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Identification of Multichannel Cardiovascular Dynamics Using Dual Laguerre Basis Functions for Noninvasive Cardiovascular Monitoring

Jin-Oh Hahn, Devin B. McCombie, Andrew T. Reisner, Horacio M. Hojman, and H. Harry Asada

Abstract—This paper presents a novel method to identify the cardiovascular (CV) system using two distinct peripheral blood pressure (BP) signals. The method can characterize the distinct arterial path dynamics that shape each of the BP signals and recover the common central-flow signal fed to them. A Laguerre series data-compression technique is used to obtain a compact representation of the CV system, whose coefficients are identified using the multichannel blind system identification. A Laguerre model deconvolution algorithm is developed to stably recover the central-flow signal. Persistent excitation, model identifiability, and asymptotic variance are analyzed to quantify the method's validity and reliability, without using any direct measurement of central-flow input signal. Experimental results based on 7000 data segments obtained from nine swine subjects show that, for all the swine subjects under diverse physiologic conditions, the CV dynamics can be identified very reliably and the waveform of the central flow can be recovered stably from peripheral BP signals.

Index Terms—Cardiovascular (CV) system, Laguerre basis function, noninvasive CV monitoring, two-channel blind system identification (ID).

I. INTRODUCTION

CENTRAL cardiovascular (CV) signals, such as aortic blood pressure (BP) and blood flow (BF), are generally more informative about cardiac dynamics and global circulation in comparison with peripheral CV signals. Measuring central signals directly, however, requires invasive procedures, which are relatively costly, uncomfortable, and risky. Therefore, there is substantial interest in alternative methods to assess the CV state using peripheral circulatory measurements, e.g., arterial BP measured in a distal extremity [1]–[6]. Many of these methods involve a population-based model, employing nominal parameter values derived from prior experimentation. In many cases, this approach yields useful estimates. In theory, however, it will be less valid for extreme physiologic conditions, during which the average relationships between the aortic and peripheral signals are less applicable [5], [6]. It would be ideal if a CV monitoring technique were independent of *a priori* knowledge of the CV dynamics.

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Efforts have been made to use multiple peripheral measurements to reduce the reliance on *a priori* knowledge and other population-based assumptions regarding the relationship between central and peripheral CV dynamics [7]–[13]. Some of the efforts involved measurements relatively close to the central aorta, e.g., the carotid [7], [8], and used it as a surrogate for a central signal. In practice, were a method required two arterial BP measurements to be ever considered practical, it would presumably involve a pair of extremity measurements rather than the carotid. Another approach used the multichannel “blind” system identification (ID) method [19], [20], which was shown capable of reproducing the central BF [9], [10] and BP [11], [12] signals as well as estimating other key CV physiologic measures [13], [14] from a couple of peripheral BP measurements. However, using these methods, which are based upon finite impulse response (FIR) [9], [12] and generalized FIR models [10], [13], [14], it is difficult to systematically determine the order of each channels' dynamics and other key parameters, due to the lack of available analytic measures and guidelines to select appropriate model structures.

Based upon the prior work by the authors' group [9]–[11], [13], [14], this paper presents a significantly improved algorithm for identifying multichannel CV dynamics and recovering the central-flow signal, and an accompanying theoretical analysis that provides analytic measures for model structure determination. Specifically, a data-compression technique using dual Laguerre basis functions will be developed, which assures identifiability and reduces estimation error by enabling a compact representation of CV dynamics. A novel deconvolution algorithm will be developed to recover an unknown central-flow signal from peripheral BP measurements. The determination of model order and other key parameters as well as data length will be based on a rigorous evaluation of persistent excitation (PE), model identifiability (MI), and asymptotic variance, without the use of any direct measurement of the central-flow input signal. The algorithm will be evaluated for 7000 data segments from nine swine subjects across diverse physiologic conditions. This analysis will demonstrate that the new algorithm is significantly more reliable and valid than the prior algorithm that required identical Laguerre basis poles for all channels [10]. A similar analysis might be constructively applied to a range of contemporary research involving dynamic physiologic models, e.g., [9], [11], and [12].

II. ALGORITHM AND THEORETICAL DEVELOPMENTS

A. Blind System ID Using Dual Laguerre Basis Functions

Consider two distinct arterial paths connected to the same central-flow input, as shown in Fig. 1. In this paper, the CV dynamics is considered to be an unknown and time-varying process. However, within a short time window, it is assumed that the system can be approximated as a time-invariant linear



Fig. 1. CV system as a two-channel dynamic system.

system. It is known that, on a beat-to-beat basis, the CV dynamics is time invariant and that the nonlinearity is negligibly small [15].

Let $G_1(z)$ and $G_2(z)$ be stable transfer functions from the common input $u(n)$ to outputs $y_1(n)$ and $y_2(n)$ observed at the ends of the first and the second arterial paths, respectively. The impulse response of each transfer function can be approximated by a finite series expansion

$$G_i(z) = \sum_{k=1}^{L_i} b_k^{(i)} V_k^{(i)}(z), \quad i = 1, 2 \quad (1)$$

where $\{V_k^{(i)}(z)\}_{k=1,2,\dots}$ is a set of orthonormal basis functions and $\{b_k^{(i)}\}_{k=1,2,\dots}$ are the associated series coefficients of the i th arterial path or channel. Given a series description of the CV system, our objective is to identify both the arterial path dynamics and the central-flow input fed to them using only the peripheral BP signals $y_1(n)$ and $y_2(n)$.

Remark 1: This formulation of the blind system ID has distinct features. If a rational function is used for the system description instead of the series expansion, at least three output BP signals are needed to identify both the denominator and numerator of the rational transfer functions [9], [19], [20]. In contrast, this series expansion method needs only two BP signals.

Since both arterial paths are driven by the same input, the following formulation between the arterial paths holds by virtue of the associative law for linear systems:

$$G_2(z)y_1(n) = G_1(z)y_2(n) \quad (2)$$

which does not include the input signal. Therefore, the arterial paths can be identified by substituting a measured time series of the peripheral BP signals for $y_1(n)$ and $y_2(n)$ into (2). This system ID technique is called the blind system ID because it does not involve the use of the input signal for system ID. It can be shown that the blind system ID problem (2) can be solved to identify $G_1(z)$ and $G_2(z)$ if $G_i(p) \triangleq \sum_{k=1}^{L_i} b_k^{(i)} p^k$, $i = 1, 2$, are coprime [9], [19], [20]. Substituting (1) into (2) yields

$$\sum_{k=1}^{L_2} b_k^{(2)} x_k^{(1)}(n) = \sum_{k=1}^{L_1} b_k^{(1)} x_k^{(2)}(n) \quad (3)$$

where $x_k^{(1)}(n) = V_k^{(2)}(z)y_1(n)$ and $x_k^{(2)}(n) = V_k^{(1)}(z)y_2(n)$, which, given time series sequences of the peripheral BP signals, can be formulated into a homogeneous matrix equation that can be readily solved for $\mathbf{b} \triangleq [\mathbf{b}_1 \ \mathbf{b}_2]$, where $\mathbf{b}_i \triangleq \{b_k^{(i)}\}_{k=1,2,\dots}$.

It is ideal that the number of series coefficients to reproduce the true impulse response be small, because a large number of series coefficients are difficult to identify unless the system input

signal is rich enough [16]. For example, the standard FIR representation with $V_k^{(i)} = z^{-k}$ requires a large number of series coefficients, particularly when the system has a slowly decaying pole. Given that we have no control over the input signal (i.e., the BF from the heart) for identifying the CV system, it is crucial to reduce the number of series coefficients. This can be achieved with an appropriate set of basis functions that can represent the CV channel dynamics with fewer terms.

A unique feature of the arterial path dynamics is that it consists of a fast decaying dynamics that is branch specific and a common slowly decaying dynamics, often referred to as the ‘‘Windkessel’’ response [10] which is often approximated by a first-order exponential decay. The Laguerre series expansion is effective for compressing the dynamic representation of systems having a dominant slow pole [17], [18]. Its basis function is given by

$$V_k(z) = \frac{\sqrt{(1-a^2)T_s}}{z-a} \left(\frac{1-az}{z-a}\right)^{k-1}, \quad k = 1, 2, \dots \quad (4)$$

where a is called a Laguerre basis pole, which is to be set close to the slowly decaying pole of the original system ($0 < a < 1$), and T_s is the sampling interval. Using the basis poles a_1 for $G_1(z)$ and a_2 for $G_2(z)$, $x_k^{(1)}(n)$ and $x_k^{(2)}(n)$ (3) becomes

$$\begin{aligned} x_k^{(1)}(n) &= \frac{\sqrt{(1-a_2^2)T_s}}{z-a_2} \left(\frac{1-a_2z}{z-a_2}\right)^{k-1} y_1(n) \\ x_k^{(2)}(n) &= \frac{\sqrt{(1-a_1^2)T_s}}{z-a_1} \left(\frac{1-a_1z}{z-a_1}\right)^{k-1} y_2(n). \end{aligned} \quad (5)$$

Remark 2: As we will demonstrate, the number of terms needed for approximating the original system depends on the choice of the Laguerre basis pole. An earlier attempt by the author’s group required that the same Laguerre basis pole must be used for both path dynamics [10], [13]. Although the use of the Laguerre basis functions was motivated by the slowly decaying common dynamics of the CV system, the estimated values of the basis poles for different arterial paths are, in general, different from each other, since the basis poles are determined in such a way that they can reproduce the gross behavior of the associated arterial paths, including the effects of both common Windkessel and branch-specific dynamics. Based on this physical intuition, the obligatory use of the same Laguerre basis pole is removed in this paper, which will allow us to further reduce the number of terms for both paths’ dynamics, which will be demonstrated using the experimental results in Section IV.

Remark 3: Considering that the denominator dynamics of the CV system is estimated using its zero-input response in diastole which is not rich in information [9], [10], it is desirable that the number of parameters involved in the denominator dynamics be small. Given this limitation, one important advantage of the Laguerre series expansion over other possible model structures for CV dynamics is that it involves only one parameter, i.e., the Laguerre basis pole in the denominator.

Remark 4: It is important to note that the blind system ID can provide the directionality of the vector \mathbf{b} , but not its actual scale. This is an inherent problem of the blind system ID caused by the unknown input signal to the system, i.e., for any $\alpha \neq 0$

$$y_i(n) = G_i(z)u(n) = [\alpha G_i(z)] \left[\frac{1}{\alpha} u(n) \right]. \quad (6)$$

Given this formulation alone, it is, in general, impossible to scale the input and the channel transfer functions from the peripheral BP signals.

B. Laguerre Model Deconvolution: Central-Flow Recovery

Once the arterial path dynamics are identified, the central-flow input may be recovered by inverting either of the identified dynamics. However, this inversion often ends up with instability due to the nonminimum-phase zeros in the arterial path transfer functions (which are often caused by the time delay associated with the central-to-peripheral transfer of BP and BF waves). An earlier attempt by the author's group [10] developed a Laguerre deconvolution filter to stably recover the input signal, but its application was limited to the situation where the Laguerre basis poles of the two distinct arterial paths are identical. It was a significant limitation since the Laguerre basis poles for different arterial paths are typically different. In this paper, we develop a Laguerre deconvolution filter that is applicable to multichannel Laguerre series systems having distinct channel basis poles by exploiting the notion of coprime transfer functions [21].

Lemma (The Generalized Bezout Identity): Two stable real rational transfer functions $M_1(z)$ and $M_2(z)$ are coprime if they have no common zeros outside the unit circle and at least one of them is of relative degree zero [21]. For coprime $M_1(z)$ and $M_2(z)$, there exist stable real rational transfer functions $W_1(z)$ and $W_2(z)$ satisfying $M_1(z)W_1(z) + M_2(z)W_2(z) = 1$. \square

Deconvolution Algorithm: The unknown input sequence $u(n)$ can be recovered from observed output sequences $y_1(n)$ and $y_2(n)$ by using the coefficients \mathbf{b} as well as the basis poles a_1 and a_2 of the Laguerre basis functions

$$u(n) = \frac{N_1(z)}{(z - a_2)^{L_2}} (z - a_1) y_1(n) + \frac{N_2(z)}{(z - a_1)^{L_1-1}} y_2(n) \quad (7)$$

where $N_1(z)$ and $N_2(z)$ are polynomials of orders L_2 and $L_1 - 1$, respectively, that satisfy the following:

$$A_1(z)N_1(z) + A_2(z)N_2(z) = (z - a_1)^{L_1-1} (z - a_2)^{L_2} \quad (8)$$

where $N_2(0) = 1$ and

$$A_i(z) = \sum_{k=1}^{L_i} b_k^{(i)} \sqrt{(1 - a_i^2) T_s} (1 - a_i z)^{k-1} \cdot (z - a_i)^{L_i-k}, \quad i = 1, 2 \quad (9)$$

if $A_1(z)$ and $A_2(z)$ are coprime¹ and

$$\sum_{k=1}^{L_1} b_k^{(1)} (-a_1)^{L_1-k} \neq 0 \quad \sum_{k=1}^{L_1} b_k^{(1)} (-a_1)^{k-1} \neq 0. \quad (10)$$

Proof: From (1) and (4), $F_1(z) \triangleq (z - a_1)G_1(z)$ is a transfer function with relative degree zero, and the transfer functions $F_1(z)$ and $G_2(z)$ are coprime. Thus, from the aforementioned lemma, there exist stable transfer functions $B_1(z)$ and

$B_2(z)$ where

$$F_1(z)B_1(z) + G_2(z)B_2(z) = 1. \quad (11)$$

Construct $B_1(z)$ and $B_2(z)$ as follows:

$$B_1(z) = \frac{N_1(z)}{(z - a_2)^{L_2}} \quad B_2(z) = \frac{N_2(z)}{(z - a_1)^{L_1-1}}. \quad (12)$$

Then, (11) reduces to the polynomial identity (8). Multiplying $u(n)$ to both sides of (11) and using $y_1(n) = G_1(z)u(n)$ and $y_2(n) = G_2(z)u(n)$ yield (7).

Since $N_2(0) = 1$ has been given, $N_1(z)$ and $N_2(z)$ together give $(L_2 + 1) + (L_1 - 1) = L_1 + L_2$ unknown coefficients. Furthermore, since both sides of (8) are polynomials of order $L_1 + L_2 - 1$, (8) yields $L_1 + L_2$ linear equations with $L_1 + L_2$ unknowns, $\mathbf{A}\boldsymbol{\eta} = \boldsymbol{\gamma}$ shown in Fig. 2, where

$$\begin{aligned} A_1(z) &= \sum_{j=1}^{L_1} b_j^{(1)} (1 - a_1 z)^{j-1} (z - a_1)^{L_1-j} \\ &\equiv \sum_{j=0}^{L_1-1} \alpha_j^{(1)} z^j \\ &= \alpha_0^{(1)} + \alpha_1^{(1)} z + \cdots + \alpha_{L_1-1}^{(1)} z^{L_1-1} \\ A_2(z) &= \sum_{j=1}^{L_2} b_j^{(2)} (1 - a_2 z)^{j-1} (z - a_2)^{L_2-j} \\ &\equiv \sum_{j=0}^{L_2-1} \alpha_j^{(2)} z^j \\ &= \alpha_0^{(2)} + \alpha_1^{(2)} z + \cdots + \alpha_{L_2-1}^{(2)} z^{L_2-1} \end{aligned} \quad (13)$$

and

$$(z - a_1)^{L_1-1} (z - a_2)^{L_2} \equiv \sum_{m=0}^{L_1+L_2-1} \gamma_m z^m + \left[\sum_{j=1}^{L_2-1} \alpha_j^{(2)} z^j \right].$$

The matrix \mathbf{A} should be nonsingular for the existence of $\boldsymbol{\eta}$, the coefficients of the deconvolution filter. Fig. 2 shows that \mathbf{A} should satisfy two conditions. First, it is obvious from the top and bottom rows of \mathbf{A} that both $\alpha_0^{(1)}$ and $\alpha_{L_1-1}^{(1)}$ should be nonzero, which, together with (9) and (13), yields (10). If $\alpha_0^{(1)}$ and $\alpha_{L_1-1}^{(1)}$ are nonzero, the first and the $(L_2 + 1)$ th column vectors are linearly independent of the remaining $L_1 + L_2 - 2$ column vectors in \mathbf{A} . Thus, \mathbf{A} is nonsingular if these remaining column vectors are linearly independent. Noting that the elimination of the first and $(L_2 + 1)$ th column vectors as well as the top and bottom rows from \mathbf{A} results in the Sylvester matrix associated with $A_1(z)$ and $A_2(z)$, $A_1(z)$ and $A_2(z)$ need be coprime (otherwise, the Sylvester matrix becomes rank deficient). \square

Fig. 3 shows the block diagram of the aforementioned deconvolution algorithm to recover the unknown input signal.

C. Validity and Quality Assurance

1) *PE and MI:* Solving (3) for $\hat{\mathbf{b}}$ hinges upon the "richness" of the observed outputs $y_1(n)$ and $y_2(n)$. If the dimension of $\hat{\mathbf{b}}$, i.e., the order of the two Laguerre series models $L = L_1 + L_2$,

¹The coprime condition can be checked by the following method. Let $\lambda_j^{(i)}$, $i = 1, 2$ and $j = 1, 2, \dots, L_i - 1$, be the roots of the polynomial equation $\sum_{k=1}^{L_i} b_k^{(i)} [\lambda^{(i)}]^{k-1} = 0$. Compute $z_j^{(i)} = 1 + a_i \lambda_j^{(i)} / \lambda_j^{(i)} + a_i$ and form $\mathbf{z}^{(i)} = \left\{ z_j^{(i)} \mid j = 1, 2, \dots, L_i - 1 \right\}$ for $i = 1, 2$. Then, $A_1(z)$ and $A_2(z)$ are coprime if $\mathbf{z}^{(1)} \cap \mathbf{z}^{(2)} = \emptyset$.

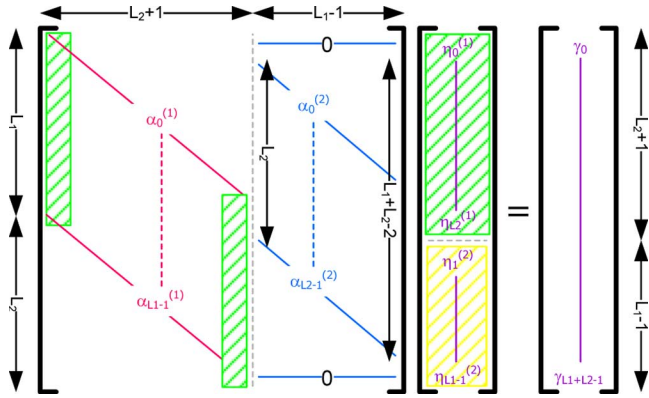


Fig. 2. Matrix equation for the design of the deconvolution filter.

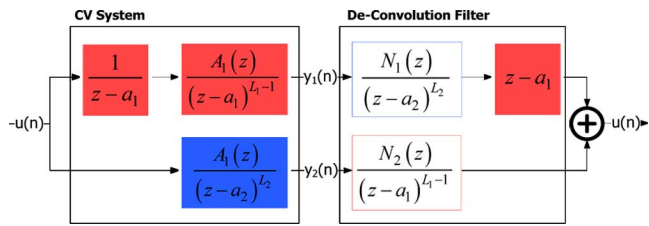


Fig. 3. Laguerre deconvolution filter.

is too large, (3) cannot be solved. Care must be taken in determining the model order, given that the output signals are passively observed and may contain limited richness. This critical issue can be addressed by examining PE and MI conditions [16].

Since the blind system ID equation is homogeneous, the scale of solution vector $\hat{\mathbf{b}}$ is undetermined. Assuming $b_1^{(2)} \neq 0$, let us scale the Laguerre series coefficients as $\bar{b}_k^{(i)} = b_k^{(i)}/b_1^{(2)}$ and rewrite (3) as

$$\begin{aligned} x_1^{(1)}(n) &= -\sum_{k=2}^{L_2} \bar{b}_k^{(2)} x_k^{(1)}(n) + \sum_{k=1}^{L_1} \bar{b}_k^{(1)} x_k^{(2)}(n) \\ &= \boldsymbol{\theta}^T \boldsymbol{\varphi}(n) \end{aligned} \quad (14)$$

where $\boldsymbol{\theta} = [\bar{b}_2^{(2)} \dots \bar{b}_{L_2}^{(2)} \bar{b}_1^{(1)} \dots \bar{b}_{L_1}^{(1)}]^T$ and $\boldsymbol{\varphi}(n)$ is specified accordingly. Therefore, $\boldsymbol{\theta}$ can be uniquely determined if $\boldsymbol{\varphi}(n)$ is persistently exciting. Once the PE condition is met, the MI issue becomes trivial. The model structure (14), which can be viewed as a moving-average model, is always identifiable [16].

Remark 5: The usefulness of the aforementioned PE condition is that it can be evaluated using a time series of the *peripheral* BP signals alone. Unlike the PE conditions for blind system ID in the previous literature (e.g., [19] and [20]), it does not include the input, i.e., the central flow, which is inaccessible.

Remark 6: When an FIR model is used for the CV dynamics, a large number of coefficients must be identified from the observed output signals [9]. This increases the dimension of the regressor vector $\boldsymbol{\varphi}(n)$ and makes the PE condition difficult to satisfy. Furthermore, as we will demonstrate, using different Laguerre basis poles a_1 and a_2 for the two distinct channels allows

us to further reduce the dimensionality [14], compared to the single basis pole model [10], [13].

2) *Asymptotic Variance:* Variance analysis is useful to assess the level of reliability in estimating parameters of unknown processes. The expected variance can be predicted by the asymptotic variance, using the model structure and the empirical measurements [16].

In the blind system ID, the model parameters are determined by minimizing the error associated with the equality (3)

$$\varepsilon(n, \hat{\mathbf{b}}) = \sum_{k=1}^{L_2} \hat{b}_k^{(2)} x_k^{(1)}(n) - \sum_{k=1}^{L_1} \hat{b}_k^{(1)} x_k^{(2)}(n) \quad (15)$$

where $\varepsilon(n, \hat{\mathbf{b}})$ is referred to as the output estimation error. Its sensitivity to the Laguerre series coefficients are given by

$$\boldsymbol{\psi}(n) = \frac{d\varepsilon(n, \mathbf{b})}{d\mathbf{b}} = [\dots x_k^{(1)}(n) \dots \dots - x_k^{(2)}(n) \dots]^T. \quad (16)$$

Following [16], it can be shown that, for a time series of the peripheral BP signals, the variance of estimating Laguerre series coefficients $\hat{\mathbf{b}}$ converges asymptotically to

$$\begin{aligned} \text{var}(\hat{\mathbf{b}} - \mathbf{b}_0) &\cong \frac{1}{N} \lambda_N(\mathbf{b}_0) \mathbf{S}_N \\ &\triangleq \frac{1}{N} \left[\frac{1}{N} \sum_{n=1}^N \varepsilon^2(n, \mathbf{b}_0) \right] \\ &\quad \times \left[\frac{1}{N} \sum_{n=1}^N \boldsymbol{\psi}(n) \boldsymbol{\psi}^T(n) \right]^{-1} \end{aligned} \quad (17)$$

where λ_N is the variance of $\varepsilon(n)$ and \mathbf{S}_N is the inverse of the sensitivity covariance matrix. This variance is not computable since it is evaluated at \mathbf{b}_0 , which is unknown. However, the following approximation using the estimate $\hat{\mathbf{b}}$ suffices in many applications

$$\text{var}(\hat{\mathbf{b}} - \mathbf{b}_0) \cong \frac{1}{N} \hat{\lambda}_N(\hat{\mathbf{b}}) \cdot \mathbf{S}_N. \quad (18)$$

From this asymptotic variance, we can obtain useful insights as to how confidently the CV system is identified.

- 1) As the number of available data N increases, the asymptotic variance decreases. This gives a guideline for selecting N to reduce the variance to a desired level.
- 2) If one or more of the Laguerre series coefficients have very low sensitivity upon the output estimation error (15), the inverse of the sensitivity covariance matrix \mathbf{S}_N in (18) blows up or it becomes extremely large as the sensitivity diminishes. This often happens when the model contains too many parameters to estimate. Low-sensitivity parameters should be eliminated from the model.
- 3) If the empirical output estimation error variance $\hat{\lambda}_N$ in (18) is large, the measurements are noisy, and/or the model does not fit the measured peripheral BP signals. The model structure must be reconsidered, and/or the quality of the BP signals must be improved.

Therefore, the asymptotic variance analysis offers a warning sign and quality measures for the validity of the CV system ID.

TABLE I
PHYSIOLOGIC SPAN OF THE PIGS: MEAN AND 95% STANDARD DEVIATION

Pigs	MAP [mmHg]	HR [bpm]	CO [lpm]	TPR [mmHg/lpm]
1	61.2+/-47.6	109+/-71.2	3.5+/-2.6	18.9+/-10.4
2	65.6+/-35.2	140+/-76.2	3.5+/-2.9	21.2+/-16.6
3	68.2+/-28.9	175+/-27.9	4.4+/-2.5	16.0+/-4.3
4	59.8+/-36.5	123+/-43.5	3.3+/-2.1	19.0+/-11.6
5	86.1+/-42.8	127+/-60.5	3.2+/-1.1	27.2+/-14.0
6	84.5+/-31.1	124+/-51.7	3.8+/-2.0	23.6+/-13.4
7	84.7+/-38.3	125+/-55.9	2.9+/-1.5	30.6+/-15.2
8	79.5+/-39.5	138+/-74.8	3.7+/-1.6	22.4+/-14.7
9	69.8+/-41.8	127+/-47.0	3.4+/-3.1	25.8+/-29.1
	73.8+/-44.0	129+/-65.6	3.4+/-2.3	23.3+/-17.7

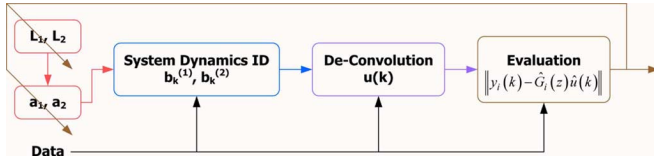


Fig. 4. Laguerre series blind CV system ID and deconvolution procedure.

III. METHODS

A. Experimental Protocol

Under the experimental protocol #01-055 approved by the Massachusetts Institute of Technology Committee of Animal Care, invasive CV data were obtained from nine anesthetized swine subjects at a sampling rate of 250 Hz. The BP signals were measured in the swine subjects simultaneously from indwelling catheters in the radial and femoral arteries, coupled via rigid fluid-filled tubing to external pressure transducers. In addition, the aortic flow signal was measured using a flow meter placed around the ascending aorta. The physiologic conditions of the swine subjects were widely altered in order to verify the proposed algorithm for a broad range of physiologic conditions. During the experiment, a combination of intravenous medication infusions, including epinephrine, nitroglycerine, dobutamine, neosynephrine, and esmolol, was made. The physiologic spans of the swine subjects are summarized in Table I. Prior to their application to the blind system ID and the input deconvolution, the BP signals were prefiltered using a FIR digital filter with 30-Hz cutoff frequency and downsampled to 125 Hz.

B. CV System ID and Central-Flow Recovery

From the experimental swine data, we obtained totally 7000 segments of data, each having approximately 2000 samples of peripheral BP and aortic flow signals. The blind system ID and the input deconvolution algorithms were applied to these data, following the procedure shown in Fig. 4. First, initial estimates of the order of the Laguerre series and the basis poles of the two arterial paths were provided², based on which the peripheral BP signals were Laguerre filtered to generate the signals $x_k^{(i)}(n)$ in (3). Then, (3) was solved for $\hat{\mathbf{b}}$, which was used to evaluate the

²If the basis poles a_1 and a_2 are unknown, the Laguerre series becomes an ARMA model, which requires at least three distinct measurements for ID [9]. In order to avoid this difficulty, the solution procedure first assumes the Laguerre basis pole values and solves the homogeneous equation (3).

polynomials $A_1(z)$ and $A_2(z)$ in (9) and solve (8) to design the Laguerre deconvolution filter (7). The central-flow signal was recovered by processing the peripheral BP signals using this deconvolution filter. The performance of the CV system ID was quantified by comparing the observed outputs to their estimated counterparts

$$J = \sum_{i=1}^2 \frac{\sum_{n=1}^N \left(y_i(n) - \hat{G}_i(z, a_i, \hat{\mathbf{b}}) \hat{u}(n, a_i, \hat{\mathbf{b}}) \right)^2}{\sum_{n=1}^N \left(y_i(n) \right)^2}. \quad (19)$$

The procedure described previously was iterated until the metric (19) was optimized.

For each data segment, the richness of the data was evaluated by examining the PE condition. Furthermore, to assess the level of reliability on the identified CV dynamics, the asymptotic variance of the identified Laguerre series coefficients $\hat{\mathbf{b}}$ was evaluated by (18). This whole procedure was repeated for different values of model orders L_1 and L_2 as well as dual basis poles a_1 and a_2 . In particular, the proposed method using dual Laguerre basis functions $a_1 \neq a_2$ was compared to the single basis pole method [10], [13] $a_1 = a_2$ in terms of both accuracy and reliability. To objectively compare the asymptotic variance of the Laguerre series coefficients $\check{\mathbf{b}}_s$ (single pole) and $\check{\mathbf{b}}_d$ (dual pole) identified with and without the constraint $a_1 = a_2$ in the absence of any reference scale, $\check{\mathbf{b}}_s$ and $\check{\mathbf{b}}_d$ were scaled in such a way that the resulting dc gains of the arterial path dynamics become identical, i.e., $\check{\mathbf{b}}_s$ and $\check{\mathbf{b}}_d$ were appropriately scaled to $\check{\mathbf{b}}_s$ and $\check{\mathbf{b}}_d$ in order to meet the following equality:

$$\sqrt{\frac{1+a_i}{1-a_i}} \sum_{k=1}^{L_i} \check{b}_{s,k}^{(i)} \Big|_{a_1=a_2} = \sqrt{\frac{1+a_i}{1-a_i}} \sum_{k=1}^{L_i} \check{b}_{d,k}^{(i)} \Big|_{a_1 \neq a_2} \quad (20)$$

where $\check{b}_{s,k}^{(i)}$ and $\check{b}_{d,k}^{(i)}$ are the elements of $\check{\mathbf{b}}_s$ and $\check{\mathbf{b}}_d$. These $\check{\mathbf{b}}_s$ and $\check{\mathbf{b}}_d$ associated with the 7000 segments of peripheral BP signals were applied to (18) for asymptotic variance analysis.

IV. RESULTS AND DISCUSSION

The examination of the PE condition for the 7000 segments of peripheral BP data showed that the PE condition was met for most data segments when the total number of model order $L = L_1 + L_2$ was less than 12. For $L = 6$, $L_1 = L_2 = 3$, the PE condition was satisfied for all the 7000 data segments. This reveals a few important points.

- 1) The observed peripheral BP data contained information rich enough to identify no more than 12 parameters.
- 2) Occasionally, some data segments were found to be rich enough to satisfy the PE condition for high model orders. However, to assure that the CV dynamics is identifiable for all the swine subjects under diverse physiologic conditions, the total model order should be kept on a conservative side, i.e., six or lower, which necessitates an effective representation of the CV system.

Fig. 5 shows a typical result of the blind CV system ID and input deconvolution. The model order was kept low: $L = 6$, $L_1 = L_2 = 3$. The two left plots are the peripheral BP signals used for ID, and the measured aortic versus recovered central-flow signals are shown on the right. Note that, although the

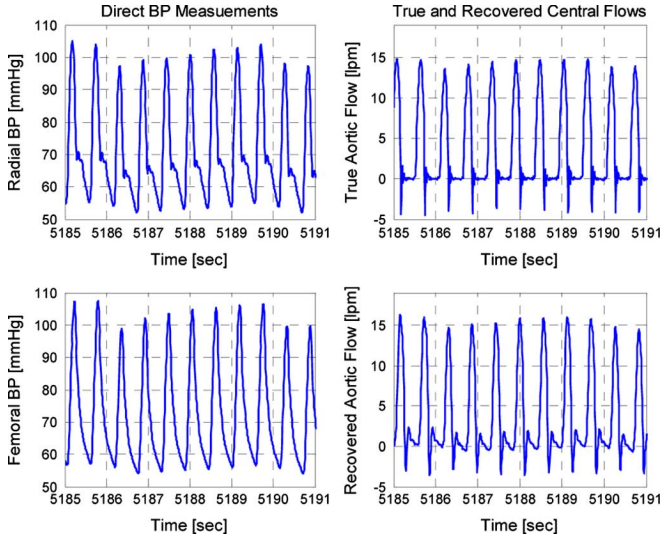


Fig. 5. Peripheral BP and true versus recovered aortic flow signals. Model order: $L_1 = 3$, $L_2 = 3$. Laguerre basis poles: $a_1 = 0.79$, $a_2 = 0.94$. Normalized squared error: $J = 0.0982$.

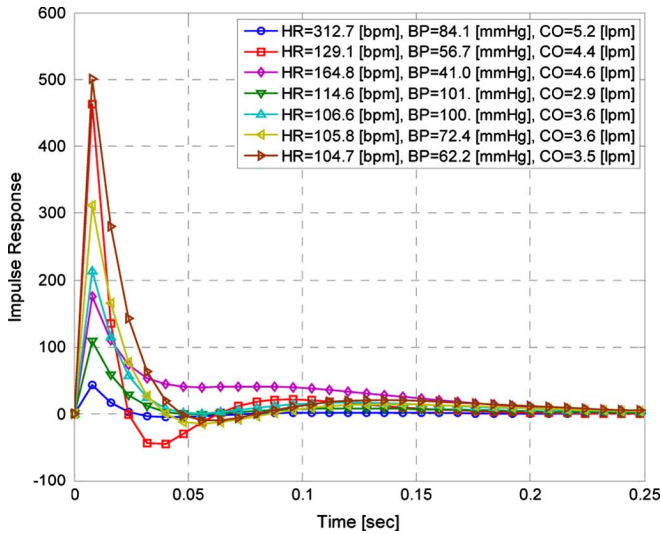


Fig. 6. Impulse responses of identified aortic-to-radial transfer functions under diverse physiologic conditions.

model order was kept low, the recovered waveform of the central flow accurately reproduced the true aortic flow.

The identified transfer functions differ significantly depending on the physiologic conditions of each swine subject. Fig. 6 shows how much variation we observed in the aortic-to-radial transfer functions even for a single swine subject. It is difficult to average these responses and replace all the diverse transfer functions by a single transfer function. Our algorithm allows for the ID of these diverse transfer functions.

The results of the asymptotic variance analysis give more insights into the accuracy and reliability of the system ID. First, Table II summarizes the asymptotic variance of the Laguerre series coefficients for the dual- and single-pole CV models, averaged over the 7000 segments of data, where the asymptotic variances for the data length of 400, 800, and 2000 show that they decrease approximately in proportion to the size of data points

TABLE II
ASYMPTOTIC VARIANCE OF LAGUERRE SERIES COEFFICIENTS ($L_1 = L_2 = 3$)

	$a_1 \neq a_2$			$a_1 = a_2$
	N=400	N=800	N=2,000	N=2,000
$\tilde{b}_1^{(1)}$	7.39 ²	5.11 ²	3.73 ²	6.17 ²
$\tilde{b}_2^{(1)}$	7.78 ²	5.65 ²	4.04 ²	5.81 ²
$\tilde{b}_3^{(1)}$	4.35 ²	3.13 ²	2.29 ²	3.64 ²
$\tilde{b}_1^{(2)}$	10.74 ²	5.16 ²	2.73 ²	6.27 ²
$\tilde{b}_2^{(2)}$	14.95 ²	5.97 ²	2.70 ²	7.32 ²
$\tilde{b}_3^{(2)}$	9.56 ²	4.04 ²	2.11 ²	5.00 ²

TABLE III
PERCENTAGE ASYMPTOTIC VARIANCE OF DC GAINS ($N = 800$)

	$a_1 \neq a_2$		$a_1 = a_2$	
	$L_1=L_2=3$	$L_1=L_2=5$	$L_1=L_2=3$	$L_1=L_2=5$
$\sqrt{\hat{\lambda}_N}$	57.3	31.7	113	93.3
$\ \mathbf{S}_N\ $	0.52	1.10	1.31	3.88
$\Delta G_1(1)/G_1(1)$	5.14%	15.2%	30.5%	188%
$\Delta G_2(1)/G_2(1)$	7.93%	28.9%	43.2%	424%

N , as predicted by (18). Hence, the confidence of the parameter estimation can be improved by increasing the data length.

Table II also compares the identified CV dynamics with dual Laguerre basis functions, $a_1 \neq a_2$, and that with a single basis pole $a_1 = a_2$, which suggests that the single-pole model does not work well for $L = 6$. The single-pole model needs more terms in the Laguerre series expansion, because constraining the Laguerre basis pole to the same value usually incurs more mismatches in the individual arterial path dynamics, resulting in a larger $\hat{\lambda}_N$ in (18). The mismatches may be reduced by using a higher model order. Unfortunately, however, this higher model order again incurs poor asymptotic variance by deteriorating the sensitivity to result in a larger \mathbf{S}_N in (18), as shown in Table III.

We also investigated how the asymptotic variance analysis can be used for the tradeoff between accuracy and reliability in CV system ID. In particular, two different model orders (six versus ten) and two different model structures (single Laguerre basis function versus dual Laguerre basis function) were considered for the 7000 segments of data. In these comparisons, $N = 800$ was used. The resulting output estimation error variance $\hat{\lambda}_N$, the magnitude of the inverse sensitivity matrix (measured by its trace norm) $\|\mathbf{S}_N\| \equiv \text{trace}(\mathbf{S}_N)$, and the resultant asymptotic variance given by (18) in identifying the individual CV path dynamics, averaged over the 7000 segments of data, are summarized in Table III. Specifically, it lists the percentage asymptotic variances of the dc gains of the first and second arterial paths normalized by its own dc gain as a representative of the asymptotic variance. Here, the dc gain of the CV dynamics $G_i(1)$ can be interpreted as the mean BP for a given mean aortic flow, and its variance includes the aggregate effects of all the variances in estimating the individual Laguerre series coefficients. First, the variance of output estimation error $\hat{\lambda}_N$ and the magnitude of the inverse sensitivity matrix $\|\mathbf{S}_N\|$ for estimating CV path dynamics were computed for different

model orders $L = 6$ and $L = 10$, respectively. As the model order increases, the output estimation error decreases. This is because more parameters can be used to fit the data as the model order increases. However, the overall asymptotic variance increases as the model order increases. The magnitude of $\|S_N\|$ increases significantly for $L = 10$, indicating that some of the Laguerre series coefficients have low sensitivities. As a result, the estimates of these series coefficients become unreliable. For $N = 800$, which corresponds to 6.4 s of data, the asymptotic variance of the dual-pole model with $L = 6$ decreases to just a few percent for both arterial paths. On the other hand, the variance is significantly larger for the single-pole model with $L = 10$, supporting again that allowing different Laguerre basis poles to different arterial paths can deliver tremendous benefit in improving the reliability of the blind system ID. In summary, the PE condition and asymptotic variance analysis provide useful insights and design guidelines as to data richness, data length, model order, and model efficiency, which allow us to guarantee the accuracy and reliability of identified CV dynamics.

V. CONCLUSION

Identifying the CV dynamics from limited peripheral BP observation is a challenging problem. In this paper, we presented several key algorithmic and experimental results. First, an effective data compression based on dual Laguerre basis functions was presented, in which the dynamics of a two-channel CV system was accurately and reliably identified from arterial BP measurements at two peripheral locations. This method allowed us to deliver compact and high-fidelity representations of the CV system. Second, the central-flow signal was recovered with our Laguerre deconvolution algorithm for stable inversion, without the use of predetermined transfer functions. Altogether, these algorithms opened up a new possibility for enhanced assessment of global CV dynamics (e.g., central and branch conditions as well as other key CV physiologic measures [13], [14]), using just peripheral BP signals measured via minimally invasive or noninvasive techniques. Third, this paper described analysis and design tools for evaluating the richness of the observed signals and quantifying the estimation error variance and parameter sensitivity, without reliance on any direct central input signal measurement. This methodology illustrated a systematic way of identifying CV dynamics based on quantitative reliability measures, which may be valuable in many physiologic applications of system ID.

In this paper, we applied these methods to 7000 data segments obtained from nine swine subjects, validating that the novel ID and deconvolution algorithm performs well under diverse physiologic conditions, given just several seconds of peripheral BP measurements. Future study of the utility of this methodology, with potentially wide-ranging clinical applications, is warranted.

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