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Modeling and Optimization for Transportation Systems Planning and Operations*

by

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ABSTRACT

In this paper, we focus on a number of applications of network optimization techniques to transportation systems analysis. In particular, network analysis problems, network design problems, and network management problems are discussed in some detail. The intent is to survey important application areas.

INTRODUCTION

Recent advances in techniques for handling large-scale network problems have found a prime area of application in the modeling and optimization of transportation systems. In this paper, transportation network problems are classified into the following three categories:

(1) network analysis problems,
(2) network design problems, and
(3) network management problems.

This classification scheme is somewhat arbitrary but valuable nonetheless. A common point of departure for dealing with these problems is to view the underlying transportation network and its operating strategies as the <u>supply</u> for transportation, whereas the demand for transportation services is generated by the persons and goods wishing to be transferred together with their motives and behavioral relationships.

Our purpose in this paper is to escort the reader on a guided tour through selected applications of optimization techniques to transportation networks; within each category we focus on a specific application. In network analysis problems one calculates the optimal distribution of flows in a given network. An example is the traffic assignment problem which concerns the assignment of origin-destination pair demands to various routes in order to minimize total travel costs. Network design problems deal with determining an optimum network configuration for a predicted demand pattern, subject to budget constraints. In particular, network improvement will be discussed. Network management problems involve the control of operations in order to make effective utilization of available resources. Vehicle routing is an important network management problem which has received widespread attention recently in the Operations Research literature. We remark that there is often a great amount of overlap between the categories listed above. Many real-world problems contain elements from each category. Potts and Oliver [45], Steenbrink [49], and Bradley [5] are recommended as general references on transportation networks. In addition, Golden and Magnanti [25] provide an extensive network bibliography.

NETWORK ANALYSIS PROBLEMS

In the jargon of transportation planners, the technique most often used for network analysis problems is called traffic assignment. Traffic assignment is a computational procedure used to aid the analyst in forecasting of future loadings on a network of transportation facilities. The result of the assignment procedure is an estimate of user volumes on each segment of a transportation network. The user volumes may be the number of vehicles, the number of persons, the number of transit riders, or any other commodity that has an origin (O), destination (D), and some quantifiable trip interchange characteristic.

The traffic assignment procedure is used for many purposes, such as:

- --Development and testing of alternate transportation systems.
- --Establishment of short range priority programs for transportation facility development.
- --Evaluating the impact of new traffic generators on an existing transportation system.
- --Location analysis of distribution and service facilities within a transportation corridor.
- --Providing input and feedback to other planning models.

The same procedure has been applied to urban area networks, statewide systems, as well as national and international transportation systems. The types of assignment that were made included: vehicles to a highway network, passengers to transit networks, passengers to air carrier routes, freight to rail and shipping lines, messages to communication channels, etc. The widest application of the traffic assignment procedure is in the urban transportation planning process, where it constitutes a fundamental step in the travel forecasting stage. It is the last in a sequence of five steps:

- --Land Use Prognostication
- --Trip Generation
- --Trip Distribution
- --Modal Split
- --Traffic Assignment

The first four steps are designed to provide the analyst with an estimate of future O-D person travel demands, by mode of travel, for a given layout of a transportation network. The assignment procedure is then used to assign persons and vehicles to the various routes in the system. A description of the transportation planning process can be found in such references as Potts and Oliver [45], Hutchinson [28], or Comsis [11].

Wardrop [52] enunciated two broad principles for determining the assignment of traffic to alternative routes: Assignment according to the first principle leads to an equilibrium situation in which travel costs on all utilized routes between any given origin-to-destination (O-D) are equal or less than those on nonutilized routes. This is a descriptive assignment emulating the traffic pattern in a transportation network when no restrictions are imposed on the route each traveler may choose. The resulting pattern has been termed a user-optimized pattern. On the other hand, assignment according to the second principle leads to a system-optimized pattern by minimizing total travel costs in the system. This is a non-equilibrium pattern and can be achieved when travelers are prescribed their travel paths so that total costs to the community are minimized. Thus, assignment according to this principle is a normative assignment.

Dafermos and Sparrow [13] show that, in the case of nonelastic demands, a traffic pattern satisfying one of Wardrop's principles is the optimal solution to the following convex problem:

$$\min \sum_j Z_j \, (f_j)$$

subject to flow conservation and nonnegativity constraints, where

$$Z_j(f_j) = \begin{cases} \int_o^{f_j} C_j(x)dx & \text{for a user-optimized pattern} \\ f_j \, C_j(f_j) & \text{for a system-optimized pattern} \end{cases} \qquad (1)$$

and $C_j(f_j)$ = travel cost on link j at flow f_j.

In general, $C_j(f_j)$ is a monotone increasing function of the link flow f_j. A typical function used in planning situations is:

$$C_j(f_j) = a_j + b_j(f_j)^4 \tag{2}$$

where a_j and b_j are constants characterizing link j.

When trip demand is elastic and given by a monotone decreasing function of the associated interzonal travel cost, the user-optimized pattern becomes the classic economic supply-demand equilibrium, where the interzonal travel cost implied by the demand function is equal to the actual travel cost of the utilized routes. The equilibrium flows are those that maximize the consumer's surplus, or equivalently (Beckman et al. [2]), minimize the function

$$\sum_j \int_o^{f_j} C_j(x)dx - \sum_i \int_o^{g_i} W_i(y)dy \tag{3}$$

where g_i represents the number of trips related to O-D pair i, and $W_i(g_i)$ is the inverse of the demand function for travel between O-D pair i.

Solution procedures for the traffic assignment problem in transportation planning may be divided into two main categories:

(1) capacity restraint methods and,
(2) equilibrium methods.

The basic idea of all these methods is to obtain an equilibrium traffic pattern, from an initial trial solution, by iteratively adjusting the travel costs and the traffic flows. The capacity restraint methods are generally based on intuitive arguments, while the equilibrium methods are based on rigorous mathematical arguments which ensure the convergence of the calculations to the solution of a convex cost minimization problem. Surveys and extensive bibliographies of existing methods are given in Comsis [11], Ruiter [47], Nguyen [43], and Assad [1]. Some of the more recent equilibrium approaches, which also report computational experience and which are based on various decomposition techniques, are discussed below.

Dafermos [12] presents an iterative procedure which begins with an initial feasible flow pattern and by means of an "equilibration operator" constructs a sequence of feasible flow patterns which converges to the optimum solution. The main drawback of this technique, which severely limits its computational effectiveness, is that it requires the enumeration of all paths between each O-D pair. Leventhal et al. [34] improve on this method by developing a column generation algorithm for the problem, which does not require the a priori generation of all O-D paths. The algorithm is capable of handling rather large networks, taking advantage of the fact that relatively few of the paths have positive flows in an optimal solution.

The most efficient computational approach to the traffic assignment problem, so far, uses an adaptation of Frank-Wolfe decomposition [54]. Given a feasible point x^k (in flow space), the objective function (1) is linearized at that point. Since the constraints are also linear, the problem turns into a linear program with an optimal solution y^k. The direction $d^k = y^k - x^k$ is then a good direction to seek a decreased value of Z. Using a search technique such as Golden Section or Bolzano, a new feasible solution x^{k+1} is derived. The procedure keeps iterating through the linearization-search stages, using x^{k+1} as a new starting point, until convergence. The main computational advantage of this decomposition method is that the LP is actually solved by a shortest-route algorithm that can be applied independently to each origin in the network. Very efficient algorithms exist for this purpose. Variants of the procedure described above were used by Bruynooghe et al. [6], and later by Cantor and Gerla [7], Golden [23], and LeBlanc et al. [32]. A similar approach with comparable computational results, was developed by Nguyen [42]. Nguyen adapts a particular form of the convex-simplex method which exploits the block-diagonal structure of the assignment problem to speed up computations.

The equilibrium techniques described above have proven to be useful for the kind of problems that transportation planners consider most often (up to, say, network sizes of 1000 nodes, 2500 links, 250 origins). It has also been shown that the equilibrium approaches provide improved predictive capability [18, 31] when compared to the traditional capacity restraint techniques [27]. These approaches fail, however, when one wishes to consider in detail very large networks such as those found in the New York or Los Angeles metropolitan areas (e.g., 10,000 nodes, 25,000 links, 1000 origins). An approach that was recently developed for this purpose uses geographic decomposition and sub-area focusing to reduce the size of the problem [38]. Geographic decomposition is based on the observation that very large networks are often only loosely connected and by deleting a small set of links the network will decompose into several disjoint subnetworks. It is shown that the equilibrium assignment techniques can also be applied in this case by linking the subnetworks together through generalized Benders decomposition as described by Maier [38].

Generalized Benders decomposition is also used by Florian and Nguyen [17] for computing network equilibrium with elastic demands. The constraints of the problem, in this case, are separable for each O-D pair. The only interaction occurs on the links and consequently in the objective function. Since the objective function is convex, a local minimum is also a global minimum, and the problem may be decomposed by each O-D pair. The entire problem is then solved by cyclical application of a special algorithm for a simple O-D pair.

In many cases the link supply functions are subject to change, e.g., in the case of signal-controlled intersections in which the link capacity is determined by the greentime apportioned to that link. Current practice is that traffic engineers, in devising control strategies for the signals, assume fixed demands. On the other hand, transportation planners, in their assignment calculations, ignore the controllability of the link capacity and assume fixed supply functions. Gartner [19] shows the potential benefits of combining the two aspects into a single optimization program which is also amenable to the decomposition techniques described above.

All models described so far presume a static situation, i.e., demand does not vary with time. This assumption is not applicable in many realistic traffic situations. Traffic assignment models are frequently used for analyzing rush-hour periods in metropolitan areas, and the dynamic behavior must be considered if congestion is to be alleviated by controlling traffic. Merchant and Nemhauser [39] present a discrete time model for dynamic traffic assignment. The model leads to a nonlinear and nonconvex mathematical programming problem. A piecewise linear version of the model can be solved for a global optimum using a one-pass simplex algorithm, without resorting to branch-and-bound. The piecewise linear program has a staircase structure and can be solved by decomposition techniques or compactification methods for sparse matrices. A somewhat similar approach akin to store-and-forward communication networks is suggested by D'ans and Gazis [14].

NETWORK DESIGN PROBLEMS

In this section, we discuss some transportation network design problems. Since it is unlikely that an entire transportation system be constructed at once, most system engineers will encounter network design problems that concern the improvement of an existing network. Therefore, we prefer to focus on network improvement problems rather than network synthesis problems.

The general network improvement problem that we will discuss has the following properties:

(1) An existing network configuration is given.

(2) There is a set of traffic flows that must be routed through the network. These flows can already be routed through the initial unimproved network. The traffic assignment can be determined by either of the two Wardrop principles.

(3) There is a set of possible improvements that can be made to upgrade the network. These network improvements include adding new arcs to the existing network or modifying arcs already in the network. These arc modifications can consist of either increasing the flow capacity or decreasing the traffic flow cost of an arc.

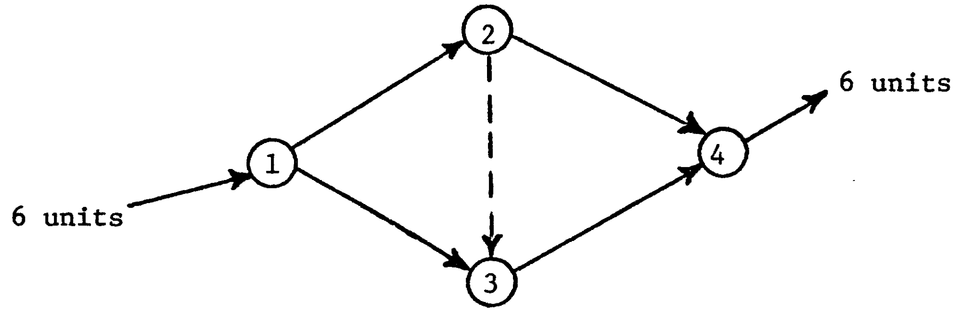
(4) There is a construction cost associated with each possible improvement to the network.

(5) The problem is to select a set of network improvements subject to a construction budget constraint so that the traffic flow costs are minimized.

This general framework encompasses a large number of network improvement problems. Next, we will discuss some interesting features of this class of network design problems. For a more comprehensive discussion of these and other network design problems see the surveys by MacKinnon [37], Stairs [48], Steenbrink [49], and Wong [53].

First, we consider a major difference between network improvement problems with user-optimized traffic flows and system-optimized traffic flows. For a network with system-optimized flows, the addition of an arc to the network can never increase the total traffic flow costs. Since we can always adopt the flow pattern that was used before the new arc was added, the traffic flow costs can never increase and will usually decrease. For a network with user-optimized flows, the addition of an arc can actually lead to an increase in the total flow assignment costs. Since Braess was the first one to recognize this phenomenon, it is known as Braess' paradox [40].

We now describe an example of Braess' paradox based on a modified form of an example reported by LeBlanc [29]. The figure below gives a sketch of the directed network that we will discuss.



Network Example of Braess' Paradox

Six units of flow must be routed from node 1 to node 4. We also have:

Traffic flow cost for arc (i, j) = $x_{ij}f_{ij}(x_{ij})$ where:

$$x_{ij} = \text{flow on arc } (i, j)$$
$$f_{12}(x) = 10 + .5(x)^4$$
$$f_{13}(x) = 150 + .9(x)^4$$
$$f_{24}(x) = 150 + .9(x)^4$$
$$f_{34}(x) = 10 + .5(x)^4$$
$$f_{23}(x) = 10.4 + (x)^4$$

path 1 = arcs (1,2) and (2,4)
path 2 = arcs (1,3) and (3,4)
path 3 = arcs (1,2), (2,3) and (3,4).

The first situation that we will analyze is when arc (2, 3) is not present in the network. By symmetry, the user-optimized traffic pattern is to send 3 units of flow via paths 1 and 2. The total flow cost is 273.4. If we consider the network with arc (2, 3) added to it, the user-optimized traffic pattern is to send 2 units of flow via paths 1, 2 and 3. The total flow cost is 302.4. With the addition of arc (2, 3) to the network, the flow cost increases by about 11%.

It is not known how prevalent this counter-intuitive behavior is in networks that have user-optimized flows. However, Murchland [40] reports on a recent experience by Knodel, "Knodel remarks that the example (of Braess) may seem contrived, but a recent experience in Stuttgart shows that it can occur in reality. Major road investments in the city centre, in the vicinity of the Schlossplatz, failed to yield the benefits expected. They were only obtained when a cross street, the lower part of Konigstrasse, was subsequently withdrawn from traffic use."

So Braess' paradox indicates that great care should be used in evaluating proposed improvements to a network with user-optimized flows.

Now we discuss an underlying issue present in every network improvement problem. All network design problems contain an implicit traffic assignment problem that must be solved. This implicit problem is the evaluation of a proposed network design. So if the underlying traffic flow problem cannot be solved efficiently then there is little hope of solving the actual network improvement problem. This could explain why most researchers in the area of network design have concentrated on problems with system-optimized flow patterns. Until recently, only very small user-optimized traffic assignment problems could be solved efficiently.

The recent advances in traffic assignment algorithms described in the previous section should enlarge greatly the range of network improvement problems that can be solved efficiently. In fact, several examples of network improvement procedures which rely upon the availability of sophisticated traffic assignment algorithms have already appeared. We will describe some of these recent efforts.

LeBlanc [29] deals with the first network improvement problem that we will consider. The design problem has a discrete set of possible improvements to an existing network. Flow patterns are assigned according to a user-optimized flow policy. The arc flow costs are convex functions of the total arc flow. LeBlanc utilizes a branch-and-bound procedure to select the optimal set of improvements. A procedure recently developed by LeBlanc, Morlok, and Pierskalla [32] is used to evaluate proposed design solutions. Also lower bounds used to limit the tree searching process are computed by the same procedure. LeBlanc tests his procedure by solving a sample problem on a network with 24 nodes, 76 arcs and 5 elements in the set of possible improvements. Finding an optimal solution to the problem required about 2 1/4 minutes of CDC 6400 computer time.

LeBlanc's work is one of the few efforts to deal with improvements for a network that has user-optimized flows. Other efforts in this area have been made by Ochoa and Silva [44] and by Barbier (whose work is described in [48]). Although Barbier's work has also been used to solve some moderate-sized problems (36 nodes, 80 arcs), the method is a heuristic one, so the quality of the solutions obtained is unknown.

Dantzig et al. [15] consider a different type of network improvement problem. For their problem the improvement variables are continuous instead of discrete as was the case in LeBlanc's problem. Therefore, the set of possible improvements has an infinite number of elements. Traffic flow is assigned according to a system-optimized policy. Arc flow costs are piece-wise linear convex functions of the total arc flow.

In order to solve their problem Dantzig et al. attach a Lagrange multiplier to the budget constraint and then place this constraint in the objective function. Then they use a decomposition technique that was developed by Steenbrink [49]. Steenbrink's method involves decomposing the problem into a master problem and a series of subproblems. Each subproblem concerns finding the optimal improvements for an arc given the total flow through it. The master problem is a traffic assignment problem which is solved using Frank-Wolfe decomposition. Steenbrink's decomposition method is applied several times with different values of the Lagrange multiplier in order to find a good solution which also satisfies the budget constraint.

Dantzig et al. tested this method on a network improvement problem with 24 nodes and 76 arcs. Their procedure required 10.68 seconds of IBM 370/168 computer time. The final solution was about 2.5% away from the optimal solution. In contrast, they report that the same problem was solved by using a linear programming formulation of the problem. The formulation contained 702 rows and 2868 variables. The simplex method, implemented in the MPS/360 package, required 40.8 minutes of IBM 370/168 computer time to obtain an optimal solution. So by utilizing a decomposition approach which requires a good traffic assignment routine, Dantzig et al. were able to obtain a near optimal solution in a fraction of the time it took a method which did not utilize a special traffic assignment algorithm. Dantzig et al. also report computational experience on a problem with 394 nodes and 1042 arcs. Their method required 5.63 minutes of IBM 370/168 computer time.

In this section, we have described a general class of network improvement problems. A major difference between design problems with system-optimized and user-optimized traffic patterns has been mentioned. In addition, we indicate that recent advances in traffic assignment methods have helped bring about new progress in network improvement procedures. The further exploitation of these traffic assignment advances appears to be a good area for future research.

NETWORK MANAGEMENT

As an illustration of a network management problem we pose the following very general problem situation. Imagine that a large organization with certain well-defined objectives must perform a number of distribution or collection activities over a transportation network in order to satisfy various demands. There are a myriad of possibilities depending upon the specific characteristics involved. Several possibilities are listed below which lead to a host of related problems:

(i) depots (single depot or multiple depots);
(ii) demand locations (pre-specified locations, random locations, or mixed);
(iii) demands (deterministic, stochastic, or mixed);
(iv) operations (pickup, delivery, or mixed);
(v) vehicle fleet (homogeneous or heterogeneous);
(vi) routing (over nodes, over arcs, or mixed).

Network management, in this example, entails the efficient utilization of central depots and vehicle fleet in order to perform the desired operations at demand locations, satisfy requirements, and maintain a cost-effective routing policy.

In this section, we discuss a special case of the above problem known as the vehicle routing problem (VRP). Vehicle routing problems, sometimes referred to as truck-dispatching problems, are almost always encountered by complex organizations in both the public and private sectors, and reliable procedures for dealing with them are needed. Recently, higher vehicle costs due to increased oil prices and rising truck drivers salaries have motivated management to study these issues more carefully.

Due to the inherent complexity of the VRP, only small problems can be solved for the optimal solution. For larger transportation networks, we use heuristic algorithms which produce near-optimal solutions. We will discuss several of the well-known heuristic approaches for the VRP in this section. Recent implementation results (see Golden, Magnanti and Nguyen [26] for details), demonstrate that large-scale problems can be solved much more efficiently than previously. Hopefully, these computational advances will result in the better management of complex logistics and transportation systems which will be more flexible and less costly than existing ones. The surveys by Bodin [4], Christofides [8], Golden [24], and Turner et al. [50] are recommended for more background regarding the VRP; here we provide an overview of vehicle routing with an emphasis on broad issues.

There may be several hundred demand points in and around a city. The vehicle routing problem is to obtain a set of delivery routes from a central depot to the various demand points, each of which has known requirements, which minimizes the total distance covered by the entire fleet. Vehicles have capacities and maximum route time constraints. In addition, the fleet of vehicles may be heterogeneous with respect to these characteristics. All vehicles depart from the central depot, make a tour of a subset of the demand nodes, and return to the central depot. All demands must be satisfied. Examples of vehicle routing problems include:

(i) municipal waste collection (see Beltrami and Bodin [3]);
(ii) fuel oil delivery (see Garvin et al. [20]);
(iii) newspaper distribution (see Golden, Magnanti, and Nguyen [26]);
(iv) routing of school buses (see Newton and Thomas [41]).

Operationally the examples may seem different, but conceptually they can be thought of as equivalent.

Proposed heuristic techniques for solving problems of this sort can be grouped into four classes: "savings" procedures [10], "sweep" procedures [22], "nearest-neighbor" procedures [51], and "r-optimal" procedures [35]. We discuss each of these approaches in this section but concentrate on the first two which seem to be more effective.

Undoubtedly, the Clarke-Wright savings method, developed in 1964, is the most widely used and cited vehicle routing algorithm. It involves first evaluating all potential savings $S_{ij} = d_{1j} + d_{1j} - d_{ij}$ from linking two nodes i and j, and then joining those nodes with the highest feasible savings at each iteration. Initially, we suppose that every two demand points i and j are supplied individually from two vehicles giving a total distance of $2d_{1i} + 2d_{1j}$. Now if instead of two vehicles, we used only one, then we would experience a savings in travel distance of $(2d_{1i} + 2d_{1j}) - (d_{1i} + d_{ij} + d_{j1}) = d_{1i} + d_{1j} - d_{ij}$.

For every possible pair of demand points i and j there is a corresponding savings S_{ij}. We order these savings from greatest to least and starting from the top of the list we link nodes i and j where S_{ij} represents the current maximum savings unless the problem constraints are violated. Christofides and Eilon found from 10 small test problems that tours produced from the savings method averaged only 3.2 percent longer than the optimal tours [9].

In 1974, Gillett and Miller [22] proposed a sweep algorithm for Euclidean networks which ranks and links demand points by their polar coordinate angle. We select a "seed" node randomly. With the central depot as the pivot, we start sweeping (clockwise or counterclockwise) the ray from the central depot to the seed. Demand nodes are added to a route as they are swept. If the polar coordinate indicating angle is ordered for the demand points from smallest to largest (with seed's angle 0) we enlarge routes as we increase the angle until capacity restricts us from enlarging a route by including an additional demand node. This demand point becomes the seed for the following route. Once we have partitioned the nodes, we can apply traveling salesman heuristics to improve tours and obtain significantly better results. In addition, we can vary the seed and select the best solution.

Tyagi [51], in 1968, presented a method which groups demand points into tours based on a nearest-neighbor concept. That is, points are added to a tour sequentially, each new addition being the closest point to the last point added to the tour. Having grouped the delivery points into m tours, we solve m traveling salesman problems to refine the tours.

Eilon et al. [16] study an r-optimal procedure for the VRP which is an outgrowth of Lin's approach to the traveling salesman problem [35]. We remark that the traveling salesman problem is a special case of the VRP which arises in many different contexts; typical applications include computer wiring, clustering, and job-shop scheduling, in addition to vehicle routing [33]. The procedure presented by Eilon et al. begins with a feasible solution and tests perturbations of r arcs at a time until we obtain r-optimality. For example, if $r = 2$ we examine each pair of arcs to see if it can be replaced by another pair such that feasibility is preserved and total distance is decreased.

Vehicle routing algorithms have recently "come of age" in the sense that they are now capable of solving some large-scale real-world problems. A 250-location problem with about 10 locations per route was solved on an IBM 360/67 in just under 10 minutes using the Gillett and Miller algorithm [22]. More recently, Golden, Magnanti and Nguyen [26] have incorporated some ideas from computer science into a modified savings procedure. A newspaper distribution problem involving 600 nodes was solved using this approach on an IBM 370/168 in 20 seconds of execution time. It should be noted that the sweep algorithm generally produces better solutions than the savings algorithm, (due to the fact that a number of seeds are considered) but running time is much greater.

These heuristic algorithms, especially the savings method, have been shown to perform very quickly. But how accurate are they? Remember that heuristic algorithms produce good solutions to given combinatorial programming problems, but not necessarily the best possible (optimal) solutions. We now construct a couple of pathological examples to indicate that there are situations in which the savings and sweep algorithms terminate with poor solutions.

<u>Example I</u>: The savings algorithm. Node 1 is the origin. At each of the four demand nodes there is a demand of 1; vehicle capacity is 2. The network is displayed below. The ratio SAVE/OPT is approximately 1.25.

<u>Savings List</u>

$S_{34} = 2.01$

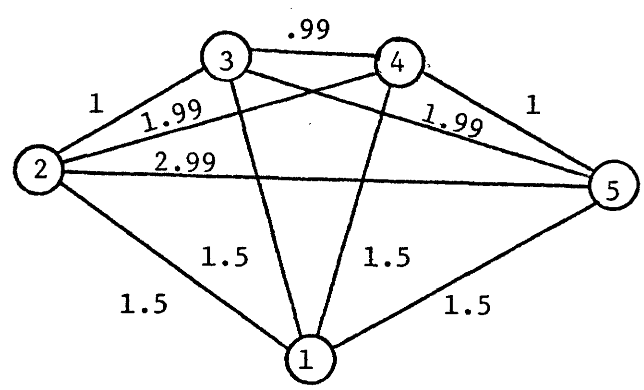
$S_{23} = 2.00$

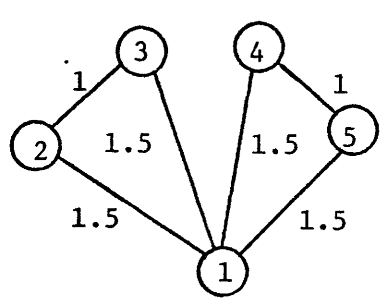
$S_{45} = 2.00$

$S_{24} = 1.01$

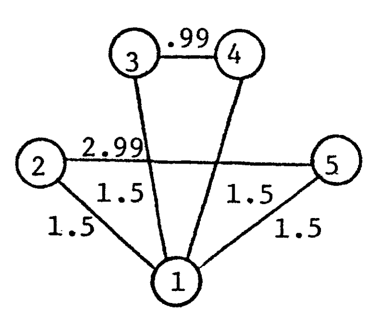
$S_{35} = 1.01$

$S_{25} = .01$



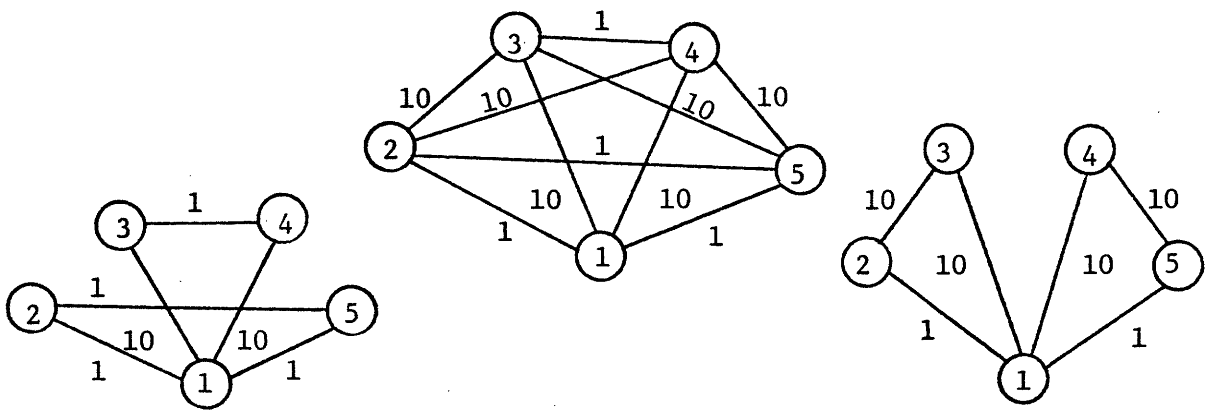


Optimal Solution
Total distance = OPT = 8



Clarke-Wright Solution
Total distance = SAVE $\approx$ 10

<u>Example II</u>: The sweep algorithm. Node 1 is the origin and node 5 is the seed. At each of the four demand nodes there is unit demand; vehicle capacity is 2. Suppose we sweep in a counterclockwise direction in the network below. The ratio SWEEP/OPT is 1.75.



Optimal Solution
Total distance = OPT = 24

Gillett-Miller Solution
Total distance = SWEEP = 42

Despite the fact that poor performance can result from these approaches, in practice these extreme cases do not arise frequently. Recently, several researchers have explored the possibility of combining several of the four methods outlined into an even more effective hybrid approach. For example, Robbins et al. [46] have assembled a tour construction-tour improvement code in which tours are initially constructed using a savings approach and then improved upon via an r-optimal procedure.

The indication is that the suggested procedures can be used as effective decision-making tools by management for large-scale vehicle routing problems encountered in many practical situations.

FINAL COMMENTS

Large-scale network problems have recently attracted a great deal of attention. As a result, network researchers have achieved substantial algorithmic advances which have direct applicability to transportation network problems. These new large-scale network techniques have enlarged greatly the size and scope of network analysis, network design, and network management problems that can be dealt with effectively. As these results become more widely recognized, the number of successful applications to actual transportation system planning problems will undoubtedly increase.

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