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A Log-Frequency Approach to the Identification of the Wiener–Hammerstein Model

Joel Goodman, *Member, IEEE*, Matthew Herman, Bradley Bond, *Member, IEEE*, and Benjamin Miller

Abstract—In this paper we present a simple closed-form solution to the Wiener–Hammerstein (W–H) identification problem. The identification process occurs in the log-frequency domain where magnitudes and phases are separable. We show that the theoretically optimal W–H identification is unique up to an amplitude, phase and delay ambiguity, and that the nonlinearity enables the separate identification of the individual linear time invariant (LTI) components in a W–H architecture.

Index Terms—Log-frequency, nonlinear system identification, power amplifier modeling, predistortion, Wiener–Hammerstein system.

I. INTRODUCTION

WIENER–HAMMERSTIEN (W–H) architectures are commonly used to represent the nonlinear response of RF analog front ends [1], or as a means of compensating for distortions imparted by high power amplifiers [2]. A W–H architecture, illustrated in Fig. 1, is a filter–nonlinearity–filter cascade that is capable of modeling a nonlinear system with memory in a computationally efficient fashion [3]. Direct identification of such an architecture is complicated by the filter preceding the memoryless nonlinearity (h_1 in Fig. 1(a)), as the discrete-time coefficients of $h_1 = [h_1(0) h_1(1) \cdots h_1(N_1 - 1)]$ at the output of a W–H system are nonlinearly coupled. This coupling can be formulated as

$$y(n) = \sum_{\ell=0}^{N_2-1} \left(g \left(\sum_{m=0}^{N_1-1} x(n-m-\ell) h_1(m) \right) h_2(\ell) \right) \quad (1)$$

where $g(\cdot)$ represents the memoryless nonlinearity. Previous approaches to identifying W–H systems assume overly restrictive parametric models, or use a combination of linear, nonlinear and

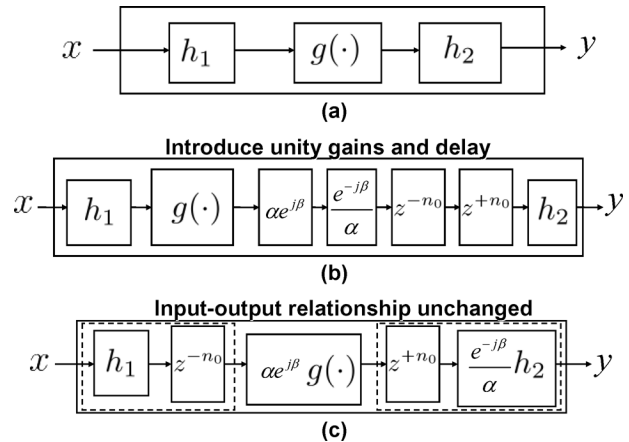


Fig. 1. The W–H architecture. (a) W–H filter-memoryless nonlinearity filter cascade architecture. Both (b) and (c) represent the non-uniqueness of the W–H architecture due to an amplitude and delay ambiguity.

iterative optimization techniques to estimate the linear time invariant (LTI) parameters h_1 and h_2 and nonlinearity $g(\cdot)$ [4].

Independent of the approach taken, the identification of a W–H system is not unique. Referring to Fig. 1(b) and 1(c), the product of $\alpha e^{j\beta}$ and its reciprocal can come before or after the nonlinearity $g(\cdot)$. Further, a delay before or after the memoryless nonlinearity is ambiguous, i.e.,

$$g(z(n - n_0)) = \sum_q g(z(q)) \delta(n - q - n_0) \quad (2)$$

where $z(n) = \sum_{q=0}^{N_1-1} x(n-q) h_1(q)$ and $\delta(n)$ is the Dirac delta function. In (2), the left hand side of the equation corresponds to delay n_0 occurring prior to the nonlinearity, while the right hand side corresponds to the delay occurring afterward. This means that the delay inherent to the system can be shifted between the two LTI components, as shown in Fig. 1(c).

It is possible, however, to model a W–H system up to an amplitude, phase and delay ambiguity, and invert this system without being adversely affected by such ambiguities. We will show that we can identify a W–H system using several 2-tone excitation signals by separating the magnitude and phase responses of the LTI components, and then using least squares in the log-frequency domain. The identification of the phase components is complicated by phase wrapping, and simple 2-dimensional (2D) phase unwrapping with residue testing [5] is used to mitigate this complication. The rest of this paper is organized as follows. In Section II, we describe the W–H log-frequency identification algorithm. In Section III, we demonstrate the performance of the algorithm on a power amplifier model, and in Section IV we provide a brief summary.

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II. WIENER–HAMMERSTEIN IDENTIFICATION

Consider the discrete-time output of a W–H system due to a rotating exponential excitation given by

$$y(n) = \sum_{p=1}^P \gamma_p |H_2(pk)| |H_1(k)|^p e^{j(2\pi \frac{k}{N} pn + p\theta_1(k) + \theta_2(pk))} \quad (3)$$

where p is the polynomial order, $H_i(k) = |H_i(k)|e^{j\theta_i(k)}$ is the magnitude and phase response of filter $i \in \{1, 2\}$ at frequency k , and γ_p represents the coefficient associated with the p th-order polynomial with $\gamma_1 = 1$. Note that any nonlinearity $g(\cdot)$ can be represented over a closed interval with arbitrary precision as $P \rightarrow \infty$ using a polynomial series expansion [6]. Unlike nonlinear systems with memory, a single tone stepped in amplitude can adequately characterize a memoryless nonlinearity [7]. Using the memoryless nonlinearity to separate the two filter responses h_1 and h_2 , we reduce the problem to the identification of two LTI systems, so by exciting the system with several tonal inputs, finely spaced in frequency, we meet the persistence of excitation criterion for linear systems. The real and imaginary components of the log of the p th-order nonlinear frequency domain output ($Y_p(pk)$) is given by

$$\begin{aligned} G_{ST}^p(k) &= \log(|\gamma_p|) + p \log(|H_1(k)|) + \log(|H_2(pk)|), \\ \phi_{ST}^p(k) &= p\theta_1(k) + \theta_2(pk) + \angle\gamma_p \end{aligned} \quad (4)$$

where $G_{ST}^p(k) = \text{Re}\{\log(Y_p(pk))\}$, and $\phi_{ST}^p(k) = \text{Im}\{\log(Y_p(pk))\}$, where

$$Y_p(pk) \stackrel{\mathcal{F}}{\longleftrightarrow} \gamma_p |H_2(pk)| |H_1(k)|^p e^{j(2\pi \frac{k}{N} pm + p\theta_1(k) + \theta_2(pk))}$$

\mathcal{F} represents the symmetric Fourier transform pair, and $\text{Re}\{\cdot\}$ and $\text{Im}\{\cdot\}$ represent taking the real and imaginary components of their arguments, respectively.

A two-tone stimulus is preferable for W–H system identification in many cases given that it is common in RF transmitters for harmonic distortions to fall outside of the band of interest where they cannot be measured digitally. Letting $\alpha_i(\cdot) = \log(|H_i(\cdot)|)$, it can be shown that a generalization of (4) with a *real* two-tone stimulus is expressed as

$$\begin{aligned} G_{DT}^{m,n}(k_1, k_2) &= \log(|\gamma_p|) + |m|\alpha_1(k_1) + |n|\alpha_1(k_2) \\ &\quad + \alpha_2(|mk_1 + nk_2|) + \log\left(\binom{p}{|m|}\right), \\ \phi_{DT}^{m,n}(k_1, k_2) &= \text{sign}(mk_1 + nk_2)\theta_2(|mk_1 + nk_2|) \\ &\quad + m\theta_1(k_1) + n\theta_1(k_2) + \angle\gamma_p \end{aligned} \quad (5)$$

where $p = |m| + |n|$, and k_1 and k_2 represent the frequencies of the two different tones, with $k_1, k_2 > 0$. In formulating (5) we use the fact that the coefficients of the LTI system and the two-tone excitation are real valued. Letting $\hat{G}_{DT}^{m,n}(k_1, k_2) = G_{DT}^{m,n}(k_1, k_2) - \log(\binom{p}{|m|})$, and $\angle\gamma_p = \log(|\gamma_p|)$, (5) can be expressed as a set of linear equations in matrix form as

$$\underbrace{\begin{bmatrix} \vdots \\ \hat{G}_{DT}^{m,n}(k_1, k_2) \\ \vdots \end{bmatrix}}_{\mathbf{g}} = \underbrace{\begin{bmatrix} \ddots & & & \\ |m| & |n| & 1 & 1 \\ & & \ddots & \\ & & & \vdots \end{bmatrix}}_{\mathbf{M}_G} \underbrace{\begin{bmatrix} \vdots \\ \alpha_1(k_1) \\ \alpha_1(k_2) \\ \alpha_2(|k_{m,n}|) \\ \vdots \\ \angle\gamma_p \end{bmatrix}}_{\mathbf{a}} \quad (6)$$

and

$$\underbrace{\begin{bmatrix} \vdots \\ \phi_{DT}^{m,n}(k_1, k_2) \\ \vdots \end{bmatrix}}_{\boldsymbol{\phi}} = \underbrace{\begin{bmatrix} \ddots & & & \\ m & n & \text{sign}(k_{m,n}) & 1 \\ & & \ddots & \\ & & & \vdots \end{bmatrix}}_{\mathbf{M}_\phi} \underbrace{\begin{bmatrix} \vdots \\ \theta_1(k_1) \\ \theta_1(k_2) \\ \theta_2(|k_{m,n}|) \\ \vdots \\ \angle\gamma_p \end{bmatrix}}_{\boldsymbol{\theta}} \quad (7)$$

where $k_{m_i, n_i}(i) = m_i k_1(i) + n_i k_2(i)$, where the row index i that indicates the tone frequencies and mixing product for each equation has been dropped for notational convenience. Using (6) and (7),

$$\begin{bmatrix} \mathbf{g} \\ \boldsymbol{\phi} \end{bmatrix} = \begin{bmatrix} \mathbf{M}_G & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_\phi \end{bmatrix} \begin{bmatrix} \mathbf{a} \\ \boldsymbol{\theta} \end{bmatrix} \quad (8)$$

where the matrices $\mathbf{M}_G \in \mathbb{Z}^{L \times K+1}$ and $\mathbf{M}_\phi \in \mathbb{Z}^{L \times K+1}$ have rank K , with $L > K$. The column rank deficiency is due to the amplitude and delay ambiguity. Note that to achieve these ranks and characterize the entire band, the linear components $\{G_{DT}^{1,0}, G_{DT}^{0,1}, \phi_{DT}^{1,0}, \phi_{DT}^{0,1}\}$ and (at least) one nonlinear component, e.g., $\{G_{DT}^{2,-1}, \phi_{DT}^{2,-1}\}$, need to be represented in (6), with the excitation frequencies chosen such that the components in \mathbf{a} and $\boldsymbol{\theta}$ uniquely cover each discrete frequency over which the LTI system is being characterized.

Once the minimum norm solution of (8) is obtained and the LTI components α_i and ϕ_i , $i \in \{1, 2\}$, are known, a lookup table (LUT) can be used to characterize the nonlinearity $g(\cdot)$ [8]. In many cases, nonlinearities are additive, i.e., the higher-order odd distortions terms have components which fall on top of lower ones, e.g., $[m, n] = [3, -2]$ and $[2, -1]$. Therefore, to isolate $G_{DT}^{m,n}$ and $\phi_{DT}^{m,n}$, we use the simple procedure of adjusting the power levels of the excitation to suppress (or enhance) higher-order distortions, although other robust procedures are possible [9], [10]. In practice, we will use a single polynomial order, e.g., only a 3rd-order distortion with $[m, n] = [2, -1]$, in (8) to characterize the linear components. This enables us to fix $\angle\gamma_3$ in (7) to 0 and identify the actual phase (either 0 or π) of the other nonlinearities during LUT training. We use only a 3rd-order distortion to separately identify h_1 and h_2 . After filter identification, we model the memoryless nonlinearity $g(\cdot)$ over a larger input amplitude range with a computationally efficient LUT. We will expand upon this further in Section III.

The digitally measured phase terms $\phi_{DT}^{m,n}(k_1, k_2)$ in (7) are wrapped, such that

$$\phi_{DT}^{m,n}(k_1, k_2) = \phi_{DT,u}^{m,n}(k_1, k_2) + 2\pi l(k_1, k_2) \quad (9)$$

where $\phi_{DT,u}^{m,n}(k_1, k_2)$ corresponds to the matrix of unwrapped phases, and $l(k_1, k_2)$ is an array of integers so that $-\pi \leq \phi_{DT}^{m,n}(k_1, k_2) \leq \pi$. In the one-tone case, (9) collapses down to a one-dimensional phase unwrapping problem. However, with a two-tone stimulus, the measured output of a W–H system at frequency k will have multiple functions in (5) that depend separately on k_1 and k_2 for differing m and n . Therefore, we must treat $\phi_{DT}^{m,n}$ as a function of two variables. To unwrap $\phi_{DT}^{m,n}(k_1, k_2)$, we first form a serpentine matrix of phases

$$\tilde{\Phi}_{DT}^{m,n} = \begin{bmatrix} \phi_{DT}^{m,n}(1, 1) & \phi_{DT}^{m,n}(N, 2) & \cdots & \phi_{DT}^{m,n}(N, N) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{DT}^{m,n}(N, 1) & \phi_{DT}^{m,n}(1, 2) & \cdots & \phi_{DT}^{m,n}(1, N) \end{bmatrix}$$

then we vectorize $\tilde{\Phi}_{DT}^{m,n}$ to get $\psi^{m,n} = \text{vec}(\tilde{\Phi}_{DT}^{m,n})$. We apply the simple phase unwrapping algorithm [11]

$$\phi_u(l) = \phi_u(0) + \sum_{r=0}^{l-1} \mathcal{W}(\psi^{m,n}(r+1) - \psi^{m,n}(r)) \quad (10)$$

for $l = 1, 2, \dots, N^2$, with $\mathcal{W}(\cdot) = \tan^{-1}(\sin(\cdot)/\cos(\cdot))$ and $\phi_u(0) = \psi(0)$. After phase unwrapping, ϕ_u is substituted for ϕ in (7).

Phase aliasing, however, may corrupt the unwrapping process. Given a reasonably high signal-to-noise ratio (SNR), phase aliasing arises when the frequency space of the phase functions in (5) are not sampled with a fine enough granularity. To test for this condition, we use the residue method in [5] with

$$\sum_{l,k \in \mathcal{S}} \text{residue}(l, k) = \sum_{l,k \in \mathcal{S}} \left(\sum_{i=1}^4 \Delta_i(l, k) \right) \quad (11)$$

where

$$\begin{aligned} \Delta_1(l, k) &= \mathcal{W}(\phi_{DT}^{m,n}(k, l+1) - \phi_{DT}^{m,n}(k, l)) \\ \Delta_2(l, k) &= \mathcal{W}(\phi_{DT}^{m,n}(k+1, l+1) - \phi_{DT}^{m,n}(k+1, l)) \\ \Delta_3(l, k) &= \mathcal{W}(\phi_{DT}^{m,n}(k, l+1) - \phi_{DT}^{m,n}(k+1, l+1)) \\ \Delta_4(l, k) &= \mathcal{W}(\phi_{DT}^{m,n}(k, l+1) - \phi_{DT}^{m,n}(k+1, l)) \end{aligned} \quad (12)$$

where $\mathcal{S} = \{1, 2, \dots, N-1\}$. If residues do not sum to zero, we sample the frequency space of (5) more finely by increasing the number of time samples used in the transform to obtain the log-frequency matrix equations in (8).

III. EXPERIMENTAL RESULTS

To test the efficacy of log-frequency identification of a W–H architecture, we apply the method derived in Section I to characterize a PA model consisting of the following two filters, $\mathbf{h}_1 = [0, -0.1183, 1, -0.12, 0.12, -0.12]$ and $\mathbf{h}_2 = [-0.25, 0, 1, 0.125, 0.125, -0.125]$ along with the nonlinearity $g(v) = \tanh(v)$. We characterize the system over the middle 2/3 of the Nyquist band. The noisy system output is given by $\hat{y}(n) = y(n) + \eta(n)$, where $\eta(n)$ is white gaussian

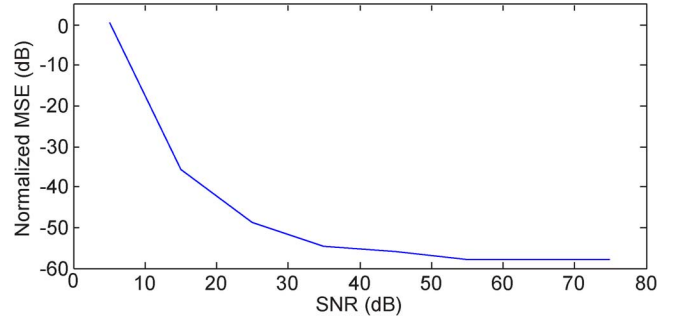


Fig. 2. Mean square error (MSE) performance of the log-frequency identification of a W–H PA model as a function of SNR with respect to the average third-order intermod power.

noise. Since we use intermodulation products to identify the W–H system, we measure SNR with respect to the average intermod power, i.e., the “signal” power is given by

$$\frac{1}{N} \sum_i \left| G_{DT}^{2,-1}(k_1(i), k_2(i)) \right|^2 \quad (13)$$

where N is the number of (k_1, k_2) pairs. In our experiments, we chose $N \approx 3000$.

Each of the two-tone input signals to the system has 256 samples and maximum amplitude 0.125, which ensures that the impact of all nonlinear distortions greater than third order is negligible. We then measure the magnitude and phase of the $2k_1 - k_2$ intermodulation product in the output sequence, ignoring all sets where this product overlaps with either of the fundamental tones or another third-order intermod. We unwrap the measured phases using the algorithm described in Section II.

We compute the least squares solution to the linear system in (8) to find $\alpha_1(k)$, $\alpha_2(k)$, $\theta_1(k)$, and $\theta_2(k)$. The frequency response of each of the filters, sampled at 256 points, is then easily found by exponentiating, with $H_1(k) = \exp(\alpha_1(k) + j\theta_1(k))$ and $H_2(k) = \exp(\alpha_2(k) + j\theta_2(k))$.

Next, we find the two filters $\{\hat{\mathbf{h}}_1, \hat{\mathbf{h}}_2\}$ that best model $\{\mathbf{H}_1(k), \mathbf{H}_2(k)\}$ in the band of interest by solving $\mathbf{W}_p \hat{\mathbf{h}}_i = \mathbf{H}_i$, with $\mathbf{H}_i = [H_i(0) \cdots H_i(255)]^T$, for $i \in \{1, 2\}$, where \mathbf{W}_p is a pruned DFT matrix consisting of only the inband frequency bins. For this experiment, we choose $\hat{\mathbf{h}}_i$ for $i \in \{1, 2\}$ to have 30 taps. After identifying and fixing $\hat{\mathbf{h}}_1$ and $\hat{\mathbf{h}}_2$, we finally construct a 128-cell look-up table (LUT) to model the memoryless nonlinearity $g(\cdot)$. As is evident in Fig. 2, mean square error modeling performance above 30 dB SNR is on the order of -50 dB, where the very small modeling error is a function of the lookup table size and training [8].

IV. SUMMARY

In this paper we have presented a simple least squares approach to Wiener-Hammerstein system identification in the log-frequency domain. The identification process is somewhat complicated by wrapped phases. We circumvent this problem using simple 2-D phase unwrapping and residue testing. Performance results indicate that the approach is robust at varying SNR levels.

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