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Lorentz force and radiation pressure on a spherical cloak

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The mechanical behavior of a transformation based spherical cloak under wave illumination is derived. We show that the equatorial region of the cloak is subject to much higher stress than the polar regions, where the polar axis is defined along the wave propagation direction. These forces do not exist before transformation but stem from the squeezed electromagnetic space. The trajectory of the ray can be interpreted as a result of the recoil force that the cloak exerts upon the ray. The total radiation pressure on an ideal cloak is shown to be exactly zero, effecting a stationary cloak.

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It is well known that electromagnetic waves carry both momentum and energy [1,2]. When light impinges onto a dielectric particle, part of the optical momentum will transfer to it [3–6], and the radiation pressure will accelerate the particle. That is the reason why we can see the comet's tail, which results from the radiation pressure of the sunlight pushing particles of the tail away from the comet. The recently proposed invisibility cloak [7–9], which neither absorbs nor reflects light, will be immune to the light radiation pressure. The idea of cloak has been proposed based on the coordinate transformation method, where a hole is created in the transformed space and an object in the hole can be concealed from detection [7–9]. Compared with the cloak design based on other approaches, such as the scattering cancellation method [10–12], the anti-object method [13], and the dipole moment cancellation method using anomalous localized resonance [14,15] etc., the transformation based design has advantages that it does not depend on the parameters of the hidden object. The ray tracing method [7,16] and the full wave method [17–22] have both shown that light can be smoothly guided around the transformation based cloak. Such interesting phenomena have been experimentally demonstrated by a two-dimensional cylindrical cloak with simplified parameters [23], a ground-plane cloak [24,25], and dielectric optical cloaks [26,27]. The reason that the rays are guided around the invisibility cloak can be interpreted as a result of the bending of electromagnetic space, which is always associated with some force appearance, such that the gravity is the appearance of the bent time and space. Therefore, a bent electromagnetic space must exhibit certain force properties and affect the mechanical behavior of light in physical space. Such mechanical behavior is a very fundamental physical problem associated with the transformed invisibility cloak but has not been revealed.

In this Rapid Communication, the mechanical behavior of a general transformation based spherical cloak under plane-wave illumination is revealed based on Lorentz force, which links the Maxwell equations to mechanics in the theory of electromagnetics. We show that the equatorial region of the cloak is subject to much higher stress than the polar regions, where the polar axis is defined along the wave propagation direction. These forces do not exist before transformation but stem from the squeezed electromagnetic space and are one of the ways that it appears in physical space. In addition, the force distribution exhibits a symmetric pattern in the cross section perpendicular to the wave propagation direction, therefore the total radiation pressure on an ideal cloak is exactly zero. The trajectory of the ray inside of the cloak can be interpreted vividly as a result of the recoil force that the cloak exerts upon the ray.

The three-dimensional spherical cloak we consider here has an inner radius R_1 and outer radius R_2. The cloak shell within $R_1 < r < R_2$ is a radially uniaxial and inhomogeneous medium with permittivity tensor $\bar{\bar{\epsilon}} = \epsilon_r \hat{r}\hat{r} + \epsilon_t \hat{\theta}\hat{\theta} + \epsilon_t \hat{\phi}\hat{\phi}$ and permeability tensor $\bar{\bar{\mu}} = \mu_r \hat{r}\hat{r} + \mu_t \hat{\theta}\hat{\theta} + \mu_t \hat{\phi}\hat{\phi}$. The constitutive parameters ϵ_t, ϵ_r, μ_t, and μ_r can be derived based on the transformation method [7]:

$$\epsilon_t/\epsilon_0 = \mu_t/\mu_0 = f'(r), \quad \epsilon_r/\epsilon_0 = \mu_r/\mu_0 = \frac{f^2(r)}{r^2 f'(r)}, \tag{1}$$

where ϵ_0 and μ_0 are the permittivity and permeability of free space, $f(r)$ is the general transformation function between the original spherical coordinate system (r', θ', φ') and the physical spherical coordinate system (r, θ, φ) [28]. An E_x polarized plane wave with unit amplitude, $E_i = \hat{x} e^{ik_0 z}$, is incident upon the spherical cloak, where $k_0 = \omega\sqrt{\mu_0 \epsilon_0}$ is the wave number in free space. The time dependence of $e^{-i\omega t}$ is assumed and is suppressed in the following part. Based on the Mie scattering model [18,28], the electromagnetic fields inside the coated layer can be calculated as follows:

$$E_r^c = f'(r) \sin\theta \cos\phi\, e^{ik_0 f(r)\cos\theta}, \quad H_r^c = \frac{E_r^c}{\eta_0} \tan\phi,$$

$$E_\theta^c = \frac{f(r)}{r} \cos\theta \cos\phi\, e^{ik_0 f(r)\cos\theta}, \quad H_\theta^c = \frac{E_\theta^c}{\eta_0} \tan\phi,$$

$$E_\phi^c = -\frac{f(r)}{r} \sin\phi\, e^{ik_0 f(r)\cos\theta}, \quad H_\phi^c = -\frac{E_\phi^c}{\eta_0} \cot\phi, \tag{2}$$

where $\eta_0 = \sqrt{\mu_0/\epsilon_0}$ is the impedance in free space. Note that the fields in the core are zero, while the fields outside of the

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cloak (air region) can be expressed using Eq. (2) with the transformation function determined by $f^{air}(r)=r$. Once the fields inside the spherical cloak are obtained, the force inside the medium can be calculated. The time-average Lorentz force density due to the harmonic wave excitation is [5]

$$\langle \bar{f} \rangle = \frac{1}{2}\mathrm{Re}\{(-\nabla \cdot \bar{P})\bar{E}^* + (-\nabla \cdot \mu_0 \bar{M})\bar{H}^* - i\omega \bar{P} \times \mu_0 \bar{H}^* + i\omega\mu_0 \bar{M} \times \epsilon_0 \bar{E}^*\}, \quad (3)$$

where Re{ } represents the real part of a complex quantity and * denotes the complex conjugate. The polarization and magnetization are defined as $\bar{P}=\bar{D}-\epsilon_0\bar{E}$ and $\mu_0\bar{M}=\bar{B}-\mu_0\bar{H}$, respectively. All of them are functions of $\bar{r}$. The leading two terms in Eq. (3) contribute via a force density on bound electric and magnetic charges, while the final two terms represent the force density on bound electric and magnetic currents and free electric and magnetic currents. First, let us focus on the final two terms. In an ideal lossless cloak, by using Eqs. (1) and (2), we can obtain that the force density on the electric and magnetic currents $\frac{1}{2}\mathrm{Re}\{-i\omega(\bar{\bar{\epsilon}}-\epsilon_0\bar{\bar{I}})\bar{E} \times \mu_0\bar{H}^* + i\omega(\bar{\bar{\mu}}-\mu_0\bar{\bar{I}})\bar{H}\times\epsilon_0\bar{E}^*\}$ is zero everywhere. Thus we can conclude that the force inside the ideal cloak is contributed only from the bound electric and magnetic charges. Second, we can calculate the first two terms in Eq. (3). In a source free region, $\nabla\cdot\bar{D}=0$ and $\nabla\cdot\bar{B}=0$, we can get $-\nabla\cdot\bar{P} = \epsilon_0\nabla\cdot\bar{E}$ and $-\nabla\cdot\mu_0\bar{M}=\mu_0\nabla\cdot\bar{H}$. In a homogenous material, both $\nabla\cdot\bar{E}$ and $\nabla\cdot\bar{H}$ are zero everywhere, therefore the force density on the bound electric and magnetic charges exist only at the boundary of two homogenous materials [3]. However, in the spherical cloak layer, the materials are both anisotropic and inhomogeneous, the force density on bound electric and magnetic charges exist everywhere. Substitution of Eq. (2) into Eq. (3) yields the bulk force density in the region of $R_1<r<R_2$:

$$\langle \bar{f}_{bulk} \rangle = \frac{1}{2}\epsilon_0 \sin\theta\left[f''(r) + \frac{2f'(r)}{r} - \frac{2f(r)}{r^2}\right]\left[\hat{r}f'(r)\sin\theta + \hat{\theta}\frac{f(r)}{r}\cos\theta\right]. \quad (4)$$

On the outer surface of the cloak, Eq. (3) is simplified to

$$\langle \bar{f}_{surf} \rangle = \frac{1}{2}\mathrm{Re}\{\rho_e\bar{E}^*_{avg} + \rho_h\bar{H}^*_{avg}\}, \quad (5)$$

where $\rho_e=\hat{r}\cdot\epsilon_0(\bar{E}^{air}-\bar{E}^c)$ and $\rho_h=\hat{r}\cdot\mu_0(\bar{H}^{air}-\bar{H}^c)$ are the bound electric and magnetic surface charge densities, respectively, and $\bar{E}_{avg}=(\bar{E}^{air}+\bar{E}^c)/2$ and $\bar{H}_{avg}=(\bar{H}^{air}+\bar{H}^c)/2$ are the average electric and magnetic fields at the surface [3,29]. Substitution of Eq. (2), the fields in the free space and the fields in the core into Eq. (5) yields the following surface force density at $r=R_2$ and $r=R_1$, respectively:

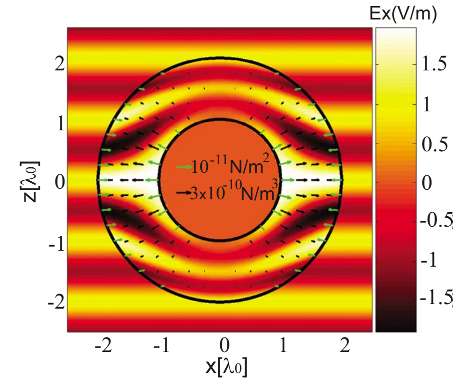

FIG. 1. (Color online) The Lorentz force density (arrows) on the ideal linearly transformed spherical cloak with $R_1=\lambda_0$ and $R_2=2\lambda_0$ results from an E_x polarized plane wave of unit amplitude incident from free space with wavelength $\lambda_0=0.1$ m. The bulk force density (unit: $\mathrm{N/m^3}$) is represented by black arrows, while the surface force density (unit: $\mathrm{N/m^2}$) is represented by green arrows. The background pattern is the electric field distribution (unit: V/m).

$$\langle \bar{f}_{r=R_2} \rangle = \frac{1}{2}\epsilon_0 \sin\theta\left\{\hat{r}\frac{1}{2}[1-f'^2(R_2)]\sin\theta + \hat{\theta}[1-f'(R_2)]\cos\theta\right\}. \quad (6)$$

$$\langle \bar{f}_{r=R_1} \rangle = \hat{r}\frac{1}{4}\epsilon_0 f'^2(R_1)\sin^2\theta. \quad (7)$$

It is interesting to see that both the bulk force density and the surface force density have no $\hat{\phi}$ component, which means that there is no angular momentum transfer along the z axis due to the rotational symmetry of the spherical cloak. In addition, while the surface force at the outer boundary has both $\hat{r}$ and $\hat{\theta}$ components, the surface force at the inner boundary has only $\hat{r}$ component because there are no tangential field components at the inner surface. The role of these forces will be illustrated in the later part of the Rapid Communication.

We take a linearly transformed spherical cloak [7] where $f(r)=\frac{R_2}{R_2-R_1}(r-R_1)$ as an example. The constitutive parameters of the linearly transformed spherical cloak are $\epsilon_t=\epsilon_0\frac{R_2}{R_2-R_1}$, $\epsilon_r=\epsilon_0\frac{R_2}{R_2-R_1}(\frac{r-R_1}{r})^2$, $\mu_t=\mu_0\frac{R_2}{R_2-R_1}$, and $\mu_r=\mu_0\frac{R_2}{R_2-R_1}(\frac{r-R_1}{r})^2$. Substituting the linear transformation function into Eqs. (4), (6), and (7), the force distribution inside the linearly transformed cloak therefore can be obtained. Assume the incident wave has a wavelength of $\lambda_0=0.1$ m and the cloak has a size of $R_1=\lambda_0$ and $R_2=2\lambda_0$, we can calculate the total electric field intensity and the force density inside the cloak. Figure 1 shows the results on the xz plane. We can see that the surface forces (green arrows) on the outer surface of the cloak point inward but not exactly toward the center, while the surface forces on the inner surface of the cloak point outward. We can also see that the bulk forces (black arrows) in the region of $R_1<r<R_2$ point outward but not exactly from the center. In order to view the force clearly, we

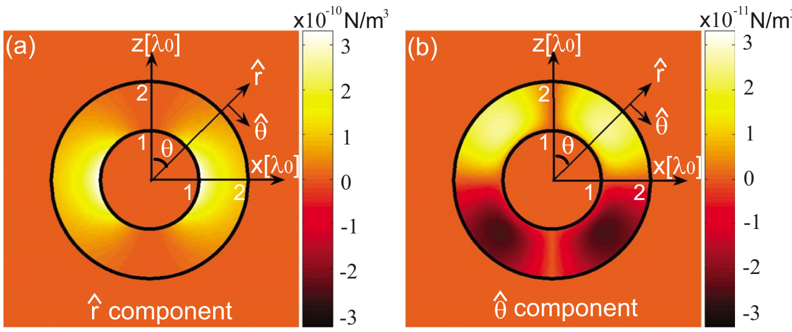

FIG. 2. (Color online) (a) $\hat{r}$ component of the bulk force and (b) $\hat{\theta}$ component of the bulk force on the *xz* plane.

plot the magnitude of each component of the bulk force in Fig. 2, where Fig. 2(a) shows the $\hat{r}$ component of the bulk force while Fig. 2(b) shows the $\hat{\theta}$ component of the bulk force on the *xz* plane. The $\hat{r}$ component of the bulk force is positive, creating tension that expands the cloak medium. The $\hat{\theta}$ component of the bulk force shown in Fig. 2(b) shows a symmetric pattern on the *xz* plane: in the region of $\theta>0$ it is positive, while in the region of $\theta<0$ it is negative. If we define the polar axis to be along the wave propagation direction, i.e., the *z* direction here, we can see that both the bulk force and surface force at the polar regions, which corresponds to $\theta=0$ and $\theta=\pi$, are exactly zero. This means that the polar regions (at $\theta=0$ and $\theta=\pi$) withstand no force while the lateral region (at $\theta=\pi/2$) withstands maximum stress.

It should be noted that all the bulk force and the surface force are rotationally symmetric along the *z* axis, as shown in Fig. 3. Therefore the total Lorentz force for an ideal cloak is exactly zero, which means there is no net momentum transfer from the electromagnetic wave to the cloak. However, the expanding force along the $\hat{r}$ direction and the shrinking force along the $-\hat{r}$ direction are not in balance. The total force on the half cloak located for $x>0$ is 1.82×10^{-13} N along $+x$ direction, while the total force on the other half cloak located for $x<0$ is the same but along $-x$ direction. The two equal forces pull the two halves of the cloak in opposite directions. Therefore, the effect is creating tension to expand the cloak but the total force on the whole cloak is zero.

The above analysis is helpful to understand the mechanical interactions of the electromagnetic wave with the cloaking material. The forces that are exerted by the electromagnetic fields upon matter change the momentum of the material object and/or create stresses within. Likewise, equal and recoil forces exist upon the electromagnetic fields. In this case, the momentum of the cloak is unchanged, while the induced material stresses shown in Figs. 1 and 2 exert equal and recoil stresses upon the wave fields. We show in Fig. 4 how the rays feel the force when propagating through the cloak. When ray 1 is incident onto the cloak at the position of P1, it feels a force with $+\hat{r}$ and $-\hat{\theta}$ components. The refracted ray therefore deviates from its original path and bend to left. If we plot the tangent of the surface illustrated as the red dashed line, we see that the angle of refraction is larger than the angle of incidence. When the ray propagates further into the cloak, for example, at position of P2, the ray is subject to a force with $-\hat{r}$ and $+\hat{\theta}$ components. We can imagine the ray enters from an outer layer to an inner layer with the tangent of the interface illustrated as a red dashed line because the cloak material are inhomogeneous along the $\hat{r}$ direction. Therefore the force upon the ray bends the refracted ray to the right, i.e., the angle of refraction is smaller than the angle of incidence at the interface of P2. At P3, the ray enters the interface from an inner layer to an outer layer and feels a force with $-\hat{r}$ and $-\hat{\theta}$ components, which force the ray to bend to the right. At P4, the ray feels a force with $+\hat{r}$ and $+\hat{\theta}$ components, which leads the ray back to its origi-

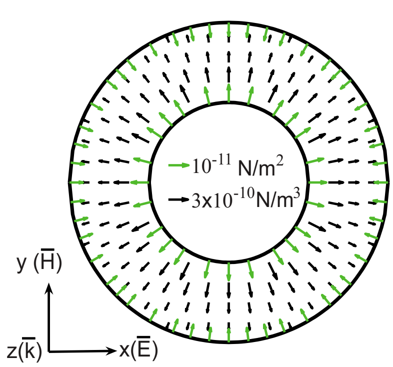

FIG. 3. (Color online) The bulk force density (unit: N/m^3) represented by black arrows and the surface force density (unit: N/m^2) represented by green arrows on the $z=0$ plane.

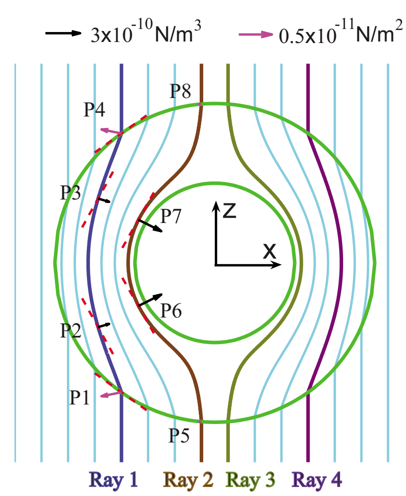

FIG. 4. (Color online) The rays are subject to equal and recoil forces when propagating through the cloak. The bulk force density at P2, P3, P6, and P7 are represented by black arrows and the surface force density at P1, P4, P5, and P8 are represented by magenta arrows. The red dashed line represent the tangent of the interface.

nal propagating direction. For a ray close to the center axis, e.g., ray 2, we can see that it is almost subject to no force when entering into the cloak, i.e., at P5. That is the reason why its path does not deviate much in the vicinity of P5. However, when it propagates around to the inner surface of the cloak, it feels an attractive force roughly pointing to the center. Therefore, the $-\hat{r}$ force plays the role of "centripetal force" under which the photons are able to do the circulating motion. Because only $\hat{r}$ component of force exists at the inner boundary of the cloak, the more the incident ray is close to the center axis, the more the ray follows a circle trajectory in the region close to the inner surface. The trajectory of other rays, e.g., rays 3 and 4, can be understood in a similar way. Our Rapid Communication therefore provides a different viewpoint to confirm the wave guidance of the cloak that was proposed in Ref. [7].

We can also see that the cloak is stressed outward at the $z=0$ plane from Fig. 3. Because the force density upon the material is approximately outward from the center, the electromagnetic wave in this vicinity is subject to stress that tends to pull the wave toward the center of the cloak. As the wave propagates around the center of the cloaked region, the expansion of the cloak tends to pull photons toward the center $(x=0,\ y=0)$ axis before exiting the cloaking material on the unilluminated side. Because we have derived the force expressions from the Chu formulation (EH representation), we believe the energy/momentum subsystem separation to be made based inherently upon what is field and what is matter [30]. In this view, the electromagnetic wave momentum decreases as it enters the cloaked region on the illuminated side, resulting in a force with a $+\hat{z}$ component. Likewise, the photons gain momentum upon exiting the unilluminated side, restoring the original momentum and exerting a $-\hat{z}$ component of force upon the surface of the cloak. Furthermore, for a spherical cloak created with a general transformation function, the trajectory of the ray inside of the spherical cloak will be different but still can be explained as a result of the opposite force that the cloak exerts upon the ray.

In conclusion, we reveal the mechanical behavior of the spherical cloak impinged on by a monochromatic plane wave. Our results show that the electromagnetic force exhibits a symmetric pattern in a cross section perpendicular to the wave propagation direction. These forces do not exist before transformation but stem from the squeezed electromagnetic space and are one of its appearance in the physical space. The trajectory of the ray can be understood as a result of the recoil Lorentz force it feels when propagating through the cloak. The total radiation pressure on an ideal cloak is shown to be exactly zero. Our Rapid Communication also provides some potential applications of the transformation based cloak in the astronomic area, such as protecting the satellite from orbital perturbations due to the solar radiation pressure.

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