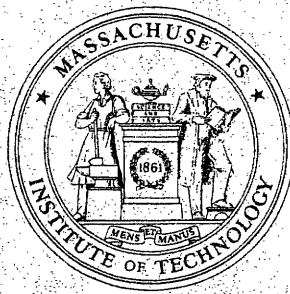


OPERATIONS RESEARCH CENTER

working paper



**MASSACHUSETTS INSTITUTE
OF TECHNOLOGY**

INTEGRATION OF NONLINEAR COAL SUPPLY MODELS
AND THE
BROOKHAVEN ENERGY SYSTEM OPTIMIZATION MODEL
(BESOM)

by

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1. Introduction

This is a report on our efforts to date to integrate BESOM, or one of the other energy planning models based on it, Hoffman (1972), Cherniavsky (1974), or Goettle et al (1977), with nonlinear supply models developed at the M.I.T. Energy Laboratory. Our experimentation and conceptualization of the integration process has thus far been primarily with BESOM and Zimmerman's coal supply model (Zimmerman (1977)). The technical difficulties in achieving the integration, and the methods for overcoming them, are applicable to similar efforts with other energy supply models (eg, the oil and gas model of MacAvoy and Pindyck (1975)).

The plan of this report is as follows: Section 2 contains the results of sensitivity analyses on primary energy supplies in the current version of BESOM. Section 3 gives a description of Zimmerman's coal supply model in mathematical programming terms, followed by a discussion of decomposition methods for solving it and integrating it with other energy models. Some concluding remarks are given in section 4. There is one Appendix giving more detail about the sensitivity analyses performed.

2. Computer Experiments

We performed some sensitivity analyses on the current version of BESOM for the year 1985 using the SESAME interactive linear programming system developed at the National Bureau of Economic Research Computer Research Center. The results are depicted in figure 1. To perform the sensitivity analysis, the supply of coal in BESOM was made a parameter which was varied from a level of near zero to a level of 28×10^{15} BTU. No cost was associated with these supply levels and the shadow price on the coal supply constraint with the parametric right hand side was calculated at the various levels. The quantity was then allowed to freely vary and the price was appropriately adjusted to determine the precise nature of the shadow price curve including the actual break points. We did this to try to measure the consistency of the actual coal supply in BESOM with estimates of coal supply at similar price levels from Zimmerman's model for the same year, 1985. The optimal coal supply in BESOM when the supply is a decision variable is 19.14×10^{15} BTU which is determined, in principle, by the unit coal price used in BESOM of $\$1.02/10^6$ BTU; not including transportation.

The indicated coal supply level in BESOM is generally consistent with Zimmerman's prediction if we make some simplifying assumptions about how to aggregate regional supplies into a national supply figure. Specifically, a coal supply of 14.42×10^{15} BTU in 1985 is indicated in Zimmerman's model by a price of $\$1.213/10^6$ BTU, including transportation costs. In BESOM, the transportation cost for coal is $\$.32/10^6$ BTU to electric thermal power units and $\$.13/10^6$ BTU to petrochemical plants and space heat. Subtracting an average of these from $\$1.213/10^6$ BTU gives us a figure near the BESOM figure of $\$1.02/10^6$ BTU.

Note, however, that the supply level of coal in BESOM is in reality not determined by price but by the lumpy structure of the model and the other

Shadow Prices for RR 22 (Coal) in BESOM BNL877

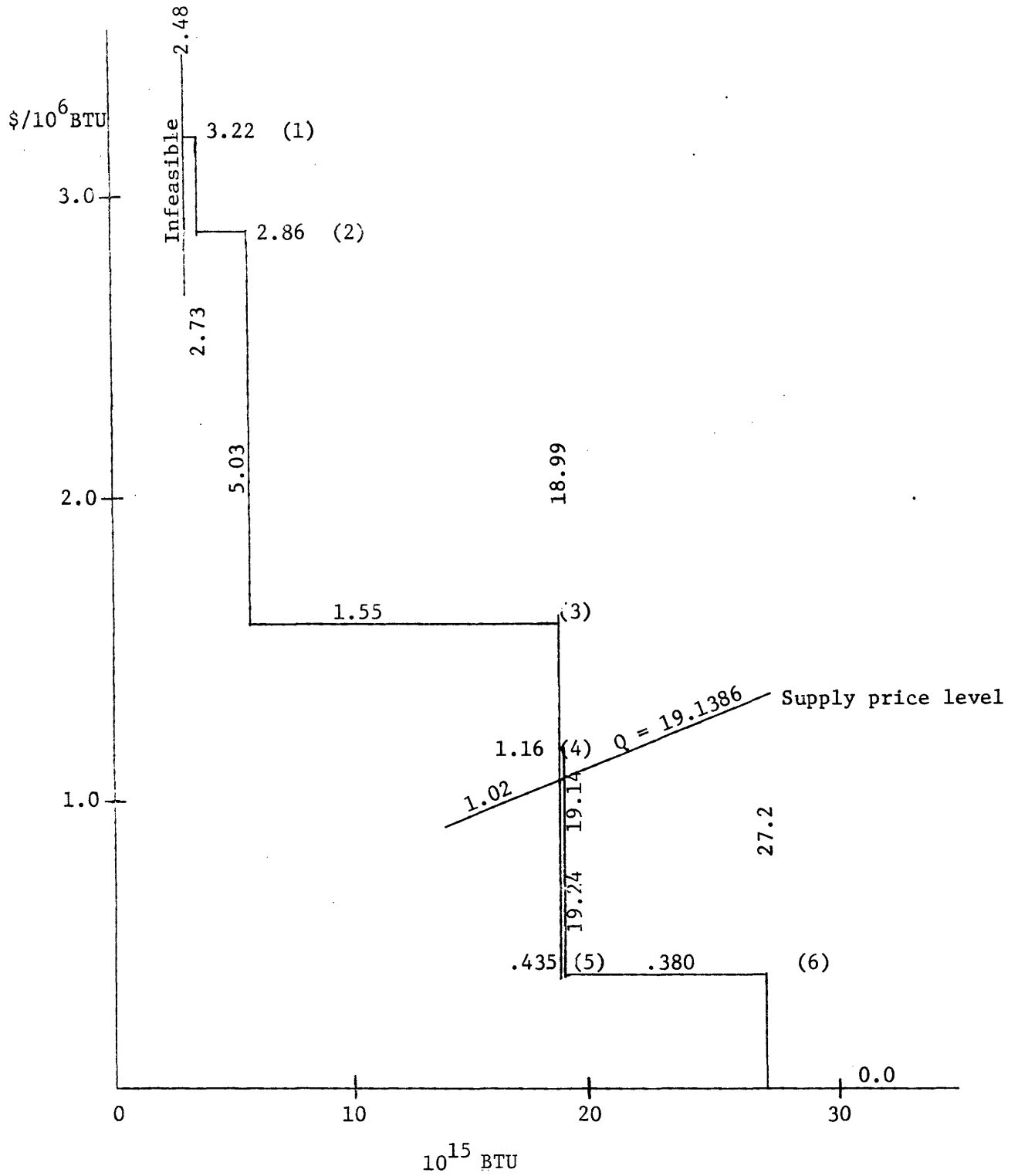


Figure 1

restrictions on the energy sector in 1985. Figure 1 indicates that for any coal price between \$.38/10⁶ BTU and \$1.55/10⁶ BTU, the variable supply selected by BESOM is approximately 19 x 10¹⁵ BTU with very little variation. Thus, the consistency we found between the two models is somewhat misleading because any unit price on coal between the two levels would produce the same supply in BESOM.

It is instructive to note the changes in activities corresponding to the linear programming basis changes as the coal supply is parametrically increased. These changes are numbered 1 through 6 on figure 1:

<u>Change</u>	<u>Explanation</u>
1	Coal used for petrochemicals
2	Coal used for miscellaneous thermal intermediate temperature processes
3	Coal used for steam electricity, reducing oil consumption
4	Coal replaces oil fired gas turbines for electricity
5	Electricity (from coal) replaces oil for some water heating
6	Coal plants replace nuclear for electricity generation
7	No further increase in use at a zero price

3. Zimmerman's Coal Supply Model and Its' Integration with BESOM

3.1 Overview

In words, Zimmerman's coal supply model is to minimize the discounted sum of extraction and transportation costs of coal supply to meet given demands for coal over T time periods, subject to constraints on the average sulfur content of the coal consumed in each demand region and in each period. The marginal cost of extraction in each supply region, and for each sulfur content, is an increasing function of the cumulative supply. The flows of coal from supply to demand regions in each period may also be subject to constraints on production, manpower, or transportation capacities.

We propose a decomposition approach to this model. The purpose of the decomposition is not simply to improve computational efficiency of an existing model. Zimmerman's model is a large scale linear programming problem which is quite difficult to solve directly, even if one desires only a single optimal solution to the supply problem. In fact, there are four main purposes for considering the decomposition approach. It would facilitate:

- (1) The calculation of optimal solutions to the overall model, or good feasible solutions with bounds on how far the solutions are from optimality;
- (2) Sensitivity analyses of the optimal solution to average sulfur content constraints, production and transportation capacities and discount factor;
- (3) The integration of the coal supply model with other energy models, such as BESOM, to test interfuel substitution effects and endogenous demands for coal;
- (4) A wide variety of model extensions because of the construction and use of matrix generation programs in the decomposition.

The idea underlying the decomposition is to fix the supplies by sulfur content in each supply region in each period. This decomposes the large scale intertemporal supply problem into T generalized transportation problems of relatively small size, one for each period. The result of solving the T generalized transportation problems is a feasible solution to the intertemporal problem, but it may not be optimal. The decomposition approach tests the solution for optimality, and if it is not optimal, the fixed supplies are changed so that the resulting new solution is closer to being optimal. The optimality test uses the gradient¹ of the total cost function at the fixed supply levels which is found by adding appropriate shadow prices from the transportation problems to the derivatives of the supply functions. If the total cost function goes up in all feasible directions of change of the supply levels, then the feasible solution is optimal. Otherwise, a direction of change of the supplies is found such that the total cost function decreases in that direction. The supplies are fixed at new levels and the process is repeated. The approach is depicted schematically in figure 2.

¹The gradient might not exist at some supply levels, but a generalization called the subgradient does exist and can be used for the same purposes.

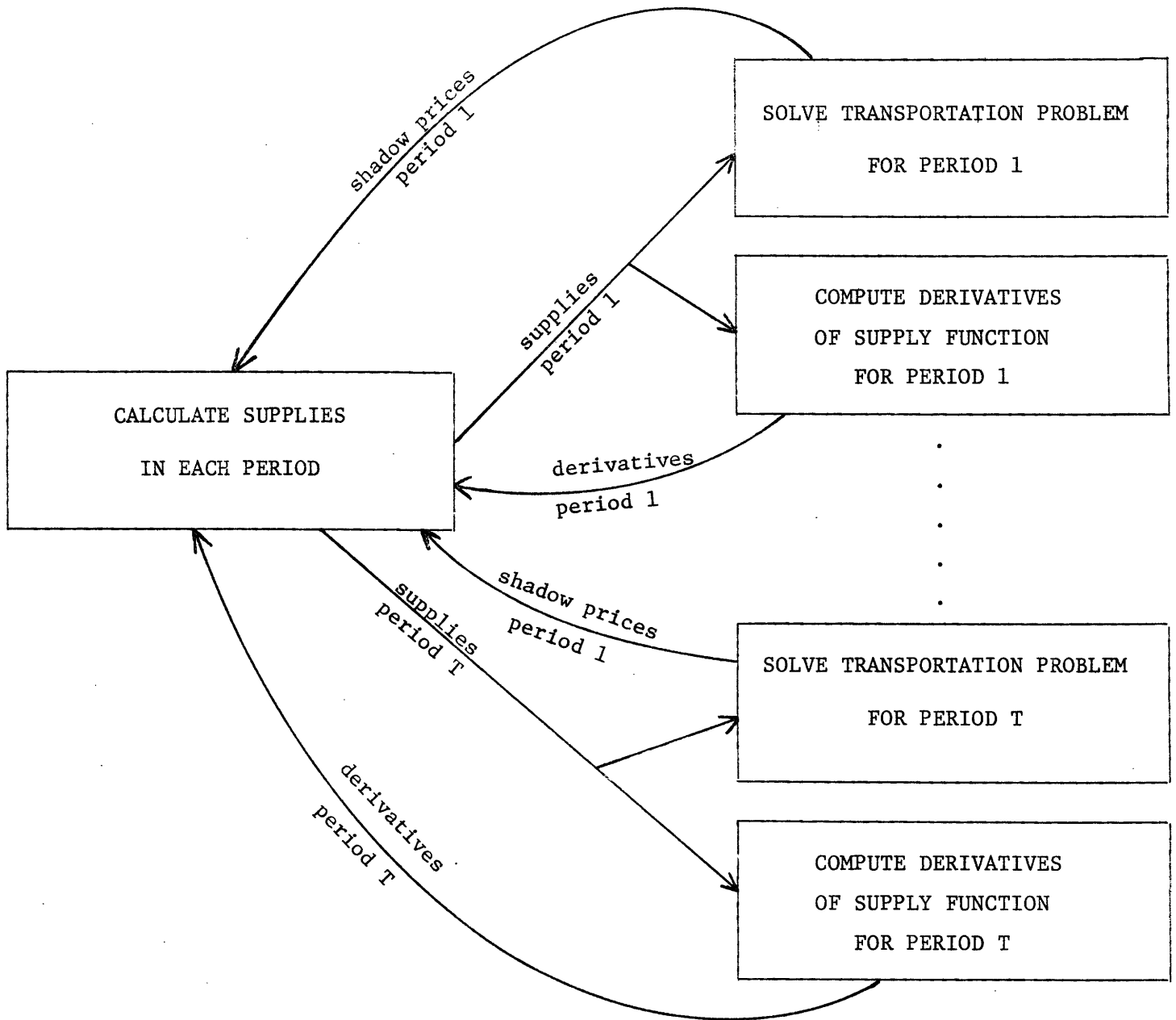


Figure 2

3.2. Details of Model

3.2a. Notation

Indices

- i supply regions (6)
- j demand regions (9)
- k sulfur contents (10)
- t time periods (20)

Variables

- x_{ijkt} = flow of coal (in tons or BTU's) from supply region i to demand region j of sulfur content k in time period t
- y_{ikt} = supply of coal (in tons or BTU's) in region i with sulfur content k in time period t
- $S_{ikt} = \sum_{w=1}^t y_{ikw}$ = cumulative supply of coal in region i with sulfur content k by the end of time period t

Parameters and Functions

- c_{ijt} = cost per unit flow from supply region i to demand region j in period t
- m_{ijt} = upper bound on flow from region i to region j in period t (transportation capacity)
- p_{ik} = sulfur content (percent) of type k coal
- r_{ikt} = upper bound on supply of type k coal in supply region i in period t (tons) (production capacity)

d_{jt} = demand for coal in region j in period t (tons)

q_{jt} = maximal allowable percentage of sulfur in coal
consumed in region j in period t

$f_{ik}(S)$ = cost of extraction of S cumulative units of coal
of sulfur type k in region i

α = discount factor

Coal Supply Model (Undecomposed)

$$v = \min \sum_{t=1}^t \alpha^{t-1} \{ \sum_{i,k} \{ f_{ik}(\sum_{w=1}^t y_{ikw}) - f_{ik}(\sum_{w=1}^{t-1} y_{ikw}) \} + \sum_{i,j,k} c_{ijt} x_{ijkt} \}$$

$$\text{s.t. } \sum_j x_{ijkt} - y_{ikt} = 0 \quad \text{for all } i,k,t \quad (\text{supply constraints 1200})$$

(*)

$$\sum_{i,k} x_{ijkt} = d_{jt} \quad \text{for all } j,t \quad (\text{demand constraints 180})$$

$$\sum_{i,k} p_{ik} x_{ijkt} \leq q_{jt} d_{jt} \quad \text{for all } j,t \quad (\text{average sulfur content constraints 180})$$

$$0 \leq \sum_k x_{ijkt} \leq m_{ijt} \quad \text{for all } j,t \quad (\text{transportation capacity constraints 180})$$

$$0 \leq y_{ikt} \leq r_{ikt} .$$

3. Decomposition

If the y_{ikt} are fixed in problem (*), say $y_{ikt} = \bar{y}_{ikt}$, then it decomposes into T linear programming subproblems to compute transportation costs

$$\tau(\bar{y}^t) = \min \sum_{i,j} c_{ij} x_{ijkt}$$

$$\text{s.t. } \sum_j x_{ijkt} = \bar{y}_{ikt} \quad \text{for all } i, k \quad (\text{LP}_t)$$

$$\sum_{i,k} x_{ijkt} = d_{jt} \quad \text{for all } j$$

$$\sum_{i,k} p_{ik} x_{ijkt} \leq q_{jt} d_{jt} \quad \text{for all } j$$

$$0 \leq \sum_k x_{ijkt} \leq m_{ijkt} \quad \text{for all } j,$$

where \bar{y}^t denotes the vector with components \bar{y}_{ikt} . The total cost associated with the solution $\bar{y} = \bar{y}^1, \dots, \bar{y}^T$ is given by the sum of extraction and transportation costs; namely

$$v(\bar{y}) = \sum_{t=1}^T \alpha^{t-1} \left\{ \sum_{i,k} \{ f_{ik} \left(\sum_{w=1}^t \bar{y}_{ikw} \right) - f_{ik} \left(\sum_{w=1}^{t-1} \bar{y}_{ikw} \right) \} + \tau(\bar{y}^t) \right\}.$$

Assuming the subproblems (LP_t) are all feasible, their solution provides a feasible solution $\bar{y}_{ikt}, \bar{x}_{ijkt}$ for the overall problem (*). Thus, $v(\bar{y})$ is an upper bound on v , the minimal cost solution of (*). The next step is to adjust the y_{ikt} so as to obtain a better solution. The best way to do this is yet to be determined by computer experimentation, but one approach is to take a descent step by looking at the gradient (strictly speaking, the sub-gradient) of v at \bar{y}_{ikt} ; namely,

$$\frac{\partial v(\bar{y}_{ikt})}{\partial y_{ikt}} = \alpha^{t-1} \left\{ (1-\alpha) \sum_{w=1}^{T-t} \alpha^{w-1} f'_{ik} \left(\sum_{z=1}^{t+w-1} \bar{y}_{ikz} \right) + \alpha^{T-t} f'_{ik} \left(\sum_{w=1}^T \bar{y}_{ikw} \right) - \Pi_{ikt} \right\}, \quad (1)$$

where $\Pi_{ikt} \geq 0$ is the optimal shadow price on the y_{ikt} constraint in (LP_t) and it measures the marginal cost reduction in (LP_t) per unit increase in supply. This partial derivative is negative if Π_{ikt} is sufficiently large to offset the discounted sum of present and future rates of increase of extraction costs.

Let $\bar{\gamma}$ denote the vector with the same dimensions as \bar{y} and with components $\bar{\gamma}_{ikt}$ equal to the right hand side in (1). This vector is a subgradient of v at \bar{y} and it equals the gradient of v if v is differentiable. It satisfies the inequality

$$v(y) \geq v(\bar{y}) + \bar{\gamma} (y - \bar{y}) \quad \text{for all } y, \quad (2)$$

or

$$v(y) \geq \bar{v}_0 + \bar{\gamma} y \quad \text{for all } y$$

where

$$\bar{v}_0 = v(\bar{y}) - \bar{\gamma} \bar{y}.$$

A systematic way to adjust the y_{ikt} is to use Benders' decomposition method (eg, see Lasdon (1970)) which decomposes (*) into the T subproblems (LP_t) and a master problem

$$\begin{aligned}
 v^L &= \min v \\
 \text{s.t. } v &\geq v_0^\ell + \sum_{i,k,t} \gamma_{ikt}^\ell y_{ikt} \quad \ell = 1, \dots, L, \\
 \sum_{i,k} y_{ikt} &= \sum_j d_{jt} \quad \text{for all } t && \text{(master)} \\
 0 &\leq y_{ikt} \leq r_{ikt} \quad \text{for all } i,k,t,
 \end{aligned}$$

where the inequalities on v have been computed from (2) at L previously generated points \bar{y} . The constraints on the sums of the y_{ikt} and d_{jt} insure that there is enough supply in each period to meet demand. The result of solving the master problem is a lower bound v^L on the minimal cost v in (*).

The \bar{y}_{ikt} optimal in the master are passed to the subproblems (LP_t) which are then optimized. If any of these subproblems are infeasible because of the sulfur content constraints, a constraint is generated on the y_{ikt} to be added to the master which prevents the infeasibility from occurring again. If all of the subproblems are feasible, then the \bar{x}_{ijkt} optimal in the subproblems, along with the \bar{y}_{ikt} , constitute a feasible solution. This solution is tested for optimality by computing the derivatives $f'_{ik}(\bar{y}_{ikt})$ for all i,k,t and by using the optimal shadow prices $\bar{\pi}_{ikt}$ on the \bar{y}_{ikt} constraints. If the solution is not optimal, then a new constraint on v is added to the master and it is reoptimized.

A related decomposition approach to the large scale coal supply model (*) is generalized programming. Pariente (1977) has used this method to compute optimal U.S. energy supply strategies based on an aggregate model somewhat similar to (*). It would appear, however, that Benders' decomposition is more appropriate for the particular structure of (*).

We have just described Zimmerman's coal supply model (*), and decomposition methods for solving it without reference to other energy models, such as BESOM. There are two perspectives to its integration with these other models. First, there is the need to incorporate inter-fuel substitution effects into the coal supply problem. Specifically, the demand constraints in (*) are for coal in various demand regions in various time periods. It is more natural to forecast energy end use demands over time, and let the fuels compete with one another on a price basis to determine the specific demands for coal. There are several means whereby the coal supply model (*) could be extended in this way. One approach is to add a gross income term to the objective function in (*) and then maximize net income rather than minimize cost. The gross income function in each time period could be estimated as a concave function derived by parametric analysis of BESOM or some other intertemporal aggregate model (the dynamic version of BESOM). We have experimented with this idea, again using decomposition methods, and found it promising (see Modiano and Shapiro (1978), Modiano (1978)). Operationally, the use of

a model like BESOM in this way could be accomplished by a straightforward extension of Benders' decomposition method. The idea is to add a third term relating to gross income to the piecewise linear functions in the master problem. Since BESOM is a national model, some regional disaggregation methods would be required to arrive at income figures for coal by region as well as by time period.

The second integration issue for the coal supply model (*) is to summarize and incorporate it into BESOM or its regionalized version (Goettle et al (1977)). A simple method for doing this is to do parametric analysis of (*) and then use the pseudo-data approach (Griffin (1977)) to summarize the results. We need to look more closely at this approach, and experiment with it, before we can be certain that it will work.

4. Conclusions

The sensitivity analyses given in section 2 indicate that BESOM is a tightly constrained model with the lumpy marginal cost structure often associated with linear programming models. This indicates the value and the need for incorporation of nonlinear supply models which may have the effect of permitting greater variation in supply levels. However, our experimentation in section 2 indicates that it is not sufficient to introduce alone a nonlinear coal supply model because the big jumps in shadow prices forces the supply of coal to 19×10^{15} BTU's even if a nonlinear supply model is used.

The central research issue in integrating nonlinear supply models is the problem of aggregating the results of intertemporal and interregional supply models in order to use them in aggregate energy sector models, and the reverse problem of disaggregating the interfuel substitution effects derived from energy sector models for use in the supply models. In section 3, we proposed some mathematical programming decomposition methods for overcoming some of the inherent difficulties. Other methods, such as the pseudo-data approach of Griffin (1977), should be tried and contrasted. In general, more research of a conceptual and experimental nature into this issue is required.

Finally, we wish to emphasize the importance of mathematical programming computational tools to the energy model construction and analysis discussed here. Flexible algorithms, mathematical programming modeling languages and interactive computation are basic necessities to significant progress in this area. The decomposition approach to the coal supply model (*), and its integration with BESOM are important practical applications which could be used as test cases for these tools.

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Appendix A: Explanation of shadow price changes for basic resources in BESOM.

As a prelude to the integration of nonlinear supply functions into BESOM, it was decided to investigate the behavior of the shadow prices for specific commodities. The one we looked at most closely was coal (variable RR22 in the model).

The following is an explanation of the changes associated with the price of coal. We will proceed in the direction of decreasing price and increasing quantity indicating at what price levels changes in quantity occur. Only the changes are indicated. Indentations represent the energy flow hierarchy of the model.

1. At a price of 99.00; Minimum coal use is fixed by the model.
 - a) Coal resource: RR22 = 2.4761
 - b) Gas from coal: 22GAS = 0.2000
 - c) Coal from coal: 22COA = 2.1667
 - d) Coal for iron ore reduction: TCOA = 2.1667

2. At a price of 3.2212; Coal is used for petrochemicals.
 - a) Coal resource: RR22 = 2.7273
 - b) Coal from coal: 22COA = 2.4167
 - c) Coal to petrochemicals: TCOA12 = 0.2500

3. At a price of 2.8597; Coal is used for miscellaneous thermal intermediate temperature processes.
 - a) Coal resources: RR22 = 5.0390
 - b) Coal from coal: 22COA = 4.7167
 - c) Coal for miscellaneous thermal intermediate temperatures: TCOA09= 2.3000

4. At a price of 1.5507; Coal is used for steam electricity.

a) Coal resource:	RR22 = 18.9947
b) Coal from coal:	22COA = 18.6026
c) Coal steam electricity:	FFCOA01 = 13.8860

At this price coal is first used for the production of electricity and less oil is used.

5. At a price of 1.1607; Coal use replaces some gas turbine production.

a) Coal resource:	RR22 = 19.1386
b) Coal steam electricity:	FFCOA01 = 14.0291
c) Petroleum for gas turbines	FFOIL05 = 0

At this price coal electricity replace oil fueled gas turbines for electricity.

*The price given for coal in the model 1.0200 falls here between the previous price 1.1607 and the next price of 0.4352.

6. At a price of 0.4352; Coal electricity replaces some oil for water heating.

a) Coal resource:	RR22 = 19.2414
b) Electric water heat:	TELE17:0.7500 → 0.7820
c) Oil water heat:	TOIL17:0.2008 → 0.1500

7. At a price of 0.3796; Coal plants replace nuclear plants for electricity.
- | | |
|-------------------------|-----------------|
| a) Coal resource: | RR22 = 27.2616 |
| b) Nuclear electricity: | TELEØ8 = 0.0000 |
8. At a price of 0.000; No change from 7 above.

- NOTES: (1) All quantities above are expressed in 10^{15} BTU (Quads).
- (2) All prices above are expressed in billions of dollars per quad, or equivalently dollars per million BTU's.
- (3) Because of the way the model is linked together a change in the bases will often produce a change in many variables. We have attempted to present here the significant ones.
- (4) Also the model is very tightly constrained. As an example, the only reason in step 1 that any coal is converted to gas at a coal price of 99.00 is because the model has rigidly fixed the amount of coal gasification and for iron ore reduction. Similarly, if the quantity of nuclear electricity remained fixed then there would be no change at all in the use of coal for any price below 0.4352, or no significant change below the price of 1.5507.

A similar analysis was done for natural gas; the results of which are shown in figure 2. The lumpy structure is apparent here as well although not quite as dramatic. In this case the price of natural gas can vary from 1.43 to 2.44 with no appreciable change in quantity demanded.

Shadow Prices for Natural Gas (RR19) in BNL8/77

