

## MIT Open Access Articles

*Exact solution to electromagnetic scattering by an impedance sphere coated with a uniaxial anisotropic layer*

The MIT Faculty has made this article openly available. **Please share** how this access benefits you. Your story matters.

**Citation:** You-Lin Geng, Cheng-Wei Qiu, and Ning Yuan. "Exact Solution to Electromagnetic Scattering by an Impedance Sphere Coated With a Uniaxial Anisotropic Layer." *Antennas and Propagation, IEEE Transactions on* 57.2 (2009): 572-576. © 2009 IEEE

**As Published:** <http://dx.doi.org/10.1109/TAP.2008.2011410>

**Publisher:** Institute of Electrical and Electronics Engineers

**Persistent URL:** <http://hdl.handle.net/1721.1/52402>

**Version:** Final published version: final published article, as it appeared in a journal, conference proceedings, or other formally published context

**Terms of Use:** Article is made available in accordance with the publisher's policy and may be subject to US copyright law. Please refer to the publisher's site for terms of use.



## REFERENCES

- [1] R. E. Collin, "Hertzian dipole radiating over a lossy earth or sea: Some early and late 20th-century controversies," *IEEE Antennas Propag. Mag.*, vol. 46, no. 2, pp. 64–79, Apr. 2004.
- [2] R. W. P. King, M. Owens, and T. T. Wu, *Lateral Electromagnetic Waves*. Berlin, Germany: Springer-Verlag, 1992.
- [3] K. A. Norton, "The propagation of radio waves over the surface of the earth and in the upper atmosphere," *Proc. IRE*, vol. 24, pp. 1367–1387, 1936.
- [4] R. W. P. King, "Electromagnetic field of a vertical dipole over an imperfectly conducting half space," *Radio Sci.*, vol. 25, no. 2, pp. 149–160, 1990.
- [5] V. A. Houdzoumis, "Vertical electric dipole radiation over spherical Earth," Ph.D. dissertation, Harvard Univ., Cambridge, MA, 1994.
- [6] J. A. Kong, *Electromagnetic Wave Theory*. New York: Wiley, 1986.
- [7] G. F. Carrier, M. Krook, and C. E. Pearson, *Functions of a Complex Variable*. New York: Hod Books, 1983.
- [8] L. B. Felsen and N. Marcuvitz, *Radiation and Scattering of Waves*. New York: IEEE Press, 1994, sec. 5.5.
- [9] P. Parhami, Y. Rahmat-Samii, and R. Mittra, "An efficient approach for evaluating Sommerfeld integrals encountered in the problem of a current element radiating over lossy ground," *IEEE Trans. Antennas Propag.*, vol. AP-28, no. 1, pp. 100–104, 1980.

## Exact Solution to Electromagnetic Scattering by an Impedance Sphere Coated With a Uniaxial Anisotropic Layer

You-Lin Geng, Cheng-Wei Qiu, and Ning Yuan

**Abstract**—Electromagnetic fields in uniaxial anisotropic media can be obtained based on spherical vector wave functions in isotropic media. Applying the boundary conditions of electromagnetic fields on each interface of an impedance sphere coated with a uniaxial anisotropic layer, the expansion coefficients of electromagnetic fields in the uniaxial anisotropic medium and free space can be derived. The present method is very general, which can be reduced to many isotropic subcases. The present method has been validated in those subcases. Scattering by an impedance sphere with an anisotropic coating is presented and discussed.

**Index Terms**—Anisotropy ratio, electromagnetic scattering, impedance sphere, spherical vector wave functions, uniaxial anisotropic media.

### I. INTRODUCTION

The electromagnetic scattering of an impedance sphere coated by complex materials has been of interest for many years [1], [2] due to its application in microwave devices, antenna, satellite communication, and wave propagation theory, etc. The electromagnetic scattering by a multilayered sphere was studied [3]. More recently, an asymptotic solution for a conducting sphere coated by complex materials was proposed

Manuscript received June 16, 2008; revised August 22, 2008. Current version published March 20, 2009. The work of Y. L. Geng was supported by the Natural Science Foundation of Zhejiang Province of China by Grant Y1080730.

C. W. Qiu is with the Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117576 and also with the Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA 02139 USA (e-mail: eleqc@nus.edu.sg).

Y. L. Geng is with the Institute of Antenna and Microwaves, Hangzhou Dianzi University, Xiasha, Hangzhou, Zhejiang 310018, China.

N. Yuan is with the Department of Electrical and Computer Engineering, University of Houston, Houston, TX 77204 USA.

Digital Object Identifier 10.1109/TAP.2008.2011410

[4] and a study of an impedance sphere coated with a chiral medium was reported [5]. The rigorous eigenfunction, physical optics (PO) and geometrical optics (GO) approximation methods for the scattering by a perfectly conducting sphere coated with a lossless or lossy ferrite material were also presented by Richmond [6], and the plane-wave scattering of arbitrary impedance conducting bodies was analyzed by the method of moments [7], [8].

In addition to scattering problems of three-dimensional isotropic mediums aforementioned, a large amount of investigations have been devoted to the interaction between waves and anisotropic mediums. Some works have been developed for scattering by two-dimensional anisotropic medium [9], [10] based on integrodifferential equations. Furthermore, the scatterings by three-dimensional anisotropic medium were studied based on the coupled-dipole approximation method [11], hybrid vector finite element method and multilevel fast multipole algorithm [12], the method of finite-difference frequency-domain [13] and mode expansion [14], [15]. However, electromagnetic scattering by an impedance sphere coated with anisotropic materials has not yet been studied. The previous results for multilayered anisotropic spheres [16] or an isotropic sphere coated by a plasma shell [17] are no longer applicable for the current anisotropic uniaxial-coated impedance sphere. The purpose of this paper is to develop an exact solution for the general case of scattering by an anisotropic uniaxial-coated impedance sphere, to study the role of the anisotropy upon far-field scattering diagrams, and to understand the mechanism of wave-medium interaction for this case. The exact solution is not only self-standing in theory but also provides more insights to understand scattering properties of the subject discussed in this paper. In this paper, the electromagnetic fields in the uniaxial anisotropic layer are expanded in terms of the first and second kinds of spherical vector wave functions in uniaxial anisotropic media in spectral domain. Enforcing continuity of the tangential components of electromagnetic fields at each interface among uniaxial anisotropic medium, free space, and the impedance sphere [7], [8], the expansion coefficients of electromagnetic fields in terms of spherical vector wave functions in uniaxial anisotropic medium are derived rigorously. Thus, the radar cross section can be obtained. Some numerical results have been presented, where two special cases are considered for verification. In what follows, some new examples, which may be of interest to the community, are discussed.

### II. FORMULATION

Consider a geometry depicted in Fig. 1 which shows the cross section of an anisotropic uniaxial-coated impedance sphere. Its outer and inner radii are  $a_1$  and  $a_2$ , respectively. The inner impedance sphere is coated with a uniaxial anisotropic material characterized by the permittivity tensor ( $\bar{\epsilon}$ ) and permeability tensor ( $\bar{\mu}$ ) with the thickness  $d(= a_1 - a_2)$ . Three distinct regions are thus defined, namely, region 0 for the free space, region 1 for the uniaxial anisotropic medium, and region 2 for the impedance sphere. This composite structure is illuminated by a plane wave (which is assumed to have a unity amplitude of the electric component, with the polarization along  $+\hat{x}$ -direction and propagation in the  $+\hat{z}$ -direction). In the following analysis, the time dependence of  $\exp(-i\omega t)$  is assumed, but suppressed throughout the paper.

The vector wave equation for the electric field in a source-free uniaxial anisotropic medium can be written in the following form [10]:

$$\nabla \times [\bar{\mu}^{-1} \cdot \nabla \times \mathbf{E}(\mathbf{r})] - \omega^2 \bar{\epsilon} \cdot \mathbf{E}(\mathbf{r}) = 0. \quad (1)$$

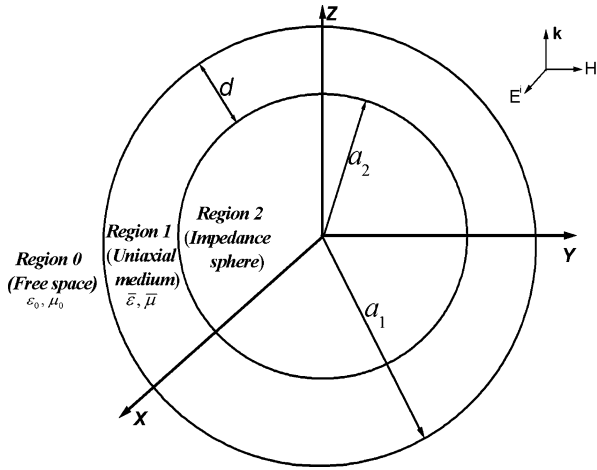


Fig. 1. Geometry of a plane wave scattered by an anisotropic uniaxial-coated impedance sphere.

$E$  denotes the electric field, while  $\bar{\epsilon}$  and  $\bar{\mu}$  represent the permittivity tensor and the permeability tensor of the uniaxial anisotropic medium, respectively. They are defined in Cartesian coordinates as

$$\bar{\epsilon} = \begin{bmatrix} \epsilon_t & 0 & 0 \\ 0 & \epsilon_t & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}, \quad \bar{\mu} = \begin{bmatrix} \mu_t & 0 & 0 \\ 0 & \mu_t & 0 \\ 0 & 0 & \mu_z \end{bmatrix}. \quad (2)$$

It is worth noting that there is another type of uniaxial anisotropic materials defined in spherical coordinates [18]. In such rotationally symmetric uniaxial material, the electromagnetic fields can be expressed in a compact manner using modified spherical vector wave functions (VWFs) [19]. Those fractional spherical VWFs take into account the anisotropy ratio systematically, resulting in great simplification in formulation. However, such technique cannot be simply employed here since the uniaxial material is with respect to Cartesian coordinates in this paper.

Using Fourier transform and the expansion of plane wave factors in terms of spherical vector wave functions in isotropic medium, one can obtain the electromagnetic fields (by the subscript <sub>1</sub>) in the uniaxial anisotropic medium ( $a_2 \leq r \leq a_1$ ) on the basis of the [15] as follows: see (3a) and (3b) at the bottom of the page, where  $n'$  and  $n$  are summed up both from 0 to  $+\infty$  while  $m$  is summed up from  $-n$  to  $n$ , and  $\mathbf{r}$  is pointing in the  $(\theta, \phi)$ -direction in the spherical coordinates.  $A_{mnq}^p(\theta_k)$ ,  $B_{mnq}^p(\theta_k)$ ,  $C_{mnq}^p(\theta_k)$  (where  $p = e$  or  $h$ ) and eigenvalues

$k_q$  ( $q = 1, 2$ ) defined in [15].  $F_{mnq}^{(l)}$  ( $l = 1, 2; q = 1, 2$ ), are unknowns to be determined by the boundary conditions.  $\mathbf{M}_{mn}^{(l)}$ ,  $\mathbf{N}_{mn}^{(l)}$ ,  $\mathbf{L}_{mn}^{(l)}$  are spherical VWFs in isotropic media [15].

Assume that the incident electric field is  $\mathbf{E} = \hat{x}e^{ik_0z}$ . The incident fields can be expanded in an infinite series of spherical vector wave functions as follows:

$$\mathbf{E}^{\text{inc}} = \sum_{mn} \left[ a_{mn}^x \mathbf{M}_{mn}^{(1)}(\mathbf{r}, k_0) + b_{mn}^x \mathbf{N}_{mn}^{(1)}(\mathbf{r}, k_0) \right] \times [\delta_{m,1} + \delta_{m,-1}], \quad r > a_1 \quad (4a)$$

$$\mathbf{H}^{\text{inc}} = \frac{k_0}{i\omega\mu_0} \sum_{mn} \left[ a_{mn}^x \mathbf{N}_{mn}^{(1)}(\mathbf{r}, k_0) + b_{mn}^x \mathbf{M}_{mn}^{(1)}(\mathbf{r}, k_0) \right] \cdot [\delta_{m,1} + \delta_{m,-1}], \quad r > a_1 \quad (4b)$$

where the expansion coefficients are defined as

$$a_{mn}^x = \begin{cases} i^{n+1} \frac{2n+1}{2n(n+1)}, & m = 1 \\ i^{n+1} \frac{2n+1}{2}, & m = -1 \end{cases} \quad (5a)$$

$$b_{mn}^x = \begin{cases} i^{n+1} \frac{2n+1}{2n(n+1)}, & m = 1 \\ -i^{n+1} \frac{2n+1}{2}, & m = -1 \end{cases} \quad (5b)$$

$$\delta_{s,l} = \begin{cases} 1, & s = l \\ 0, & s \neq l. \end{cases} \quad (5c)$$

According to the radiation condition of an outgoing wave (attenuating to zero at infinity) and the asymptotic behavior of spherical Bessel functions, only  $h_n^{(1)}$  should be retained in the radial functions. Therefore the expansions of scattered fields (designated by the superscript  $s$ ) are

$$\mathbf{E}^s = \sum_{mn} \left[ A_{mn}^s \mathbf{M}_{mn}^{(3)}(\mathbf{r}, k_0) + B_{mn}^s \mathbf{N}_{mn}^{(3)}(\mathbf{r}, k_0) \right] \quad (6a)$$

$$\mathbf{H}^s = \frac{k_0}{i\omega\mu_0} \sum_{mn} \left[ A_{mn}^s \mathbf{N}_{mn}^{(3)}(\mathbf{r}, k_0) + B_{mn}^s \mathbf{M}_{mn}^{(3)}(\mathbf{r}, k_0) \right] \quad (6b)$$

where the coefficients,  $A_{mn}^s$  and  $B_{mn}^s$  ( $n$  varies from 0 to  $+\infty$  while  $m$  changes from  $-n$  to  $n$ ), are unknowns to be determined by the boundary conditions.  $\epsilon_0$ ,  $\mu_0$ , and  $k_0 = \omega(\epsilon_0\mu_0)^{1/2}$  identify the permittivity, permeability and wave number of free space, respectively.

Now, the unknown expansion coefficients in uniaxial anisotropic medium and scattered fields in free space can be thus determined by matching the boundary conditions at each interface. The boundary conditions at the interface between uniaxial anisotropic medium and free space ( $r = a_1$ ) are

$$\mathbf{E}_1 \cdot \hat{\theta} = \mathbf{E}^{\text{inc}} \cdot \hat{\theta} + \mathbf{E}^s \cdot \hat{\theta} \quad (7a)$$

$$\mathbf{E}_1 \cdot \hat{\phi} = \mathbf{E}^{\text{inc}} \cdot \hat{\phi} + \mathbf{E}^s \cdot \hat{\phi}. \quad (7b)$$

$$\mathbf{E}_1 = \sum_{l=1}^2 \sum_{q=1}^2 \sum_{mnn'} F_{mnq}^{(l)} \times \int_0^\pi \left[ A_{mnq}^e(\theta_k) \mathbf{M}_{mn}^{(1)}(\mathbf{r}, k_q) + B_{mnq}^e(\theta_k) \mathbf{N}_{mn}^{(1)}(\mathbf{r}, k_q) + C_{mnq}^e(\theta_k) \mathbf{L}_{mn}^{(1)}(\mathbf{r}, k_q) \right] P_n^m(\cos \theta_k) k_q^2 \sin \theta_k d\theta_k, \quad a_2 \leq r \leq a_1 \quad (3a)$$

$$\mathbf{H}_1 = \sum_{l=1}^2 \sum_{q=1}^2 \sum_{mnn'} F_{mnq}^{(l)} \times \int_0^\pi \left[ A_{mnq}^h(\theta_k) \mathbf{M}_{mn}^{(1)}(\mathbf{r}, k_q) + B_{mnq}^h(\theta_k) \mathbf{N}_{mn}^{(1)}(\mathbf{r}, k_q) + C_{mnq}^h(\theta_k) \mathbf{L}_{mn}^{(1)}(\mathbf{r}, k_q) \right] P_n^m(\cos \theta_k) k_q^2 \sin \theta_k d\theta_k, \quad a_2 \leq r \leq a_1 \quad (3b)$$

The boundary conditions at the interface ( $r = a_2$ ) are as follows [7], [8]:

$$\mathbf{E}_1 - (\mathbf{E}_1 \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} = \eta_s \eta_0 (\hat{\mathbf{r}} \times \mathbf{H}_1) \quad (8)$$

or in its dual form, i.e.

$$\mathbf{H}_1 - (\mathbf{H}_1 \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} = \frac{-1}{\eta_s \eta_0} (\hat{\mathbf{r}} \times \mathbf{E}_1) \quad (9)$$

where  $\eta_s$  and  $\eta_0$  denote the relative impedance of the impedance sphere and the impedance of vacuum, respectively.

Substituting (3), (4), and (6) into (7), one yields the following relations at ( $r = a_1$ ):

$$\begin{aligned} \sum_{l=1}^2 \sum_{q=1}^2 \sum_{n'=0}^{\infty} F_{mn'lq}^{(l)} \int_0^{\pi} Q_{mnq}^l P_{n'}^m(\cos \theta_k) k_q^2 \sin \theta_k d\theta_k \\ = [\delta_{m,-1} + \delta_{m,-1}] a_{mn}^x \frac{i}{(k_0 a_1)^2}, \end{aligned} \quad (10a)$$

$$\begin{aligned} \sum_{l=1}^2 \sum_{q=1}^2 \sum_{n'=0}^{\infty} F_{mn'lq}^{(l)} \int_0^{\pi} R_{mnq}^l P_{n'}^m(\cos \theta_k) k_q^2 \sin \theta_k d\theta_k \\ = [\delta_{m,-1} + \delta_{m,-1}] b_{mn}^x \frac{i}{(k_0 a_1)^2} \end{aligned} \quad (10b)$$

where

$$\begin{aligned} Q_{mnq}^l = A_{mnq}^e \frac{1}{k_0 r} \frac{d}{dr} \left( r h_n^{(1)}(k_0 r) \right) z_n^{(l)}(k_q r) \\ - \frac{i\omega\mu_0}{k_0} \left[ B_{mnq}^h \frac{1}{k_q r} \frac{d}{dr} \left( r z_n^{(l)}(k_q r) \right) \right. \\ \left. + C_{mnq}^h \frac{z_n^{(l)}(k_q r)}{r} \right] \cdot h_n^{(1)}(k_0 r) \end{aligned} \quad (11a)$$

$$\begin{aligned} R_{mnq}^l = \frac{i\omega\mu_0}{k_0} A_{mnq}^h \frac{1}{k_0 r} \frac{d}{dr} \left( r h_n^{(1)}(k_0 r) \right) z_n^{(l)}(k_q r) \\ - \left[ B_{mnq}^e \frac{1}{k_q r} \frac{d}{dr} \left( r z_n^{(l)}(k_q r) \right) \right. \\ \left. + C_{mnq}^e \frac{z_n^{(l)}(k_q r)}{r} \right] \cdot h_n^{(1)}(k_0 r). \end{aligned} \quad (11b)$$

Similarly, the substitution of electromagnetic fields in uniaxial anisotropic medium into the boundary condition [i.e., (8) or (9)] at the surface of  $r = a_2$  leads to

$$\sum_{l=1}^2 \sum_{q=1}^2 \sum_{n'=0}^{\infty} F_{mn'lq}^{(l)} \int_0^{\pi} D_{mnq}^l P_{n'}^m(\cos \theta_k) k_q^2 \sin \theta_k d\theta_k = 0 \quad (12a)$$

$$\sum_{l=1}^2 \sum_{q=1}^2 \sum_{n'=0}^{\infty} F_{mn'lq}^{(l)} \int_0^{\pi} H_{mnq}^l P_{n'}^m(\cos \theta_k) k_q^2 \sin \theta_k d\theta_k = 0 \quad (12b)$$

where

$$\begin{aligned} D_{mnq}^l = A_{mnq}^e z_n^{(l)}(k_q r) \\ + \eta_s \eta_0 \left[ B_{mnq}^h \frac{1}{k_q r} \frac{d}{dr} \left( r z_n^{(l)}(k_q r) \right) + C_{mnq}^h \frac{z_n^{(l)}(k_q r)}{r} \right] \end{aligned} \quad (13a)$$

$$\begin{aligned} H_{mnq}^l = \eta_s \eta_0 A_{mnq}^h z_n^{(l)}(k_q r) \\ - \left[ B_{mnq}^e \frac{1}{k_q r} \frac{d}{dr} \left( r z_n^{(l)}(k_q r) \right) + C_{mnq}^e \frac{z_n^{(l)}(k_q r)}{r} \right]. \end{aligned} \quad (13b)$$

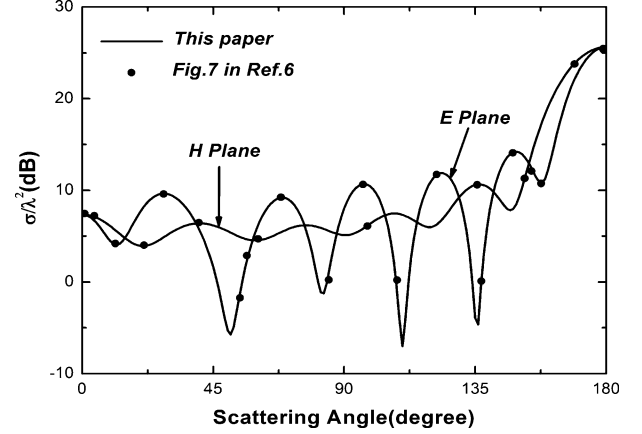


Fig. 2. Radar cross sections (RCSs) versus scattering angle  $\theta$  (in degrees) for a conducting sphere coated by an isotropic medium. The radius of the impedance sphere and the coating thickness of uniaxial anisotropic medium are chosen as  $a_2 = \lambda$ , and  $d = 0.05\lambda$ , respectively. The permittivity and permeability tensorial elements of the uniaxial anisotropic medium are assumed to be  $\epsilon_t = \epsilon_z = 2.5\epsilon_0$ , and  $\mu_t = \mu_z = 1.6\mu_0$ , respectively. The relative impedance of the inner-most conducting sphere is  $\eta_s = 0$ .

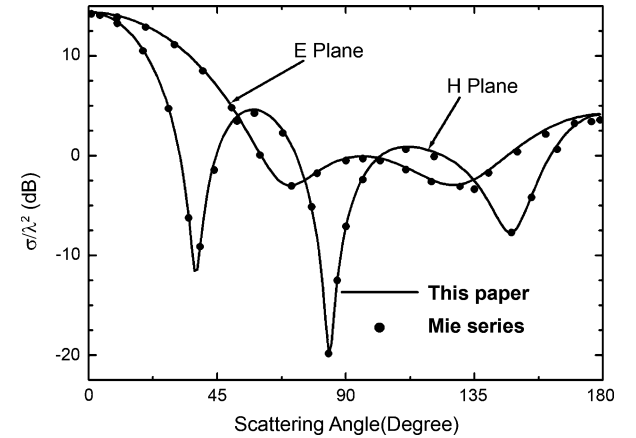


Fig. 3. RCSs versus scattering angle  $\theta$  (in degrees) for a normal impedance sphere coated by a lossy isotropic medium. The radius of the impedance sphere and the coating thickness are chosen as  $a_2 = 0.45\lambda$  and  $d = 0.05\lambda$ , while material parameters of the uniaxial anisotropic medium and relative impedance are assumed to be  $\epsilon_t = \epsilon_z = (3 + 0.2i)\epsilon_0$ ,  $\mu_t = \mu_z = (2 + 0.2i)\mu_0$ , and  $\eta_s = 0.1 + 0.1i$ , respectively.

From (10) and (12), we can see that those unknown coefficients of electromagnetic fields in the anisotropic uniaxial-coated impedance sphere can be determined analytically for the first time. The solution has only one-dimensional integral which can be easily calculated using the Gauss integral [20].

### III. NUMERICAL RESULTS AND DISCUSSION

In the previous section, we have presented the necessary theoretical formulation of the electromagnetic fields in the presence of an anisotropic uniaxial-coated impedance sphere. To gain more physical insights into the problem, we will provide in this section some numerical solutions to the problem of electromagnetic scattering by an anisotropic uniaxial-coated impedance sphere.

In order to verify the correctness of the newly obtained results, we compare the bistatic radar cross sections (RCSs) in  $E$ -plane

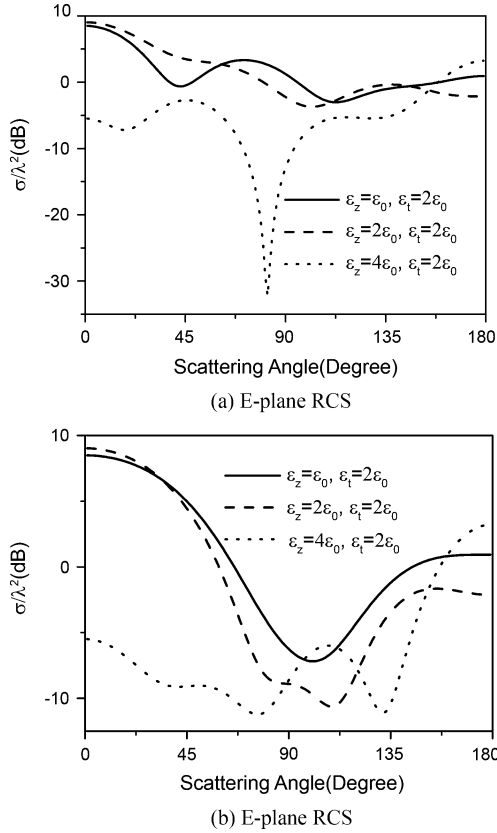


Fig. 4. The effect of anisotropy ratio in RCSs of a uniaxial-coated impedance sphere: (a) E-plane RCS and (b) H-plane RCS. The other parameters for the coated sphere remain unchanged. The radius of the impedance sphere and the coating thickness of the lossless uniaxial anisotropic medium are chosen as  $a_2 = 0.2\lambda$  and  $d = 0.25\lambda$ , respectively. The material parameters of the anisotropic coating are assumed to be  $\mu_t = 4\mu_0$ , and  $\mu_z = 2\mu_0$ . The relative impedance of the inner sphere is  $\eta_s = 0.5$ . Three electric anisotropy ratios  $A_e$  are considered in particular where the transversal permittivity is fixed at  $\epsilon_t = 2\epsilon_0$ : 1)  $A_e = 2$  (solid line); 2)  $A_e = 1$  (dashed line); 3)  $A_e = 0.5$  (dotted line).

( $xoz$ -plane) and  $H$ -plane ( $yo z$ -plane) [16] for the structure in Fig. 1 with the Mie series results for the cases of isotropic coatings [6].

In Fig. 2, the comparison is demonstrated for a special case of the impedance sphere (a conducting sphere which has  $\eta_s = 0$ ) coated by an isotropic medium. The solid curves are the results from this paper and the circle dots are obtained from [6, Fig. 7]. In Fig. 3, we compute the RCSs of a normal impedance sphere ( $\eta_s = 0.1 + 0.1i$ ) coated by a lossy isotropic medium. The maximum number  $n'$  in (10a)–(10b) and (12a)–(12b) is found to be 6 in order to achieve good convergence. From Figs. 2 and 3, it can be seen that, if the coating is isotropic, the radar cross sections calculated by the present method and the Mie series are in very good agreement in both the  $E$ - and  $H$ -planes for conducting and impedance inner spheres. It partially verifies the correctness and applicability of our theorem as well as the Fortran codes. In what follows, we will obtain and discuss some interesting results unavailable elsewhere, based on the approach in this paper.

Fig. 4 represents RCSs of an anisotropic uniaxial-coated impedance sphere, where the uniaxial anisotropic medium is lossless. The effect of electric anisotropy ratio in the scattering is studied, which is defined as the ratio of transversal permittivity over radial permittivity, i.e.,  $A_e = \epsilon_t/\epsilon_z$ . The maximum number  $n'$  in (10) and (12) to achieve good convergence is found to be 10 (slightly larger than the quantity

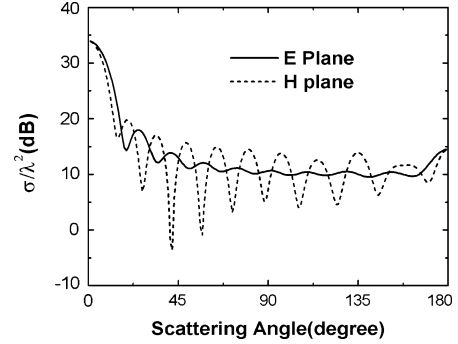


Fig. 5. RCSs versus scattering angle  $\theta$  (in degrees) in the  $E$ -plane (solid curve) and in the  $H$ -plane (dashed curve). The radius of the impedance sphere and coating thickness of the uniaxial anisotropic medium are chosen to be  $a_2 = 1.75\lambda$  and  $d = 0.05\lambda$ , while the parameters for the anisotropic medium and relative impedance are assumed to be  $\epsilon_t = \epsilon_z = (1.5 + 0.01i)\epsilon_0$ ,  $\mu_t = (2 + 0.01i)\mu_0$ ,  $\mu_z = (4 + 0.02i)\mu_0$ , and  $\eta_s = 0.1 + 0.2i$ , respectively.

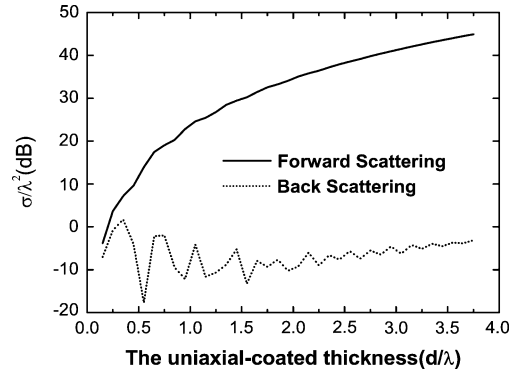


Fig. 6. RCSs versus the thickness of the uniaxial anisotropic medium in the forward (solid curve) and in the back (dashed curve) directions, when inner radius is fixed at  $a_2 = 0.1\lambda$ . The parameters of the uniaxial coating and relative impedance of the inner sphere are assumed to be  $\epsilon_t = (3 + 0.2i)\epsilon_0$ ,  $\epsilon_z = (1.5 + 0.1i)\epsilon_0$ ,  $\mu_t = \mu_z = (2 + 0.1i)\mu_0$ , and  $\eta_s = 0.1$ , respectively.

used in Fig. 3 due to the change in the thickness and parameters of the uniaxial anisotropic medium).

Interestingly, in Fig. 4(a), it is found that for the case of  $A_e = 0.5$  there exists a nearly *blind area* at about  $20^\circ$  in the scattering angle from  $73.2^\circ$  to  $95^\circ$  where the intensity of  $E$ -plane RCS is lower than  $-10$  dB. However, the RCS for  $H$ -plane does not hold such feature. Also, the role of anisotropy in the far-field scattering is characterized. It is found that the anisotropy ratio less than unity (which corresponds to the isotropic case) will drastically reduce both of the  $E$ -plane and  $H$ -plane RCSs particularly when the scattering angle is smaller than  $90^\circ$ .

To further demonstrate the applicability of the proposed method, the case of an electrically large impedance sphere coated by a shell made of a lossy ferrite is investigated in Fig. 5. The radar cross sections are obtained for both the  $E$ -plane and the  $H$ -plane. Due to the increased electrical dimension, the maximum number of  $n'$  must be significantly increased to 18 in order to achieve good convergence. The  $H$ -plane RCS is found to be highly oscillating against the observation angle, while the  $E$ -plane is quite smooth except in the region near the backward direction. For instance, when the observation angle is about  $45^\circ$ , a fine change in the angle would result in a drastic variation (at about 20 dB) in the scattering efficiency on the  $H$  plane.

Of particular interest are the monostatic RCSs, for example, the forward RCS (the scattering angle  $\theta = 0$ ) and the back RCS (the scattering angle  $\theta = \pi$ ). The parameters of the coated uniaxial medium and the

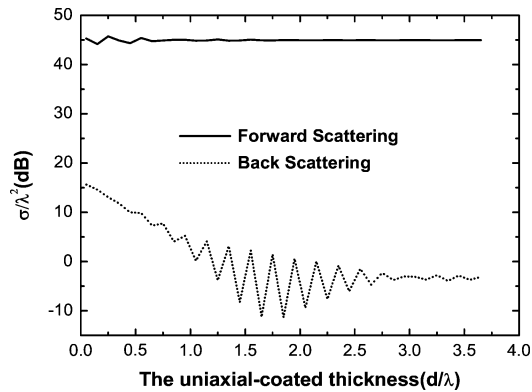


Fig. 7. RCSs versus the thickness of uniaxial anisotropic medium in the forward (solid curve) and in the back (dashed curve) directions, when outer radius is fixed at  $a_1 = 3.75\lambda$ . The uniaxial coating and the impedance sphere are identical to those in Fig. 6.

relative impedance in Figs. 6 and 7 are the same. In Fig. 6, the radius of the impedance sphere is constant:  $a_2 = 0.1\lambda$ . Thus, it is straightforward to understand and characterize the effects of the thickness of the uniaxial coating upon the forward and back scatterings in particular. In Fig. 7, the outer radius of the coated sphere is constant:  $a_1 = 3.75\lambda$ . The radius of the inner impedance sphere varies from  $a_2 = 3.7\lambda$  to  $a_2 = 0.1\lambda$  with an interval at  $0.1\lambda$ , namely, the thickness of the uniaxial anisotropic medium is changing from  $d = 0.05\lambda$  to  $d = 3.65\lambda$  at the interval of  $0.1\lambda$  in Fig. 7.

If a given bare impedance sphere is to be coated with uniaxial anisotropic media (e.g., the case in Fig. 6), the forward RCS of the core-shell system always increases as the thickness of the anisotropic coating increases. However, it is interesting to note that the back scattering will be highly oscillating especially when the ratio of  $d/a_2$  is not too large (i.e.,  $d/a_2 < 15$ ). In Fig. 7, when the outer radius of the coated sphere is fixed, the forward scattering is nearly a constant value no matter how much space is filled by the anisotropic shell. It coincides with the findings in Fig. 6, leading to the conclusion that the forward RCS is primarily determined by the outermost size  $a_1$ . Also, Fig. 7 shows that the back scattering is quite oscillating when  $0.36 < d/a_1 < 0.67$  (i.e.,  $0.27 < d/a_2 < 2$ ). The RCS in the forward direction is always larger than that in the back direction, if the real parts of the parameters of the uniaxial anisotropic layer are all positive.

#### IV. CONCLUSION

The spherical vector wave function expansion solution to the plane wave scattering by an anisotropic uniaxial-coated impedance sphere is obtained analytically in this paper. First, our general solution is reduced to its subcase (the coating material is isotropic, and the inner sphere is conducting), and results obtained by the present method are compared with those by Mie theory. A fairly good agreement is observed. After validating our method, general numerical results are presented and discussed. The present analysis is believed to be useful to characterize the antenna and radiowave propagation in coated impedance spheres involving complex media.

#### REFERENCES

- [1] W. Swarner and L. Peters, Jr, "Radar cross sections of dielectric or plasma coated conducting spheres and circular cylinders," *IEEE Trans. Antennas Propag.*, vol. 11, pp. 558–569, Sep. 1963.
- [2] D. Colton and P. B. Monk, "Target identification of coated objects," *IEEE Trans. Antennas Propag.*, vol. 54, pp. 1232–1242, 2006.
- [3] Z. S. Wu and Y. P. Wang, "Electromagnetic scattering for multilayered sphere: Recursive algorithms," *Radio Sci.*, vol. 26, no. 6, pp. 1393–1401, 1991.
- [4] J. Shim and H. T. Kim, "An asymptotic solution of EM backscattering from a conducting sphere coated with a composite material," *IEEE Trans. Antennas Propag.*, vol. 52, pp. 1465–1472, 2004.
- [5] P. L. E. Uslenghi, "Scattering by an impedance sphere coated with a chiral layer," *Electromagn.*, vol. 10, pp. 201–211, 1990.
- [6] J. H. Richmond, "Scattering by a ferrite-coated conducting sphere," *IEEE Trans. Antennas Propag.*, vol. 35, pp. 73–79, 1987.
- [7] L. N. Medgyesi-Mitschang and J. M. Putnam, "Integral equation formulations for imperfectly conducting scatterers," *IEEE Trans. Antennas Propag.*, vol. 33, pp. 206–214, Feb. 1985.
- [8] L. N. Medgyesi-Mitschang and D. S. Wang, "Hybrid solutions for large-impedance coated bodies of revolution," *IEEE Trans. Antennas Propag.*, vol. 34, pp. 1319–1329, Nov. 1986.
- [9] R. D. Graglia and P. L. E. Uslenghi, "Electromagnetic scattering from anisotropic material Part I: General theory," *IEEE Trans. Antennas Propag.*, vol. 32, pp. 867–869, 1984.
- [10] X. B. Wu and K. Yasumoto, "Three-dimensional scattering by an infinite homogeneous anisotropic cylinder: An analytical solution," *J. Appl. Phys.*, vol. 82, no. 1, pp. 1996–2003, 1997.
- [11] V. V. Varadan, A. Lakhtakia, and V. K. Varadan, "Scattering by three-dimensional anisotropic scatterers," *IEEE Trans. Antennas Propag.*, vol. 37, pp. 800–802, 1989.
- [12] P. Wang, L. Z. Zhou, M. Y. Xia, and Y. H. Tan, "Analysis of electromagnetic scattering from objects coated with arbitrarily magnetized lossy ferrite materials," *IEEE Trans. Magn.*, vol. 42, no. 4, pp. 791–794, 2006.
- [13] C. M. Rappaport and B. J. McCartin, "FDFD analysis of electromagnetic scattering in anisotropic media using unconstrained triangular meshes," *IEEE Trans. Antennas Propag.*, vol. 39, pp. 345–349, 1991.
- [14] R. J. Tarento, K. H. Bennemann, P. Joyes, and J. Van de Walle, "Mie scattering of magnetic spheres," *Phys. Rev. E*, vol. 69, p. 026606/1–5, 2004.
- [15] Y. L. Geng, X. B. Wu, L. W. Li, and B. R. Guan, "Mie scattering by a uniaxial anisotropic sphere," *Phys. Rev. E*, vol. 70, no. 5, p. 056609/1–8, 2004.
- [16] Y. L. Geng, X. B. Wu, L. W. Li, and B. R. Guan, "Electromagnetic scattering by an inhomogeneous plasma anisotropic sphere of multilayers," *IEEE Trans. Antennas Propag.*, vol. 53, pp. 3982–3989, 2005.
- [17] Y. L. Geng, X. B. Wu, and L. W. Li, "Characterization of electromagnetic scattering by a plasma anisotropic shell," *IEEE Antennas Wireless Propag. Lett.*, vol. 3, no. 6, pp. 100–103, 2004.
- [18] C. W. Qiu, L. W. Li, T. S. Yeo, and S. Zouhdi, "Scattering by rotationally symmetric anisotropic spheres: Potential formulation and parametric studies," *Phys. Rev. E*, vol. 75, p. 026609, 2007.
- [19] C. W. Qiu, S. Zouhdi, and A. Razek, "Modified spherical wave functions with anisotropy ratio: Application to the analysis of scattering by multilayered anisotropic shells," *IEEE Trans. Antennas Propag.*, vol. 55, pp. 3515–3523, Dec. 2007.
- [20] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*. New York: Dover, 1972, pp. 916–919.