizing factor, $Z_{1/3}$ is an appropriate Bessel or Hankel function of order 1/3, and

$$u = \frac{1 - \sin^2\theta - 2by \sin\theta}{\left(\frac{2bc \sin\theta}{\omega}\right)^{2/3}}$$

Now, if the approximation $\sin\theta \ll 1$ is made again, the asymptotic form of the Hankel function can be used to obtain

$$H_{1/3}^{(1)}\left(\frac{2}{3} u^{3/2}\right) \rightarrow \frac{A(\theta, \omega)}{u^{3/4}} e^{(i\omega y)/c}$$

If the initial condition is a plane wave at $y = 0$ with a specified value of θ, Eq. 3 becomes

$$\frac{p(y, \theta, \omega)}{p(0, \theta, \omega)} = \left(1 - \frac{M \sin\theta}{2}\right) \exp\left(- i\omega t + \frac{i\omega x \sin\theta}{c} + \frac{i\omega y}{c}\right) \tag{5}$$

Because the exponential factor merely indicates a plane wave, this result agrees with the transmission coefficient of Eq. 2. (In the present approximation, $y \cos\theta$ is represented by y in the exponential factor of Eq. 5.)

If the method of Keller (1) is used, the following expression, rather than Eq. 1 is obtained:

$$\frac{p_1}{p_o} = T = \frac{2\delta_1^{1/2}}{\delta_1^{1/2} + \frac{(\delta_1 - \sin^2\theta)^{1/2}}{\cos\theta}}$$

This leads to

$$\frac{p_n}{p_o} = 1 + 0\left(M_n^2\right)$$

The M-term drops out entirely, which is clearly in disagreement with the results given in Eqs. 2 and 5. Thus, only the method that leads to Eq. 2 is correct, and the solution does have applications to physical situations.

P. Gottlieb

References

1. J. B. Keller, Reflection and transmission of sound by a moving medium, J. Acoust. Soc. Am. 27, 1044-1047 (1955).

2. H. S. Ribner, Reflection, transmission, and amplification of sound by a moving medium, J. Acoust. Soc. Am. 29, 435-441 (1957).

D. CALCULATION OF GROUND-LOSS FACTOR

Figure XV-6 shows an acoustic monopole in an absorbing plane that is assumed to have a normal admittance β. Calculation of the pressure amplitude as a function of distance from the source at constant ϕ shows that the sound pressure decreases as $1/r$ for sufficiently large distances from the source, but that the amplitude is always less than that which would be calculated in the absence of the plane. The ratio of these two amplitudes, expressed in terms of decibels, is called the "ground-loss factor."

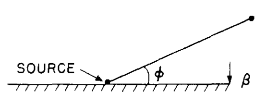

Fig. XV-6. A sound source in an absorbing plane.

The ground-loss factor for a constant-strength monopole in an absorbing plane can be easily calculated from the theory developed by Ingard (1) concerning the reflection of a spherical sound source from an infinite plane. The purpose here is to present the results of the calculation and to show the limits within which the ground-loss factor is applicable.

A source whose pressure field is given by $p_s = [\exp(ikr)]/(kr)$ in the absence of a plane will have a field in the presence of a plane (1) given by

$$|p_t| = \left| \frac{\exp(ikr)}{kr} (1+Q) \right| \tag{1}$$

The source must be in the boundary for this result to be valid. In Eq. 1,

$$Q = R_o + (1 - R_o) F$$

$$F = 1 - (\beta + \sin\phi) \int_0^\infty \frac{\exp(-krt)}{[(1+\beta\sin\phi + it)^2 - (1 - \sin^2\phi)(1-\beta^2)]^{1/2}}$$

$$R_o = \frac{\sin\phi - \beta}{\sin\phi + \beta}$$

The quantity F can be expanded in an asymptotic series whose value is

$$F = \frac{1}{2\rho} - \frac{1.3}{(2\rho)^2} + \ldots \tag{2}$$

where $\rho = ikr(\sin\phi + \beta)^2/[2(1 - \beta\sin\phi)]$. The ground-loss factor, G, is defined as the ratio $|p_t|/|p_s|$ when $|p_t|$ decreases as $1/r$, hence it must be calculated when F is small. Therefore

$$G = 20\log(1 + R_o) = 6 - 20\log(1 + \beta/\sin\phi) \tag{3}$$

The ground-loss factor is plotted as a function of ϕ for several values of normal admittance β in Fig. XV-7.

The quantity F is important only in the calculation of the value of kr at which the ground-loss factor becomes applicable. For our purposes, $(kr)_{min}$ will be calculated from the value of F that results in a pressure at $(kr)_{min}$ that is within 0.5 db (6 per cent) of the value that would be obtained if F were zero. Since, for real β, F is

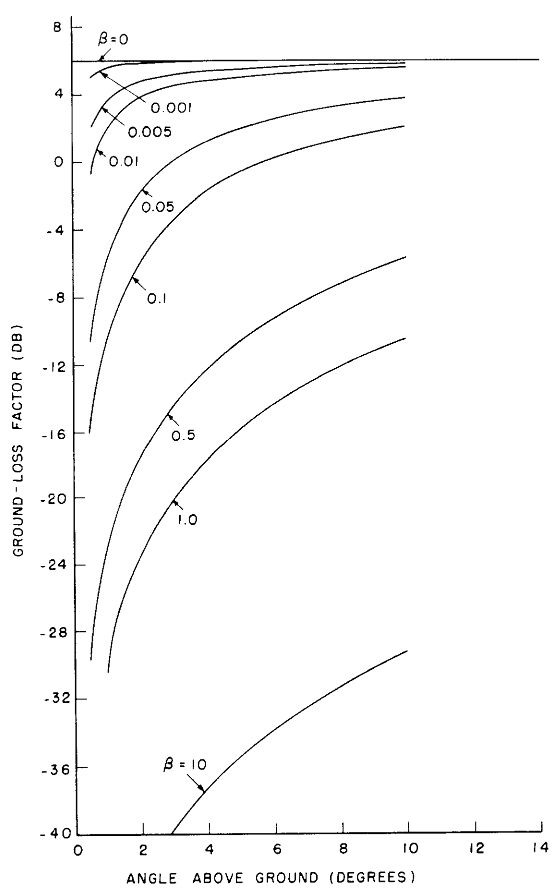

Fig. XV-7. Theoretical ground-loss factor.

imaginary, as Ingard (2) has shown, the sound pressure can be written

$$|p_t| = \frac{1}{kr}\left|(1 + R_o) + i(1 - R_o)\,|F|\right| \tag{4}$$

so that the limit on $|F|$ according to the foregoing criterion is found from

$$\left[(1 + R_o)^2 + (1 - R_o)^2\,|F|^2\right]^{1/2} \leq 1.06(1 + R_o) \tag{5}$$

If R_o is replaced by $(\sin\phi - \beta)/(\sin\phi + \beta)$, Eq. 5 can be solved for F:

$$F \leq \frac{0.35 \sin\phi}{\beta} \tag{6}$$

The magnitude of F is always less than unity (3), and therefore the minimum value of kr must be zero (within 0.5 db) for all β and ϕ for which $\sin\phi/\beta \geq 2.85$. By using Eq. 6 and Ingard's results (4), we can easily calculate $(kr)_{min}$ for any β and ϕ. The results of these calculations are given in Table XV-1.

Table XV-1.

$(kr)_{min}$ for Various Values of ϕ and β.

ϕ (degrees) \ β	0.001	0.005	0.01	0.05	0.1	0.5	1.0	10.0
0.5	0	1800	5720	5220	2880	628	316	35.0
1	0	0	450	2200	1160	300	159	19.0
2	0	0	0	417	440	142	78	11.1
3	0	0	0	133	165	91.5	53.5	8.50
4	0	0	0	42	105	64.2	39.2	7.10
5	0	0	0	18	57.2	48.3	30.4	6.20
6	0	0	0	8.4	33.5	37.4	24.6	5.40
7	0	0	0	0	20.4	30.2	20.5	5.00
8	0	0	0	0	10.6	25.8	17.6	4.60
9	0	0	0	0	7.6	22.5	15.6	4.50
10	0	0	0	0	4.3	19.2	13.6	4.30

For $\sin\phi < \beta$, the reflection coefficient R_o is negative, so that, according to Eq. 4, the reflected wave cancels out part of the pressure that would be present in the absence of reflections. Therefore, the pressure field decreases as 1/r only at very large distances from the source, at least for small values of β. For $\sin\phi > \beta$, R_o is positive, and the field decreases as 1/r at much smaller values of kr. For small

β and $\sin\phi \gg \beta$, total pressure varies as $1/r$ (within 0.5 db) for all values of kr. These conclusions are illustrated by Table XV-1.

G. C. Maling, Jr.

References

1. U. Ingard, On the reflection of a spherical sound wave from an infinite plane, J. Acoust. Soc. Am. 23, 329-335 (1951).

2. Ibid., see Fig. 4.

3. Ibid., see Fig. 3.

4. Ibid., see Fig. 3 and Eq. 14.

E. ATTENUATION AND REGENERATION OF SOUND

It is well known that in many systems designed to attenuate sound in the presence of a steady air flow the over-all attenuation is often much less than would be predicted on the basis of attenuation measurements in the absence of flow. This effect can generally be attributed to regeneration of sound in the system because of the presence of air flow, but in practical cases may be difficult to predict owing to the complicated geometry of the system and the absence of a precise knowledge of the source distribution.

In order to see some of the main features of the problem, it is convenient to look at an idealized situation, a duct of length ℓ with attenuation $\alpha(x)$ along the length of the duct. The sound intensity at the inlet (x=0) is assumed to be I_o, and a regeneration function s(x) is assumed to exist, so that the intensity generated inside the duct in a distance dx is s(x) dx.

The differential equation that describes the intensity as a function of distance along the duct is readily found to be a general first-order equation

$$\frac{dI(x)}{dx} + \alpha(x)\, I(x) = s(x) \tag{1}$$

The solution to this equation is well known, and for $I = I_o$ at $x = 0$, the intensity at $x = \ell$ is

$$I(\ell) = I_o \exp\left[-\int_0^{\ell} \alpha(x)\, dx\right] + \int_0^{\ell} s(x) \exp\left[-\int_x^{\ell} \alpha(x)\, dx\right] dx \tag{2}$$

or, with $\bar{\alpha} = \frac{1}{\ell}\int_0^{\ell} \alpha(x)\, dx$, we have

$$I(\ell) = I_o e^{-\bar{\alpha}\ell} + e^{-\bar{\alpha}\ell} \int_0^{\ell} s(x) \exp\left[\int_0^{x} \alpha(x)\, dx\right] dx \tag{3}$$

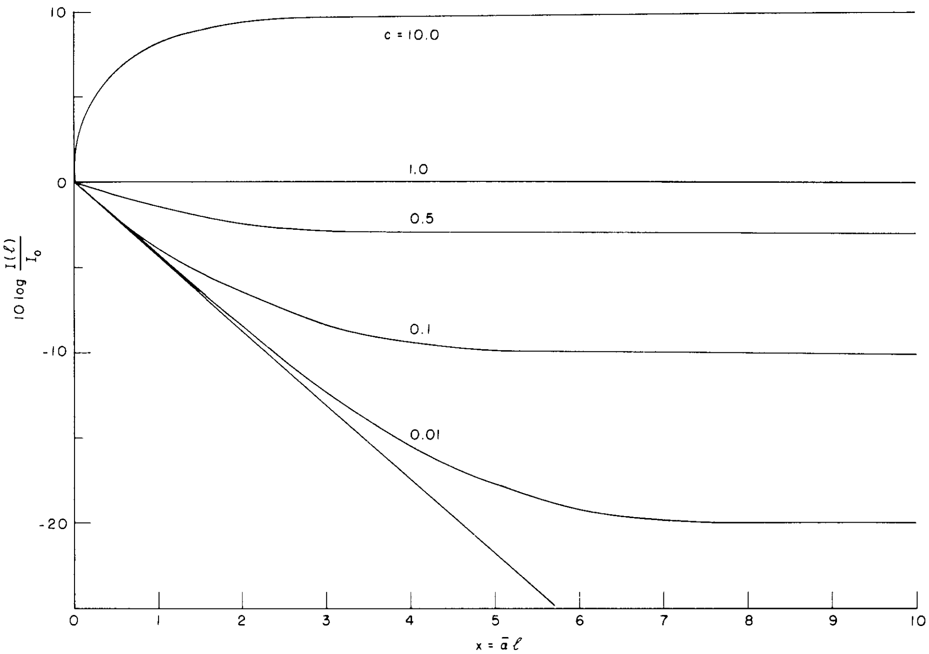

Fig. XV-8. Intensity level as a function of duct length for $s(x) = \frac{s_o}{a_o} a(x)$.

In order to evaluate the integrals, it is necessary to introduce some relation between $a(x)$ and $s(x)$. The simplest exact solution occurs if it is assumed that $s(x)/s_o = a(x)/a_o$. Under these conditions, $I(\ell)$ is easily found to be

$$I(\ell) = I_o\left[e^{-\bar{a}\ell} + c\left(1 - e^{-\bar{a}\ell}\right)\right] \qquad c = \frac{s_o}{I_o a_o} \tag{4}$$

This equation has been plotted in Fig. XV-8 as a function of $\bar{a}\ell$ for several values of c, and shows directly the importance of the regenerated sound in the effectiveness of the duct as an attenuator.

Other exact solutions to the problem can be found if it is assumed that the attenuation and regeneration can be expressed as a power of x/ℓ:

$$a = a_o\left(\frac{x}{\ell}\right)^m \qquad s = s_o\left(\frac{x}{\ell}\right)^n \tag{5}$$

Using these forms in Eq. 3 leads to exact solutions when n and m are related by $(n-m)/(m+1) = k$, where k is an integer. For $k = 0$, the result is the same as Eq. 4,

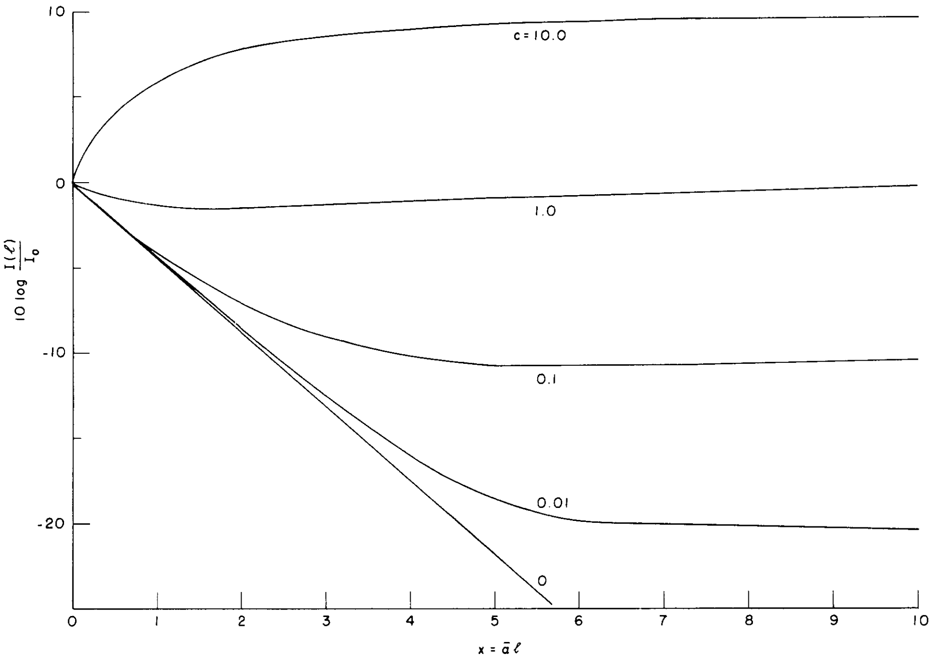

Fig. XV-9. Intensity level as a function of duct length for k = 1.

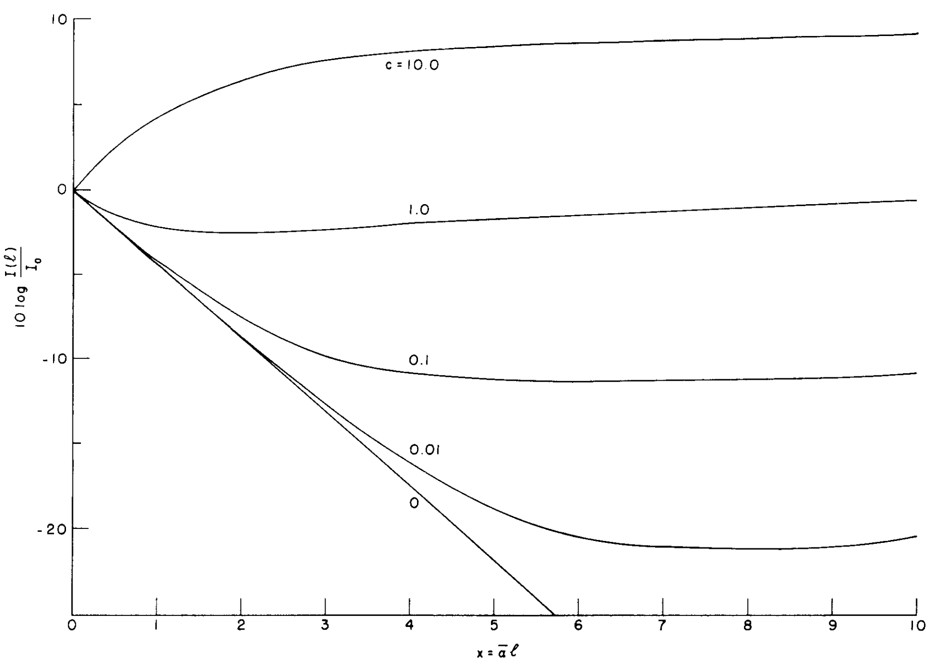

Fig. XV-10. Intensity level as a function of duct length for k = 2.

and for k = 1 and 2 we have

$$I(\ell) = I_o\left[e^{-\bar{a}\ell} + c\left[1 - \frac{1}{\bar{a}\ell}(1 - e^{-\bar{a}\ell})\right]\right] \qquad k = 1 \tag{6}$$

$$I(\ell) = I_o\left[e^{-\bar{a}\ell} + c\left[1 - \frac{2}{\bar{a}\ell} + \frac{2}{(\bar{a}\ell)^2}(1 - e^{-\bar{a}\ell})\right]\right] \qquad k = 2 \tag{7}$$

Solutions for larger values of k can be obtained by successive integrations of Eq. 3 by parts. These functions have been plotted in Figs. XV-9 and XV-10, and show the same general behavior as Eq. 4 except for the presence of a slight minimum for certain values of $\bar{a}\ell$. In all cases, lengthening the duct may provide very little increase in attenuation, and, in some cases, may decrease the over-all attenuation.

It should be noted that for constant attenuation (m=0) an exact solution can be obtained for any value of n, so that if s(x) can be found as a Taylor series in (x/ℓ), the problem can be solved immediately with any desired accuracy.

U. Ingard, G. C. Maling, Jr.

F. SCATTERING OF SOUND BY SOUND

Equipment for the generation and reception of sound pulses under water has been built and tested. A quartz crystal is driven by an oscillator and gated amplifier to produce pulses of 50- to 500-μsec duration at the resonant frequency of the crystal. The crystal frequency may be chosen anywhere between 500 kc and 5mc. The receiver is a small hollow cylinder of barium titanate, polarized to give it piezoelectric properties.

This equipment will be used for studying the scattering of one sound pulse by another under water.

L. W. Dean

G. FINITE WAVE TRANSMISSION IN LIQUID HELIUM

An experiment is being set up to measure finite wave distortion in liquid helium. Equipment has been constructed that will generate 1-mc pulse trains of approximately 100-μsec duration, and will deliver up to several hundred volts to a resonant quartz transducer. This same transducer, in addition to a transducer that is resonant at 2 mc, will be used for measuring pressure amplitudes at 1 mc and 2 mc, and at odd multiples thereof. The apparatus allows changing the orientation and the separation of crystals and/or reflectors while they are immersed in liquid helium under reduced pressure.

One possible use of the data that have been obtained is for calculation of the first few virial coefficients in the equation of state of helium II.

H. L. Willke, Jr.