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Bremsstrahlung and $K\alpha$ fluorescence measurements for inferring conversion efficiencies into fast ignition relevant hot electrons

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The Bremsstrahlung and *K*-shell emission from $1 \times 1 \times 1$ mm³ planar targets irradiated by a short-pulse $3 \times 10^{18} - 8 \times 10^{19}$ W/cm² laser were measured. The Bremsstrahlung was measured using a filter stack spectrometer with spectral discrimination up to 500 keV. *K*-shell emission was measured using a single photon counting charge coupled device. From Monte Carlo modeling of the target emission, conversion efficiencies into 1–3 MeV electrons of 3%–12%, representing 20%–40% total conversion efficiencies, were inferred for intensities up to 8×10^{19} W/cm². Comparisons to scaling laws using synthetic energy spectra generated from the intensity distribution of the focal spot imply slope temperatures less than the ponderomotive potential of the laser. Resistive transport effects may result in potentials of a few hundred kV in the first few tens of microns in the target. This would lead to higher total conversion efficiencies than inferred from Monte Carlo modeling but lower conversion efficiencies into 1–3 MeV electrons. © 2009 American Institute of Physics. [DOI: 10.1063/1.3183693]

I. INTRODUCTION

The fast ignition¹ concept promises higher gains, lower sensitivity to hydrodynamic instabilities, and reduced driver energy when compared to conventional inertial confinement fusion (ICF). This is achieved using a short-pulse laser to ignite a hot spot in a precompressed, 300 g/cc fusion capsule. The laser interacts with the plasma near the critical density surface, generating hot electrons that propagate into the core to heat the hot spot. In cone-guided fast ignition, $^{2-4}$ a high Z cone keeps a channel open in the blowoff plasma, reducing the distance the electrons have to travel to reach the core. The success of fast ignition is primarily dependent on the coupling of the short-pulse laser energy to the hot spot. This can be broken down into three components: the coupling of the laser into relativistic electrons $(\eta_{I \rightarrow e^{-}})$, the transport efficiency of the electrons to the core, which depends on the divergence angle and collimation effects (η_{trans}), and the deposition of the electron energy in the hot spot, which is a function of the electron energy spectrum ($\eta_{\text{deposition}}$). Simulations⁵ found that to achieve ignition at 300 g/cc, 18 kJ of energy must be deposited in a 40 μ m diameter hot spot in 20 ps, requiring laser intensities of $2-3 \times 10^{20}$ W/cm². For a 40 μ m diameter hot spot, electrons of 1–3 MeV have the ideal range to couple efficiently, making the conversion efficiency to 1-3 MeV electrons a critical parameter for fast ignition. For intensities of $2-3 \times 10^{20}$ W/cm², scaling of the electron energy with the ponderomotive potential of the laser⁶ gives temperatures of 6–8 MeV, reducing the deposition efficiency and driving up the driver requirements. Recent modeling suggests, however, that the slope temperature may be colder than ponderomotive scaling due to the steepening of the density gradient.^{7–9}

Previous measurements of the electron spectrum relied on techniques such as vacuum electron spectrometers,^{10,11} nuclear activation,¹² Bremsstrahlung spectrometers,¹³ buried fluorescent foils,¹⁴ and proton emission.¹⁵ Measurements using electron spectrometers and nuclear activation found that the electron slope temperature scales with the laser intensity in proportion to $(I\lambda^2)^{1/2}$ and is close to the ponderomotive potential of the laser. The energies of escaping electrons are however modified by megavolt potentials and significant modeling is required to accurately infer the source spectrum at 1-3 MeV energies of most interest to fast ignition. Nuclear activation is sensitive to photons above 8 MeV and provide limited information about 1-3 MeV electrons. Other measurements using Bremsstrahlung and rear surface proton emission found a $(I\lambda^2)^{1/3}$ scaling, usually referred to as Beg scaling.¹⁵

Conversion efficiencies have been determined by measuring and modeling the $K\alpha$ yield in very thin foils with strong refluxing. Myatt *et al.*¹⁶ and Nilson *et al.*¹⁷ found coupling efficiencies of $20\% \pm 10\%$, independent of the laser intensity for $I=10^{17}-10^{20}$ W/cm² using a hybrid particle in cell model. This model did not include energy transfer to fast ions, making these measurements lower bounds on the conversion efficiency. Yasuike *et al.*¹⁴ used the fluorescence

16, 082705-1

yields from buried layer $K\alpha$ emitters in thick nonrefluxing foils to infer both the slope temperature and conversion efficiency, estimating conversion efficiencies scaling from 10% to 50%, for $I=10^{18}-10^{20}$ W/cm² from Monte Carlo modeling. However, the slope temperature inferred using Monte Carlo modeling is sensitive to collective electric and magnetic field effects, which were not included in the analysis. Davies¹⁸ has shown that neglect of the Ohmic potential results in an underestimate of the conversion efficiency.

In this paper we describe measurements of the number of 1–3 MeV electrons using absolute Bremsstrahlung and *K*-shell fluorescence emission from planar foil targets irradiated by a short-pulse laser. From Monte Carlo transport modeling, the laser conversion efficiency into fast ignition relevant 1–3 MeV electrons is inferred by fitting both the Bremsstrahlung measurements and the fluorescence yield. The sensitivity of the inferred conversion efficiency to the details of the method of analysis is also discussed. The results are compared to scaling laws and modeling estimates commonly used in simulations of fast ignition.

This paper starts with an overview of the experiment and Bremsstrahlung and *K*-shell diagnostics. It then describes the Monte Carlo simulation of the target and techniques for unfolding the electron spectrum from the simulation. This is followed by comparisons of the data to different scaling laws and summaries of the inferred conversion efficiencies. The paper concludes with a discussion on the role of the assumed electron cone angles and resistive transport effects on the deduction of the conversion efficiencies.

II. EXPERIMENT OVERVIEW

Experiments were performed on the Titan laser at Lawrence Livermore National Laboratory.¹⁹ Titan is a 1.06 μ m laser with a maximum energy of 150 J at 0.7 ps pulse length. Focusing is with an f/3 off-axis parabola. The focal spot at target chamber center was imaged at low power with a 16-bit charge coupled device (CCD) camera, which shows a full width at half maximum (FWHM) of 7 μ m and $\sim 15\%$ of the laser energy within the FWHM. The prepulse was measured on each shot with a water cell protected fast diode and the preformed plasma was diagnosed with a $2\omega_0$ interferometer. Prepulse levels for the data shown here ranged from 5 to 80 mJ in a 3 ns pedestal before the main pulse with a 1-30 mJ parasitic short pulse 1.4 ns ahead of the main pulse.²⁰ From hydrodynamic modeling of the prepulse, preformed plasma scale lengths are estimated at up to 10 μ m for the critical surface and 50 μ m for 1/10th critical. The pulse length was measured at 0.7 ± 0.3 ps using a second order autocorrelator.

The targets consisted of $1 \times 1 \text{ mm}^2$ sandwich targets, layered with 10 μ m Al at the interaction surface, 25 μ m Cu for a fluor, and a 1 mm Al layer to range out the electrons and prevent multiple passes through the Cu (refluxing). They were irradiated at 16° horizontal off normal *s*-polarization with 5–150 J of laser energy for intensities averaged over the FWHM region (containing 15% of the laser energy) from 3 $\times 10^{18}$ to 8×10^{19} W/cm². Two filter stack Bremsstrahlung spectrometers²¹ measured the x-ray emission at rear target



FIG. 1. Schematic of the diagnostic layout.

normal (TN) and at 23° horizontal to rear TN, about 80 cm from the target. A Spectral Instruments SI-800 CCD operating in single hit mode²² was used to measure the *K*-shell emission from the buried Cu layer at 24° horizontal to the front TN. A schematic of the setup is shown in Fig. 1.

III. BREMSSTRAHLUNG SPECTROMETER DESCRIPTION AND CALIBRATION

The Bremsstrahlung spectrometer consists of a stack of 13 filters from 100 μ m Al to 4 mm Pb alternating with image plate (IP) dosimeters. It is able to differentiate photons up to 500 keV. A magnet was used to eliminate electron contamination (<100 MeV) and a 5 in. Pb collimator with a 1/2 in. diameter hole limited the field of view to 5 cm at the target plane to minimize fluorescence from other diagnostics and the chamber walls. The filter stack is also enclosed within a 6 mm thick interlocking Delrin cartridge loaded into a 1.8 cm Pb housing. This spectrometer is described in greater detail elsewhere.²¹

The spectrometer was calibrated at the High Energy X-ray (HEX) Laboratory (NSTec, Livermore, CA). The HEX facility produces K-shell line energies by exposing fluorescent foils to a 160 kV x-ray source. The spectrometer was calibrated with 11 K-shell sources from Zr to Pb, representing line energies of 16 (Zr $K\alpha$) to 85 (Pb $K\beta$) keV. The photon flux was measured using a Canberra high-purity germanium detector previously calibrated with NIST-traceable sources. Additionally, the spectrometer was calibrated with a Cs-137 source (662 keV photon) at the Radiation Calorimetry Laboratory at the Lawrence Livermore National Laboratory to provide a single high energy calibration point. The dosimeter signals were compared to one-dimensional Monte Carlo simulations of the filter stack calculated using the Monte Carlo code Integrated Tiger Series 3.0 (ITS 3.0).²³ The simulations were used to generate a spectrometer response matrix (SRM), representing the dose on each IP for incident photons from 1 keV to 100 MeV energies. Figure 2 shows the calibration for four of these data points, at 22, 57, 75, and 662 keV. The data points are the IP readouts and the solid lines represent the predictions of the Monte Carlo model, normalized to the data. The responses are scaled to show them on the same plot. The calibration data show that the Monte Carlo model is a good representation of the response



FIG. 2. (Color online) The Bremsstrahlung spectrometer is calibrated from 15 to 85 keV and at 662 keV. The points are the measured dosimeter signals and the solid lines represent the Monte Carlo model predictions. The signals are scaled to show them on the same plot. The absolute calibration factor is consistent across the spectrum.

function. Additionally, the absolute calibration factor determined from scaling the model to the data was found to be consistent for each of the exposures. This calibration factor scales the readout from scanner units to an absolute energy deposition in the IPs.

IV. EXPERIMENTAL DATA

The data from the Bremsstrahlung spectrometer are scanned images of the 13 IPs with a projection of the collimator hole on each one. Figure 3(a) shows a sample of six of these channels. For each of the IP channels the mean is taken as the signal level for that channel. The error in the signal level for each channel is quantified as the quadrature addition of three different parameters: the standard deviation in each channel, the gradient across the projection, and a 3% response variability in the IP scanner. The standard deviation about the mean in each channel is related to the uniformity of the dose and thus the statistics of the deposition. The gradient is taken as the difference in the mean across different parts of the image (away from the boundary) and is a measure of three-dimensional (3D) effects in the spectrometer modeling. The 3% IP response variability is empirically determined from shot to shot calibration exposures. Figure 3(b) shows sample data for a 18 J shot and a 121 J shot. The cutoff energy and the slope of the dosimeter signals clearly indicate a trend toward an increasing Bremsstrahlung slope temperature with increasing laser intensity. The signals and errors from each of the spectrometer channels, together with the Monte Carlo modeled SRM, constitute a "few channel spectrometer" problem. The unfolding of the photon spectrum is described later in this paper.

The $K\alpha$ data from the single hit spectrometer are shown in Fig. 4. The CCD image was processed using a "single event" algorithm where only photons which deposit all of their energy in a single pixel are counted. The yield is calculated by factoring in the detection efficiency of the chip (as a function of chip crowding), the solid angle, and filter trans-



FIG. 3. (Color online) Data extraction from the HXBS channels. (a) The mean signal in each channel is taken as the PSL signal value. The error bar on each channel is the quadrature sum of the standard deviation in each channel, the gradient across the channel, and a 3% IP scanner response variation. (b) Sample data for a 18 J shot and a 121 J shot. The cutoff energy and the slope of the dosimeter signals clearly indicate a trend toward an increasing Bremsstrahlung slope temperature with increasing laser intensity.



FIG. 4. (Color online) (a) The $K\alpha$ data from the single hit spectrometer. These yields are not corrected for the opacity of the target and are considered the number of photons escaping from the target at the view angle of the spectrometer. The blue diamonds show the yield in units of photons/SR and the red squares represent the same data normalized to the laser energy. The $K\alpha$ yield scales with the laser energy. The normalized yields are plotted on a linear axis to highlight a shot to shot variation up to 30%.

missions. The calculated yields have an error bar of 20% based on the statistics of the signal and on the error in the detection efficiency as calibrated by Maddox *et al.*²² These yields are not corrected for the opacity of the target and are considered the number of photons escaping from the target at the view angle of the spectrometer. The blue diamonds show the yield in units of photons/SR and the red squares represent the same data normalized to the laser energy. The normalized yields are flat with the laser energy, which shows that the $K\alpha$ yield scales linearly with the laser energy. The normalized yields are plotted on a linear axis to highlight the shot to shot variation, which is up to 30%.

V. TARGET SIMULATION

The Bremsstrahlung and K-shell emission from the electron-target interaction was modeled in 3D using an ITS simulation. 81 narrow spectral bins of electrons logarithmically spaced from 10 keV to 100 MeV are injected at the target surface in a 30 μ m spot. The electron beam directionality and electron cone angle are variable parameters in the simulation. The beam direction is varied between 0° and 16° , consistent with experiments by Santala et al.,²⁴ who found that the beam direction varied between TN to along the laser axis (LA) depending on the preformed plasma scale length. The electron cone angle was assumed to have a distribution based on the classical electron ejection angle of electrons in a laser field²⁵ ($\theta_{half} = \tan^{-1}\{[2/(\gamma-1)]^{1/2}\}$). This assumption follows work by Stephens,²⁶ where images from buried fluorescent foils showed a broad 70-100 μ m K α spot up to 100 μ m depth, followed by a 40° (full) divergence angle. From Monte Carlo simulations the assumption of the classical ejection angle was consistent with the measured cone angle distribution. For reference a constant 40° full cone angle response was also simulated. Recent hybrid-PIC simulations by Honrubia and Meyer-ten-Vehn²⁷ found that initial electron cone angles consistent with the classical ejection angle reproduced mean divergence angles of 30°-40° seen in experiments due to magnetic collimation effects. The net propagation angle in Monte Carlo simulations is not reduced by collimation so the initial cone angle is expected to be bounded by these two parameter choices.

For each combination of simulation parameters and spectrometer locations, a target response matrix (TRM) is generated, representing the Bremsstrahlung emission from the target for the injected electron energies. The Bremsstrahlung spectrum is averaged over 5° polar angular bins and 20° azimuthal bins for the off-axis directionality. The $K\alpha$ emission detected by the single hit spectrometer is also calculated, generating a $K\alpha$ response matrix ($K\alpha$ RM) for each parameter combination.

From this model, the Bremsstrahlung emission for various electron energies is plotted in Fig. 5 up to the 500 keV differential photon sensitivity of the spectrometer. There is a clear distinction between 200 keV, 1 MeV, and 2 MeV. Between 2 and 3 MeV the photon spectrum starts to look similar in the energy range of the spectrometer. The 500 keV differential photon sensitivity of the spectrometer thus maps a 2–3 MeV differential electron sensitivity.



FIG. 5. (Color online) The Bremsstrahlung emission for various electron energies up to the 500 keV differential sensitivity of the spectrometer. There is a distinction between 200 keV, 1 MeV, and 2 MeV. Between 2 and 3 MeV the photon spectrum converges in the energy range of the spectrometer. The 500 keV differential photon sensitivity of the spectrometer maps a 2–3 MeV differential electron sensitivity.

The SRM and TRM are multiplied together for the overall response matrix, representing the response of the dosimeter layers to electrons injected into the target. This constitutes a classic few channel spectrometer problem and the electron spectra can be unfolded using a number of techniques, such as fitting test distributions, maximum entropy methods, or singular value decomposition. Here, one and two temperature test distributions seen in previous experimental and computational work^{13,15,28} are used to determine the band of spectra consistent with the data.

VI. ELECTRON SPECTRUM UNFOLDING AND COMPARISONS TO SCALING LAWS

A. One temperature parametrizations

Previous comparisons to intensity scaling laws^{10,12,13} typically compared a single electron slope temperature (T_{hot}) to a single intensity parameter used to characterize the laser. If this is done with the data here, a single electron slope temperature provides a good fit to the Bremsstrahlung data. The fit is characterized by the weighted, reduced χ^2 fitting parameter, where $\chi^2 < 1$ means on average the predictions fit the measured data within their error bars. The single slope temperature distributions fit the data within 1 χ^2 . The beam directionality is set by the requirement that the electron slope temperature must be simultaneously consistent for both spectrometers. If the assumed directionality is along the TN, the spectrometer along TN will infer a lower temperature than the spectrometer along the LA. For a given measured spectrum, the LA spectrometer assumes the actual spectrum is harder since it is not measuring the center of the beam. If the beam directionality is taken along the LA, the TN spectrometer likewise assumes a harder spectrum. The beam direction is taken as the angle at which the predicted temperatures are equal. The best fit temperature is thus taken as the temperature that is simultaneously consistent with the measurements from both spectrometers. This works better for higher intensities since the electron cone angle is more directional. For low intensities and consequently lower energy electrons, the assumed cone angle is almost 2π and the beam directionality does not matter. The calculated beam directionality varied from 6° to 16°.

Using the FWHM-averaged peak intensity as previously discussed, the slope temperatures are consistent with Beg scaling $[T_{hot}=215(I_{18}\lambda^2)^{1/3} \text{ keV}]$ up to $2 \times 10^{19} \text{ W/cm}^2$ and 20%-40% higher than Beg scaling for intensities up to $8 \times 10^{19} \text{ W/cm}^2$. For the 121 J shot shown in Fig. 3(b), the single-temperature fit has a T_{hot} of 1.3 ± 0.1 MeV. Beg scaling predicts a 0.9 ± 0.1 MeV T_{hot} (error bar from the pulse length uncertainty), about 40% lower (χ^2 =10). Ponderomotive scaling predicts 3.3 MeV, significantly hotter ($\chi^2 \approx 70$) than the single temperature fit.

This seems to suggest that Beg scaling provides a better fit to the data than ponderomotive scaling. However, this analysis is misleading for two reasons. First, the electron distributions that fit the data are not unique. Parametrization with two temperature components show that different spectra consistent with the data can be drastically different, as will be discussed shortly. Second, comparisons to scaling laws using a single intensity parameter are simplistic and do not properly account for the intensity distribution. The ponderomotive potential is a local effect and proper estimates of the electron spectrum must account for the focal spot intensity distribution rather than just a single peak intensity. Empirical scalings like the Beg scaling law correlate the slope temperature with a defined intensity, as long as a consistent intensity definition is used (which is not always the case). This should more consistently predict the slope temperature. However, it is not clear that the intensity scaling law should necessarily translate to other laser systems where the focal spot profile may be very different. Scaling law comparisons using the focal spot intensity distribution will be discussed later in this section.

B. Two temperature parametrizations

The spectral space of the electron distributions can be expanded by parametrizing the distribution using hot and cold temperatures and a ratio between the two components, taking the form $f(E) \propto R\varphi(E|T_c) + \varphi(E|T_h)$, where T_c and T_h are varied from 10 keV to 10 MeV, *R* from 0.1 to 1000, and φ is a normalized Boltzmann or one-dimensional relativistic Maxwellian distribution. The target response matrix with a beam directionality selected from the one-temperature fits is used. The matrix response model simplifies testing of the entire parameter space. The fitting parameter is calculated for 16×10^6 distributions per shot, providing highly resolved variances of the distribution. The electron distributions that simultaneously fit both the spectrometers within 1 χ^2 are selected as valid fits.

Figure 6(a) shows a sample subset of allowed distributions for a 121 J shot, represented by the color lines, along with the envelope of fits, represented by the solid black lines. A broad range of electron distributions is consistent with the data, with almost an order of magnitude difference in the number of electrons at any given energy. The straight red line in Fig. 6(a) represents a single temperature distribution with



FIG. 6. (Color online) (a) Various 2-T electron spectra are consistent with the spectrometer data. A sample subset of these curves is plotted, with the black lines representing the envelope of these curves. (b) The black lines represent the resultant Bremsstrahlung spectra from the electron spectra in (a). The Bremsstrahlung is significantly more constrained, with the different electron spectra in (a) producing the same Bremsstrahlung emission. The red lines represent the Bremsstrahlung spectra from the $K\alpha$ constrained electron energy spectra. (c) 2-T electron spectra consistent with both the Bremsstrahlung data and the *K*-shell emission. The *K*-shell emission acts as an electron counter that further constraints the set of possible distributions.

a 1.3 MeV slope temperature. The other sample distributions show, however, that this single temperature is not unique and depends on the energy range in which the slope is measured. The solid black lines define the envelope in which the twotemperature (2-T) solutions fall. The envelope itself is not a solution. The envelope serves to bound the spectral space for this shot.

All of these electron distributions generate similar photon spectra up to the differential sensitivity limit of the spectrometer. The solid black lines in Fig. 6(b) represent the envelope of Bremsstrahlung distributions generated by the different electron spectra. This envelope is significantly narrower; similar Bremsstrahlung spectra can be generated with larger numbers of colder electrons or smaller numbers of hotter electrons. Above 500 keV the spectrometer has no differential sensitivity, but the Bremsstrahlung spectrum is restricted by the assumption of an exponentially falling electron distribution (in the 2-T parametrization).

The band of unfolded electron spectra was reduced by using the Cu $K\alpha$ emission from the fluorescence layer as an additional constraint. For this shot the measured $K\alpha$ yield was $5.3 \times 10^{11} \pm 20\%$ photons/SR. This yield is not corrected for the target opacity since the opacity is already factored into the $K\alpha$ RM through the Monte Carlo simulation. The $K\alpha$ signal effectively acts as a counter of electrons above 50 keV; these have sufficient energy to reach the copper layer and efficiently stimulate fluorescence. With the $K\alpha$ constraint, the range of possible electron distributions is further reduced in the lower energy part of the spectrum, as shown in Fig. 6(c). The photon spectrum is only mildly affected, as seen in the solid red lines in Fig. 6(b).

C. Scaling law comparisons using synthetic electron distributions

A better comparison to analytic scaling laws can be done by generating a synthetic spectrum from the vacuum focal spot intensity distribution. Any modifications due to selffocusing in the preformed plasma are not included here. The intensity distribution is binned in time and space using the focal spot image and a 0.7 ps Gaussian temporal profile as measured from the autocorrelator trace. A conversion efficiency and electron energy spectrum are then assigned to each intensity element in space and time using an exponential distribution with a slope temperature from the scaling law and a given conversion efficiency model. Here the flat coupling inferred by Myatt et al. and Nilson et al. is used, with the coupling efficiency scaled to the measured data (alternatively Yasuike's 10%-50% coupling efficiency would make the hot tail of the electron distribution slightly hotter and would increase the discrepancy shown later between the measured data and the ponderomotive model for temperature). The synthetic distribution is then generated by integrating these electron distributions in space and time. This spectrum does not have a single temperature and the slope temperature is higher when measured at higher energies.

Synthetic electron distributions were generated for three intensity scaling models: ponderomotive scaling, ponderomotive scaling reduced by a 75% scale factor, and the parametrization of density gradient steepening by Chrisman *et al.*⁸

The original 1992 PIC simulations of Wilks *et al.*⁶ on ponderomotive scaling show a +/-25% difference in elec-



FIG. 7. (Color online) The synthetic spectra derived from different T_{hot} scalings using the focal spot intensity distribution are fit to the measured Bremsstrahlung data for the 121 J shot. The synthetic ponderomotive spectrum is slightly hotter than the data. The Chrisman parametrization and 75% synthetic ponderomotive spectrum fit the Bremsstrahlung data within the error bars. The inset shows the $K\alpha$ predictions of the different spectra relative to the measured data, given by the orange line and error bars. The ponderomotive and 75% ponderomotive spectra underpredict the $K\alpha$ signal by a factor of 2.

tron temperatures about the ponderomotive potential for *p*and *s*-polarized light, respectively. A synthetic spectrum can then be generated by reducing the ponderomotive temperature by a scale factor, i.e., $T_h = fT_{\text{POND}}$. Since this experiment was performed with a near-normal, *s*-polarized laser, a synthetic spectrum using f = 75% is thus considered within the "error bar" of ponderomotive scaling.

Recent simulations by Chrisman *et al.*⁸ and Kemp *et al.*⁷ have shown a reduction in the slope temperature due to light pressure induced steepening of the density gradient. This arises from shortening of the $j \times B$ acceleration distance by a factor of $\sqrt{\gamma n_c/n_s}$, where n_s is the density of the steepened shelf. From two-dimensional PIC simulations, the energy spectrum was parametrized by splitting the hot electrons into two components, one with a slope temperature equal to the ponderomotive potential and the other reduced by the factor $\sqrt{\gamma n_c/n_s}$, with each component containing half of the electron energy.

The synthetic spectra generated from these three models are compared to the experimental data in Fig. 7. The main figure shows the comparison to the Bremsstrahlung data, while the inset shows the comparison to the measured $K\alpha$ signal. The synthetic ponderomotive spectrum is still slightly hotter than the data ($\chi^2 = 8$), but the fit is significantly better than the 3.3 MeV single temperature ponderomotive distribution. The Chrisman parametrization and the 75% ponderomotive model fit the measured data within the error bars. The Bremsstrahlung spectrum predicted by the Chrisman and 75% ponderomotive models are almost exactly the same up to the differential sensitivity limit of the spectrometer. The $K\alpha$ signal is shown in the insert by the horizontal orange line, with its associated 20% error bar. The ponderomotive spectrum and the 75% ponderomotive spectrum underpredict the $K\alpha$ signal by about a factor of 2. The Chrisman parametrization is consistent with the measured $K\alpha$ signal within its error bars.



FIG. 8. (Color online) (a) The conversion efficiency into 1–3 MeV electrons is plotted vs laser intensity. The gray bars represent the predicted conversion efficiencies from the Bremsstrahlung data. The orange bars represent the conversion efficiencies also consistent with the $K\alpha$ measurement. Above 10^{19} W/cm², about 3%–12% of the laser energy goes into 1–3 MeV electrons. For reference, the black and blue lines represent conversion efficiencies into 1–3 MeV electrons given ponderomotive scaling applied to a single peak intensity and a representative vacuum focal spot distribution, respectively, and a 25% total conversion efficiency. (b) Laser conversion efficiencies into all electrons range from 20% to 40% for intensities above 10^{19} W/cm².

The Chrisman parametrization provides the best fit to both the Bremsstrahlung and $K\alpha$ data in this Monte Carlo model. Resistive transport effects neglected in the Monte Carlo simulations are a potential source of uncertainty and will be assessed later in this paper.

VII. CONVERSION EFFICIENCY SCALINGS WITH INTENSITY

The total conversion efficiency $(\eta_{L\to e^-})$ and the conversion efficiency into 1–3 MeV $(\eta_{L\to 1-3 \text{ MeV}e^-})$ electrons is calculated for each of the 2-T distributions previously described. Figure 8(a) shows the scaling of $\eta_{L\to 1-3 \text{ MeV}e^-}$ with laser intensity. The solid gray bars represent the predicted conversion efficiencies for the spectra that fit the Bremsstrahlung data with 1 χ^2 ; the orange bars also fit the $K\alpha$ constraint within its 20% error bars. $\eta_{L\to 1-3 \text{ MeV}e^-}$ peaks around 2 $\times 10^{19}$ W/cm² and then falls off. The conversion efficiency into 1–3 MeV electrons is banded between 3%–12%. For reference, the black and blue lines represent conversion efficience

TABLE I. Conversion efficiencies for the synthetic electron spectra for the 121 J shot.

Model	$egin{array}{c} oldsymbol{\eta}_{L ightarrow e^-} \ (\%) \end{array}$	$egin{array}{c} oldsymbol{\eta}_{L ightarrow 1-3} {}_{\mathrm{MeV}e^-} \ (\%) \end{array}$
2-T parametrization	20-31	2.5-7.7
Ponderomotive	16	5.3
75% ponderomotive	20	6.9
Chrisman	25	4.6

ciencies into 1-3 MeV electrons given ponderomotive scaling applied to a single peak intensity and a representative vacuum focal spot distribution, respectively, and a 25% total conversion efficiency. With standard ponderomotive scaling, $\eta_{L \to 1-3 \text{ MeV}e^-}$ peaks at too low an intensity compared to the data, suggesting that the calculated spectrum needs to take into account both the focal spot distribution and a possible reduction in the temperature. The blue line takes the vacuum focal spot into account, and predicts a flat coupling to 1-3 MeV electrons for laser intensities of 2×10^{19} - 8×10^{19} W/cm². The total conversion efficiencies are shown in Fig. 8(b). The total conversion efficiencies are banded between 20% and 40%. This is slightly higher than the 10%–30% minimum conversion efficiencies measured by Nilson *et al.*, which is probably due to their neglect of energy transfer to fast ions.

The conversion efficiencies predicted from the synthetic spectra from the various scaling models are shown in Table I. For the 121 J shot, the electron distributions consistent with both the Bremsstrahlung spectrum and the $K\alpha$ constraint bound $\eta_{L\to e^-}$ between 20% and 31% and $\eta_{L\to 1-3 \text{ MeV}e^-}$ between 2.5% and 7.7%. The synthetic ponderomotive spectrum results in a total conversion efficiency lower than these bounds (while the distribution shape is given by the models, the absolute conversion efficiency is scaled to the data). The Chrisman conversion efficiencies are calculated to be $\eta_{L \to e^-} = 25\%$ and $\eta_{L \to 1-3 \text{ MeV}e^-} = 4.6\%$. The higher conversion efficiency is due to the larger number of prescribed colder electrons, which contribute to the energy of the spectrum but do not produce significant levels of Bremsstrahlung. At these intensities, the Chrisman parametrization has essentially the same energy fraction of 1-3 MeV electrons as the ponderomotive scaling. At higher intensities required for ignition, more of the low energy component of the spectrum would fall in the energy range of interest so the useful fraction would increase.

The 75% ponderomotive spectrum results in more energy going into 1-3 MeV electrons by reducing the temperature of the higher energy tail. This model does not prescribe a large bulk of low energy electrons that inflate the total conversion efficiency of the Chrisman model.

VIII. DISCUSSION AND SYSTEMATIC UNCERTAINTIES

The requirements for ignition have been the subject of several modeling studies.^{5,27,29} Atzeni *et al.*⁵ assumes for example a 25% total conversion efficiency and a hot electron temperature consistent with ponderomotive scaling, resulting

in electrons with too large a range for optimal coupling to the hot spot. The total conversion efficiency measured here is consistent with that assumption, with electron temperatures giving a more optimal range, suggesting somewhat more favorable conditions. The conversion efficiency into 1–3 MeV electrons reported here is low. In this experiment, this is due to the large amounts of energy in the low intensity wings of the focal spot, suggesting the focal spot profile is an important consideration in a fast ignition driver. The cone angle assumed here, however, ranges from 90° to 60° for 1–3 MeV electrons. This would result in a transport cone angle worse than usually assumed in modeling of ignition. Magnetic focusing of the electron beam at ignition scale conditions mitigates the effect of divergence of the source electrons and is a subject of ongoing study.^{30,31}

There are two primary sources of error that systematically bias this analysis: the assumption of the electron cone angle and the neglect of resistive transport effects. In contrast with the classical ejection angle assumption, if all the electrons are launched into a 40° cone angle whether by magnetic collimation or otherwise, the calculated conversion efficiencies for the 121 J shot would be $\eta_{L \to e^-} = 16\% - 22\%$ and $\eta_{L \to 1-3 \text{ MeV}e^-} = 1.6\% - 5.4\%$ (with the $K\alpha$ constraint). The full conversion efficiency is 25% lower and the conversion efficiency into 1-3 MeV electrons is reduced by 30%-40%. The angular distribution of the electrons is the main uncertainty in unfolding the electron spectrum from the Bremsstrahlung measurements. A full spectral and angular measurement, similar to work by Schwoerer et al.,³² or imaging of multiple fluorescent layers buried in the target could help provide additional constraints on the angular distribution.

The other source of error is the neglect of collective electric and magnetic effects in the transport model. While collective fields are not present in a basic Monte Carlo model, the impact can be estimated using a Bell-like³³ model where an Ohmic potential drives a return current of thermal electrons. The magnitude of the potential in these experiments is estimated using an initial electron spectrum from the Chrisman parametrization applied to the focal spot intensity distribution and a 25% conversion efficiency, as described above. The electrons are binned into energy groups and are launched from a 30 μ m diameter spot into a cone angle given by the classical ejection angle. The electric field is given by

$$E(z) = \eta(z)j(z) = \eta(z)\sum_{i} \frac{N_{i}e}{\pi\tau} \frac{1}{(r_0 + z \tan \theta_i)^2} H(R_i - z),$$

where η is the resistivity of the material, N_i is the number of electrons in each energy group, r_0 is the initial spot size, θ_i is the divergence angle of each energy group, H is the Heaviside step function, and R_i is the range of the electrons in each energy group.

The electric field is calculated as a function of depth with a cutoff at the electron range. The electrons lose energy through collisional, radiative, and resistive effects. The collisional and radiative losses are taken from tabulated values for cold matter.³⁴ Scattering is included in a rough way by taking the electron path length as two times greater than the

linear penetration depth (in Al), which has been shown for energies from 10 keV to several MeV.^{35,36} The electron range is thus approximated by integrating the energy loss against the potential into the target along with twice the collisional and radiative losses. The electric field is calculated and summed for all energy groups and integrated for a potential across the target. This procedure is then iterated to converge upon a self-consistent solution of the potential and electron penetration depth.

Using this formulation, the potential across the target is calculated at 700 kV if a peak aluminum resistivity of about $1.5 \times 10^{-6} \ \Omega$ m is used for the entire bulk, assuming that most of the interactions occur at temperatures between 10 and 100 eV for which the resistivity of Al is close to the peak value. Half of the potential is in the first 10 μ m and $\frac{3}{4}$ is in the first 30 μ m, before the electrons have had a chance to spread. The calculated potential is temperature dependent and may be lower depending on the actual temperature distribution in the target. 3D currents may also serve to partially lower the front surface charge buildup and thus reduce the potential felt by the hot electrons. Regardless, given the electron currents in these experiments, the potential is likely to be at least a couple hundred kV.

Neglecting this potential in a Monte Carlo analysis may influence the interpretation of the conversion efficiencies. Since the majority of the potential is in the first few tens of microns and the Bremsstrahlung is produced throughout the bulk, all of the electrons producing Bremsstrahlung in the target pass through this potential. The electron spectrum is essentially downshifted by this potential before it produces Bremsstrahlung. The electron spectrum inferred from the measured Bremsstrahlung emission just needs to be upshifted by the potential. If the mean energy is taken as the 1.3 MeV single temperature fit, a shift of a couple hundred kV would slightly perturb the inferred conversion efficiencies. The synthetic ponderomotive spectrum discussed in Sec. VI C is also within range of this potential Ohmic correction. Transport simulations similar to those by Davies¹⁸ and Honrubia and Meyer-ten-Vehn²⁷ are necessary to quantify the shift in the spectrum. The $K\alpha$ emission is strongly influenced by the Ohmic potentials, as previously discussed by Davies. With the Monte Carlo analysis, the fluor acts as a counter of electrons above 50 keV. If there is a potential of a few hundred kV in the first ten microns before the fluor, the energy at which this counter acts is upshifted to a few hundred keV. This could slightly downshift the number of $K\alpha$ photons predicted by the different models.

The impact of the Ohmic potentials on the $K\alpha$ constraint is mixed. Some of the electron spectra in Fig. 6(a) with larger numbers of electrons in the hundred of keV range produce too much $K\alpha$ in the Monte Carlo model to be consistent with measured $K\alpha$ emission. With an Ohmic model, these distributions would be consistent with the data. This would result in an upward revision of the total conversion efficiency since there would be larger numbers of low energy electrons. The conversion efficiency into 1–3 MeV electrons is likely to drop since fewer high energy electrons would be needed to produce the measured Bremsstrahlung emission. Graphically, the orange bars in Fig. 8(b) will shift upwards,

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slightly increasing $\eta_{L\to e^-}$, while still subject to the Bremsstrahlung constraint indicated by the gray bars. The orange bars in Fig. 8(a) will shift downwards, reducing $\eta_{L\to 1-3}$ MeVe⁻.

Magnetic fields may enhance the energy loss near the front surface of the target and reduce the penetration lengths by increasing the curvature of the electron path.¹⁸ The impact of magnetic fields on the conversion efficiency would thus be analogous to that of the Ohmic potential.

IX. CONCLUSIONS

The Bremsstrahlung spectrum and $K\alpha$ emission of a fluorescent layer have been measured in nonrefluxing targets. Through Monte Carlo modeling, conversion efficiencies into 1-3 MeV electrons of 3%-12% have been inferred. This represents a total conversion efficiency of 20%-40%. Assumptions of the angular distribution model are a major source of uncertainty in the analysis. Better comparisons to scaling laws used synthetic energy spectra generated from the intensity distribution of the focal spot. Forward calculations of the electron spectra for various intensity scaling models were compared to the Bremsstrahlung data. The forward calculations are suggestive of energy spectra with lower slope temperatures than scaling with the ponderomotive potential of the laser. Resistive effects result in potentials of a few hundred kilovolts in the first tens of microns in the target. This may result in higher total conversion efficiencies than inferred from Monte Carlo modeling but lower conversion efficiencies into the 1-3 MeV band of primary interest to fast ignition.

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