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Hickey Replies: The authors of the preceding Comment [1] correctly assert that the full spin orbital coupling Hamiltonian stated by Hickey *et al.* in a recent Letter [2] is Hermitian. The original article on which the Comment is based contained the same observation: ''Note that, while neither spin-orbital term is Hermitian, the two terms taken together are Hermitian.'' Let us recall the spin-orbital terms:

$$H^\Sigma = -\frac{e\hbar}{4m_0^2 c^2}\Sigma \cdot \mathbf{E} \times \mathbf{p} - \frac{ie\hbar^2}{8m_0^2 c^2}\Sigma \cdot (\nabla \times \mathbf{E})$$
$$= H_1 + H_2. \tag{1}$$

This Hamiltonian describes the spin-orbital coupling of single particle states in the presence of a static electric field, and it is Hermitian. In most ferromagnets, the lattice electron spins are orbitally quenched in the inversion symmetric crystal. Therefore, the first term is unimportant (spin-orbital splitting energies are negligible in most transition metal based ferromagnets). In the static case, the second term is zero (the curl of any scalar field gradient is always zero). When the magnetization evolves dynamically, however, H_2 becomes nonzero due to the nonvanishing electric field curl, commensurate with the time varying magnetic induction. We import a Maxwell relation to describe this time-dependent perturbation. Let us recall the Hamiltonian in the reduced spinor basis $\mathbf{S} = (\hbar/2)\boldsymbol{\sigma}$, as follows:

$$H^\mathbf{S} = -\frac{e\hbar}{4m_0^2 c^2}\mathbf{S} \cdot \mathbf{E} \times \mathbf{p} + i\frac{e\hbar^2 \mu_0}{8m_0^2 c^2}\mathbf{S} \cdot (1 + \chi_m^{-1}) \cdot \frac{d\mathbf{M}}{dt} = \tag{2}$$

$$H^\mathbf{S} = H_0^\mathbf{S} + H^\mathbf{S}(t). \tag{3}$$

The Hamiltonian is now a time-dependent, anti-Hermitian interaction which is never canceled by the single particle energy of H_1 and which acts as a perturbation on the system and couples to the time varying magnetization. Therefore, Gilbert damping is an emergent feature of a collective of magnetization spins. In our Letter [2], we calculate the time evolution of the magnetization in the nonequilibrium case of an open system of spins. Non-Hermiticity of the interaction Hamiltonian leads to irreversible, nonadiabatic and nonunitary time evolution of the density matrix [3], giving rise to dissipation, which is precisely what our theory on Gilbert damping purports to describe.

In Fig. 1 of our original Letter, we have plotted the value of the Gilbert constant α at $\omega = 0$ (the dc limit) using the absolute value of the transverse susceptibility tensor, and

not its imaginary part. However, we need to clarify that the Re[α] drops to zero when $\omega \rightarrow 0$ in the basis in which we have calculated it, as implied by the final equation of page 3 of our Letter. We conclude that a finite perturbation frequency is required—$\omega \neq 0$ (where ω corresponds to the Fourier component of the applied field perturbation) for precessional damping to take place, otherwise the system of spins (the magnetization) is in equilibrium with the field.

The dimensionless Gilbert damping tensor, which determines the intrinsic linewidth of FMR resonances, reduces to [2]:

$$\hat{\alpha} = \frac{ie\hbar\mu_0 M_s}{8m^2 c^2}(1 + \hat{\chi}^{-1}), \tag{4}$$

$$\chi_{\perp_\pm}^{-1} = \frac{(\frac{\omega_0 - \omega}{\gamma\mu_0 M_s} - i\frac{\sigma_\perp \mathbf{q}^2 \omega_0}{\mu_0 M_s^2})(1 + i\gamma\sigma_\perp \mathbf{q}^2/M_s)}{1 + \gamma^2\sigma_\perp^2 \mathbf{q}^2/M_s}, \tag{5}$$

where the susceptibility was written without the longitudinal response and in a basis in which the Levi-Civita tensor is diagonal and the parameters in the above equation are given by [2].

This result confirms the empirically held picture that the damping, and hence the FMR linewidth scales linearly with frequency (without an intercept) as demonstrated by Urban *et al.* [4]. We agree with Widom *et al.* that, in the limit of static electric and magnetic fields, the spin-orbital coupling is Hermitian and has a vanishing second term. However, Widom *et al.* did not take into account that, when the magnetization varies in time, the second term cannot be neglected, and we believe that this gives rise to dynamical Gilbert damping.

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