SYNOPTIC-SCALE WATER BUDGETS FOR QUANTITATIVE PRECIPITATION DIAGNOSIS AND FORECASTING

by

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ABSTRACT

In an attempt to examine the causes of persistently poor quantitative precipitation forecasts by dynamic models, routine soundings are used to evaluate the synoptic-scale water budget in a diagnostic study of New England rainfall. Precipitation minus evaporation is obtained as a residual from the sum of storage, advection, and divergence of water vapor in a column over an area determined by three radiosonde stations, about 10,000 km. The results are compared to observed precipitation, with RMS errors varying from about 5 mm in winter to 15 mm in summer. Days without observed rain give an indication of the noise level of the calculations, since the uncertainty in determining area averaged rainfall is not present. A typical noise level is 8 mm. A persistent upward motion is calculated at 100 mb, perhaps an indication of systematically non-representative data at a particular station. The divergence term is found to dominate the calculations, both in magnitude and expected error.

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Introduction

In spite of advances made in numerical forecasting by dynamic models, one type of forecast remains grossly inaccurate: quantitative precipitation forecasting. In a study made at the State University of New York at Albany connected with their forecasting game (Bosart, 1978) it was determined that the forecast of 24 hr precipitation amounts made for Albany by the LFM were often worse than climatology over a period of three months. Regression analysis indicated that in the mean the forecast by the LFM was too large by a factor of 1.6. In contrast, the same study indicated that the MOS forecast of minimum temperature for the next 24 hr period improved upon climatology by 25% or more. Similar results have been discovered at M.I.T. The intent of this paper is to provide a basis for the use of a kinematic model to examine the problems involved in forecasting precipitation.

Consider a volume of the atmosphere, with the surface of the earth as the bottom boundary. A knowledge of the water content of the volume, and the flux through the non-surface boundaries would be sufficient to determine the flux through the surface boundary, as rain and evaporation. The flux and storage ice and liquid water will not be considered for practical reasons, and because they are relatively small compared to water vapor. Several problems associated with the kinematic method itself will be discussed.

Inaccuracies are created by finite resolution. Ideally, one would desire measurements of water content at every point within the volume, and measurements of wind at all points on the boundaries. Instead, the only data available are from radiosonde stations spaced at approximately 200 km intervals. Furthermore, the time resolution is limited to two soundings at the beginning and end of a 12 hr period. This lack of resolution raises a question of how the model is to be applied. Should rainfall rates be determined by using a set of radiosonde soundings at a particular time, or should 12 hour amounts be determined by averaging the fluxes from the beginning and end of the period, and adding the change in storage? The former choice would be more accurate if storage changes were much smaller than fluxes, and small scale variations in wind and moisture were important. It will be shown that one of these conditions holds while the other does not. The second method of applying the kinematic model was chosen, although no determination was made about the relative accuracies of the two methods.

The calculation of precipitation minus evaporation (P - E) contains three terms: storage, advection, and divergence. The relative magnitude of each of these terms was determined. In addition to the problems associated with finite resolution, there are problems associated with the data. In an early study Charney (1948) stated that the observed winds are never accurate enough to calculate large scale divergence because the divergences of small scale features which influence the wind observations are often greater than the large scale divergence. This is a problem of non-representativeness of the data: In a synoptic scale study we want the winds to represent synoptic scale features. They often do not. This problem is associated with resolution: We can not differentiate between small and large scale components of the wind. Furthermore, the instruments in the radiosonde may not measure the wind, temperature and dewpoint accurately. The effect of these inaccuracies on the calculation of P - E will be considered. An attempt will be made to determine the relative importance of non-representative and inaccurate wind measurements.

Finally, results of the calculation of P - E by the model will be examined. Cases will be studied from 1977 and 1979 for southern New England. The standard deviation of the results is found to be large, indicating that the model's use lies mainly in statistical examinations, and that application of the model to individual cases is fraught with pitfalls.

The Kinematic Model

Showalter (1944) used a simple kinematic model to calculate the precipitation in river basins. After determining that southerly winds are required for rainfall in river basins, his method basically consisted of calculating the in flow of moisture on the southern boundary and the out flow at the northern boundary of the basin. Furthermore, he assumed that little moisture was transported above 10,000 ft in order to be able to use existing wind measurements. Calculating water transport is similar to calculating divergence, and Charney's gloomy diagnosis of such calculations was undoubtedly inhibiting to other investigators. Nevertheless, Landers (1955), in a case study, noted a good correlation between areas of convergence and areas of precipitation. To the extent that moisture convergence is associated with mass convergence, such results offer support to the use of the kinematic method.

The kinematic method is often used to calculate vertical motions, mainly by default, since the other two common methods, adiabatic and omega equation, cannot be used in certain cases. The calculations support Charney's conclusion. In this study, vertical motions of 500 mb/12 hrs at 100 mb were regularly encountered, enough to exhaust the atmosphere in a day. Such calculations are not realistic. In an attempt to solve this problem, some authors (e.g. Lateef, 1967) have developed methods for forcing the 100 mb vertical motion to be a prescribed value, often zero. There has been dispute about the correct method to use, but the simplest is to solve the pressure differentiated continuity equation, to which two boundary conditions can be applied. This method is equivalent to applying a constant correction to the divergence at all levels. This method was found to improve the upper troposphere vertical motions (Fankhauser, 1969), where the most unrealistic vertical motions were calculated, but reduce the accuracy of lower troposphere calculations. O'Brien (1970) suggested using a scheme which corrects the divergences at upper levels more than at lower levels, which would seem to preserve the accuracy of the lower troposphere calculations.

Burpee (1979) was able to calculate the advent of low level sea breeze convergence in Florida. While the magnitude of the convergence in this case is great due to the geography of the situation, such calculations indicate the problem of divergence calculations is not hopeless. Burpee did not calculate moisture convergence, and was unable to find a consistent relationship between convergence and precipitation. Consequently, including moisture in the calculations seems necessary to adequately examine precipitation.

The basis for the model is the integration of the water budget for an infinitesimal volume

$$W = -\frac{\partial q}{\partial t} - \nabla\cdot q\underset{\sim}{v} - \frac{\partial q_l}{\partial t} - \nabla\cdot q_l\underset{\sim}{v} - \frac{\partial q_i}{\partial t} - \nabla\cdot q_i\underset{\sim}{v}$$

q, q_l and q_i are the mixing ratios for water vapor, liquid water and ice respectively, $\underset{\sim}{v}$ is wind velocity and t is time. All quantities are dependent on pressure, time, and horizontal coordinates. The last four terms deal with liquid water and ice, and will be ignored for practical reasons. No routine measurements are made of q_l and q_i, so they cannot be considered on a regular basis. If the model were to be run on a scale the size of a thunderstorm the last four terms would often be important, since storage and transport of ice and water in thunderstorms is substantial. Their effects diminish on the synoptic scale. To determine the importance of ignoring liquid water, assume a value of 3 gm/m^3 liquid water is a cumulus cloud (Fletcher, 1962) of 5 km depth. This is about 15000 gms/m^2, or an equivalent depth of 15 mm. The value 3 gm/m^3 is an extreme case, and if by some miracle all of southern New England were covered with such moist clouds, one could at least also expect a relatively high value of precipitable water, say 50 mm. The liquid water in this case amounts to only 20%, and in a more realistic case would be much less.

The domains of the model conform to the synoptic scale. The region of horizontal integration is determined by three radiosonde stations. In New England, a typical spacing is 200 km, although the spacing is greater in other regions. The domain of the pressure integral is surface pressure to 100 mb. The time integral is over the 12 hr period between radiosondes.

To make the 12 hour time integration, no values of the variables other than the initial and final ones are available. Since only two pieces of information are available, we must assume linear variation with time. This may cause several problems. For example, the passage of a front is often accompanied by a sudden shift in wind direction.

The accuracy of representing such an event by a gradual wind shift over a 12 hour period is problematical. Small variations not associated with synoptic scale features are even more bothersome. Under static conditions, the wind observations at 00GMT and 12GMT often do not represent the actual winds near that time. Examination of surface hourlies reveal this problem. Furthermore, observations at 00GMT and 12GMT may not catch such effects as the sea breeze, commonly experienced in spring and summer in coastal locations. These problems cannot be rectified in any way using existing data. Using extra radiosonde observations at times surrounding the integration period in order to perform a higher order fit would be wrong because important variations occur on time scales shorter than 24-36 hours. Increasing the frequency of radiosonde observations would be useful, as will be shown more clearly below.

The situation with horizontal integration is similar. The horizontal boundaries of the integration are determined by three radiosonde stations. The four southern New England radiosonde stations determine four separate triangles. Within each triangle linear variation of the data must be assumed. In the case of $\underset{\sim}{v}$, Schaefer and Doswell (1979) point out that there is an ambiguity as to the form of that variation, e.g. whether u and v or r and θ are chosen to be linear. They describe a method by which an interpolated field can be created which preserves both the divergence and the vorticity of the original data. In this study the only explicit interpolation of $\underset{\sim}{v}$ is used to calculate an area mean $\underset{\sim}{v}$, and it is deemed sufficient to assume linear variation in u and v , the westerly and southerly components. Improvements on the data may be made by smoothing via map analysis to eliminate undesirable variations. The smoothing may remove relevant variations also, so its effectiveness is not certain. Small scale variations are also present, although less obvious than their analagous time variations, because no routine observations are taken at less than 20 km spacing. A valley may create this sort of effect at low levels, and a front at levels up to the mid-troposphere. Mesoscale or smaller waves will cause small scale variations in both time and space. Finally, the usefulness of doing the calculations for geometric figures formed from four or more radiosonde stations should be considered. Increasing the number of radiosonde stations within a given area is clearly desirable, but using more existing radiosonde stations increases the area. In an extreme case, we could use all the radiosonde stations in North America to get a good measurement of the average rainfall over the continent in a 12 hour period. The usefulness of such data is not clear.

Integration over pressure does not have the same limitations, since many observations are available between the surface and 100 mb. Fifty or more levels of wind reports, and thirty or more of thermodynamic reports are routinely made. These include reports at all levels at which significant changes in the thermodynamic variables occur. The high resolution makes linear interpolation between observations reasonable in this case. Interpolations to 1 mb intervals of pressure are made, and from these 50 mb averages are computed. The integration is performed over the 18 resultant layers, with the lowest layer, 950 mb - P_{SFC} given a weight of

$$\frac{P_{SFC} - 950}{50}$$

compared to a weight of 1. for the other layers. If the surface pressure is less than 950 mb, the values of the variables for pressures between P_{SFC} and 950 are taken to be zero. This gives the layer in which the surface pressure falls the proper weight with respect to the other layers.

Before attempting to integrate the water budget equation, it is useful to determine whether the mass continuity equation is satisfied for the mass of air in the volume. Observations and theory indicate that the vertical transport across the 100 mb surface is small, consequently the vertical motion at this level must be nearly zero. In pressure coordinates, the mass continuity equation is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial \omega}{\partial p} = 0$$

Integrating with respect to pressure yields:

$$\omega(p) = \omega_{SFC} + \int_{p}^{p_{SFC}} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \, dp$$

$$= \omega_{SFC} + \int_{p}^{p_{SFC}} \nabla_H \cdot \underset{\sim}{V}_H \, dp$$

To determine ω_{SFC}, expand

$$\omega_{SFC} = \left(\frac{dp}{dt}\right)_{SFC} = \left(\frac{\partial p}{\partial t}\right)_{SFC} + \left(\underset{\sim}{V_H} \cdot \nabla_H p\right)_{SFC} + \left(w \frac{\partial p}{\partial z}\right)_{SFC}$$

Of the three terms on the right hand side, $w \frac{\partial p}{\partial z}$, the orographic term, dominates. To show that the model correctly calculates this term, examine Figure 2. The darkly outlined area represents a volume of the atmosphere is sloping terrain. For simplicity, variables will be assumed constant in the y direction (into the paper). Since only the surface effects are to be examined, the divergence at all levels will be taken to be zero, so v_x = constant = v. Given a constant $\underset{\sim}{v}$, after a time t the air inside the solid volume will be displaced to the dashed volume. The flux into the solid volume, represented by the cross-hatched area, is $v \delta t \, (P_{SFC2} - P_{TOP})$. The outflow out the opposite side, the dotted area, is $v \delta t \, (P_{SFC1} - P_{TOP})$. The net inflow from the sides is $v \delta t \, (P_{SFC2} - P_{SFC1}) = v \delta t \, \frac{\partial p}{\partial x} \delta x$. This must be balanced by the outflow out the top, $\omega \delta t \delta x$. At the surface, $\underset{\sim}{v}$ is parallel to the surface, so $w_{SFC} = v_{SFC} \frac{dz}{dx}$. Also, at the surface, $\frac{\partial p}{\delta x} = \frac{\partial p}{\delta z} \frac{dz}{dx}$. Thus

$$\omega \delta t \delta x = v \delta t \frac{\partial p}{\partial x} \delta x$$

$$\omega = v \frac{\partial p}{\partial x} = \frac{w_{SFC}}{\frac{dz}{dx}} \frac{\partial p}{\partial z} \frac{dz}{dx} = w \frac{\partial p}{\partial z}_{SFC}$$

This ω is due only to surface effects, since the wind was assumed nondivergent. In addition, the smaller pressure tendency has been approximated as:

$$\frac{1}{3} \cdot \frac{P_2^{\,1} + P_2^{\,2} + P_2^{\,3} - P_1^{\,1} - P_1^{\,2} - P_1^{\,3}}{12 \text{ hours}}$$

where the subscript refers to time, and the superscript to station (vertex). Therefore, the orographic term is included implicitly in the calculation of mass flux, and the pressure tendency is calculated explicitly.

In this case, the desired quantity is actually the area averaged $\omega(p)$.

$$\frac{1}{A}\int_A \omega(p)\,dA = \frac{1}{A}\int_A \left(\frac{\partial p}{\partial t}\right)_{SFC} dA + \frac{1}{A}\int_A\int_p^{p_{SFC}} \nabla_H \cdot \underset{\sim}{v}_H \, dp\, dA$$

This amounts to integrating the divergence over a volume extending from p_{SFC} to p, covering an area A. As a preliminary to doing this integration, 50 mb averages of $\underset{\sim}{v}$ are formed. The order of integration is switched on the divergence term. For triangular areas with linearly varying velocity components, the divergence can be calculated by a method described by Bellamy (1949) (see Appendix 1). The result is:

$$\nabla \cdot \underset{\sim}{V} = \frac{V_{n1} \ell_1 + V_{n2} \ell_2 + V_{n3} \ell_3}{2A}$$

where v_{n1} is the component of velocity at vertex #1 normal to the opposite side, and ℓ_1 is the length of that side. If the pressure at which the divergence calculation is being made is below the surface at any station, v_n is assumed to be zero at that station. This accounts for the effects of location dependent surface pressure. In order to calculate the vertical velocity at any level, simply sum the surface pressure tendency and the divergences for all pressures below that level.

Vertical motions at 100 mb calculated in this manner invariably differ significantly from zero. In extreme cases, the calculated motions would be sufficient to exhaust the atmosphere in 12 hours. Clearly, such a situation is neither realistic nor desirable. The sources of this difficulty are manifold. A primary problem is error in measurement of winds. Such error may be related to low elevation angles, which in turn are caused by high wind velocities, and large distances between the balloon and observer (Duvedal, 1962). Both causes are generally greater at lower pressures, which would imply that errors in measurement are more important at high altitudes. Small scale variations in wind speed and direction are also present. These variations cause the measured winds to be non-representative of the synoptic scale wind field. This effect is conceivably more important than the previous one, and may even be significant at higher levels.

In order to correct the erroneous 100 mb vertical motion (ω_{100}) caused by these effects, a correction wind field $\underset{\sim}{v}_c(x,y,p,t)$ can be created such that:

$$\int_{100}^{p_{sfc}} \nabla \cdot \underset{\sim}{v}_c(x,y,p,t)\, dp = \omega_{100}(t) \tag{1}$$

This wind field can be subtracted from the actual winds, forcing $\omega_{100} = 0$. The most desirable form of $\underset{\sim}{v}_c$ is debatable. O'Brien (1970) suggests that the correction at each layer increases linearly with decreasing pressure, because he believes the errors in $\underset{\sim}{v}$ are greater at higher levels. We have chosen to use a constant $\underset{\sim}{v}_c$, since the relative magnitudes of the two sources of error described in the preceeding paragraph are not known. In order to apply the constant $\underset{\sim}{v}_c$ in this study, we have used what would be more accurately described as a constant magnitude $\underset{\sim}{v}_c$, which is irrotational. (1) reduces to

$$\int_{100}^{p_{sfc}} \nabla \cdot \underset{\sim}{v}_c(t)\, dp = \omega_{100}(t) = (p_{sfc} - 100)(\nabla \cdot \underset{\sim}{v}_c(t))$$

To find $\nabla \cdot \underset{\sim}{v}_c$, again resort to Bellamy. $\underset{\sim}{v}_c$ was chosen such that at every vertex it is normal to the opposite side.

$$\nabla \cdot \underset{\sim}{V_c} = \frac{V_c l_1 + V_c l_2 + V_c l_3}{2A} = \frac{V_c P}{2A}$$

(P is length of perimeter)

$V_c = |\underset{\sim}{V_c}|$

Thus,

$$V_c(t) = \frac{\omega_{100}(t)}{(P_{SFC} - 100)} \frac{2A}{P},$$

the orientation of $\underset{\sim}{v_c}$ at each vertex is determined by the geometry of the triangle, that is, $\underset{\sim}{v_c}$ is always normal to the opposite side. The corrected wind field $\underset{\sim}{v} - \underset{\sim}{v_c}$ can then be used instead of $\underset{\sim}{v}$ to calculate the moisture budget. This result can be compared to the result using raw data.

The integration of

$$W = -\frac{\partial q}{\partial t} - q \nabla \cdot \underset{\sim}{v} - \underset{\sim}{v} \cdot \nabla q \qquad (2)$$

is similar to the integration of the mass continuity equation. At this point a note should be made about the method of integrating the equation. Using Bellamy's method of calculating divergence, the value obtained is ($\overline{Q}$ is area averaged Q)

$$\overline{\nabla \cdot q \underset{\sim}{v}} = \overline{q}\,\overline{(\nabla \cdot \underset{\sim}{v})} + \overline{\underset{\sim}{v}} \cdot \overline{\nabla q} + \overline{q'(\nabla \cdot v')} + \overline{v' \cdot \nabla q'} \qquad (3)$$

where $\mathbf{v} = \bar{\mathbf{v}} + \mathbf{v}'$ and $q = \bar{q} + q'$.

On the other hand, separating the terms prior to integration, as was actually done, produces (see below)

$$\overline{\nabla \cdot q\mathbf{v}} \approx \bar{q}\,\overline{(\nabla \cdot \mathbf{v})} + \bar{\mathbf{v}} \cdot \bar{\nabla} q \qquad (4)$$

The difference, the final two terms in (3), is not necessarily due to a lack of accuracy in (4) but to a difference in assumptions. In (3), the product $q\mathbf{v}$ is assumed to be linear, while q and $\mathbf{v}$ are each linear in the derivation of (4). The most accurate assumption of the two has not been determined. The second method was chosen because the divergence and advection terms (the RHS of (4)) are calculated, while $\overline{\nabla \cdot q\mathbf{v}}$ is calculated directly using the first method. The relative importance of these two terms can then be examined. Each of the three terms on the RHS of (2) will be dealt with separately. First, note the following conventions: Subscripts 1 and 2 refer to initial final times respectively, so $Q_z = Q(tz)$. Since all quantities are linear in x and y , triangular area averages are simply the average of the values at the vertices, so

$$\bar{Q} \equiv \frac{1}{A}\int_A Q(x,y)\,dx\,dy = \frac{1}{3}\left(Q^1 + Q^2 + Q^3\right)$$

where superscripts are the index of the station position, or vertex.

1 Storage change

$$\int_{\text{time}} \int_{\text{area}} \int_{\text{pressure}} \frac{\partial q}{\partial t}\, dp\, dA\, dt$$

$$= \int_T \int_A \int_P \frac{q_2(x,y,p) - q_1(x,y,p)}{t_2 - t_1}\, dp\, dA\, dt$$

since q is linear in time.

Now do the area integral:

$$= \int_T \int_P \left[\frac{\int_A q_2(x,y,p)\,dA - \int_A q_1(x,y,p)\,dA}{t_2 - t_1} \right] dp\, dt$$

$$= \int_T \int_P \frac{\overline{q_2}(p) - \overline{q_1}(p)}{t_2 - t_1}\, dp\, dt$$

Next do the pressure integral

$$\int_P \overline{q_2}(p) \cong \sum_{i=1}^{18} WF_i \cdot \Delta p_i \cdot \overline{q_2}(p_i)$$

$$\equiv \overline{\overline{q_2}}$$

where: $p_i = 1000 - i \cdot 50$

$$\Delta p_i = \begin{cases} 50 & \text{if } i = 1 \\ \min(50, p_{sfc} \dot{-} p_i) & \text{otherwise} \end{cases}$$

$$\left(a \dot{-} b \equiv \begin{cases} a - b & \text{if } a > b \\ 0 & \text{if } a \leq b \end{cases} \right)$$

weighting factor $WF_i = \begin{cases} 1 & \text{if } i = 2, 18 \\ \dfrac{p_{sfc} \dot{-} 950}{1000 - 950} & \text{if } i = 1 \end{cases}$

and $\bar{q}_2(p_i)$ is the average $q_2(p)$ for the layer

$p = [p_i, p_i + 50 \cdot WF_i]$

So,

$$\iiint_{T\,A\,P} \frac{\partial q}{\partial t} \, dp \, dA \, dt = \int_T \frac{\bar{q}_2 - \bar{q}_1}{t_2 - t_1} \, dt$$

$$= \frac{\bar{q}_2 - \bar{q}_1}{t_2 - t_1} (t_2 - t_1)$$

$$= \bar{q}_2 - \bar{q}_1$$

2 Divergence

Since $\underset{\sim}{v}$ is linear in x and y , $\nabla \cdot \underset{\sim}{v}$ is constant over the area of the triangle. Thus:

$$\iiint_{T\,A\,P} q\, \nabla \cdot \underset{\sim}{v}\, dp\, dA\, dt = \iint_{T\,P} \overline{\nabla \cdot v}(p,t) \left[\int_A q(x,y,p,t)\, dA \right] dp\, dt$$

$$= \iint_{T\,P} \overline{\nabla \cdot \underset{\sim}{v}}(p,t)\, \bar{q}(p,t)\, dp\, dt$$

from Bellamy,

$$\overline{\nabla \cdot v}(p,t) = \left[\frac{V_n^1 l^1 + V_n^2 l^2 + V_n^3 l^3}{2A} \right] A$$

(The multiplication by A on the right side is because the $\overline{\nabla \cdot \underset{\sim}{v}}(p,t)$ desired is area integrated, not area averaged.

$$= \frac{1}{2} \sum_{i=1}^{3} V_n^i l^i \equiv Div(p,t)$$

V_n^i is the component of v normal to the side opposite vertex i

l^i is the length of that side

Next do the pressure integral

$$\int_P Div(p,t) \cdot \bar{q}(p,t)\, dp$$

$$= \sum_{i=1}^{18} \bar{q}(p_i,t) \cdot Div(p_i,t) \cdot \Delta p_i \cdot WF_i$$

$$\equiv DIV(t)$$

with p_i, Δp_i, WF_i as before.

Now do the time integral.

$$\int_T DIV(t)\, dt = \frac{DIV_2 + DIV_1}{2} \cdot 12 \text{ hours}$$

This term represents the divergence of water vapor from the volume integrated over the twelve hour time period.

3 Advection

In the advection term, q is linear in x and y, so ∇q is constant over the area of the triangle.

Thus:

$$\iiint_{T\,A\,P} \underset{\sim}{V} \cdot \nabla q \, dp\, dA\, dt$$

$$= \iint_{T\,P} \left[\int_A \underset{\sim}{V}\, dA\right] \cdot \overline{\nabla q}\, dp\, dt$$

Split this integral into x and y components:

$$= \iint_{T\,P} \left\{ \left[\int_A V_x\, dA\right] \overline{\frac{\partial q}{\partial x}} + \left[\int_A V_y\, dA\right] \overline{\frac{\partial q}{\partial y}} \right\} dp\, dt$$

All quantities are functions of t and p. V_x and V_y are also functions of x and y. $\overline{\frac{\partial q}{\partial x}}$ and $\overline{\frac{\partial q}{\partial y}}$ are calculated from q^1, q^2 and q^3 in the following manner. Assume the x' axis is the longest side of the triangle. Then $\frac{\partial q}{\partial x'}$ is simply the difference of q at one endpoint of that side minus q at the other endpoint, divided by the length of that side (with proper sign). The value of q is determined at the intersection of the altitude and the side by linear interpolation, subtracted from the value of q at the third vertex, and divided by the altitude (and given proper sign). This is $\frac{\partial q}{\partial y'}$. Now perform an orthogonal rotation transformation on the computed derivatives, namely, $\nabla q = A \nabla' q$, where ∇' is the grad operator in the primed coordinate system, and A is a matrix such that

$$\underset{\sim}{x} = A \underset{\sim}{x}'$$

($\underset{\sim}{x}$ is the position vector.)

After doing the area averaging of the velocities, the advection term is

$$\int_T \int_P \bar{V}_x \overline{\frac{\partial q}{\partial x}} + \bar{V}_y \overline{\frac{\partial q}{\partial y}} \, dp\, dt$$

Now do the pressure integral

$$\sum_{i=1}^{18} \left[\bar{V}_x(p_i,t) \overline{\frac{\partial q}{\partial x}}(p_i,t) + \bar{V}_y(p_i,t) \overline{\frac{\partial q}{\partial y}}(p_i,t) \right] \cdot \Delta p_i \cdot WF_i \equiv GRAD(t)$$

Finally, do the time integral

$$\int_T GRAD(t)\, dt = \frac{GRAD_1 + GRAD_2}{2} \cdot 12 \text{ hours}$$

This term represents the advection of water vapor into the volume, integrated over the twelve hour time period.

The computer program written to perform the above computations has been included as an appendix.

Data

The data used in this study came from two time periods. Real time studies were done in the spring of 1979 to determine the accuracy of the model; these data have been assembled to form one group for statistical analysis. Data archived at the National Center for Atmospheric Research were used to run the model throughout 1977. These results form another group.

Data used in the model calculations is collected by radiosondes from the four stations surrounding southern New England: Portland ME, Chatham MA, Albany NY, and Fort Totten NY (nominally JFK Airport) (Figure 1). These stations were chosen because the distance between stations is less than other locations, and because the investigation was done at M.I.T., so southern New England was a particularly interesting location. Soundings are made at each of the four stations at 12 hour intervals, at 00GMT and 12GMT. The model requires observations from three stations at two successive times. There are four possible triangles in southern New England, referenced by the three initials of the stations used, e.g. APC.

The soundings include observations from the surface up to 100 mb. Wind speed and direction, temperature, and dew point temperature are determined at standard levels SFC, 1000, 850, 700, 500, 400, 300, 250, 200, 150 and 100 mb. For the purpose of this study, the sounding was assumed to be missing if data from 100 - 200 mb was not present. This is because the lowest pressure at which data was reported was assumed to be the "top" of the atmosphere, where the vertical motion was forced to be zero. This is a very reasonable procedure if the "top" of the atmosphere is 100 mb, marginally reasonable if the tip is 200 mb, and not reasonable below that. Further thermodynamic data is reported at levels where significant deviations from linear variation between standard levels occur, and winds are reported at specified heights, multiples of 100 ft above sea level. The 1974 data was transcribed directly from teletype to computer cards; in order to facilitate the real time studies only the standard levels were used with a consequent reduction of resolution.

Wind directions are reported to the nearest 5°. Wind speeds are reported to the nearest knot. Temperatures are reported to the nearest 2°C. Dewpoint temperatures are reported to the nearest .1°C if T - Td 5°C , and to the nearest 1°C otherwise. When the relative humidity is less than 20%, no dewpoint is reported. In such cases a nominal relative humidity of 20% has been assumed, although the temperatures involved are so low that 20% relative humidity corresponds to very little water vapor.

The 1979 data were examined for errors by comparison to surrounding data in time and space, and to map analyses whenever available. The great volume of data precluded a detailed synoptic evaluation of every wind and temperature. Hydrostatic checks were done on the 1977 data before it was archived, and the data were used as archived in this study. Undoubtedly in a great number of cases obviously inaccurate observations could have been corrected by careful examination. For the purpose of this study examining a large number of cases with correctable errors was deemed superior to dealing with a small number of cases with no obvious errors. Clearly if the model is to be applied to an individual case all efforts must be taken to make corrections to the data, especially the winds, when a synoptic analysis of the situation warrants them.

Incomplete and missing soundings posed a constant problem. Out of 730 possible cases in 1977 for APC, in only 479 did all six necessary soundings extend to 200 mb. Performance for the other three triangles was worse. APC was also the most reliable triangle in 1979. There is a danger that this sample may be biased, in the sense that the radiosondes may fail more often on rainy days than on non-rainy days. This sort of bias is not necessarily bad, although it would tend to enhance the already great inbalance between the number of non-rainy and rainy cases. There is no hard evidence to suggest such a bias, but from many abortive attempts to run the model on interesting rainy day cases, I believe that this bias does exist.

Probability of Detection

The ideal scenario for use of this model is a large scale feature which encompasses the entire volume, and does not change rapidly over a 12 hour period. Such a situation is unusual in New England, especially in summer. When any discernible feature is smaller in scale than the sides of the traingle, and/or 12 hours, there is a finite possibility that it will not be detected by any radiosonde. A thunderstorm is an example of this type of feature.

According to Gleeson (1959), the probability that one station will detect the effects of a feature of area S is

$$P = 1 - [\, 1 - S/R \,]^{N}$$

R is the area containing N randomly distributed stations. In the case of this study, the radiosonde stations are not randomly distributed, but are located at the vertices of a triangular area. This may make the effective R greater than the area of the triangle. This result can be extended to include the probability that the radiosonde will detect a time dependent feature of duration t as follows. If the time period is T , then the probability that a given radiosonde will detect the feature is $\frac{S}{R}\frac{t}{T}$, the probability that it will be at the proper place at the proper time. The probability of non-detection is $1 - \frac{S}{R}\frac{t}{T}$. The probability that the feature will not be detected by any station at a given time is $(1 - \frac{S}{R}\frac{t}{T})^{N}$ for N stations, and the probability of non-detection at any of M times is $(1 - \frac{S}{R}\frac{t}{T})^{NM}$. Thus, the probability of detection is

$$P = 1 - [\,1 - (S/R)(t/T)\,]^{NM}$$

where T is 12 hours, and M is 2 representing 2 observations within the 12 hour period. Again, since these observations are at the end points of the 12 hour period, the effective T may be greater than 12 hours. Increasing R and T have the effect of decreasing the probability of detection, if S and t are not increased proportionally.

In order to detect a type of feature in 80% of the cases:

$$(1 - \frac{S}{R}\frac{t}{T})^6 < .2$$

$$1 - \frac{S}{T}\frac{t}{T} < .76$$

$$\frac{S}{R}\frac{t}{T} > .24$$

if $S = R$ then $t > .24T$

if $t = T$ then $S > .24R$

or $t > 166$ minutes

and $S > 2300\ km^2$ (corresponding to a diameter of 54 km)

Since larger features are generally longer lived, both conditions usually must apply simultaneously. Clearly no individual thunderstorm will be detected with 80% certainty. For 50% certainty, the values are $t > 72$ min, $S > 10^3\ km^2$ (36 km diameter). This is also much larger than a thunderstorm. If a thunderstorm has an area of 20 km^2 and a duration of 20 min, the chance of detection is 10003, virtually no chance at all.

This is a rather pessimistic result, although there are mitigating aspects to the problem. Thunderstorms usually occur in groups; and the group covers an area much larger than any individual storm. Furthermore, features which generate thunderstorms, such as fronts and squall lines, endure several hours, or even days. The problem then reduces to determining how much area at any time is covered by thunderstorms, and how many of the 12 hours that area is covered. From this point of view, detecting effects of thunderstorms is not a hopeless problem. Furthermore, even if a thunderstorm is not detected directly, it would have the effect of reducing the average mixing ratio in the volume. If the air is mixed rapidly enough, this may be detected at one or more of the stations, and influences the result through the storage term.

The truth of the matter is that we probably do not want to detect a thunderstorm directly. Sending a weather balloon in or near a thunderstorm would cause measurement of values of q and $\underset{\sim}{v}$ which are not synoptically representative, exactly what we are trying to avoid. The best chance of incorporating thunderstorms is through the storage term. Alternatively, the problem could be viewed as trying to detect the edge or tongue of a large scale feature which happens to encroach on the triangle. Such an encroachment must occur along the boundary of the triangle, just where the radiosonde stations are located. The prospects for detecting this type of feature are good. Finally, take the same thunderstorm from above, and try to detect it with a network of

30 rain gauges observing continuously. The probability of detection is .06, also very slim. The only good way to detect a thunderstorm is through storage, the model may actually do a better job than the rain gauges!

Rainfall Rates

In order to verify the computations of precipitation made by the model, measurements of actual precipitation were necessary. Obtaining these amounts is not a trivial problem, since the spatial correlation of rainfall amounts is often extremely low, and the network of rain gauges in southern New England is sparse. The actual amounts for all 1 hour periods in 1977 were determined by weighted averages of the 1 hour amounts reported by 70 rain gauges in southern New England. Since 1 hour resolution is not possible in the model, 12 hour totals were obtained by summing the 1 hour amounts. Real time studies were done with the 1979 data, and a simpler scheme was used to determine the rainfall amounts. Six hour observations were added for the 20 synoptic stations in southern New England, and averaged without weighting. The resulting amount favors certain areas in terms of number of stations reporting, e.g. southern Connecticut and eastern Massachusetts. Furthermore, the rainfall amounts apply to the entire quadrilateral APCJ, not to any individual triangle. This problem, plus the problem of detection discussed in the previous section, cast some doubt on the accuracy of the rain gauge data. So while rain gauge precipitation will often be referred to as "actual precipitation" in the following paragraphs, observed precipitation is a much more accurate term.

Budyko (1974) gives a figure of 320 mm/year for evaporation in North America. This works out to about 1 mm/day. He also states that evaporation is proportional to vertical mixing ratio and temperature gradients, and that terms containing these variables are greater during days and in summer. The motivation for adding estimates of evaporation to the rain gauge data is to provide a correspondence to the model's ability to calculate negative P - E. For this reason, a parameterization was chosen as an attempt to minimize the effects of the calculation of evaporation by the model. It is difficult to ascertain whether the 1 mm/day figure applied to New England. An approximate figure of about 2 mm/day was used because it was close to both Budyko's figure, and the mean evaporation calculation made by the model. The parameterization differentiated night from day: At night evaporation is considered to be 0 if any precipitation was observed, and to vary sinusoidally from 0 in winter to .5 mm in summer. The daytime rate was also assumed to be 0 if rain was observed, and to vary sinusoidally from 1 mm to 3 mm from winter to summer. It was found that all of these values were well within the noise level of the model (e.g. 7 mm), so the effect of adding them is most likely minimal. Furthermore, no difference was discovered between daytime and nighttime evaporation by the model, so including that in the parameterization was ineffective!

The results of the model on days without rain are important. They are nominally a determination of evaporation amounts, but since there is no uncertainty in the amount of actual precipitation, the only uncertainty is the amount of actual evaporation. Since this uncertainty is small the calculation of P - E by the model is virtually all noise. It will be shown that the greatest error in the computation of P - E is the error in calculating $\nabla \cdot v$. Unless the divergence is significantly different (in the sense of being harder or easier to calculate) on days without rain and days with rain, the noise determined for no rain cases applied to all cases.

The results for the evaporation amounts have been summarized in Table A. Uncorrected (unc.) refers to calculations made with raw winds, corrected (corr.) to calculations made with winds corrected by a constant magnitude wind field in order to force $_{100}$ to be 0. The standard deviations range from 5.81 mm to 11.75 mm. The latter is from a sample of only 41 cases. The noise levels of 7.4 mm and 9.2 mm for APC in 1977 are probably representative. With only one minor exception, the mean evaporation results of corrected data are algebraically less than the uncorrected data. This is the result of a constant upward motion at 100 mb in the uncorrected data for the no rain cases. This spurious upward motion is less intense for the triangle PCJ, indicating some recurrent problem with reports from Albany, causing convergence. If this spurious vertical motion is due solely to the Albany data, a non-representative WNW to NW component is indicated. The corrected winds give better results for evaporation in 4 of the 8 cases (i.e., negative as opposed to positive values, or smaller positive values), and uncontrovertibly worse results in only one case (CJA 77, 4.6 mm is too much). The corrected data also has smaller standard deviations in six of the eight cases, one of the exceptions is the statistically unreliable JAP 1979. The noise levels indicated by the standard deviations show that inaccurate determination of actual evaporation is not of paramount importance in verification of the model.

Even if the area averaged rainfall rate at 00GMT and 12GMT could be determined exactly by the model, there would be an error in the 12 hour amount due to the fact that the rates at the two end points do not represent the rates at intermediate hours. The assumption of linearity necessitated by having only two calculations of the rainfall rate within the 12 hour period is often inaccurate. A study by Hall and Suhickedanz (1973) indicated that lag correlations for rainfall rates was less than .1 for lags of 15 minutes or more. The time variation is likely to be extremely nonlinear, and values at times within the 12 hours may not be related to end point values. For example, in a case when rainfall is found at the beginning of the period but not at the end, an exponential decay may be a more accurate use of the two points than a linear decay. Other nonlinear variations are likely, and attempts to simulate all of them by a single combination of the two end points would be futile.

Since hourly amounts, area averaged over the quadrilateral APCJ, are available for 1977, an objective study of this effect is possible. Assume that the hourly amount for 00GMT represents the rate at 00GMT, and similarly for 12GMT. This assumption must be tempered with the observation that the lag correlation for 15 minutes is small, so the hourly amount most likely does not represent the rate at all times within that hour. On the other hand, the fact that the radiosonde does not rise to 100 mb instantaneously, and instead takes about an hour, means that the rates calculated by radiosonde data in the model are more representative of average hourly rates anyway.

The results which the model would calculate if rates were exactly determined can be simulated by adding the 00Z and 12Z amounts and multiplying by 6. Better accuracy may be achieved by having observations at the midpoint of the 12 hour period also. There would then be the option of assuming linearity between two successive pairs of observations, the trapezoidal rule, and weighting the rates 1-2-1, or assuming a quadratic variation and weighting the rates 1-4-1 (Simpson's rule). Results are tabulated in Table B. The Simpson's rule calculation gives superior results in 50% of the cases, and both three point calculations are within 20% of the actual 12 hour amounts in 47% of the cases. The two point calculation is within 20% in only 17% of the cases, and leans toward over predicting in measurable rain cases. The root mean square error is halved by adding the midpoint data. The standard deviation corresponding to RMC 4.13 mm and near 1.02 mm is 4.03 mm. This may be the cause of a large portion of the 7mm standard deviation observed in the evaporation study. While these numbers may not be directly related, the indication is that the addition of 6 GMT and 18 GMT radiosondes would greatly improve the results of the model.

Magnitude of Errors

Given the equation $P - E = -\frac{\partial q}{\partial t} - \bar{q}\nabla \cdot \underset{\sim}{v} - \bar{\underset{\sim}{v}} \cdot \nabla q$ it is useful to determine the effects of non-representative values of $\underset{\sim}{v}$ and q , either through instrumental error, or small scale variations. For the purposes of this discussion $\bar{q}$ will be considered to be the equivalent depth of water vapor in the column above a particular area, ∇q will be the variation of that equivalent depth within that area. Similarly, $\bar{v}$ is veolcity averaged throughout the column and $\nabla \cdot v$ the vertically averaged divergence within the column. The error in P - E is due to errors in the determination of either $\underset{\sim}{v}$ or q . Each will be considered separately. Finally, all of the computations are made with the intent of assessing the relative magnitude of the errors induced, not the actual magnitudes. All the calculations will be made assuming that the worst possible errors occur; for example, winds too fast at all levels, q too low at all levels. While such circumstances sometimes occur due to instrument malfunction, a more likely condition is that some of the errors compensate each other, and others are smaller than the assumed values.

To find the effect of errors in the measurement of v , assume that q is accurately determined. This means that the storage term $\frac{\partial q}{\partial t}$ is exact, and does not contribute to the error. Let E(Q) be the error in quantity Q , and assume E(ab) = aE(b) + bE(a) , i.e. E is linear. Now determine the error necessary in the divergence term to produce an error of 10 mm/12 hrs.

$$E(\bar{q}\ \nabla\cdot v) = 10\ mm/12\ hrs$$

assume $\bar{q}$ = 20 mm, a typical spring value, and $E(\bar{q})=0$

then $E(\nabla\cdot v) = .5/12\ hrs.$

$$E\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right) = 1.16 \times 10^{-5} s^{-1}$$

Now dx and dy are about 200 km for New England radiosonde stations, so

$$E(\delta v_x) \approx 1.64\ m/s\quad (1.16 \times \sqrt{2} \times 200000\ m \times 10^{-5})$$

Now $\delta v_x = v_{x_2} - v_{x_1}$, and errors in determining V_{x_2} and V_{x_1} are similar, so:

$$E(v_x) = .82\ ms^{-1}$$

The implication is that an error in measuring $\bar{v}$ of less than 1 m/s is enough to affect the kinematic results by 1 cm/12 hrs. This would seem to be a catastrophic effect, since the winds are recorded to the nearest m s^{-1}, and thus only accurate to within 1 to 2 m s^{-1}. This does not necessarily mean the error in $\bar{v}$ is that large, but it is clear that errors in the divergence term are extremly important.

For the advection term,

$$E(\bar{v}\cdot\nabla q) = 10\ mm/12\ hrs$$

Again assume q is accurately determined. Set $\nabla q = \frac{20 \text{ mm}}{200 \text{ km}} = 1 \times 10^{-6}$

Then

$$E(\bar{v}) = \frac{10 \text{ mm}/12 \text{ hrs}}{1 \times 10^{-6}} = \frac{1.16 \times 10^{-6} \text{ cm s}^{-1}}{1 \times 10^{-6}} = 2.32 \text{ ms}^{-1}$$

This is much larger than the divergence term result, indicating that a larger error in wind measurement is required to create a similar error in the advection term. Thus, errors in v do not affect the results through the advection term as much as through the divergence term.

The same type analysis can be done to evaluate the effect of error in determination of q . If the result is wrong by 10 mm/12 hrs, then $\bar{q}$ must be wrong by about 5 mm at each time if the storage term is at fault. This is about 25% of the total amount. Temperatures are reported to .2°C, dewpoints to .1°C. Given these conditions, it is unlikely that instrumental uncertainty could account for the entire error. Most of the 5 mm must be accounted for by inaccurate calibration of the instruments, or small scale variations.

In the divergence term,

$$E(\bar{q}\, \nabla \cdot v) = 10 \text{ mm}/12 \text{ hrs}$$

Assume $E(\nabla \cdot v) = 0$, so $E(\bar{q}) = (10 \text{ mm}/12 \text{ hrs})/\nabla \cdot v$

A reasonable Δv is 20 m/s in 200 km, so $\nabla \cdot v \approx 1 \times 10^{-4} s^{-1}$

Thus

$$E(\bar{q}) = \frac{10\ mm/12\ hrs}{1 \times 10^{-4} s^{-1}} = 2.31\ mm$$

This term is of greater importance than the storage term.

In the advection term,

$$E(\bar{v} \cdot \nabla q) = 10\ mm/12\ hrs$$

Use $\bar{v}$ = 20 m/s, a modest estimate. Then

$$E(\nabla q) = 1.16 \times 10^{-7}$$

$$E(\delta q) = 1.16 \times 10^{-8} \times 200\ km$$

$$= 2.32\ mm$$

and $E(\bar{q})$ is about 1.16 mm if the magnitude of error in both determinations of q is the same. This is the most important of the three terms.

To summarize, errors in determination of $\underset{\sim}{v}$ affect the divergence term most, and errors in q affect the advection term most. The magnitude of the error in $\overline{\underset{\sim}{v}}$ necessary to cause an error in P - E of 10 mm is entirely within the bounds of reasonably expected errors of any indivudual measurement of $\underset{\sim}{v}$. We must count on compensating errors, which are luckily present in balloon soundings, where errors at one level are often balanced by opposite errors at the next level. Nevertheless, the divergence term is the most likely of the three terms to be the source of an important error.

An observation can be made using the calculations of the model. When the velocity is corrected by a certain amount to make the vertical motion equal to 0 at 100 mb, the added velocity is similar to an added (or subtracted) error. In most cases the velocity correction is about .2 m/s. When the correction is near 1 m/s, the resulting change in P - E is often about 10 mm. This is approximately the error one would expect if the winds were in fact in error by 1 m/s.

In order to examine the error from another point of view, consider the calculation of P - E on days without reported precipitation. The standard deviation of these measurements can be considered to be noise, as described above. Since we have discovered that errors in the divergence term due to uncertainty in $\underset{\sim}{v}$ dominate other errors, assume this type of error is the only error. In 1977, for triangle APC, there are 20 no rain cases, with standard deviation 7.40 mm, and the mean value of $\bar{q}$ is 15.5 mm. Using the error function,

$$E(\nabla \cdot v) = \frac{7.4 \text{ mm}/12 \text{ hrs}}{15.5 \text{ mm}} = 1.11 \times 10^{-5} s^{-1}$$

A separate indication of $E(\nabla \cdot \underset{\sim}{v})$ can be determined by examining the RMS 100 mb vertical motion calculated using raw data. For the 200 cases. it is 492.44 mb/12 hrs. This is equivalent to an error in $\nabla \cdot \underset{\sim}{v}$ of $1.27 \times 10^{-5} s^{-1}$. Thus, the error in P - E is close to, but not quite as large as, the error indicated by mass divergence. The RMS 100 mb vertical motion above corresponds to a velocity correction of

1 m/s, and represents a few large errors and many small ones. Unless we count on compensating errors, not a futile hope, the results represent the limit of the accuracy of the model.

Results

According to Duvedal (1962) wind measurements from a GMD-1A system deteriorate with decreasing elevation angles. This may be the result of strong winds and sounding duration. These effects often combine to cause low elevation angles at high altitudes. Consequently, errors in wind measurements are greatest at low pressures. O'Brien (1970) suggests that this is good reason to adjust the winds at higher altitudes by a larger amount than at lower altitudes by using a divergence correction that increases linearly with decreasing pressure.

An additional point in favor of such a correlation scheme is the observation that vertical motions computed with raw data are often relatively accurate at lower levels of the atmosphere (e.g. Fankhauser, 1969). Any correction to the vertical motion at these levels would have the effect of decreasing the accuracy of the calculations. In a study involving water vapor this is an important point, since most of the water vapor is at the levels at which raw data results are presumably accurate. In this case such an argument is misleading. One of the reasons the lower layer vertical motions are more accurate is that the vertical motions deteriorate with height due to an accumulation of errors calculated at the lower layers in the vertical integration. This does not mean that the actual divergence calculated at any level is any better or worse than another level. While the vertical motion calculated at a low pressure may be significantly worse than at a high pressure, the divergences at each level may be equally accurate. This is the primary factor in this study, since precipitation is dependent on the divergence of a given layer, and not the vertical motion.

The importance of decreasing elevation angles can be examined with results from the model. Small elevation angles are the result of strong winds in the same direction at many successive levels. This is a characteristic winter condition in New England, when the core of the westerly jet is over the Northern United States. In summer, relatively slow upper air winds predominate over New England. Consequently, one would expect winter wind measurements to be generally more inaccurate than summer measurements. If this effect is the major cause of inaccuracy in the model, summer results should be superior. Figures 3 and 4 indicate the opposite to be true. It must be noted that precipitable water is also greater in the summer (25 mm as opposed to 10 mm), and an error in the divergence calculations could cause greater errors in the precipitation calculation in summer than winter. On the other hand, small scale effects, exemplified by thunderstorms, could be more important in summer, and cause greater divergence term errors. In fact, the RMS 100 mb vertical motion for the quadrilateral, obtained by averaging results from APC and CJA, is 447 mb/12 hrs in winter and 418 mb/12 hrs in summer. This represents vertically integrated divergence, and is not much greater in winter than summer. Thus, even allowing for the factor of precipitable water, one cannot say the divergence calculations are any worse in winter, as would be the case if large upper air inaccuracies were the major cause of error.

A further verification of this hypothesis can be found by examining results of the model using both types of correction schemes, constant and linearly varying divergence corrections. The RMS error for 479 cases in 1977 for the triangle APC is 10.26 mm for the linearly varying correction, and 9.79 mm for the constant correction. These results are virtually equal, and do not offer much support for the use of the linearly varying method. While the unrealistic vertical motions at 100 mb indicate that some correction scheme is necessary, there is no indication that a scheme more complicated than a constant correction at all levels is warranted. Figures 3 and 4 indicate that the constant correction scheme does offer improvement over raw data. In the next section, all results are calculated using corrected wind data.

There are three terms involved in the calculation of precipitation - advection, divergence, and storage. Of the three, the divergence term has the greatest magnitude. In the 479 cases of 1977 for APC, it has a range from -35 mm to +25 mm. The storage term is next largest, with range -20 mm to +25 mm. The advection term is smallest, with range -10 mm to +10 mm. None of those terms can be assumed to dominate either of the other terms.

Comparison of the results of the model to actual precipitation is not encouraging. Consider Figure 5, a scatter diagram of actual vs. calculated precipitation for APC in 1977. A simple parameterization for evaporation, described in a previous section, has been added. As shown before, RMS error for the calculations are on the order of 10 mm. This is enough to cause many of the calculated precipitation values to be of opposite sign to the actual precipitation. A linear regression fit to the data produces a coefficient of calculated precipitation of 105, indicating severe over precipitation, and the variables have a correlation coefficient of .12. Removing the cases with no actual precipitation does not improve either of these values (Figure 6). The fact that there are many more cases with little or no precipitation than much precipitation, and the fact that the standard deviation of calculated precipitation for any given value of actual precipitation is large, tends to reduce the coefficient on the linear regression. If a linear regression is done on the median calculated precipitation for a given actual precipitation, the coefficient is .82, slight over-prediction.

An indication of the problems involved is found in the scatter diagram of calculated precipitation (Figure 2). These two variables have a correlation coefficient of .92, indicating that most of the calculation of precipitation is error. A comparison of corresponding graphs for the divergence and advection terms (Figures 8 and 9) shows that the divergence term produces more of the error, as expected. The results for the other three triangles are similar.

The results for 1979 are considerably more encouraging than 1977. There are two factors to be considered. There are more cases examined in 1977 than 1979, so the results for 1977 should be statistically more convincing. In contrast, the data used in the 1979 cases were examined for errors. It is possible that this contributed to the improved results for 1979.

In the triangle APC, the correlation between actual precipitation is .44 (Figure 10). In order to form a larger data set, results from all four triangles were grouped together. This composition resulted in a correlation coefficient of .35 (Figure 11). These numbers are improved to .52 and .56 respectively when only the cases with rain are considered (Figures 12 and 13). With or without the no-rain cases, the regression coefficient is less than .3, again indicating severe over predicting by the model. If the linear regression is done on the medians of calculated precipitation for a given actual precipitation amount for the composition of the four triangles, the coefficient is .78, much closer to the ideal value of 1, and close to .82 for APC 1977. This method may be interesting to use if more cases were available. There are so few cases with large amounts of rain in the present sample that only very tenuous conclusions can be drawn from regression analysis. The scatter diagram of calculated precipitation vs. actual minus calculated precipitation, or error, is a great improvement over the 1977 cases. The correlation is much lower, indicating that less of the calculation is error (Figure 14). Comparison of error vs. divergence and advection terms (Figures 15, 16) shows that the correlation of error to divergence is much less than the correlation of error to advection. The indication is that less of the error is due to divergence than in the 1977 cases. The implication is that the results of the model are extremely sensitive to the accuracy with which the model calculates the divergence of the winds.

The graphs of RMS precipitation error (Figures 3,4) divide into two regimes in all four cases, Dec. to May and June to Oct. A month was chosen from each regime to examine its characteristics. April and August for APC 1977 seemed representative of surrounding months, and had more periods in which all stations reported than other possibilities.

In April 1977 there were three groups of 12 hour periods in which rain fell, the remainder of the month was virtually dry (Figure 17).

The first group, April 2 and 3, had a total rainfall of 11 mm. The second group, April 5 and 6, had a total rainfall of 32 mm. The third group, April 23 to 25, had a total rainfall of 45 mm, with three periods of more than 10 mm and one of 9 mm.

Neither of the first two groups can be compared to the calculated results, since data was not present from all stations at those times. The model calculates 6 mm on April 8, which corresponds to a trace of actual precipitation, but also 12 mm on April 12, when no rain fell (Figure 18). The model does calculate a group of precipitation events from April 23 to 26. The total rainfall for this group is 73 mm, much more than the actual amount. The calculated peak precipitation amount is 12 hours after the actual peak. Nevertheless, this storm is fairly well predicted. Of the 73 mm, 41 mm is due to divergence, 14 mm to advection and 18 to storage (Figures 19 - 21). There is a definite association of the raw 100 mb vertical motion to the calculated precipitation (Figure 22). For most of the month the raw vertical motion is random, centering at about -400 mb/12 hrs. By April 21 the raw vertical motion is +400 mb/12 hrs, then decreases almost monotonically to -800 mb/12 hrs on April 25, when the maximum precipitation was calculated, then increases monotonically to +500 mb/12 hrs on April 28. Most of this is caused below 550 mb, as indicated by Figure 23. A coastal front existed in the lower layers at this time in southern New England. Perhaps the synoptic scale winds associated with the front were strong enough to mask the effects of small scale variations, and create an organized pattern of divergence in the lower layers of the model. No such organization appears at any other time in the month. The effect of correcting the winds for the negative 100 mb vertical motion during the rainy period is to reduce the convergence, and thereby reduce the calculated rainfall. Even with this correction, the calculated amount is too great. The corrected 550 mb vertical motion differs significantly from the raw pattern (Figure 24). While there is generally rising motion during the period April 23 to 26, the monotonic pattern is not present. There are two interesting aspects to the graph of precipitable water (Figre 25). Just as the 100 mb vertical motion developed into a pattern of April 21, two days before rainfall occurred, so does the precipitable water begin to increase on the 21st. Also, the precipitable water on April 12 is relatively high, which would tend to magnify an error in the calculation of the divergence of winds and account for the error in the calculation of the April 12 precipitation.

The pattern of actual precipitation in August is different from that in April. Some rain fell in almost half of the periods. Most of these periods were scattered throughout the month, except for the period from August 5 to 15 when precipitation was observed in most of the periods (Figure 26). The model correctly calculates the existence of many of these events, but overstates their amount by a factor of 2 or more (Figure 27). The model correctly calculates the mostly dry middle of the month, but calculates large precipitation amounts at the end of the month when little rain occurred. The graph of advection (Figure 28) indicates that moist advection was happening for most of the month. The erroneous calculations at the end of the month are due mostly to divergence (Figure 29). Large and rapid changes in storage occur throughout the month (Figure 30).

The 100 mb vertical motion was predominantly upward at the end of the month (Figure 31), indicating a spurious convergence. In most cases, this convergence was present below 550 mb also (Figure 32), which would account for the calculated rainfall during that period. In these cases, the lower level divergence error was greater than the upper level divergence error, since the constant velocity correction was insufficient to remove the upward vertical motion at 550 mb (Figure 33).

The precipitable water was much greater in amount than in April, with RMS value of 31 mm (Figure 34). The period from August 5 to 15 was replete with water, allowing positive calculations of precipitation, even though the divergence was small during that period. There was also much precipitable water at the end of the month, magnifying the divergence errors notee above. The large storage changes noted earlier are encouraging, since storage changes are the only way to detect the effects of thunderstorms, which are common in summer. The indication is that the model can detect this type of change.

Conclusions

A kinematic quantitative precipitation model has been developed. This model may be useful in examining the failure of present dynamic models to accurately predict precipitation amounts. This paper has examined the shortcomings of the kinematic model. Keeping these shortcomings in mind, it may be possible to use the model to determine the conditions that exist in cases when the dynamic models are inaccurate. These conditions may in turn give an indication of the aspects of the dynamic models which need improvement.

The primary problem with the kinematic model is the data used in it. The data may be non-representative due to either inaccurate measurements, or contamination by features of a scale smaller than the resolution of the model. The effects of the non-representative data are exemplified by the 100 mb vertical motions calculated by the model. Instead of virtually no vertical motion at 100 mb, the raw winds yield very large vertical motions there, often enough to exhaust the atmosphere in 12 hours. For certain groups of stations, this problem even exists in the average over several hundred cases. In order to provide better data, the velocities have been corrected to give zero vertical motion at 100 mb. The convection used cannot be said to rid the winds of small scale variations, but it does result in improved calculation of vertical motion, especially in the upper troposphere. The model is also more successful in calculating precipitation with the corrected winds, the ultimate measure of the effectiveness of the correction. The evidence presented suggests that a constant magnitude correction of the winds is as good as, if not better than, a more complicated scheme.

Examination of actual rainfall rates indicate that with 12 hour time resolution, on the average the model will calculate more rain than actually occurred. The April 1977 case examined is an example of this problem. Other than the amount of rain calculated, the results of the model for this case are very close to the actual rainfall. This suggests that application of the model to individual cases is not hopeless. In general, the large standard deviations of error over a year's data implies that only a study of many cases is reliable, comparison of 1979 and 1977 results reveals the importance of careful examination of the data used in the model. A study of many more cases should be done to provide more results for days with large amounts of observed rain. In all long term situations examined in this study, the no rain cases far out-numbered the large rain cases. A method was suggested to overcome this bias, but a much larger sample is required to use it with any confidence.

The largest error contributions to erroneous calculations of P - E is the error in the divergence term. It was calculated that a mean error in v over the volume of 1 m s^{-1} is enough to cause an error in the computation of P - E of about 10 mm/12 hrs. Using balloon data, we have good reason to believe that compensating errors exist in the measurements of wind, and that the mean error in v may not be related closely to any individual error. This must be carefully considered when using data from other sources, or data from several sources in implementing the kinematic model.

Tables and Figures

Explanation of Figures 5 - 16: Scatter diagrams.

Each number represents that number of cases with the specified values of x and y . (A = 10, Z = 36, @ = 37 or more). For example, in Figure 5 the A at the origin represents 10 cases when zero precipitation was calculated and zero precipitation was observed. The stars represent a least squares linear regression fit to the data, the correlation between x and y is found in the lower right corner of the plot. The diagonal solid line in the upper right quadrant represents a perfect calculation of precipitation (where applicable). Circled numbers (where applicable) represent the median value for that row, and the dashed line is a least squares linear regression fit to the circled numbers.

TABLE A

Cases without observed precipitation

		APC		PCJ		CJA		JAP	
1977	# Cases	200		162		135		141	
	Raw ω_{100}	-205.5 $\frac{mb}{12hrs}$		-92.5		-140.0		-279.0	
	Uncr. Precip / St. Dev.	1.29mm	9.17 mm	-1.28	5.81	-1.99	8.28	-0.29	9.41
	Cor. Precip / St. Dev	-1.64mm	7.40mm	-3.00	6.30	-4.55	7.98	-3.99	7.82
1979	# Cases	76		68		45		41	
	Raw ω_{100}	-161.5 $\frac{mb}{12hrs}$		-7.5		-112.5		-262.6	
	Uncr. Precip / St. Dev	0.85mm	9.29mm	0.70	9.16	1.19	7.06	4.78	10.04
	Cor. Precip / St. Dev.	-1.01mm	8.11 mm	-0.65	8.14	-0.39	6.83	2.46	11.75

TABLE B

Comparison of approximate determinations of precipitation

1 – 1: Trapezoidal rule

1 – 2 – 1: Trapezoidal rule – 3 points

1 – 4 – 1: Simpson's rule – 3 points

88 cases with observed rain Jan — Jun 1977

	most accurate determination	Low by 20%	within 20%	High by 20%	Trace observed
1 – 1	9 (12%)	17 (19%)	17 (19%)	39 (44%)	15 (17%)
1 – 2 – 1	26 (35%)	9 (10%)	41 (47%)	23 (26%)	15 (17%)
1 – 4 – 1	38 (52%)	18 (20%)	41 (47%)	14 (16%)	15 (17%)

Errors for 73 cases with measurable rain (mm)

	mean $\overline{(\text{approximate} - \text{observed})}$	Rms $\sqrt{\overline{(\text{approximate} - \text{observed})^2}}$
1 – 1	1.022	4.13
1 – 2 – 1	0.15	1.26
1 – 4 – 1	-0.17	1.96

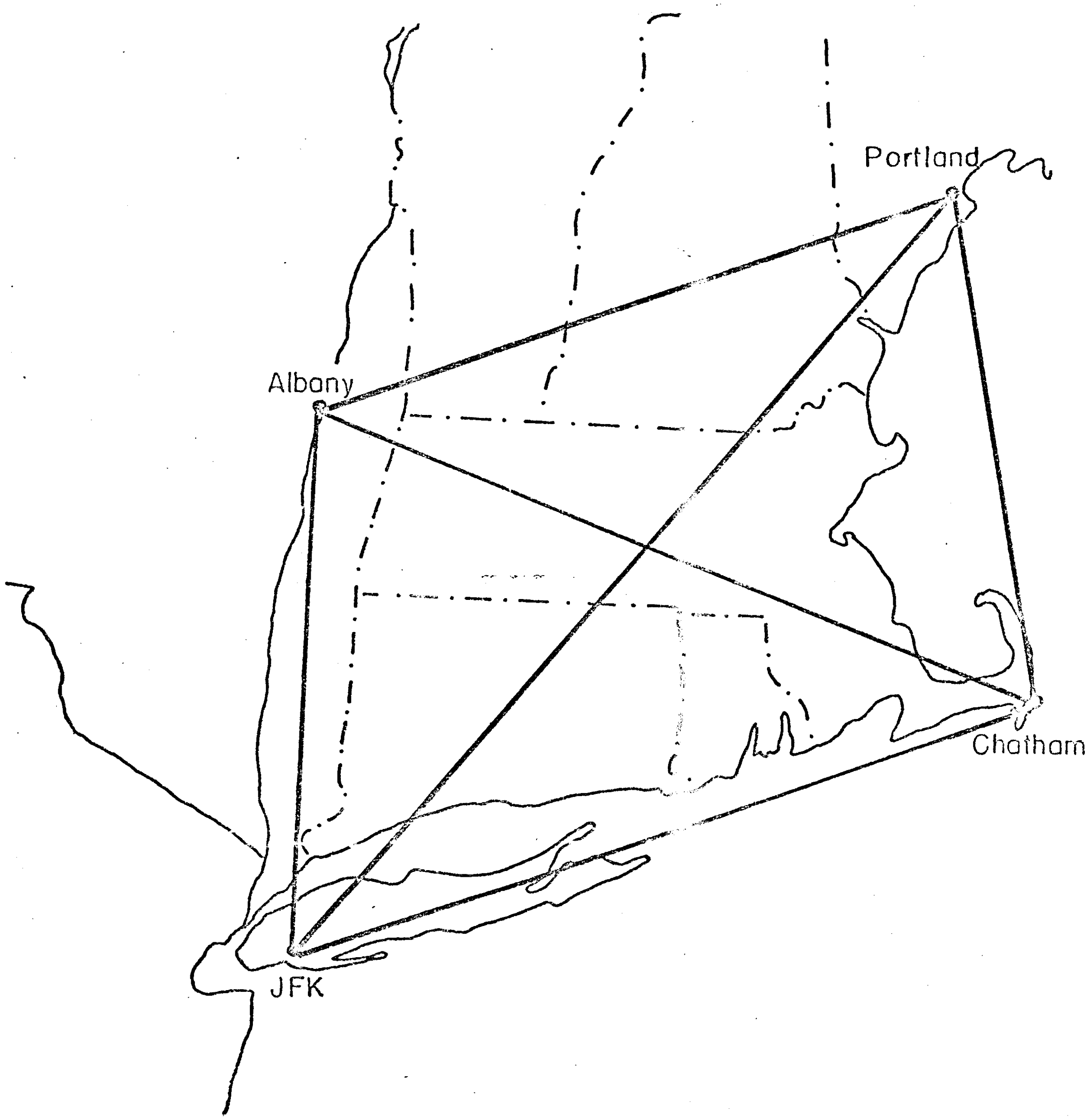


Fig. 1

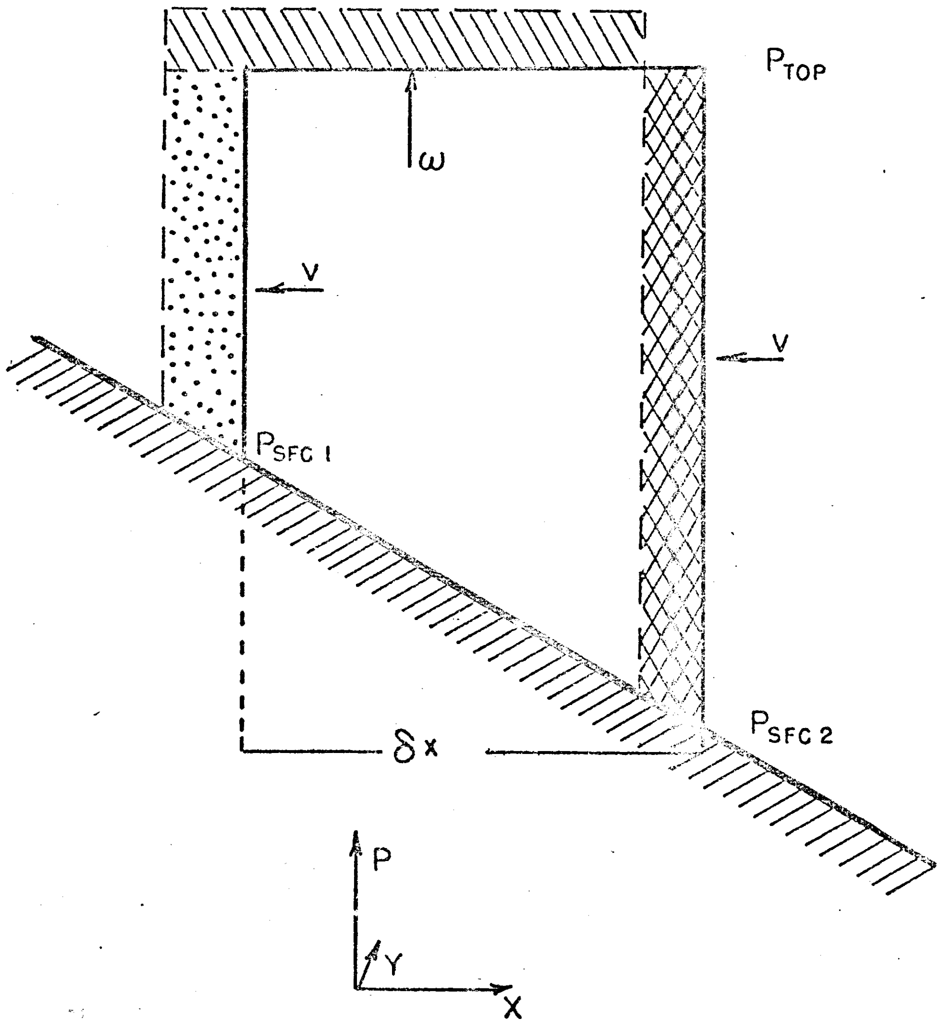


Fig. 2. See text for description

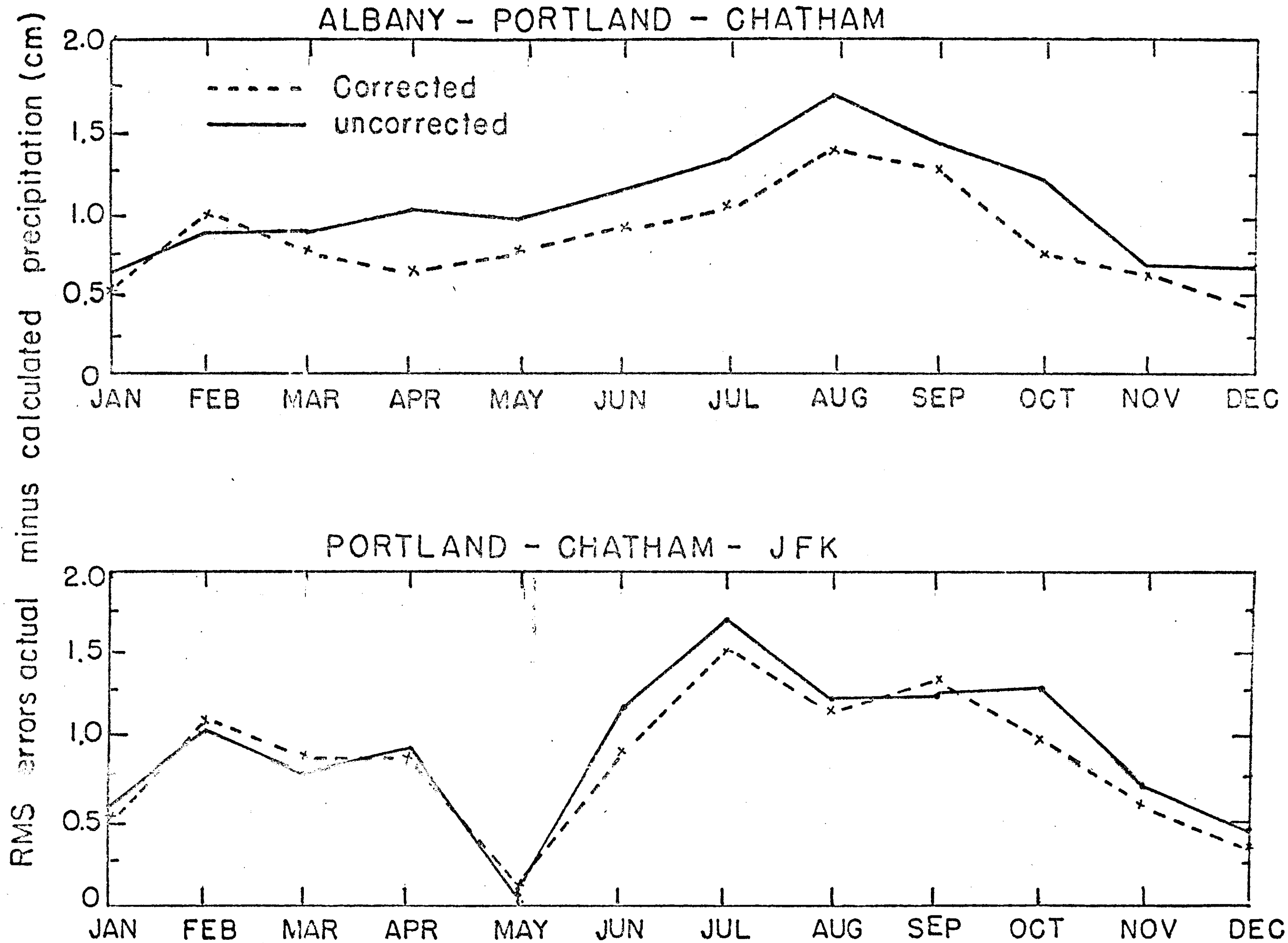

ALBANY - PORTLAND - CHATHAM
Corrected
uncorrected
RMS errors actual minus calculated precipitation (cm)
2.0
1.5
1.0
0.5
0
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC
PORTLAND - CHATHAM - JFK
2.0
1.5
1.0
0.5
0
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC

Fig. 3

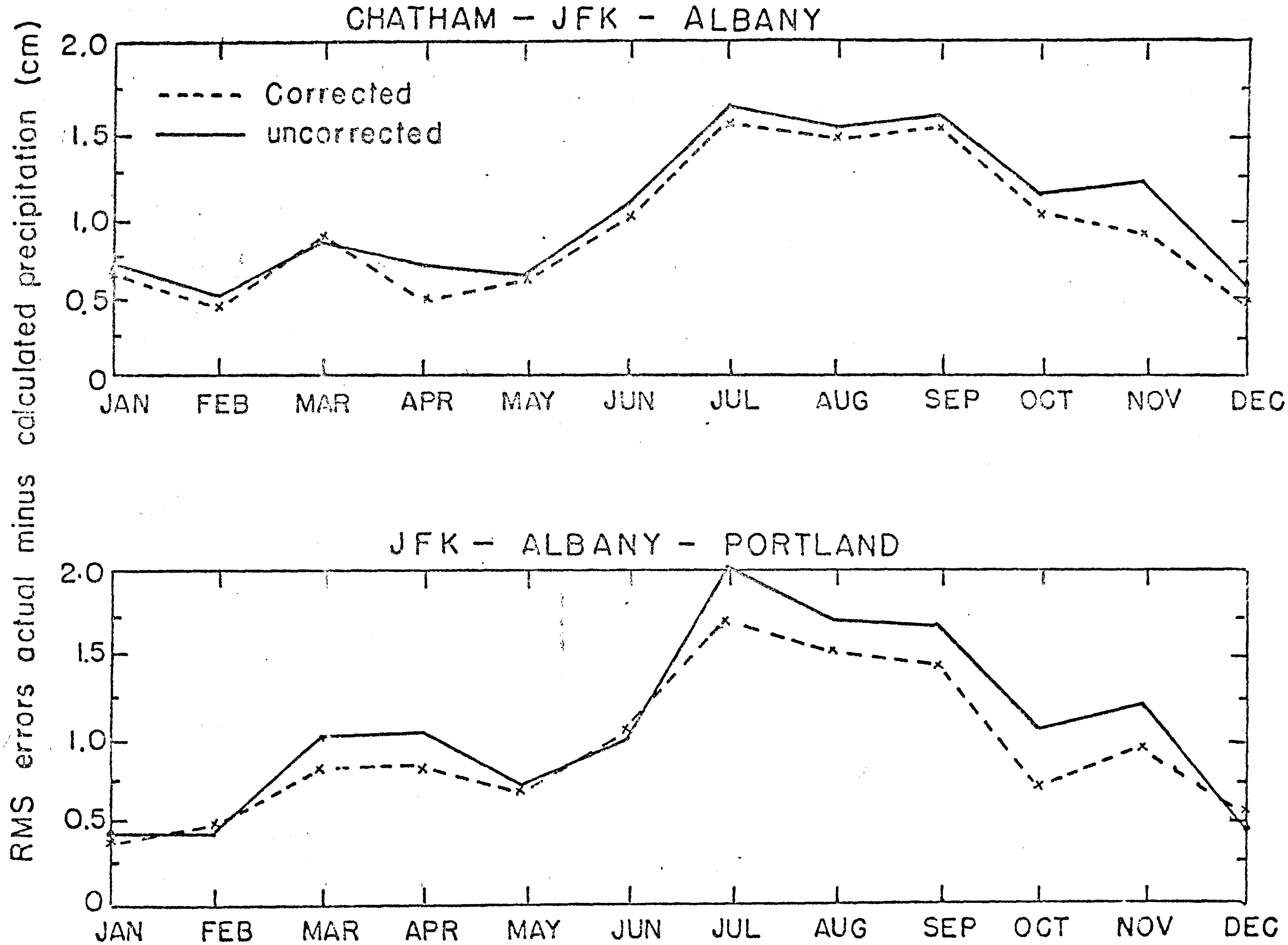

CHATHAM - JFK - ALBANY
Corrected
uncorrected
JFK - ALBANY - PORTLAND
RMS errors actual minus calculated precipitation (cm)
2.0
1.5
1.0
0.5
0
JAN
FEB
MAR
APR
MAY
JUN
JUL
AUG
SEP
OCT
NOV
DEC

Fig. 4

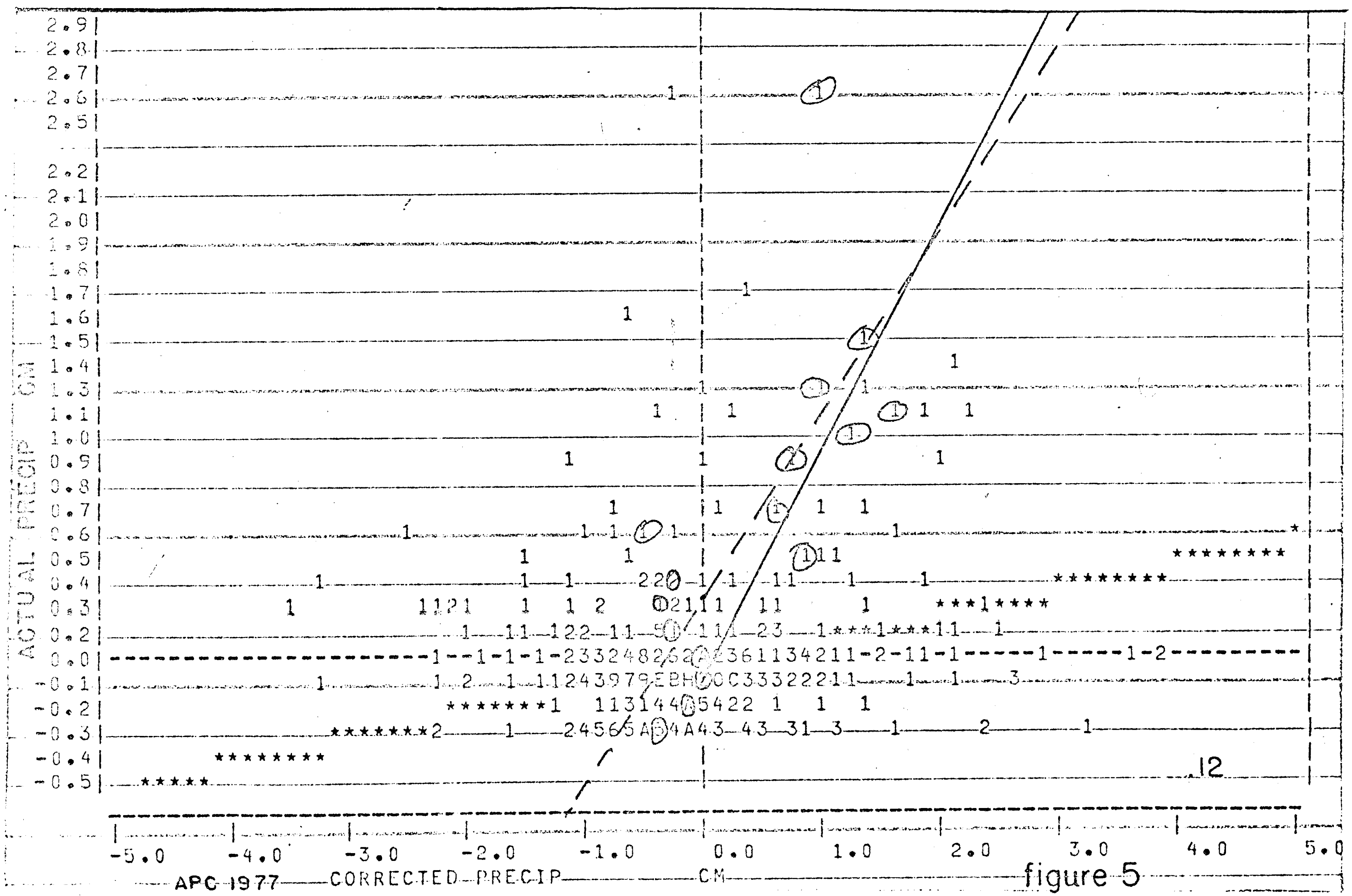
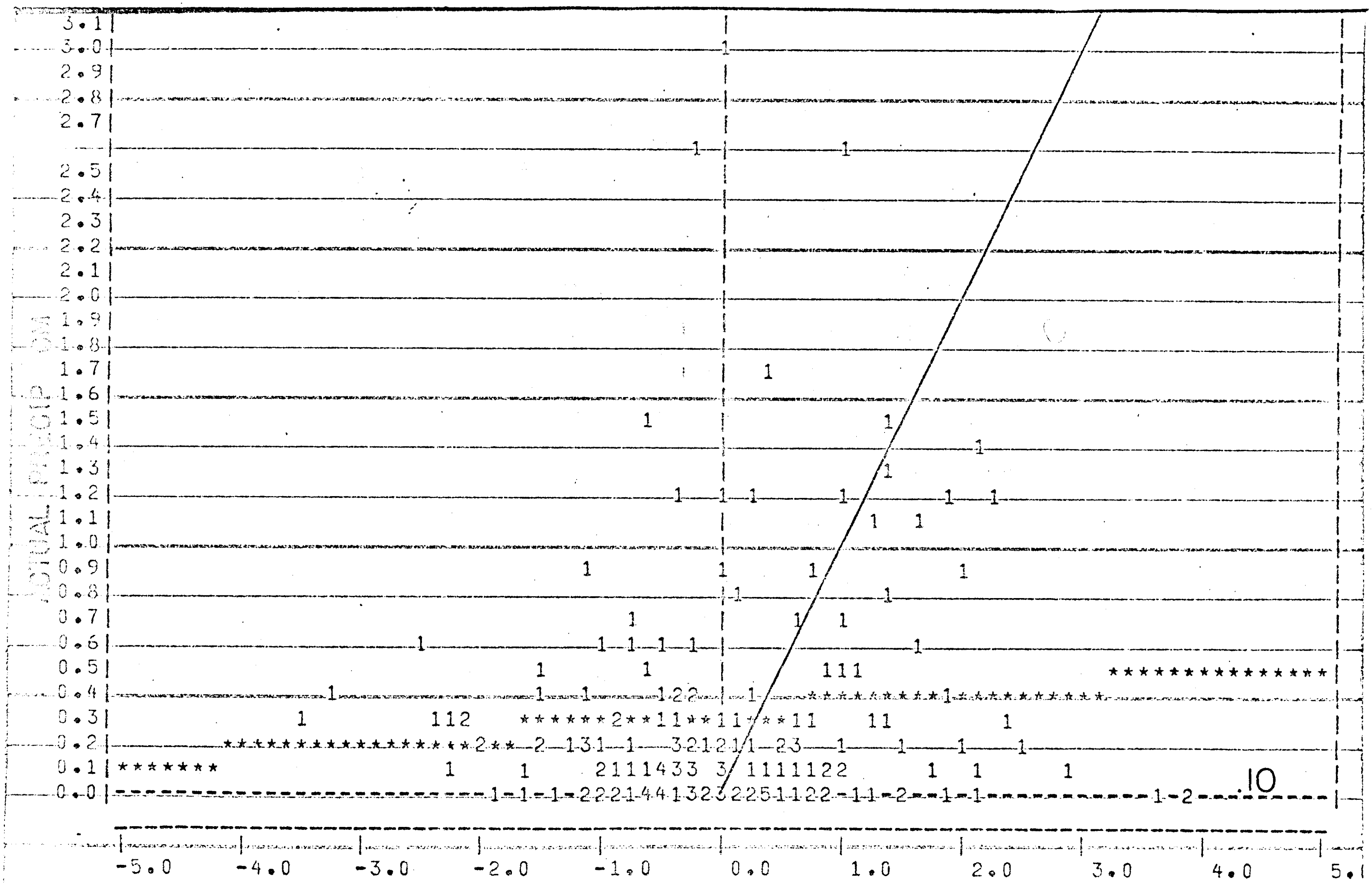

ACTUAL PRECIP CM
APC 1977 CORRECTED PRECIP CM
-5.0
-4.0
-3.0
-2.0
-1.0
0.0
1.0
2.0
3.0
4.0
5.0
.12

figure 5



APG 1977 CORRECTED PRECIP CM \ RAIN CASES ONLY

figure 6

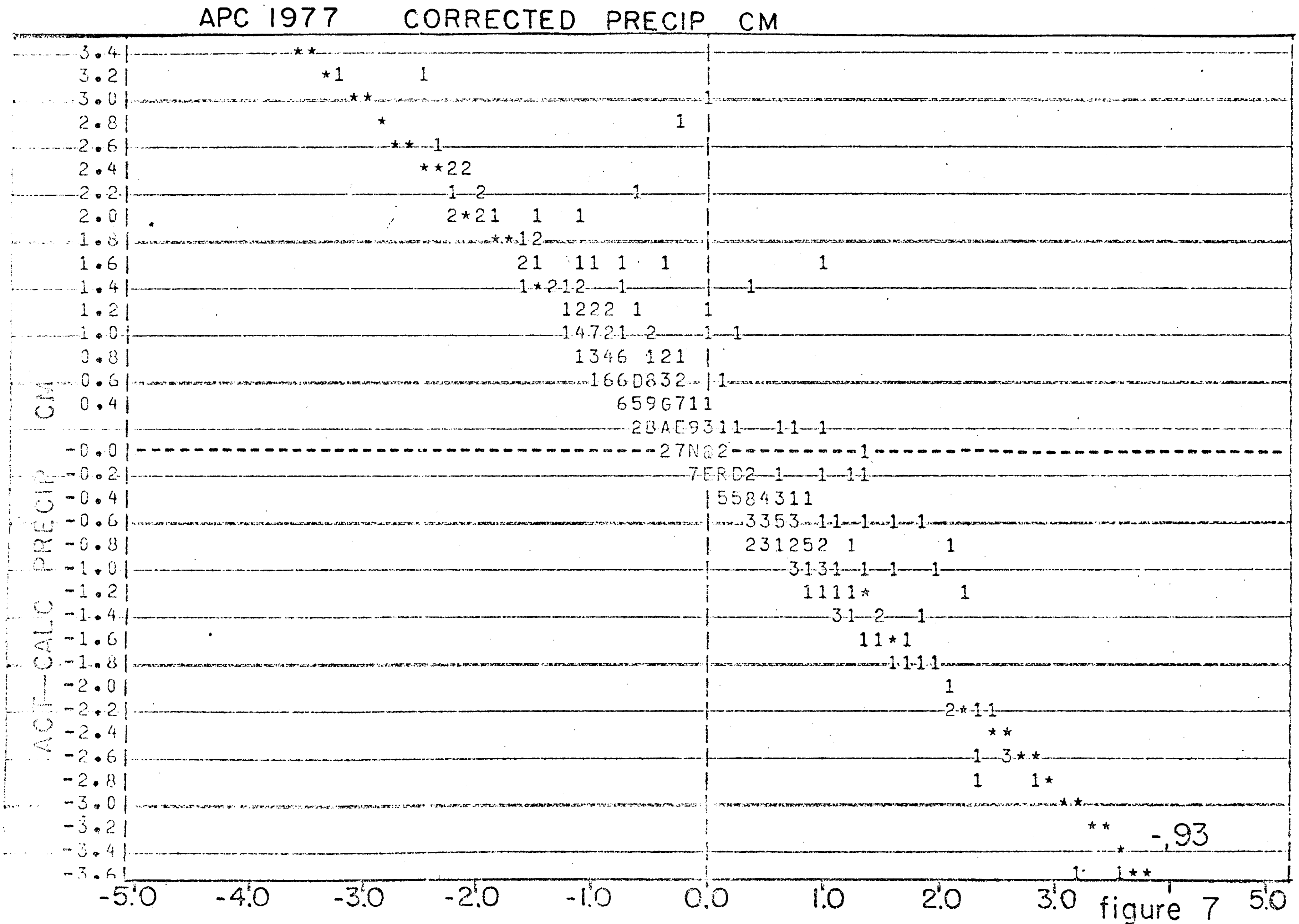

APC 1977 CORRECTED PRECIP CM
ACT—CALC PRECIP CM
-5.0 -4.0 -3.0 -2.0 -1.0 0.0 1.0 2.0 3.0 5.0
-.93

figure 7

APC 1977 DIVERGENCE CM

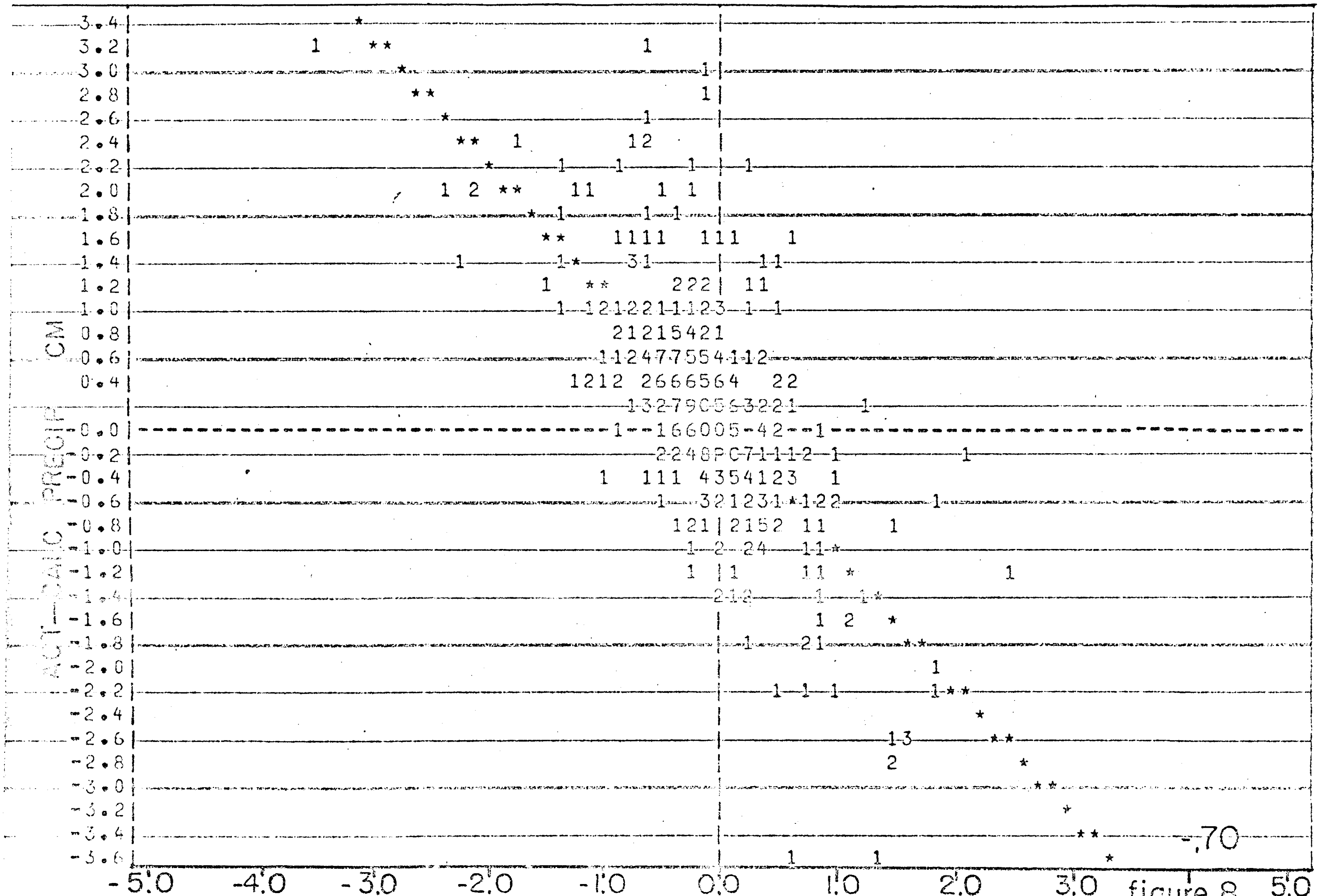


figure 8

APC 1977 ADVECTION CM

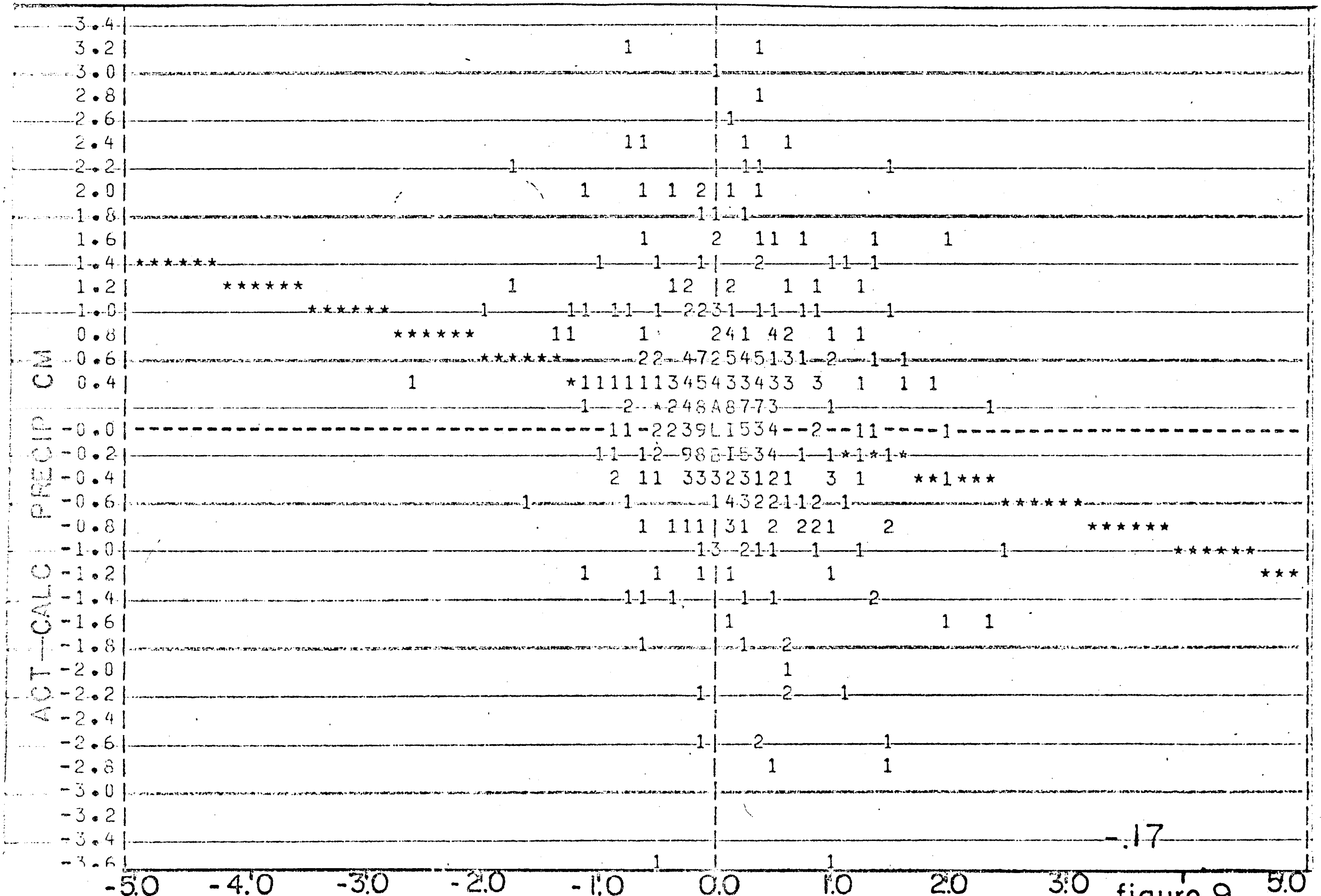


figure 9

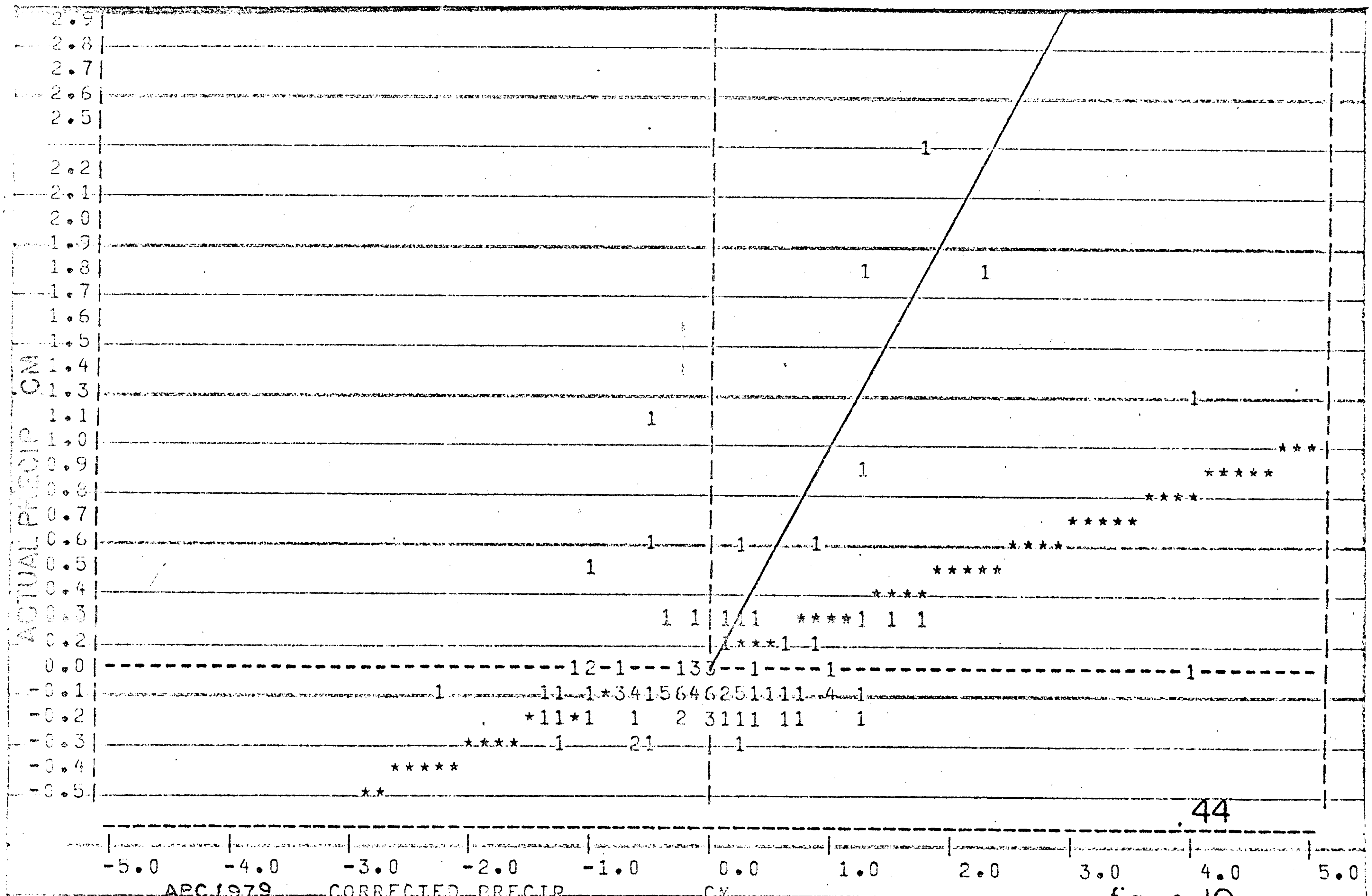

ACTUAL PRECIP CM
.44
-5.0 -4.0 -3.0 -2.0 -1.0 0.0 1.0 2.0 3.0 4.0 5.0
APC 1979 CORRECTED PRECIP CM

figure 10

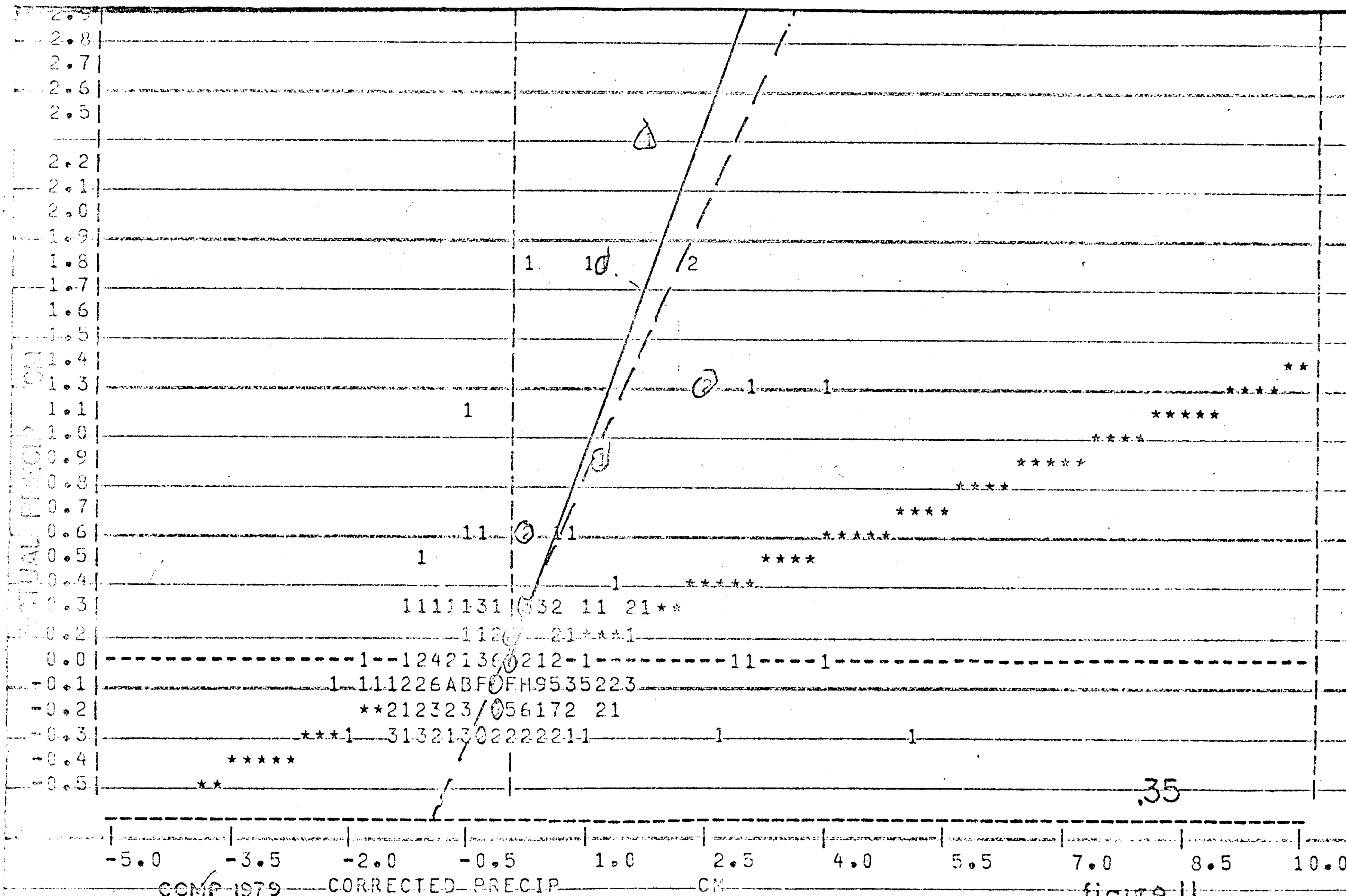

2.8
2.7
2.6
2.5
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
-5.0
-3.5
-2.0
-0.5
1.0
2.5
4.0
5.5
7.0
8.5
10.0
COMP 1979 CORRECTED PRECIP CM
.35

figure 11

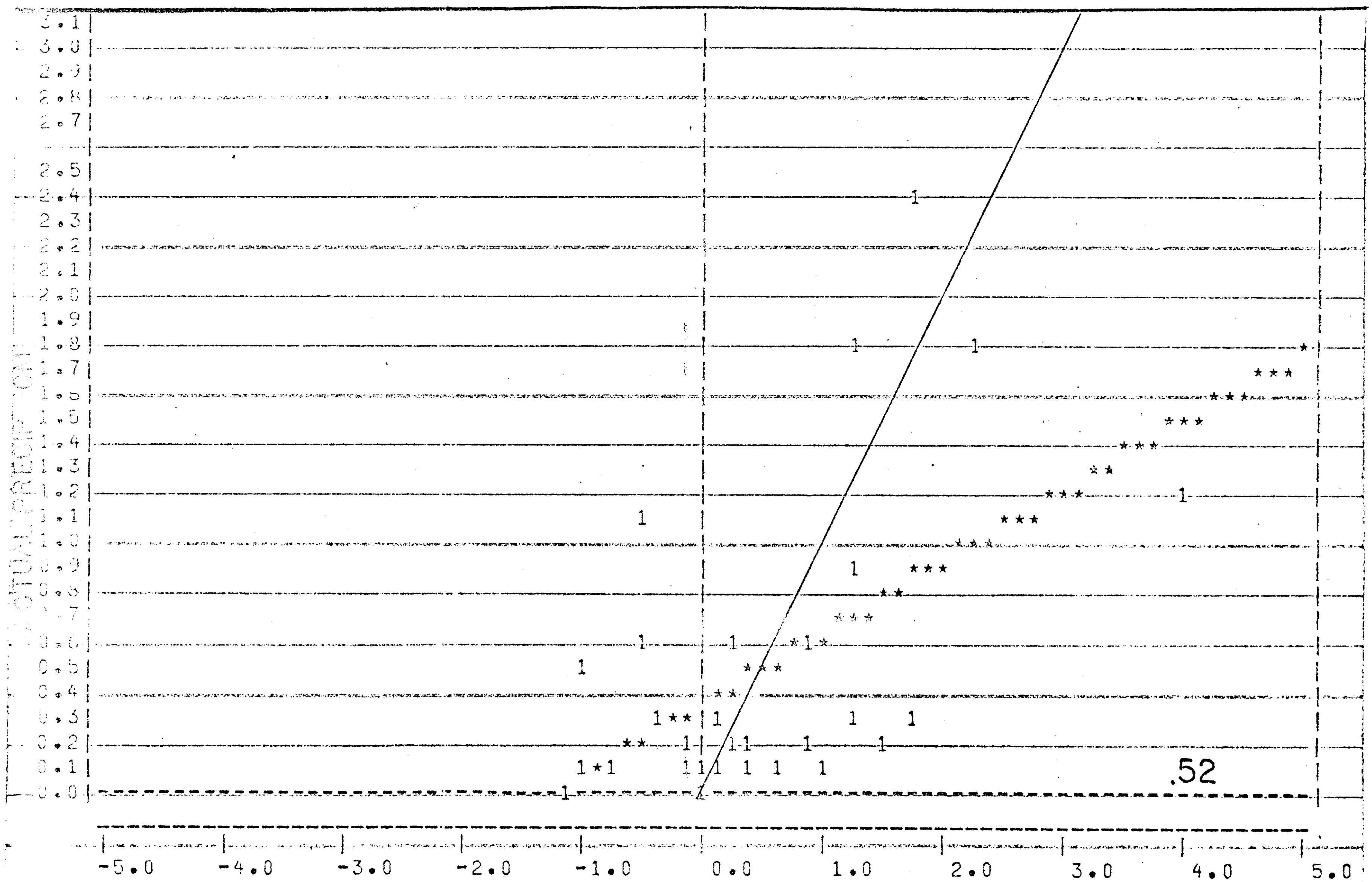
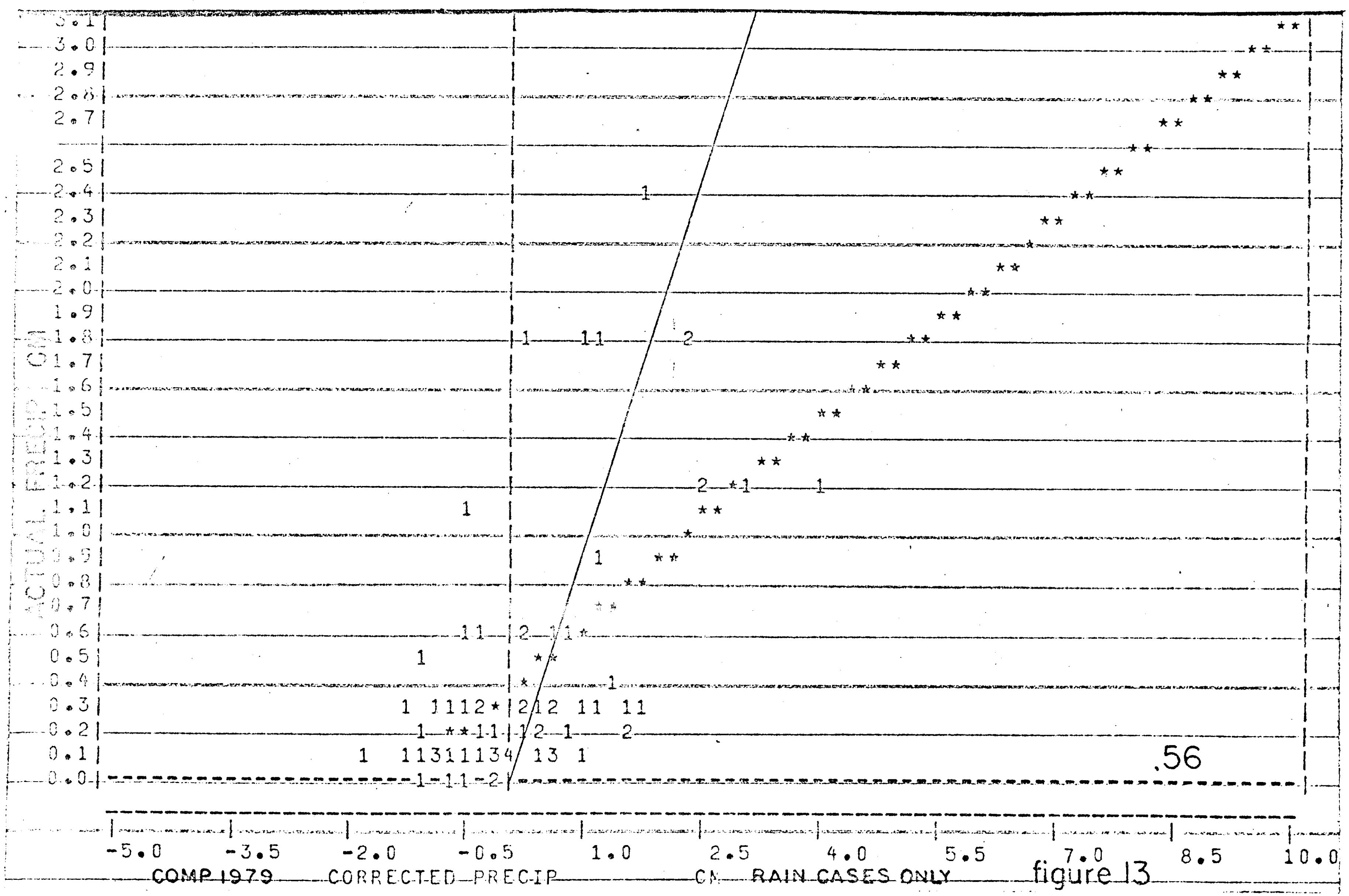

ACTUAL PRECIP CM
.52
-5.0 -4.0 -3.0 -2.0 -1.0 0.0 1.0 2.0 3.0 4.0 5.0
APC 1979 CORRECTED PRECIP CM RAIN CASES ONLY

figure 12



ACTUAL PRECIP. CM
3.1
3.0
2.9
2.8
2.7
2.5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-5.0
-3.5
-2.0
-0.5
1.0
2.5
4.0
5.5
7.0
8.5
10.0
COMP 1979 CORRECTED PRECIP CM RAIN CASES ONLY
.56

figure 13

APC 1979 CORRECTED PRECIP CM

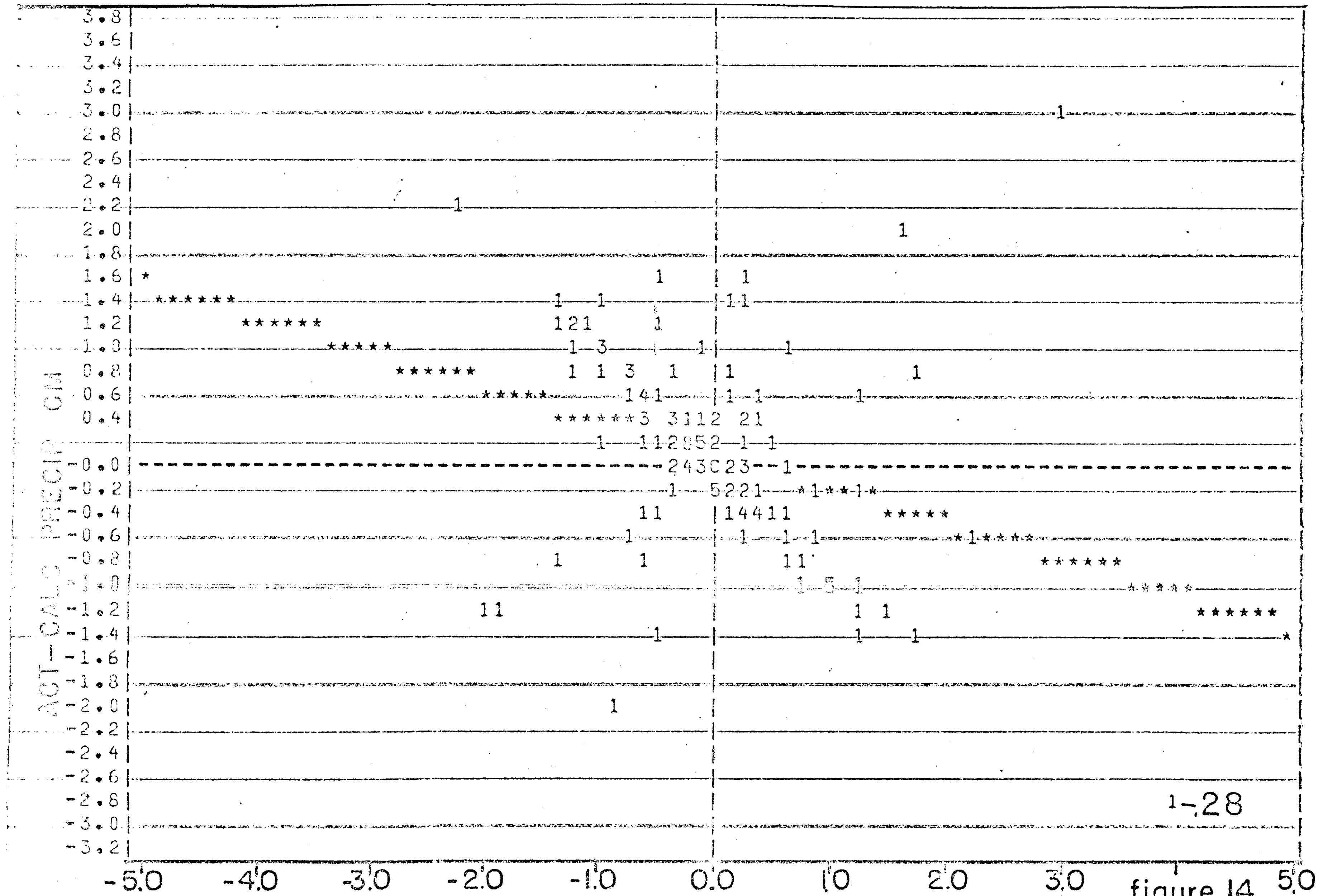


figure 14

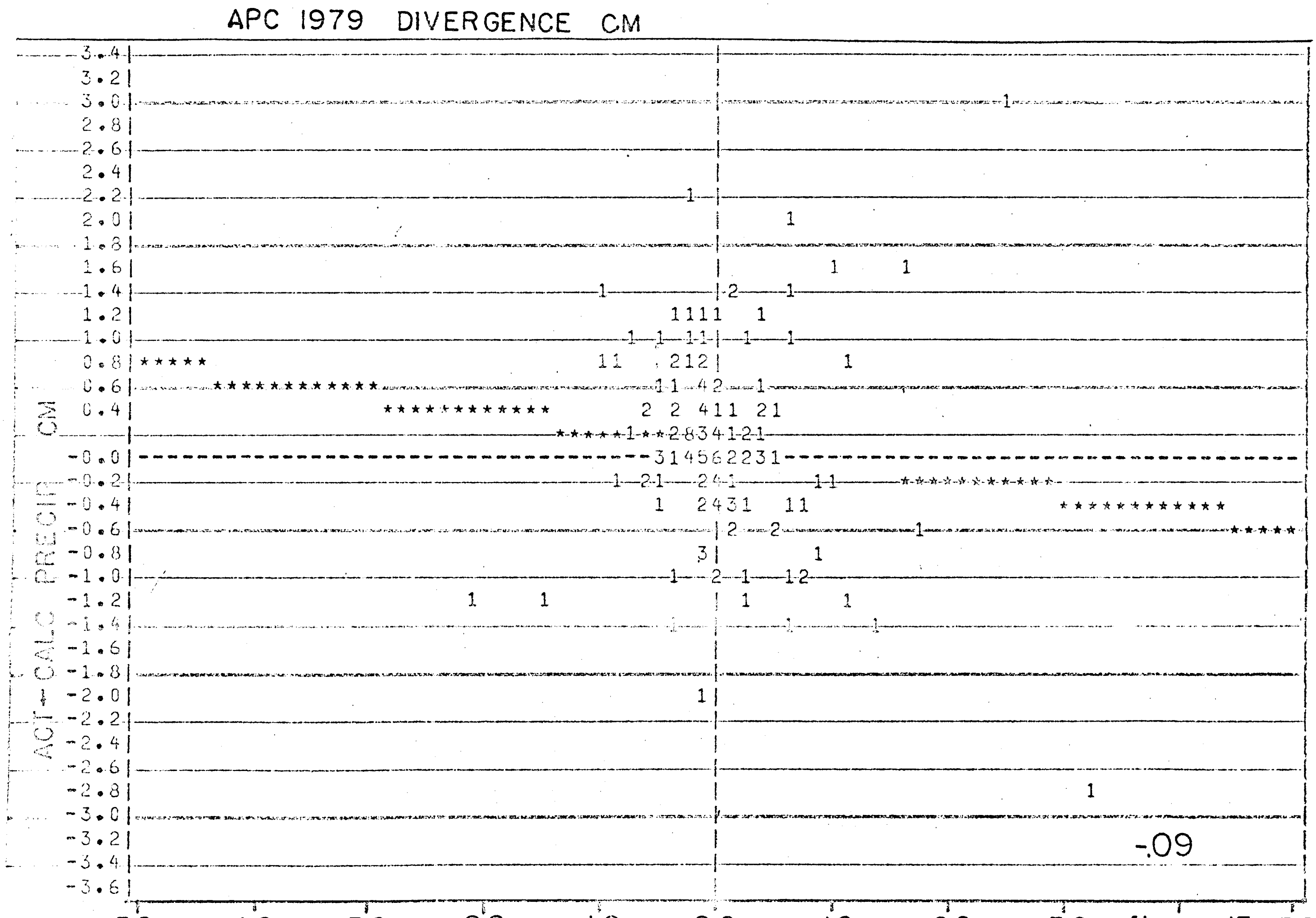

APC 1979 DIVERGENCE CM
ACT - CALC PRECIP CM
-.09
-5.0 -4.0 -3.0 -2.0 -1.0 0.0 1.0 2.0 3.0 5.0

figure 15

APC 1979 ADVECTION CM

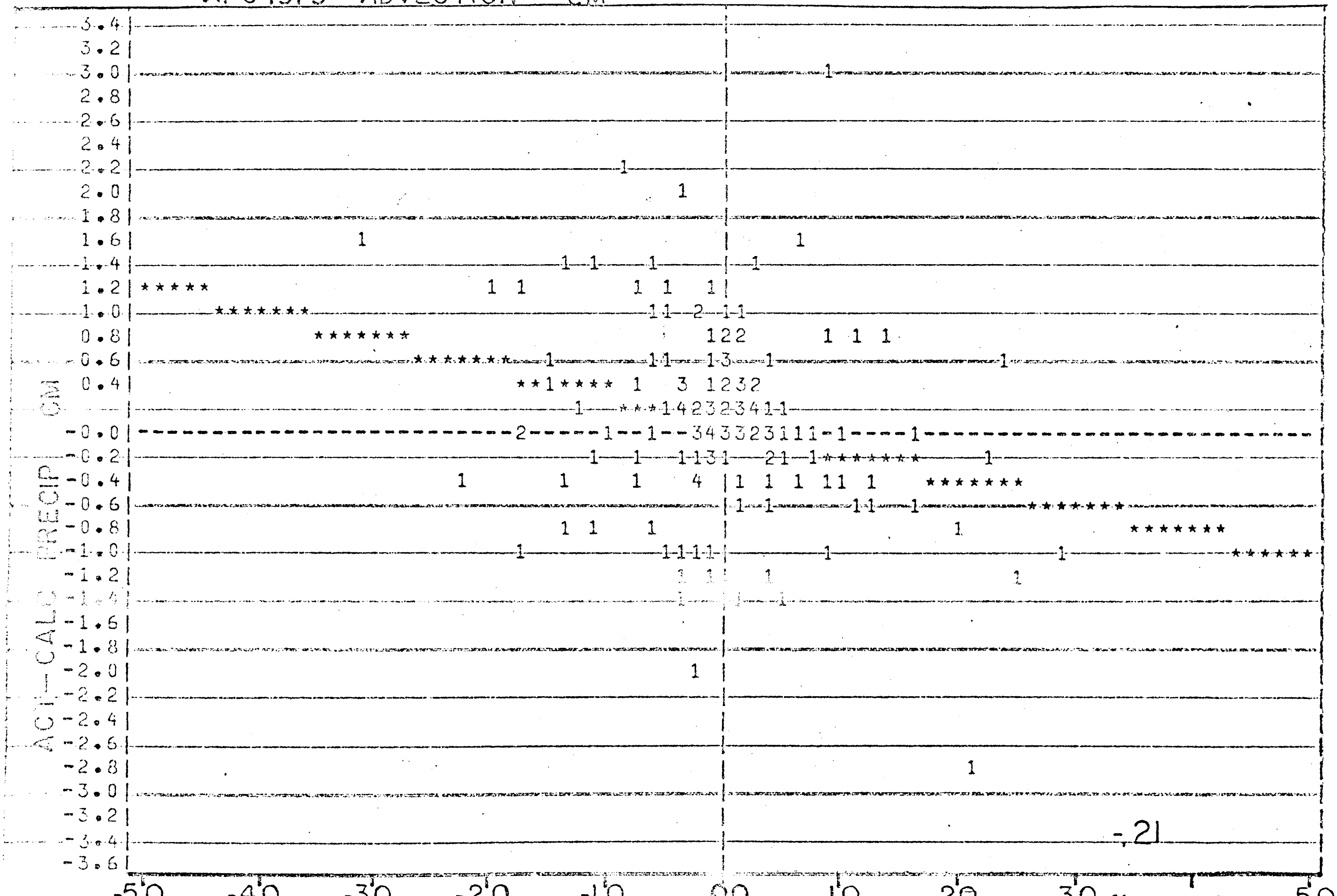


figure 16

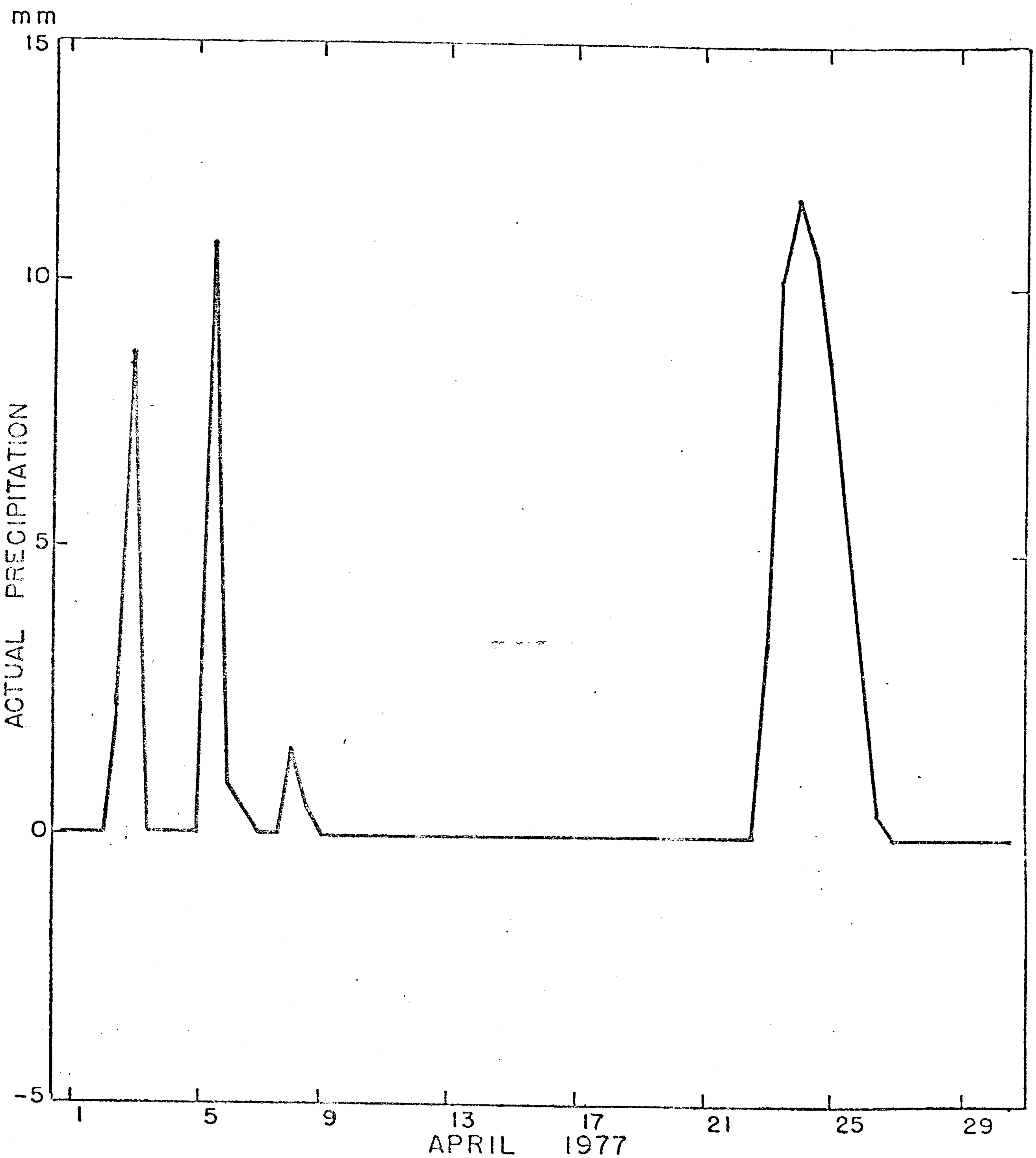

mm
15
10
5
0
-5
ACTUAL PRECIPITATION
1
5
9
13
17
21
25
29
APRIL 1977

Fig. 17

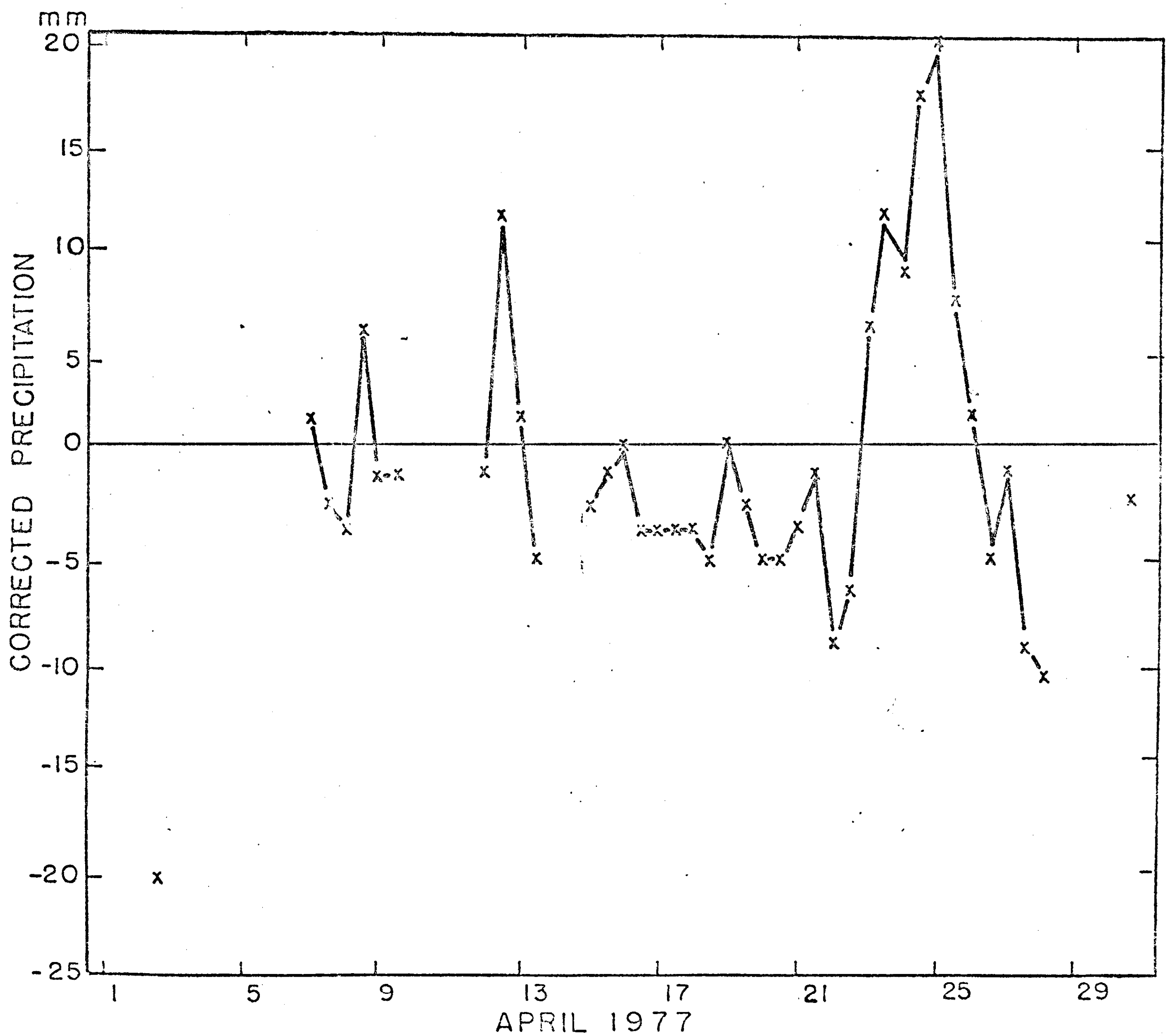

mm
20
15
10
5
0
-5
-10
-15
-20
-25
CORRECTED PRECIPITATION
1
5
9
13
17
21
25
29
APRIL 1977

Fig. 18

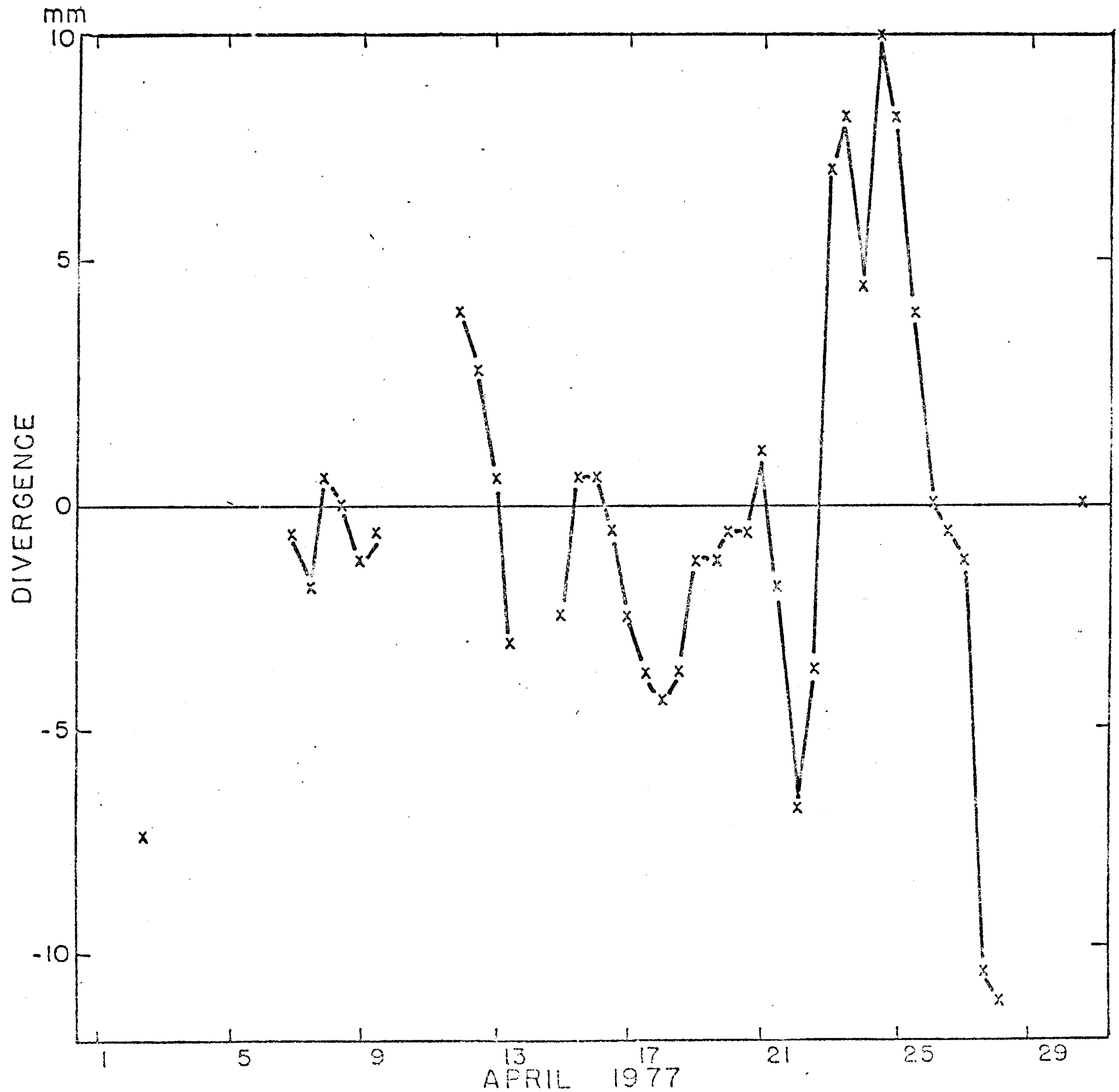


Fig. 10

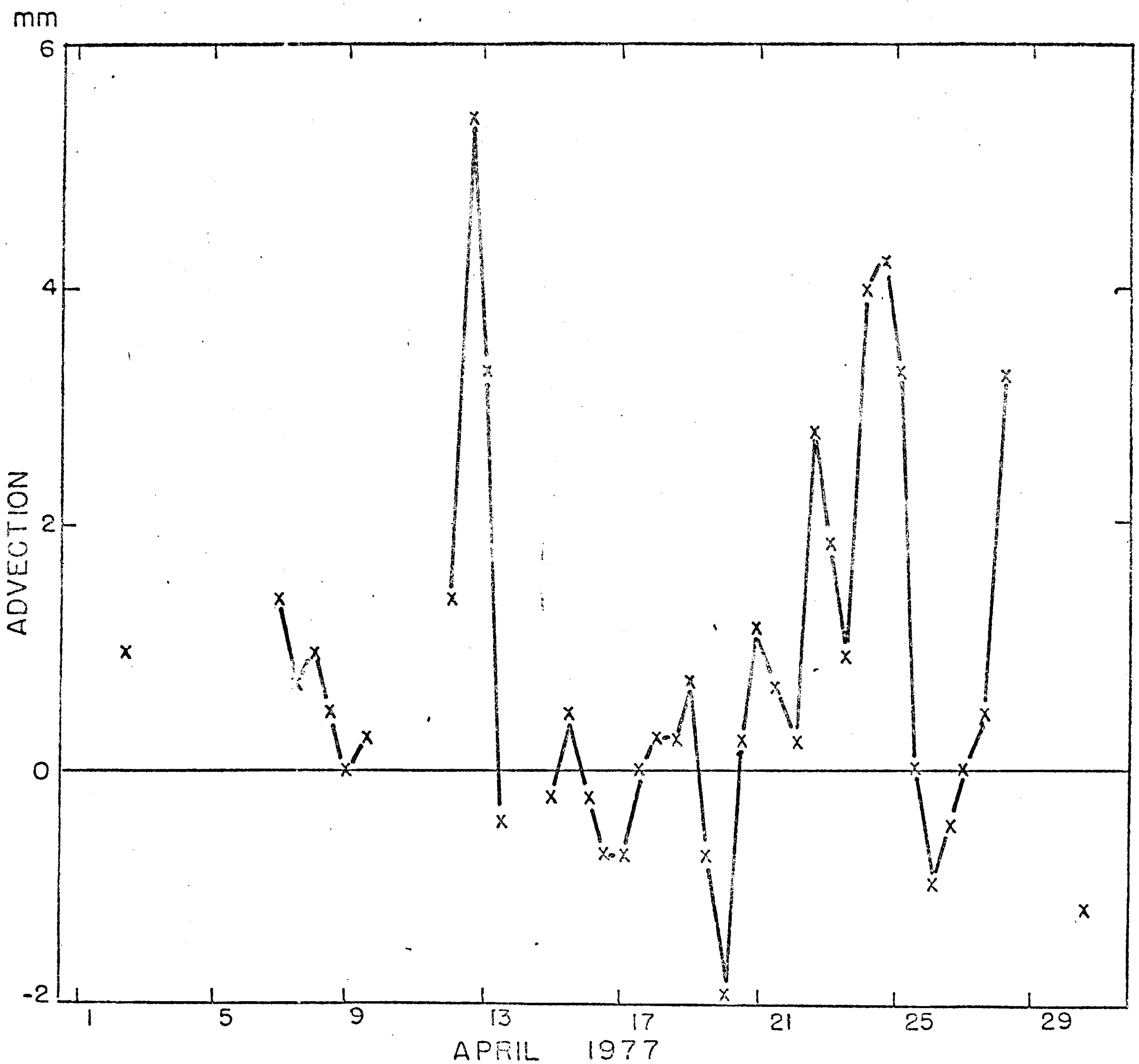


Fig. 20

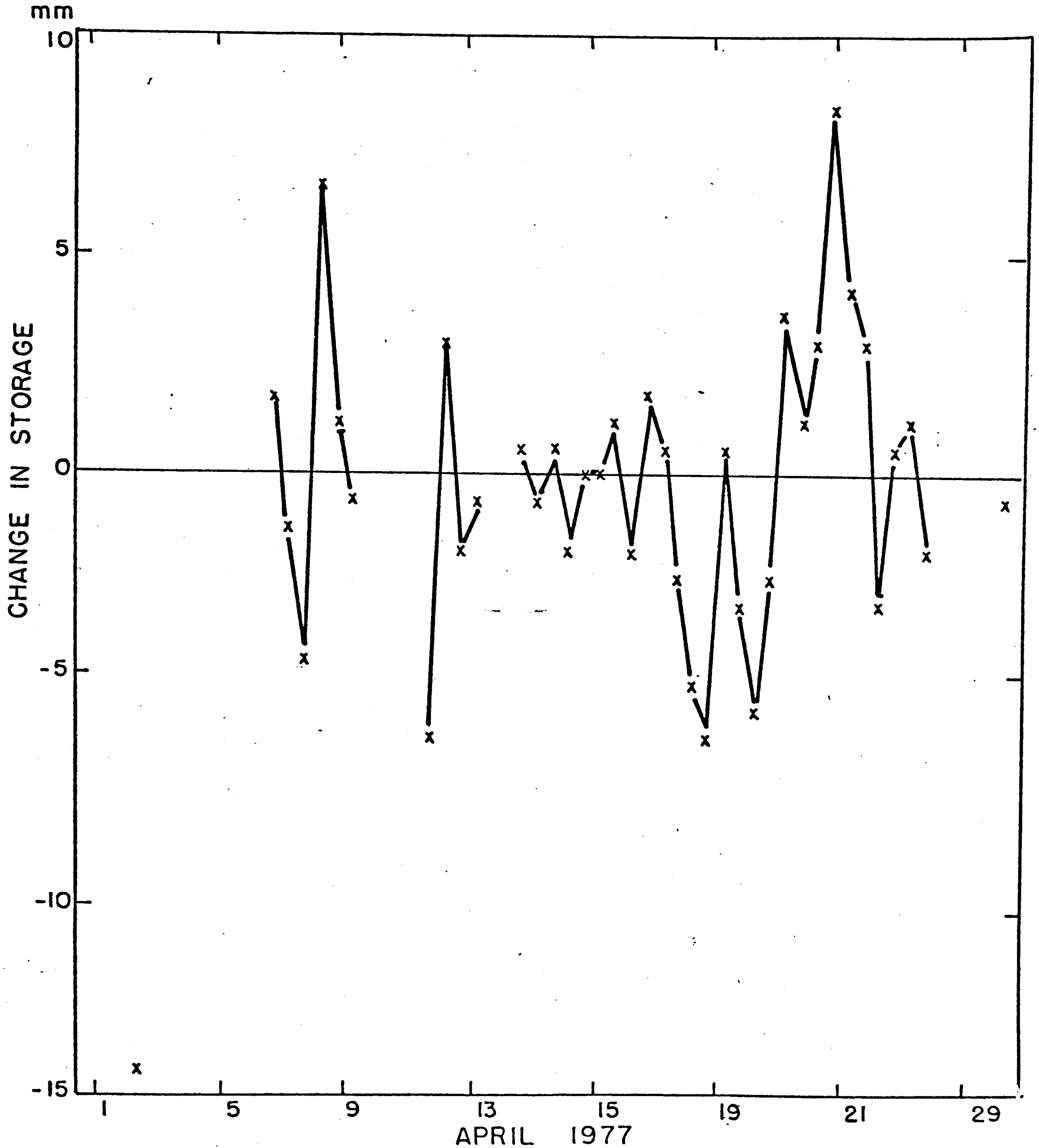


Fig. 21

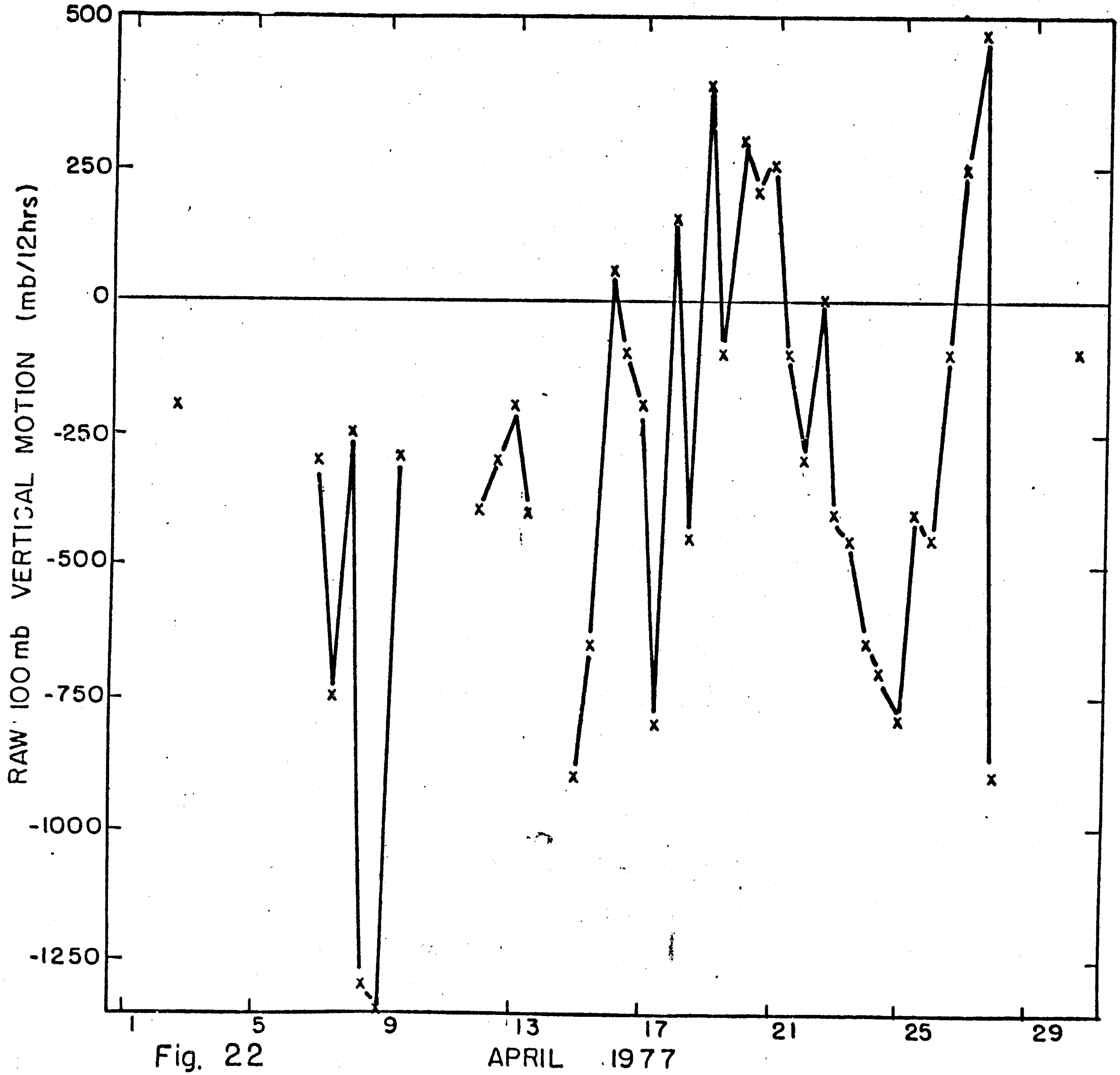

RAW 100 mb VERTICAL MOTION (mb/12hrs)
500
250
0
-250
-500
-750
-1000
-1250
1
5
9
13
17
21
25
29
APRIL 1977

Fig. 22

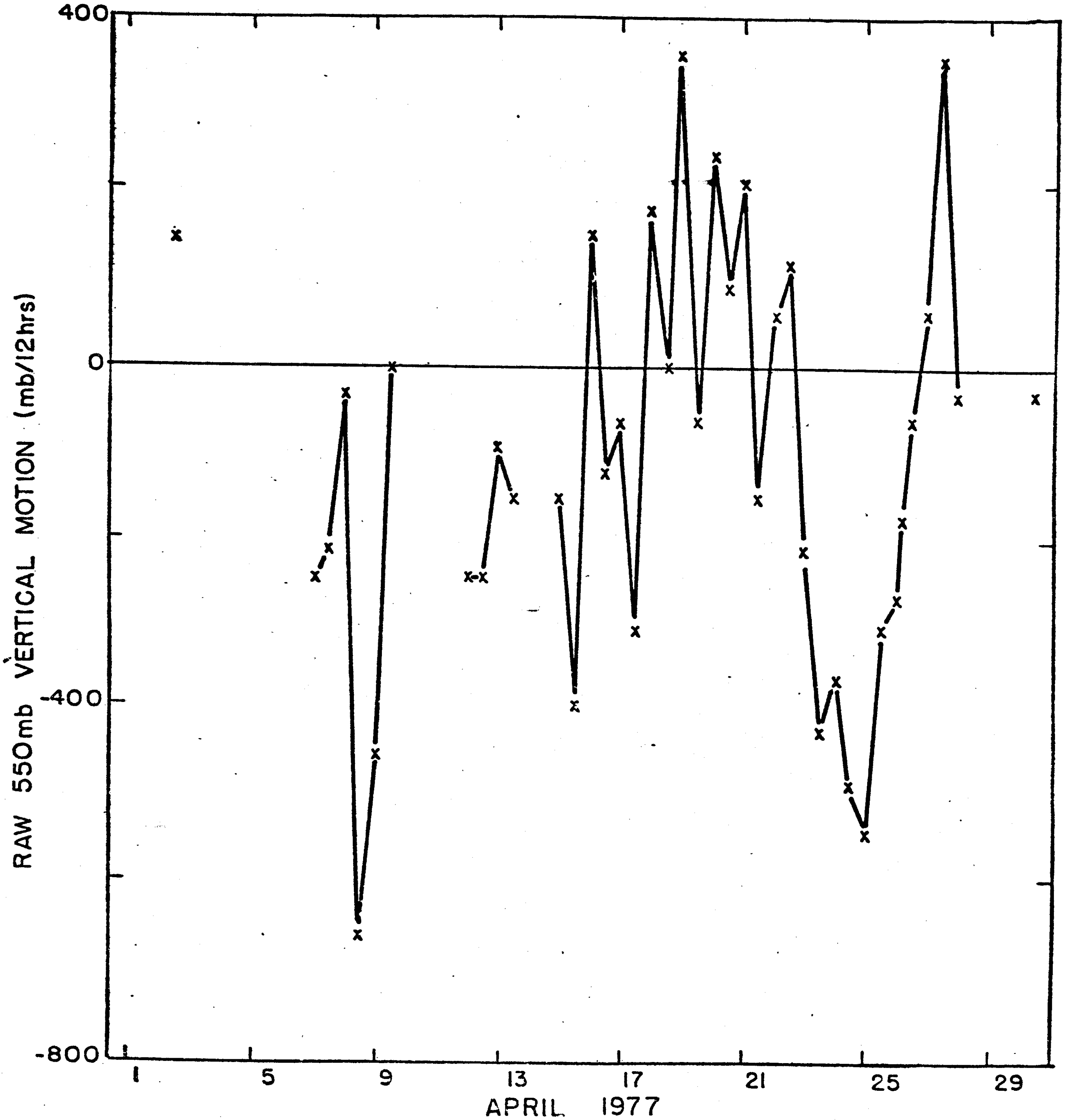
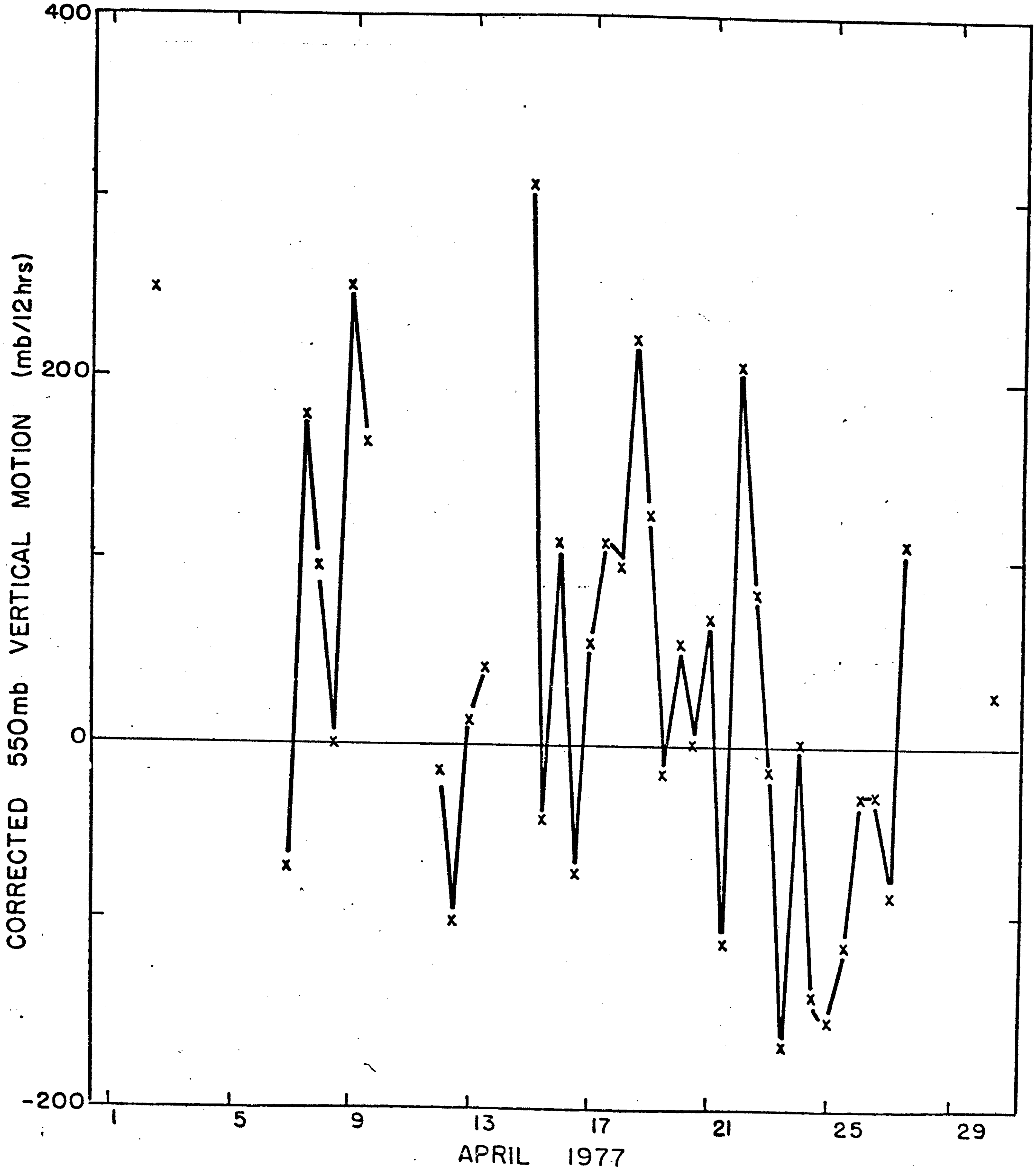

400
0
-400
-800
RAW 550mb VERTICAL MOTION (mb/12hrs)
1
5
9
13
17
21
25
29
APRIL 1977

Fig. 23



400
200
0
-200
CORRECTED 550mb VERTICAL MOTION (mb/12hrs)
1
5
9
13
17
21
25
29
APRIL 1977

Fig. 24

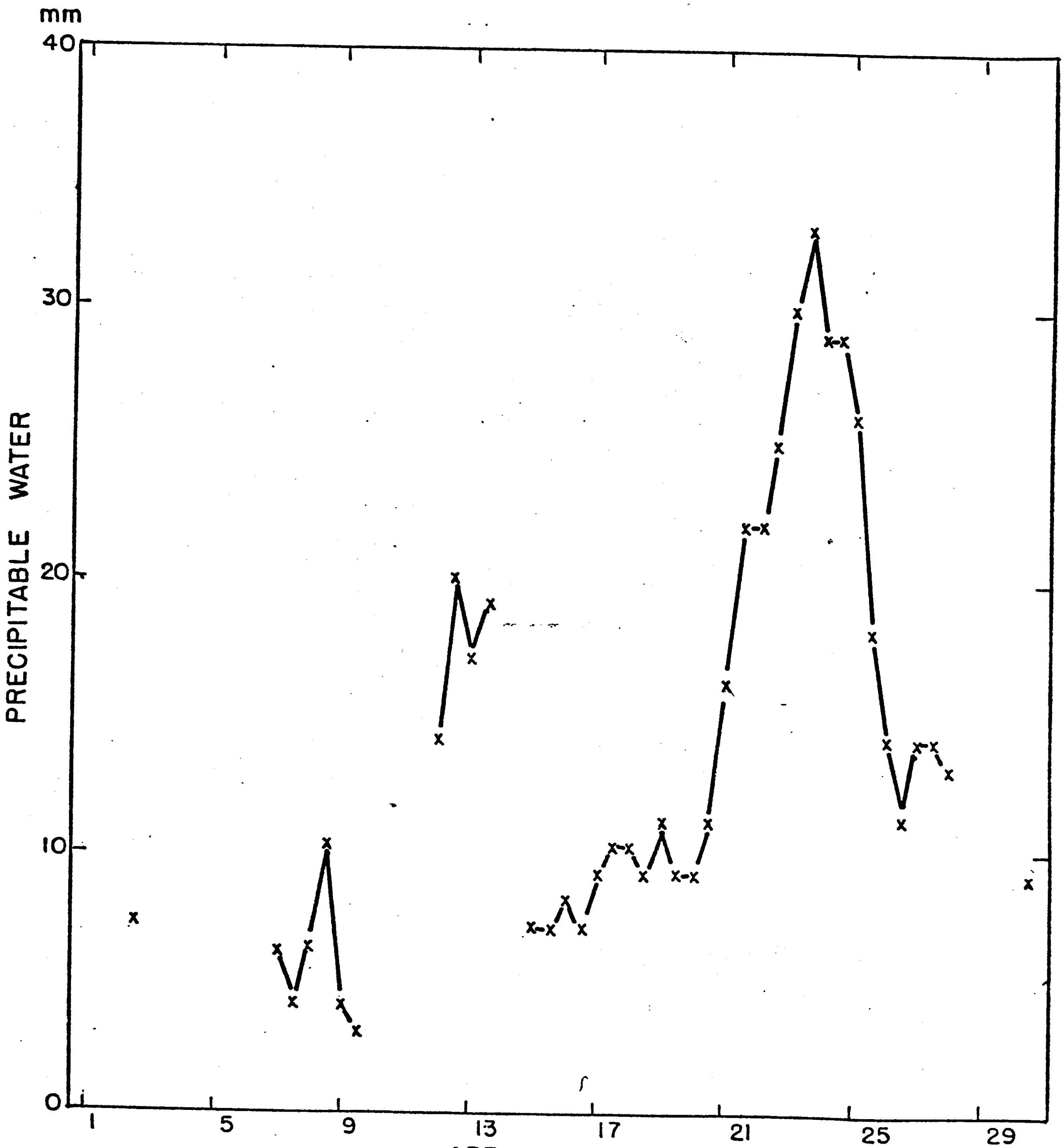

mm
40
30
20
10
0
PRECIPITABLE WATER
1
5
9
13
17
21
25
29
APRIL 1977

Fig. 25

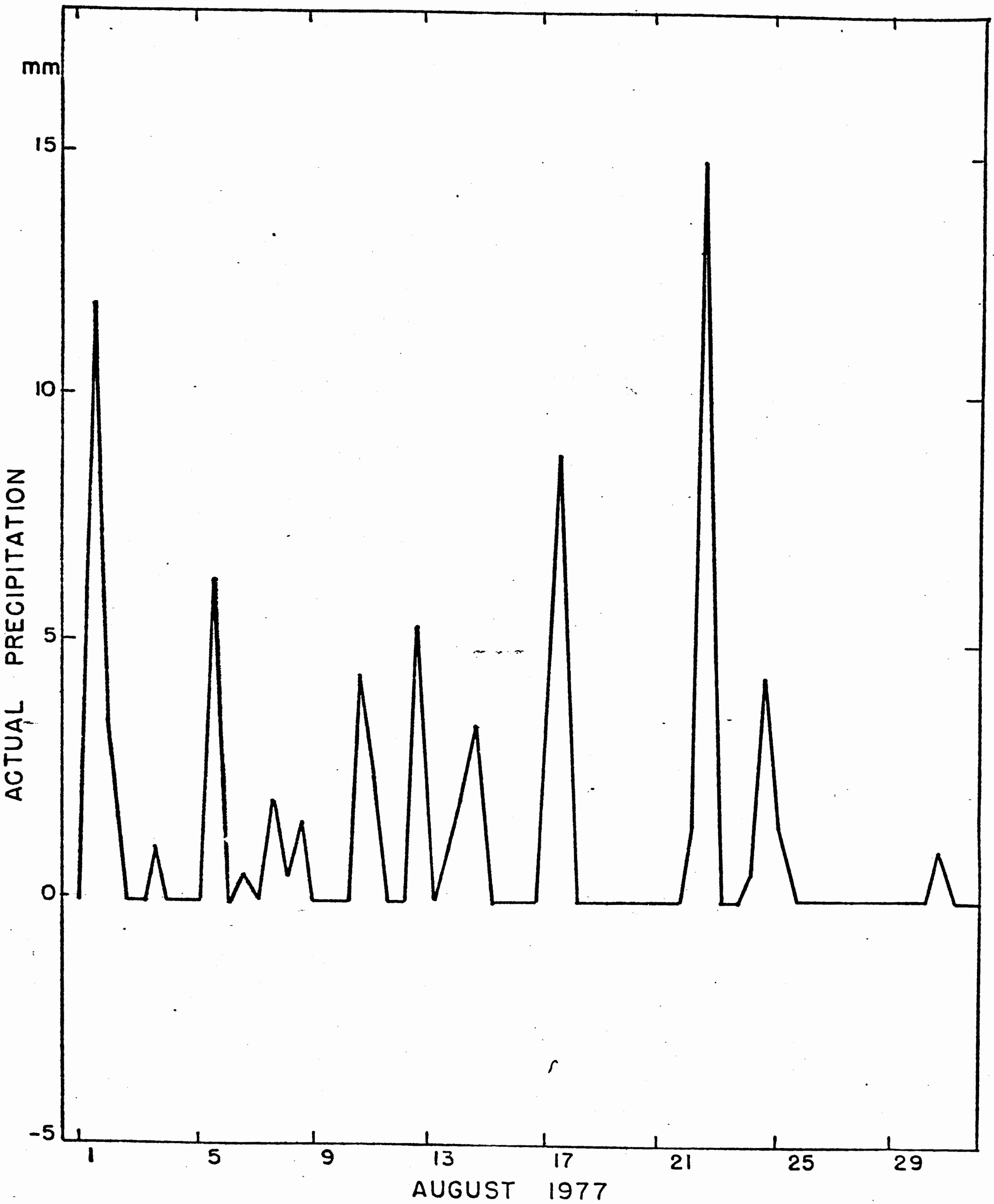
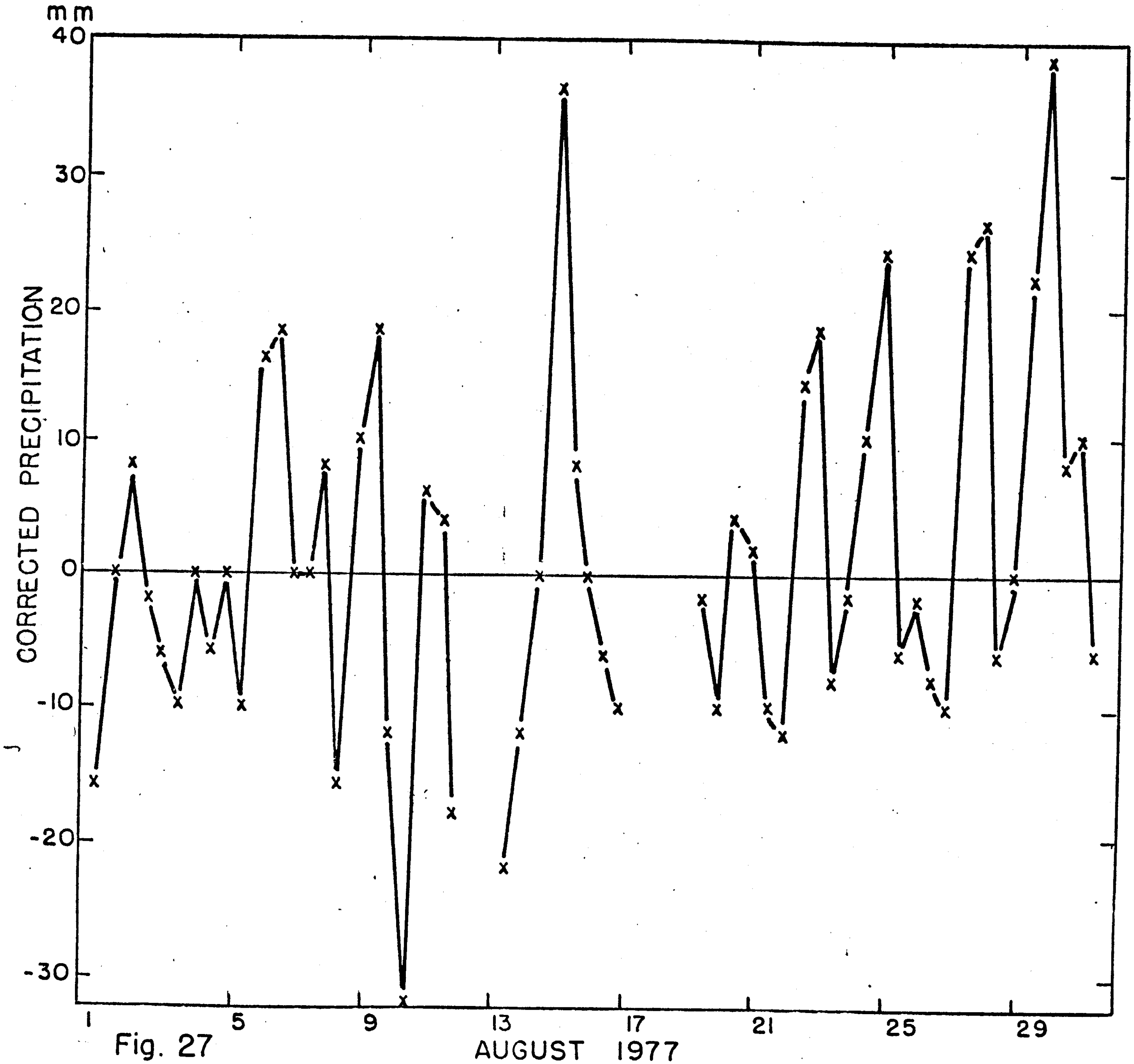

mm
15
10
5
0
-5
ACTUAL PRECIPITATION
1
5
9
13
17
21
25
29
AUGUST 1977

Fig. 26


mm
40
30
20
10
0
-10
-20
-30
CORRECTED PRECIPITATION
1
5
9
13
17
21
25
29
AUGUST 1977
Fig. 27

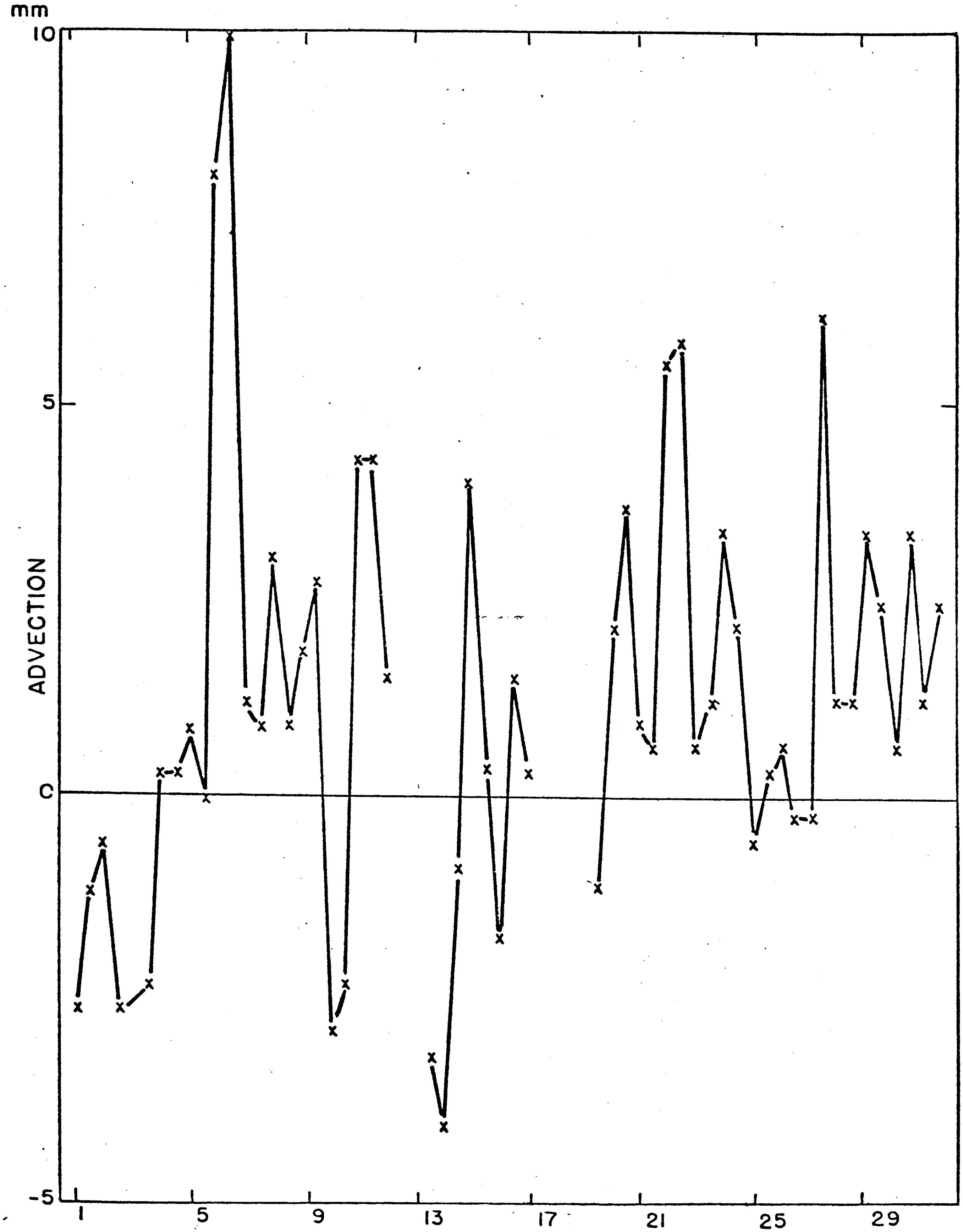


Fig. 28

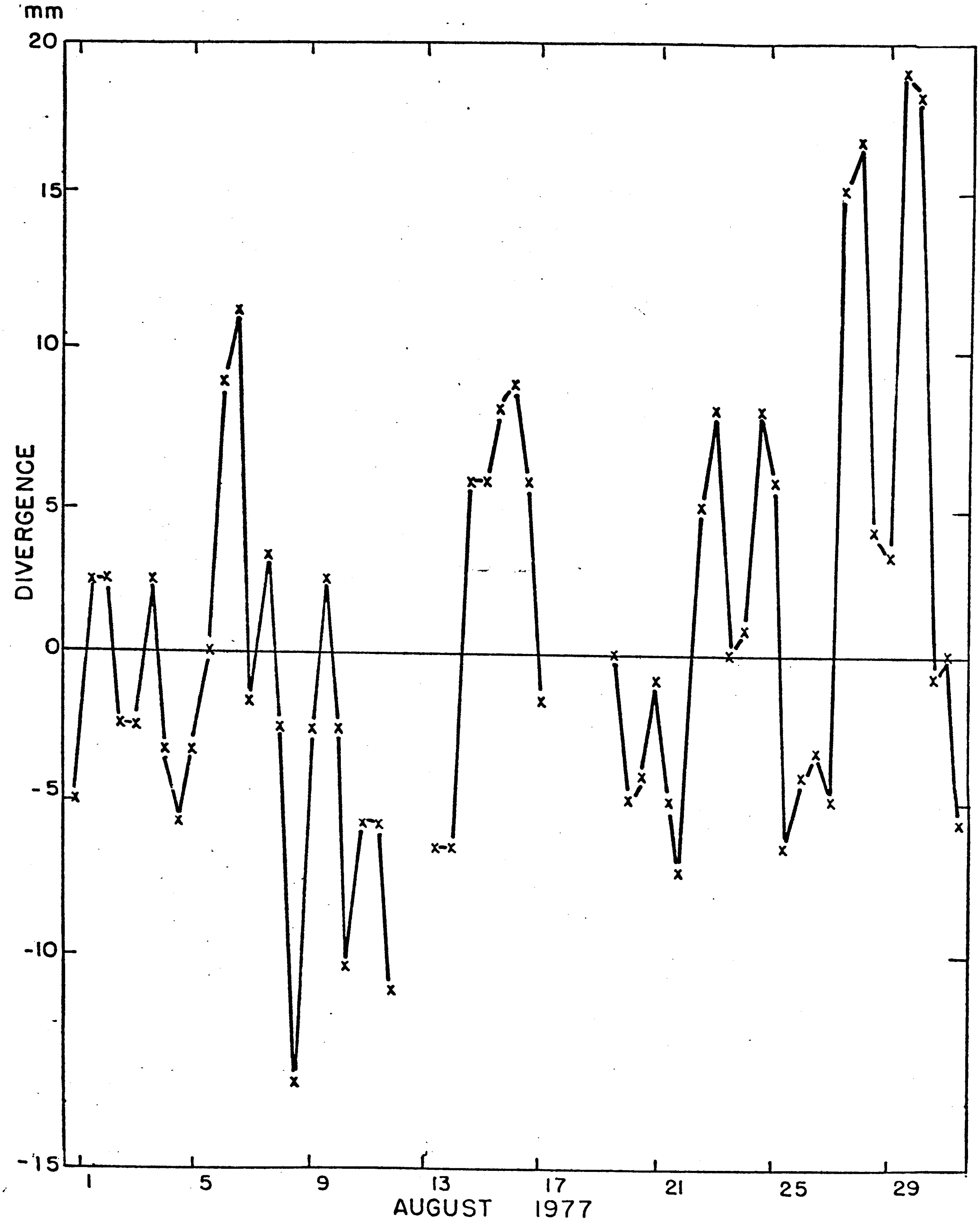


Fig. 29

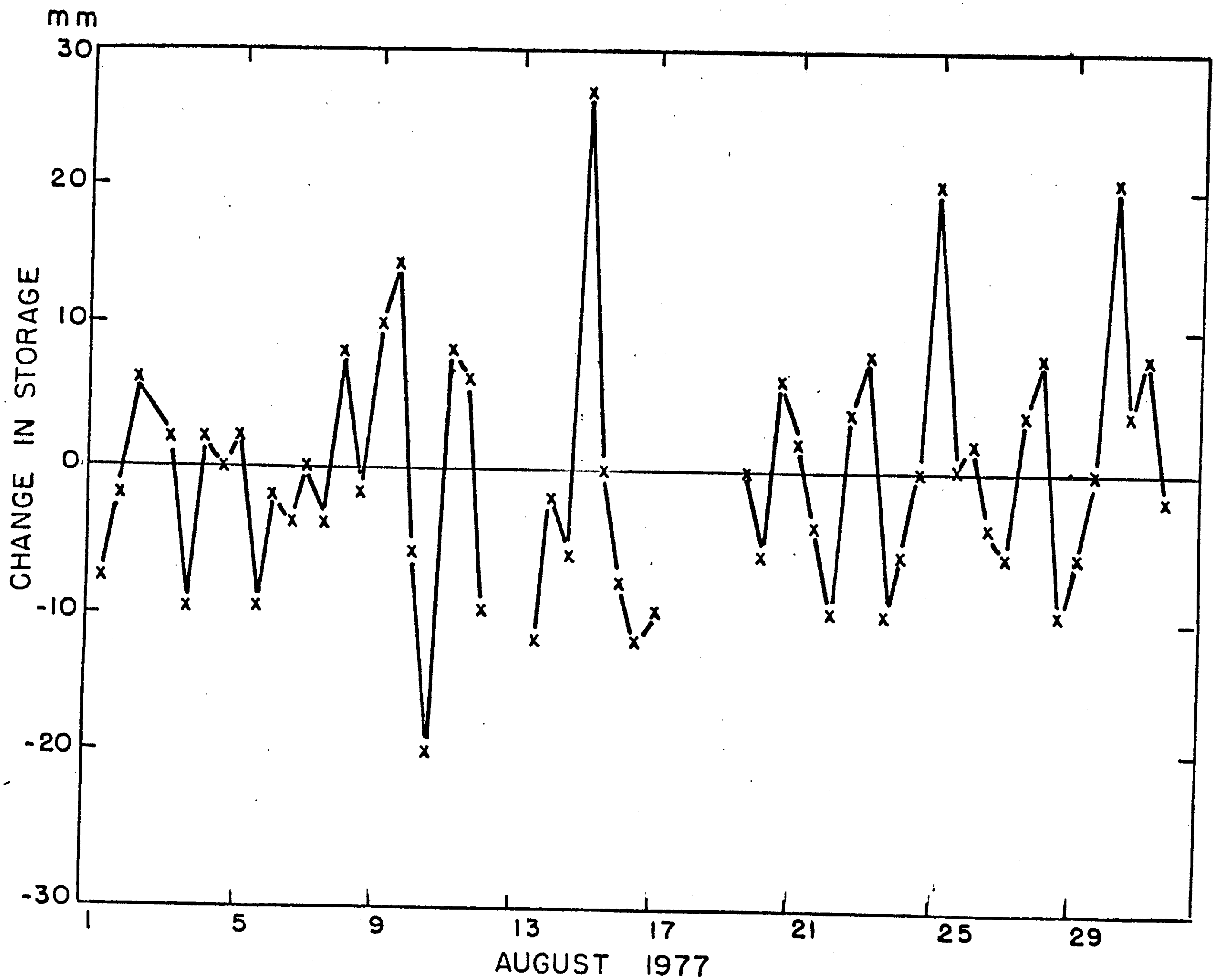

mm
30
20
10
0
-10
-20
-30
CHANGE IN STORAGE
1
5
9
13
17
21
25
29
AUGUST 1977

Fig. 30

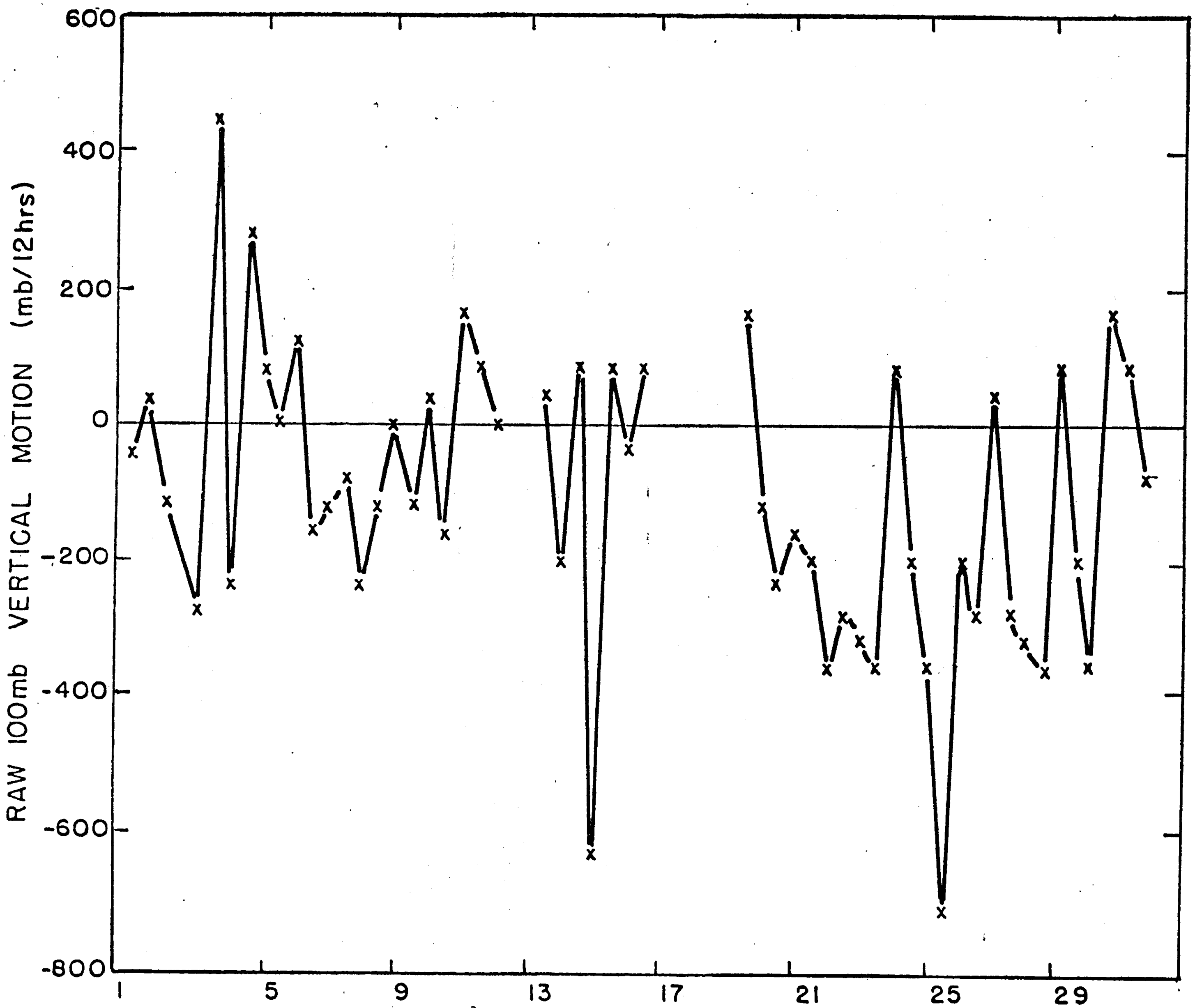


Fig. 31 AUGUST 1977

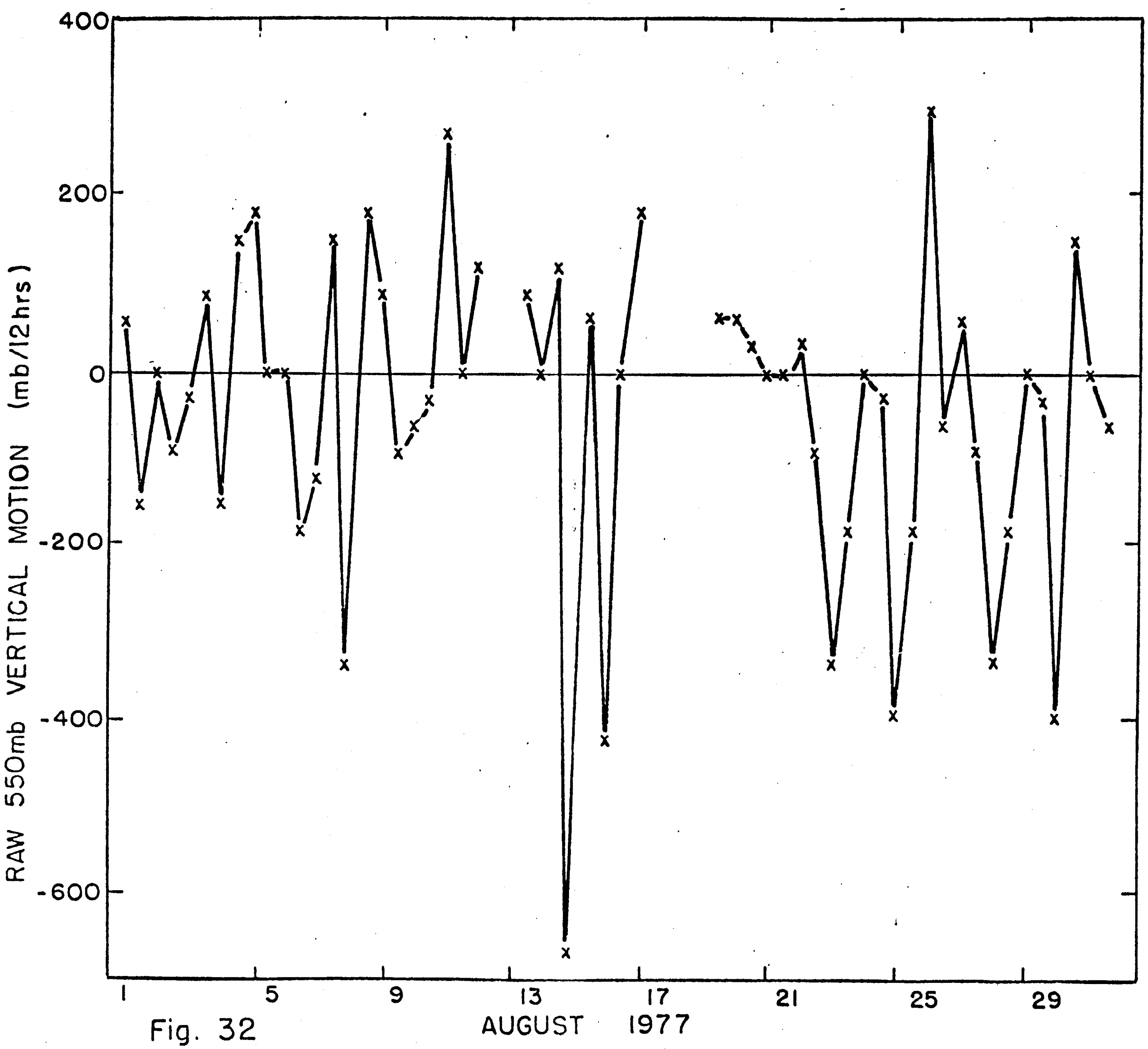

400
200
0
-200
-400
-600
RAW 550mb VERTICAL MOTION (mb/12hrs)
1
5
9
13
17
21
25
29
AUGUST 1977

Fig. 32

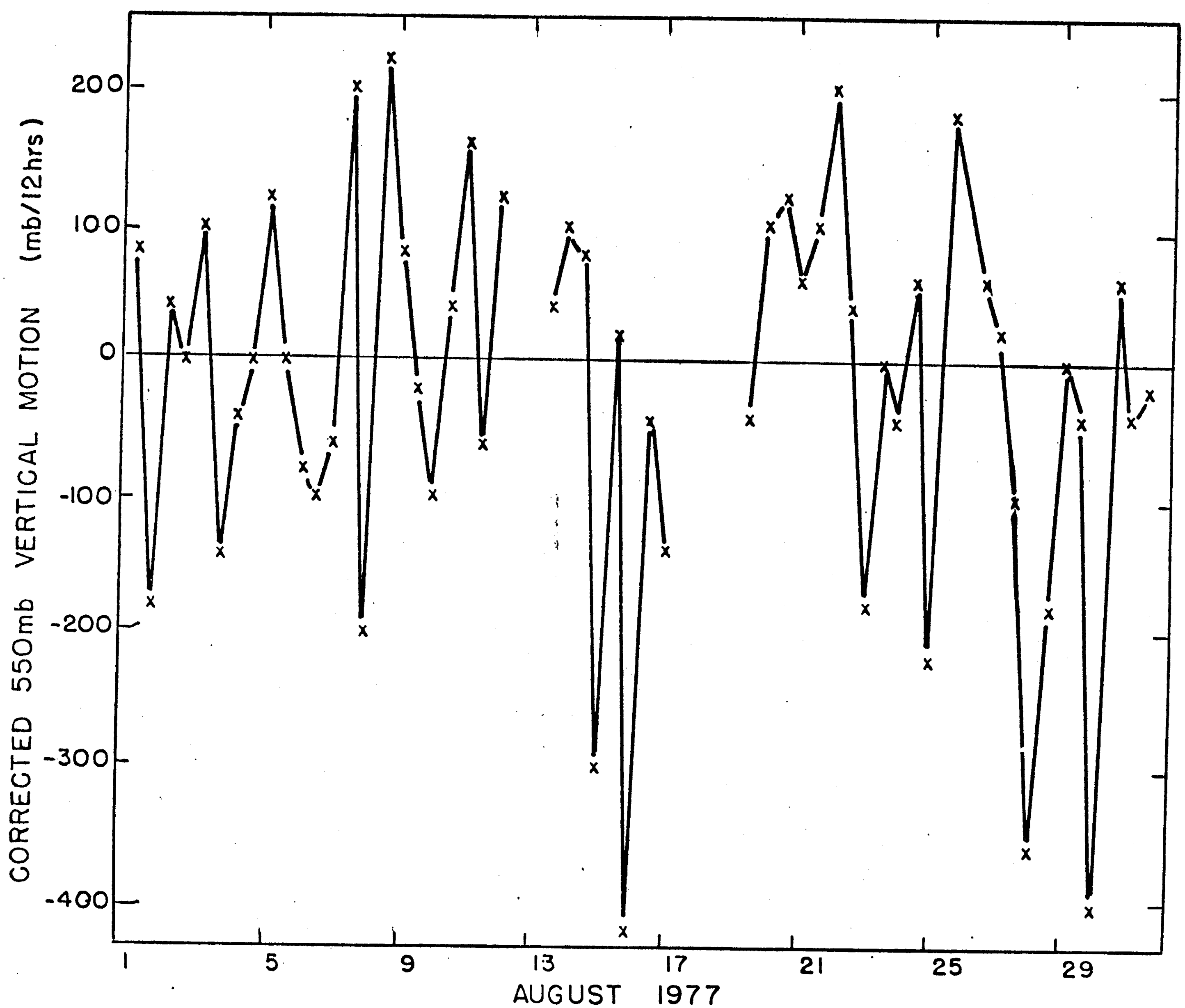

CORRECTED 550mb VERTICAL MOTION (mb/12hrs)
200
100
0
-100
-200
-300
-400
1
5
9
13
17
21
25
29
AUGUST 1977

Fig. 33

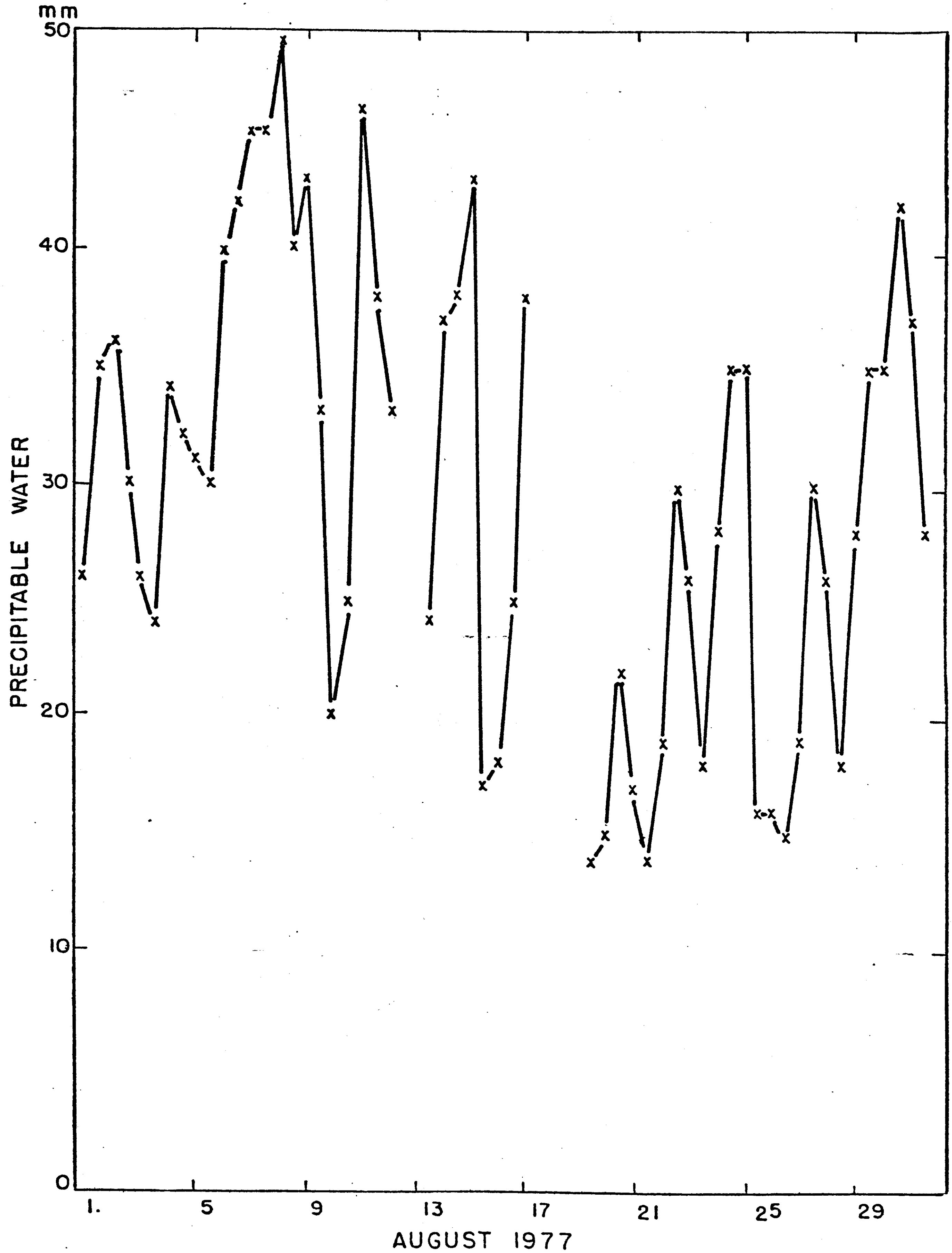
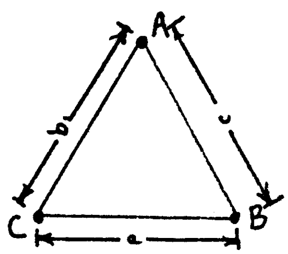

mm
50
40
30
20
10
0
PRECIPITABLE WATER
1.
5
9
13
17
21
25
29
AUGUST 1977

Fig. 34

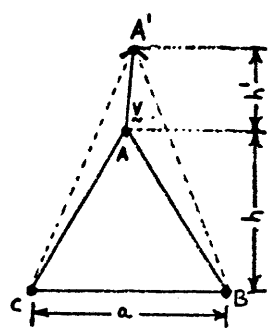
APPENDIX I

Calculation of Divergence. Bellamy 1949

Given a triangular area, and winds which can be assumed to vary linearly between vertices of the triangle, the following geometric approach can be used to calculate the divergence per unit height.



Consider triangle ABC, with winds reported at A, B & C. Since the winds vary linearly, the effect of each wind can be considered separately, for example by considering the winds at B and C to be calm and computing ∇_A , the partial divergence due to the wind at A.



The wind vector at A is represented by v . The flow of air out of triangle ABC per unit time is the area ACA' + ABA' = A'BC-ABC the area ABC = $\frac{1}{2}$ ah and the area

$$A'BC = \frac{1}{2} a(h+h').$$

Thus the outflow is $\frac{1}{2} a(h+h') - \frac{1}{2} ah'$

But h' is the component of $\underset{\sim}{v}$ normal to side BC, $v_{A\perp}$. Therefore, the divergence is $\frac{1}{2} a\, v_{A\perp}$. Now normalize this for the area of ABC ; divide this by area $\mathcal{A}$. After adding the divergences from all three winds,

$$\nabla \cdot v = \nabla_A + \nabla_B + \nabla_C = \frac{1}{2\mathcal{A}}\left(a V_{A\perp} + b V_{B\perp} + c V_{C\perp}\right).$$

APPENDIX II

Water Vapor Budget Computer Program

```
C       ***  PROGRAM BELTR                                                   ***
C       ***  WRITTEN JUNE 1979 BY GEOFFREY S. DOMM                           ***
C       ***  AT MASSACHUSETTS INSTITUTE OF TECHNOLOGY                        ***
C       ***  THE PROGRAM IS BASED IN PART ON A PAPER BY BELLAMY              ***
C       ***  IN THE BULLETIN OF THE AMERICAN METEOROLOGICAL                  ***
C       ***  SOCIETY, 1949.  THE PAPER DEALS WITH A METHOD TO CALCULATE THE  ***
C       ***  MASS DIVERGENCE FROM A =ERTICAL TRIANGULAR PRISM.               ***
C       ***  DATA IS AVAILABLE AT THE THREE VERTICES OF THE TRIANGLE AT ANY L***
C       ***  EVEL.  THE ASSUMPTION IS THAT THE VARIATION IN THE WINDS IS LINE***
C       ***  AR, SO THE TOTAL DIVERGENCE CAN BE CONSIDERED TO BE THE SUM OF T***
C       ***  HE DIVERGENCES OBTAINED BY ASSUMING THE WINDS AT ALL BUT ONE VER***
C       ***  TEX ARE ZERO.  THIS CAN BE GENERALIZED TO WATER VAPOR DIVERGENCE***
C       ***  BY MULTIPLYING THE WINDS BY THE WATER VAPOR MIXING RATIO.        ***
C       ***  CONSIDER THE TRIANGLE SHOWN BELOW, PQR:                         ***
C       ***                                                                  ***
C       ***                                                                  ***
C       ***                            R                                     ***
C       ***                            *                                     ***
C       ***                           /|                                     ***
C       ***                          / |                                     ***
C       ***                         /  |                                     ***
C       ***                        /   |                                     ***
C       ***                       /    |                                     ***
C       ***                      /     |                                     ***
C       ***                     /      |                                     ***
C       ***                    /       |                                     ***
C       ***                   /        |                                     ***
C       ***                 Q/         P                                     ***
C       ***                 *----------*--->                                 ***
C       ***                              V                                   ***
C       ***                                                                  ***
C       ***  THE DIVERGENCE CAUSED BY THE WIND V AT POINT P IS THE RATIO OF  ***
C       ***  THE COMPONENT OF V NORMAL TO SIDE QR TO THE ALTITUDE FROM P TO  ***
C       ***  QR.  SINCE THE LENGTH OF QR MULTIPLIED BY THIS ALTITUDE IS TWICE***
C       ***  THE AREA OF THE TRIANGLE, THE DIVERGENCE CAN ALSO BE EXPRESSED A***
```

```
C         ***  S THE COMPONENT OF V NORMAL TO SIDE QR TIMES THE LENGTH OF QR DI***
C         ***  VIDED BY TWICE THE AREA OF THE TRIANGLE.                        ***
C         ***  THE BASIC PROCEEDURE OF THE PROGRAM:                            ***
C         ***  1) SUBROUTINE COOR: READS THE COORDINATES OF THE THREE VERTICES ***
C         ***  OF THE TRIANGLE, CALCULATES THE HEADING AND LENGTH OF EACH OF TH***
C         ***  E SIDES AND THE AREA OF THE TRIANGLE.  SHPERICI.Y HAS BEEN USED ***
C         ***  TO CALCULATE THE LENGTHS AND HEADINGS, WHICH WERE THEN USED TO  ***
C         ***  CALCULATE THE AREA S IF ON A PLANE.  THIS WAS FOUND TO BE ACCURA***
C         ***  TE BY ABOUT 1% OR LESS, AND ONLY AFFECTS THE MAGNITUDE OF THE DI***
C         ***  VERGENGE AND NOT ITS SIGN..                                     ***
C         ***  2) SUBROUTINE INPUT READS THE DATA.  VARIOUS VERSIONS OF INPUT A***
C         ***  RE AVAILABLE, AND WILL BE DESCRI!): WITHIN EACH ONE.            ***
C         ***  3) SUBROUTINE AVERAG COMPUTES 50 MB AVERAGES OF THE NORMAL COMPO***
C         ***  NENT OF THE WINDS AND MIXING RATIOS.  WIND DATA IS STORED IN MAT***
C         ***  RIX ASBN, WATER VAPOR DATA IN MATRIX ASWS.                     ***
C         ***  4) SUBROUTINE DIVEG:  THE DIVERGENCE FOR EACH LEVEL IS CALCULATE***
C         ***  DAND STORED IN MATRIX SUM.                                      ***
C         ***  5)  THE VERTICAL VELOCITY AT THE TOP OF THE VOLUME IS COMPUTED B***
C         ***  Y ADDING THE DIVERGENCES AT EACH LEVEL.  THE PROGRAM THEN HAS TH***
C         ***  E OPTION OF COMPUTING A CORRECTION VELOCITY, WHICH IF ADDED TO T***
C         ***  HE ACTUAL VELOCITY AT EACH POINT WOULD CAUSE THE VERTICAL VELOCI***
C         ***  TY AT THE TOP OF THE VOLUME TO BE ZERO.                         ***
C         ***  6)  THE (CORRECTED) WIND VELOCITIES ARE USED TO CALCULATE THE CO***
C         ***  NVERGENCE OF WATER VAPOR.                                       ***
C         ***  THIS IS DONE BY SUBROUTINE DIVEG2.  IF CORRECTED WIND VELOCITIES***
C         ***  ARE BEING USED, THE CORRECTION PROCESS IS ITERATED  TO ACHIEVE  ***
C         ***  BETTER ACCURACY.                                                ***
C         ***  7)  SUBROUTINE SUMMAR PRINTS OUT IMPORTANT RESULTS FROM EACH    ***
C         ***  CASE.                                                           ***
C         ***  A NOTE ON DIMENSIONS OF ARRAYS:                                 ***
C         ***  THE PROGRAM IS MODIFIED FROM A PROGRAM SOLVING THE DIVERGENCE PR***
C         ***  OBLEM FOR A QUADRILATERAL.  IN ARRAYS DIMENSIONED 20,4,4        ***
C         ***  20 IS THE VERTICAL COORDINATE.  THE HIGHEST LEVEL IS #3, THE LOW***
C         ***  EST IS #20, WITH 1 AND 2 NOT USED.  THE FIRST FOUR IS TIME, #1 T***
C         ***  HE INITIAL TIME, #3 THE FINAL TIME, 2 AND 4 NOT USED.  THE      ***
```

```
C         ***  SECOND 4 IS THE STATION NUMBER.  STATIONS 1,2, AND 3 MUST BE     ***
C         ***  CLOCKWISE, 4 IS NOT USED.  SIMILAR CONVENTIONS HOLD FOR OTHER AR***
C         ***  RAYS, WITH ANY EXCEPTION EXPLAINED BELOW                         ***
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/WATER/QWAT(2)
      COMMON/SUM2/GRAD(20)
      COMMON/BAR/VXBAR(20,4),VYBAR(20,4)
      COMMON/CREEP/J
      COMMON/TOPS/IPTOP(20)
      COMMON/VORT/ASBT(20,4,4),VORTI(2,18)
      COMMON/ICOOR/ISTAT(3,3)
      COMMON/RCOOR/RLON(3),RLAT(3),ELEV(3),DIST(3),RHEAD(3),AREA
      COMMON/MORON/I
      COMMON/SUMINT/INDEX
      COMMON/SUMMA/       IDATS1(20,3),IDATS2(20,3),RAIN(20),DELS(20),
     ,DIVE(20),WATER(20,2),Q10(20,2),Q7(20,2),UOM100(20,2),
     ,UOM550(20,2),COM550(20,2),VELCOR(20,2)
      REAL TIPTOP
      INTEGER PIP(18)
      REAL*8 PREWAT(2),VCOR(2),DUBU(18)
      REAL*8 VCYY(2,20)
      REAL*8 STORE(2),FLUX(2)
      REAL*8 SUM(20)
      REAL*8 Y(4)
      REAL*8 QER(4),WOF(20),VCNY(20,4)
      REAL*8 OMEGA(18)
      REAL*8 ASBN(20,4,4),ASWS(20,4,4)
      REAL*8 STT(18)
      INTEGER TOP,J,I,K
      INTEGER PTS(8),PTZ(8)
      INTEGER PS,PZ
      REAL DUMP(50),DUMT(50),DUMD(50)
      REAL DUMP2(50),DUMDD(50),DUMFF(50)
      REAL PRES(8,50),TEM(8,50),DWPT(8,50)
      REAL PRESS(8,50),DD(8,50),FF(8,50)
```

```
      REAL P1(2,4),P2(2,4)
      INTEGER E,G
      INTEGER IDAT(7)
      DATA STT/11.6,8.8,6.5,3.3,-1.0,-4.4,-8.1,-12.2,-16.5,-21.1,
     ,-26.3,-31.7,-37.6,-44.4,-52.2,-56.5,-56.5,-56.5/
      DATA IDON/'DONE'/
      DATA PRES/400*0./,TEM/400*0./,DWPT/400*0./
      DATA PRESS/400*0./,DD/400*0./,FF/400*0./
      DATA DUMP/50*0./,DUMT/50*0./,DUMD/50*0./
      DATA DUMP2/50*0./,DUMDD/50*0./,DUMFF/50*0./
      DATA P1/8*0./,P2/8*0./
      DATA WOF/20*0./,VCNY/80*0./,OMEGA/18*0./
      DATA ASBN/320*0./,ASWS/320*0./
      DATA QER/4*0./
      PGRT(N)=(1000.-50.*N)*9.8/(287.*(STT(N)+273.))
      R=287.
109   CONTINUE
      IDAT(1)=0
C       ***  SUBROUTINE COOR IS DESCRIBED ABOVE, AND IN MORE DETAIL IN THE SU***
C       ***  BROUTINE ITSELF.  ALL VARIABLES ARE PASSED VIA COMMON.          ***
      CALL COOR
C       ***  INDEX COUNTS THE NUMBER OF CASES DEALT WITH.                    ***
C       ***  IF INDEX IS 20 OR GREATER, SUBROUTINE SUMMAR IS CALLED.         ***
C       ***  SUBROUTINE SUMMAR IS CALLED ANY TIME THE END OF DATA IS INDICATE***
C       ***  D BY A CARD SAYING 'DONE' IN COLUMNS 1-4 IS READ FOLLING THE DAT***
C       ***  A.                                                              ***
      INDEX=0
110   IF(INDEX.NE.0) CALL SUMMAR
      INDEX=0
      IF(IDAT(1).EQ.IDON) GO TO 109
1000  CONTINUE
      DO 800 JUL=1,2
      UPRIME=0.
      INDEX=INDEX+1
      IF(JUL.EQ.2) GO TO 801
```

```
      IF(INDEX.EQ.21) GO TO 110
      READ(5,1) (IDAT(IY),IY=1,7)
1     FORMAT(7A4)
      IF(IDAT(1).EQ.IDON) GO TO 110
801   CONTINUE
      DO 125 IKE=1,3
      IDATS1(INDEX,IKE)=IDAT(IKE)
125   IDATS2(INDEX,IKE)=IDAT(IKE+4)
      IF(JUL.EQ.2) GO TO 802
CXQ   WRITE(6,2) (IDAT(IY),IY=1,7)
2     FORMAT(5X,'PERIOD FROM ',7A4)
      DO 10 J=1,2
      DO 20 I=1,3
C        ***  SUBROUTINE INPUT READS THE DATA.                                   ***
C        ***  DATA IS OF TWO TYPES, THERMODYNAMIC AND WIND.  EACH VARIABLE IS ***
C        ***  STORED AS A FUNCTION OF AVAILABLE PRESSURE LEVELS.                 ***
C        ***  DUMP ARE PRESSURES AT WHICH THERMODYNAMIC INFORMATION IS AVAILAB***
C        ***  LE, WITH PS THE NUMBER OF LEVELS.  DUMP IS IN MILLIBARS, WITH TH***
C        ***  E HIGHEST PRESSURE IN DUMP(1).  CORRESPONDING TEMPERATURES ARE S***
C        ***  TORED IN DUMT, TEMPERATURES IN DEGREES KELVIN, AND CORRESPONDING***
C        ***  RELATIVE HUMIDITIES IN DUMD, IN PERCENT.                           ***
C        ***  DUMP2 ARE THE PRESSURES AT WHICH WIND INFORMATION IS AVAILABLE, ***
C        ***  WITH PZ THE NUMBER OF SUCH LEVELS.  PRESSURES ARE IN MB, WITH TH***
C        ***  E HIGHEST PRESSURE STORED IN DUMP2.  CORRESPONDING DIRECTIONS, I***
C        ***  N DEGREES IN THE CONVENTIONAL METEOROLOGICAL SENSE, AND SPPEDS I***
C        ***  N METERS PER SECOND ARE STORED IN DUMDD AND DUMFF RESPECTIVELY. ***
      CALL INPUT(DUMP,DUMT,DUMD,PS,DUMP2,DUMDD,DUMFF,PZ)
C        ***  P1(2,3) HOLDS A RECORD OF THE STATION PRESSURE OF ALL ATATIONS    ***
C        ***  AT BOTH TIMES.                                                     ***
      P1(J,I)=DUMP2(1)
      DO 5 K=1,PS
4         FORMAT(1X,3F12.4)
C        ***  PRES,TEM,DWPT,PRESS,DD,FF ARE USED TO ASSEMBLE THE INFORMATION F***
C        ***  ROM EACH STATION IN ONE ARRAY.  INPUT READS THE DATA IN THE FOLL***
C        ***  OWING ORDER; STATION 1-TIME1, STATION 2-TIME1, STATION 3-TIME 1,***
```

```
C         ***  STATION 1-TIME28 STATION 2-TIME 28 STATION 3-TIME29  PRES(6,50) ***
C         ***  IS FILLED IN THIS ORDER.                                          ***
      PRES(2*I+J-2,K)=DUMP(K)
      TEM(2*I+J-2,K)=DUMT(K)
5     DWPT(2*I+J-2,K)=DUMD(K)
      DO 6 K=1,PZ
      PRESS(2*I+J-2,K)=DUMP2(K)
      DD(2*I+J-2,K)=DUMDD(K)
6     FF(2*I+J-2,K)=DUMFF(K)
      L=I-1-((I-4)/3)*3
      P2(J,L)=P1(J,I)
      PTS(2*I+J-2)=PS
      PTZ(2*I+J-2)=PZ
20    CONTINUE
10    CONTINUE
C         ***                                                                    ***
C         ***  TIPTOP IS THE LOWEST PRESSURE AVAILABLE AT ALL THREE STATIONS AT***
C         ***  BOTH TIMES, AND IS USED AS THE TOP OF THE ATMOSPHERE IF WIND COR***
C         ***  RECTIONS ARE USED.  IPTOP STORES THESE VALUES FROM EACH CASE FOR***
C         ***  USE IN SUMMAR.                                                    ***
      TIPTOP=100.
      DO 24 JON=1,6
      DUM7=PTZ(JON)
24    TIPTOP=AMAX1(TIPTOP,PRESS(JON,DUM7))
      TIPTOP=(TIPTOP-1.)/50.
      IPTOP(INDEX)=50*(IFIX(TIPTOP)+1)
      IPTOP(INDEX+1)=IPTOP(INDEX)
C         ***
C         ***  DELTA IS THE STATION PRESSURE TENDENCY, TAKEN AS THE AVERAGE STA***
C         ***  TION PRESSURE AT TIME 2 MINUS THE AVERAGE STATION PRESSURE AT TI***
C         ***  ME 1.                                                             ***
C         ***  DELTA IS IN MB/SEC.                                               ***
      DELTA=0.
      DO 25 LL=1,3
25    DELTA=DELTA+P1(2,LL)-P1(1,LL)
```

```
      DELTA=DELTA/3.
      DELTA=DELTA/(12.*3600.)
C       ***                                                                    ***
C       ***  SUBROUTINE AVERAGE COMPUTES 50 MB AVERAGES OF MIXING RATIOS        ***
C       ***  AND NORMAL COMPONENTS OF THE WIND.  IN ADDITION TO THE             ***
C       ***  VARIABLES PASSED EXPLICITLY, SEVERAL ARE PASSED VIA COMMON.        ***
C       ***                                                                    ***
      CALL AVERAG(PRES,TEM,DWPT,PTS,PRESS,DD,FF,PTZ,ASBN,ASWS)
802   CONTINUE
      VCOR(1)=0.
      VCOR(2)=0.
      I=1
      E=1
      G=2
C       ***                                                                    ***
C       ***  SUBROUTINE DIVERG COMPUTES THE DIVERGENCE FOR EACH 50 MB LAYER.   ***
C       ***                                                                    ***
29    CALL DIVERG(ASBN,P1,P2,VCNY,QER,YTOT,E,G,I,SUM)
      WF=0.
      DO 30 I=1,3
30    WF=WF+QER(I)
      OM=DELTA
      DO 35 JJ=1,18
35    PIP(JJ)=1000.-JJ*50.
      DO 40 I=1,19
40    WOF(I)=1.
      WOF(20)=WF
      DO 50 II=3,20
      I=23-II
      OM=OM+(50./AREA)*WOF(I)*SUM(I)
      OMEGA(II-2)=OM*60.*60.*12.
C          OMEGA IN MB/12 HOURS
50    CONTINUE
      DO 45 N=1,18
      DUBU(N)=-OMEGA(N)/PGRT(N)*100./(12.*60.*60.)
```

```
45    CONTINUE
      IF(JUL.EQ.1) GO TO 803
C     SET OMEGA AT TO  EQUAL TO ZERO
      TOP=18-(IPTOP(INDEX)-100)/50
      UPRIME=AREA*OMEGA(TOP)/(60.*60.*12.*YTOT*F1(TOP,WF))
                                                  UPRIME=UPRIME*2.
CXZ   UPRIME=0.
CXQ   WRITE(6,2) (IDAT(IY),IY=1,7)
CXQ   WRITE(6,500)
CXQ   WRITE(6,499)
CXQ   WRITE(6,501) ((PIP(JJ),OMEGA(JJ),DUBU(JJ)),JJ=1,18)
      WRITE(6,2)(IDAT(IY),IY=1,7)
      WRITE(6,500)
      WRITE(6,499)
      WRITE(6,501)((PIP(JJ),OMEGA(JJ),DUBU(JJ)),JJ=1,18)
499   FORMAT(1X,'P(MB)',2X,'OMEGA(MB/12HR)',2X,'W(CM/SEC)')
500   FORMAT(5X,'OMEGA')
501   FORMAT(2X,I3,3X,F12.6,2X,F12.6)
803   CONTINUE
      IF(E.EQ.1) GO TO 70
      VC2=-UPRIME
      VCOR(2)=VCOR(2)+VC2
      UOM100(INDEX,E-1)=OMEGA(18)
      UOM550(INDEX,E-1)=OMEGA(9)
      GO TO 90
70    VC1=-UPRIME
      VCOR(1)=VCOR(1)+VC1
      UOM100(INDEX,E)=OMEGA(18)
      UOM550(INDEX,E)=OMEGA(9)
      E=3
      G=4
      I=2
      GO TO 29
90    CONTINUE
      ITER=2
```

```
      INK=0
72    INK=INK+1
CXQ   WRITE(6,2) (IDAT(IY),IY=1,7)
      CALL DIVEG2(ASBN,ASWS,P1,P2,VC1,VC2,VCYY,STORE,FLUX)
      DO 95 IO=1,2
           OM=DELTA
           DO 96 IF=3,20
                IN=23-IF
                OM=OM+(50./AREA)*WOF(IN)*VCYY(IO,IN)
96              OMEGA(IF-2)=OM*60.*60.*12.
      DO 94 N=1,18
      DUBU(N)=-OMEGA(N)/PGRT(N)*100./(12.*60.*60.)
94    CONTINUE
      IF(JUL.EQ.1) GO TO 805
      IF(INK.LE.ITER) GO TO 97
CXQ   WRITE(6,2) (IDAT(IY),IY=1,7)
CXQ        WRITE(6,502)
502   FORMAT(1X,'CORRECTED OMEGA')
CXQ   WRITE(6,511)
511   FORMAT(1X,'P(MB)',2X,'OMEGA(MB/12HR)',2X,'W(CM/SEC)',2X,
     2  'ZETA(10E-5/S)')
CXQ   WRITE(6,512)((PIP(JJ),OMEGA(JJ),DUBU(JJ),VORTI(IO,JJ)),JJ=1,18)
512   FORMAT(2X,I3,3X,F12.6,3X,F9.3,4X,F9.3)
      WRITE(6,2)(IDAT(IY),IY=1,7)
      WRITE(6,502)
      WRITE(6,511)
      WRITE(6,512)((PIP(JJ),OMEGA(JJ),DUBU(JJ),VORTI(IO,JJ)),JJ=1,18)
      IF(ITER.EQ.0) GO TO 975
      IF(INK.GT.ITER) GO TO 95
97    UPRIME=AREA*OMEGA(TOP)/(60.*60.*12.*YTOT*F1(TOP,WF))
                                                   UPRIME=UPRIME*2.
CXZ   UPRIME=0.
975   CONTINUE
805   CONTINUE
      IF(IO.EQ.1) GO TO 98
```

```
      VC2=-UPRIME
      VCOR(2)=VCOR(2)+VC2
      COM550(INDEX,IO)=OMEGA(9)
      VELCOR(INDEX,2)=VCOR(2)
      IF(JUL.EQ.1) GO TO 95
      IF(INK.GT.ITER) GO TO 95
      GO TO 72
98    VC1=-UPRIME
      VCOR(1)=VCOR(1)+VC1
      COM550(INDEX,IO)=OMEGA(9)
      VELCOR(INDEX,1)=VCOR(1)
95         CONTINUE
100   CONTINUE
CXQ   WRITE(6,505)
505   FORMAT(5X,'WIND CORRECTIONS:')
CXQ   WRITE(6,506)((NZ,VCOR(NZ)),NZ=1,2)
506   FORMAT(10X,'TIME ',I1,' - ',F12.4,'M/SEC')
      AFLUX=12.*60.*60.*100.*(FLUX(1)+FLUX(2))/19.6
      BFLUX=12.*60.*60.*100.*(QWAT(1)+QWAT(2))/19.6
C         AFLUX IN KG
      STOR=90000.*AREA*(STORE(2)-STORE(1))/9.8
C          STOR IN KG
      EMP=AFLUX+BFLUX+STOR
C          EMP IN KG
      DO 101 NZ=1,2
      PREWAT(NZ)=9000.*STORE(NZ)/9.8
      PINCH=PREWAT(NZ)/2.54
      WATER(INDEX,NZ)=PREWAT(NZ)
CXQ   WRITE(6,504)NZ,PREWAT(NZ),PINCH
101   CONTINUE
504   FORMAT(15X,'PRECIPITABLE WATER AT TIME ',I2,1X,'IS 'F6.2,' CM (',
     ,F6.2,' IN)')
      AFLUX=-.1*AFLUX/AREA
      BFLUX=-.1*BFLUX/AREA
      STOR=-.1*STOR/AREA
```

```
      DEPTH=-.1*EMP/AREA
      RAIN(INDEX)=DEPTH
      DELS(INDEX)=STOR
      DIVE(INDEX)=AFLUX
      GRAD(INDEX)=BFLUX
C             ALL QUANTITIES NOW IN CM
CXQ   WRITE(6,503) DEPTH,AFLUX,STOR
503   FORMAT(5X,'THE NET RESULT:',/,25X,'P - E =',F10.3,1X,'CM',/,
     ,30X,'OF WHICH ',F10.3,' CM IS DUE TO DIVERGENCE',/,
     ,35X,'AND ',F10.3,' CM IS DUE TO INCREASED STORAGE')
800   CONTINUE
      GO TO 1000
1001  CONTINUE
      STOP
      END
```

```
      FUNCTION QSAT(TA,P)
C       ***  FUNCTION QSAT -                                         ***
C       ***      TA - TEMPERATURE IN DEGREES KELVIN                  ***
C       ***      P - PRESSURE IN MILLLIBARS                          ***
```

```
C         ***  QSAT - SATURATION MIXING RATIO (NO UNITS)                      ***
C         ***                                                                 ***
      PS=1013.246
      TS=373.16
      E1=11.344*(1.0-TA/TS)
      E2=-3.49149*(TS/TA-1.0)
      F1=-7.90298*(TS/TA-1.0)
      F2=5.02808*ALOG10(TS/TA)
      F3=-1.3816*(10.0**E1-1.0)/10000000.0
      F4=8.1328*(10.0**E2-1.0)/1000.0
      F5=ALOG10(PS)
      F=F1+F2+F3+F4+F5
      ES=10.00**F
      QSAT=.62197*ES/(P-ES)
      RETURN
      END
```



```
      FUNCTION F1(TOP,WF)
C         ***  FUNCTION F1 -                                                   ***
C         ***     TOP - LOWEST PRESSURE AVAILABLE AT ALL STATIONS              ***
C         ***     WF -  WEIGHTING FACTOR VECTOR                                ***
C         ***    F1 - FACTOR FOR CONSTANT DIVERGENCE CORRECTION                ***
```

```
C        ***  FOR A LINEARLY VARYING DIVERGENCE CORRECTION USE              ***
C        ***          F1=(TOP-1)*25                                         ***
      INTEGER TOP
      F1=(TOP-1+WF)*50
      RETURN
      END
```

```
      FUNCTION F2(W,FAC)
C        ***  FUNCTION F2 -                                                 ***
C        ***     W - INDEX OF LAYER, 20 IS THE LOWEST                       ***
C        ***     FAC - NOT USED                                             ***
C        ***  F2 - CORRECTION FACTOR FOR CONSTAND DIVERGENCE CORRECTION     ***
C        ***  FOR A LINEARLY VARYING DIVERGENCE CORRECTION , REPLACE F2=1.  ***
C        ***  WITH THE FOLLOWING STATEMENTS -                               ***
C        ***          COMMON/SUMMINT/INDEX                                  ***
C        ***          COMMON/TOPS/IPTOP(20)                                 ***
C        ***          F2=0.                                                 ***
C        ***          IF(W.EQ.20) RETURN                                    ***
C        ***          F2=(W*50.-975.)/(IPTOP(INDEX)-950.)                   ***
C        ***                                                                ***
C        ***  IPTOP(INDEX) IS THE LOWEST PRESSURE BEING USED IN THE PRESENT CA***
C        ***  LCULATION.                                                    ***
C        ***                                                                ***
      INTEGER W
```

```
      F2=1.
      RETURN
      END
```

```
      SUBROUTINE INPUT(PRES,TEM,DWPT,PTS,PRESS,DD,FF,PTZ)
C        ***  SUBROUTINE INPUT :                                                  ***
C        ***  THIS SUBROUTINE IS USED TO INTERPRET TELETYPE DATA TRANSCRIBED D***
C        ***  IRECTLY ONTO COMPUTER CARDS IN THE PRESCRIBED FASHION.              ***
C        ***  THE CORRECT METHOD IS DESCRIBED IN DETAIL ON A SEPARATE SHEET,      ***
C        ***  IN BREIF : TTBB DATA FIRST, ONE LEVEL PER CARD                      ***
C        ***     TTAA DATA SECOND, ONE LEVEL PER CARD                             ***
C        ***    PPBB DATA THIRD, ONE GROUP OF THREE LEVELS PER CARD               ***
C        ***  ONLY HTE TTAA DATA ARE NECESSARY, ALTHOUGH RESOLUTION IS            ***
C        ***  IMPROVED BY INCLUSION OF ALL DATA                                   ***
C        ***                                                                      ***
C        ***  PRES,TEM,AND DWPT ARE CONNECTED VECTORS.                            ***
C        ***  PRES - PRESSURE LEVELS FOR THERMODYNAMIC DATA IN MB.                ***
C        ***    TEM - TEMPERATURE IN KELVIN AT CORRESPONDING PRESSURE             ***
C        ***  DWPT - RELATIVE HUMIDITY AT CORRESPONDING PRESSURE.                 ***
C        ***    PTS - NUMBER OF LEVELS OF THERMODYNAMIC DATA                      ***
C        ***  PRESS,DD,AND FF ARE CONNECTED VECTORS.                              ***
C        ***    PRESS - PRESSURE AT WHICH WIND DATA IS REPORTED IN MB             ***
C        ***    DD - WIND DIRECTION IN DEGREES IN CONVENTIONAL METEOROLOGICAL ***
C        ***  SENSE FOR CORRESPONDING PRESSURE                                    ***
```

```
C          ***    FF - WIND SPEED IN METERS PER SECOND FOR CORRESPONDING PRESSUR***
C          ***  E                                                              ***
C          ***    PTZ - THE NUMBER OF LEVELS FOR WHICH WIND DATA IS REPORTED    ***
C          ***  OTHER INPUT PROGRAMS CAN BE USED FOR DIFFERENT TYPES OF         ***
C          ***  DATA AS LONG AS THE FINAL RESULTS ARE RETURNED TO THE MAIN PROGR***
C          ***  AM IN THE ABOVE FORM.                                           ***
C          ***                                                                  ***
        REAL*8 RLON,RLAT,ELEV,DIST,RHEAD,AREA
        COMMON/RCOOR/RLON(3),RLAT(3),ELEV(3),DIST(3),RHEAD(3),AREA
        COMMON/MORON/IDIOT
        INTEGER PTS,PTZ,PRE,TEN,DW
        REAL HITE(50),TE(50),DPT(50)
        INTEGER DDR(3),VVL(3)
        INTEGER DUMP
        REAL HIGH(50),DDD(50),FFF(50),P(50),T(50),Z(50)
        INTEGER A,B(3)
        REAL PRES(1),TEM(1),DWPT(1),PRESS(1),DD(1),FF(1)
        INTEGER LEVEL,HEIGHT,TEMPE,DT,DIR,VEL
        DO 70 LM=1,50
        HITE(LM)=0.
        TE(LM)=0.
        DPT(LM)=0.
        HIGH(LM)=0.
        DDD(LM)=0.
        FFF(LM)=0.
        T(LM)=0.
        Z(LM)=0.
        P(LM)=0.
        PRES(LM)=0.
        TEM(LM)=0.
        DWPT(LM)=0.
        PRESS(LM)=0.
        DD(LM)=0.
        FF(LM)=0.
70      CONTINUE
```

```
C      STEP 1   INPUT TTBB
C      WRITE(6,11)
C      WRITE(6,12)
11     FORMAT(5X,'TTBB')
12     FORMAT(40X,'PRESSURE',2X,'TEMP',2X,'DWPT DEPRESSION')
13     FORMAT(41X,F5.0,4X,F5.1,4X,F4.1)
       READ(5,9999)
       I=0
100    I=I+1
       READ(5,1)PRE,TEN,DW
1      FORMAT(2X,I3,1X,I3,I2)
       IF (PRE.EQ.999) GO TO 130
       PRES(I)=PRE
       IF(PRES(I).LT.100.) PRES(I)=PRES(I)+1000.
       IF(MOD(TEN,2).EQ.0)  TEM(I)=TEN/10.+273.2
       IF(MOD(TEN,2).NE.0) TEM(I)=-TEN/10.+273.2
C          ERROR MESSAGE
110    IF (DW.LE.50) DWPT(I)=DW/10.
       IF (DW.EQ.-1) DWPT(I)=-1
       IF (DW.GE.56) DWPT(I)=DW-50.
       IF(DWPT(I).EQ.-1) GO TO 120
       TD=TEM(I)-DWPT(I)
       DWPT(I)=100.*QSAT(TD,PRES(I))/QSAT(TEM(I),PRES(I))
120    CONTINUE
       DVM=TEM(I)-273.2
C      WRITE(6,13)PRES(I),DVM,DWPT(I)
       GO TO 100
130    I=I-1
C      STEP 2    INPUTTTAA
C      WRITE(6,22)
C      WRITE(6,23)
22     FORMAT(5X,'TTAA')
23     FORMAT(10X,'PRESSURE',2X,'HEIGHT',2X,'TEMP',3X,'DWPT DPN',2X,
      ,'DIRECTION',2X,'SPEED')
24     FORMAT(12X,F5.0,3X,F6.0,2X,F5.1,3X,F4.1,6X,F4.0,6X,F4.0)
```

```
      READ(5,9999)
      J=0
2100  J=J+1
      READ(5,21) LEVEL,HEIGHT,TEMPE,DT,DIR,VEL
21    FORMAT(I2,I3,1X,I3,I2,1X,I3,I2)
      IF(LEVEL.EQ.-1) GO TO 2160
      PRESS(J)=10.*LEVEL
      IF (LEVEL.EQ.0) PRESS(J)=1000.
2110  IF(LEVEL.EQ.99) PRESS(J)=HEIGHT
      IF(PRESS(J).LT.100.) PRESS(J)=PRESS(J)+1000.
      IF(LEVEL.EQ.99) GO TO 2111
      IF (LEVEL.EQ.0) GO TO 2112
      IF (LEVEL.EQ.85) GOTO 2113
      IF (LEVEL.EQ.70) GO TO 2114
      IF (LEVEL.EQ.50) GO TO 2115
      IF (LEVEL.GT.25) GO TO 2116
      HITE(J)=HEIGHT*10.+10000.
      GO TO 2120
2111  HITE(J)=ELEV(IDIOT)
      GO TO 2120
2112  HITE(J)=HEIGHT
      GO TO 2120
2113  HITE(J)=HEIGHT+1000.
      GO TO 2120
2114  IF (HEIGHT.GT.500) HITE(J)=HEIGHT+2000.
      IF (HEIGHT.LE.500) HITE(J)=HEIGHT+3000.
      GO TO 2120
2115  HITE(J)=HEIGHT+5000.
      GO TO 2120
2116  HITE(J)=HEIGHT*10.
      GO TO 2120
2120  IF(MOD(TEMPE,2).EQ.0) TE(J)=TEMPE/10.+273.2
      IF(MOD(TEMPE,2).NE.0) TE(J)=-TEMPE/10.+273.2
      GO TO 2130
C     ERROR MESSAGE
```

```
2130   IF (DT.LE.50) DPT(J)=DT/10.
       IF (DT.EQ.-1) DPT(J)=-1
       IF (DT.GE.56) DPT(J)=DT-50.
       IF(DPT(J).EQ.-1) GO TO 2140
       TD=TE(J)-DPT(J)
       DPT(J)=100.*QSAT(TD,PRESS(J))/QSAT(TE(J),PRESS(J))
       GO TO 2140
C      ERROR MESSAGE
2140   IF(MOD(DIR,5).NE.0) GO TO 145
       DD(J)=DIR
       FF(J)=VEL*.514
       GO TO 2150
145    DD(J)=DIR-1
       FF(J)=(VEL+100.)*.514
2150   CONTINUE
       DVM=TE(J)-273.2
       DVVM=FF(J)/.514
C      WRITE(6,24)PRESS(J),HITE(J),DVM,DPT(J),DD(J),DVVM
       GO TO 2100
2160   J=J-1
       PRES(1)=PRESS(1)
C      STEP 3    READ IN PPBB
C      WRITE(6,32)
C      WRITE(6,33)
32     FORMAT(5X,'PPBB')
33     FORMAT(70X,'HEIGHT',2X,'DIRECTION',2X,'SPEED')
34     FORMAT(70X,F6.0,5X,F4.0,5X,F4.0)
       READ(5,9999)
9999   FORMAT(1X)
       K=1
3100   CONTINUE
       READ(5,31)A,B(1),B(2),B(3),((DDR(L),VVL(L)),L=1,3)
       IF (A.EQ.9) GO TO 3150
31     FORMAT(1X,4I1,1X,3(I3,I2,1X))
       M=3
```

```
      IF (B(3).EQ.0) M=2
      IF (B(2).EQ.0) M=1
      DO 3110 L=1,M
         IF(MOD(DDR(L),5).EQ.0) GO TO 3105
         DDR(L)=DDR(L)-1
         VVL(L)=VVL(L)+100
3105     CONTINUE
3110     CONTINUE
      DO 3120 L=1,M
         HIGH(K+L-1)=A*10000+B(L)*1000
         DDD(K+L-1)=DDR(L)
         FFF(K+L-1)=VVL(L)*.514
3120  CONTINUE
      K=K+M
      GO TO 3100
3150  CONTINUE
      DO 3160 ILK=1,K
      DVVM=FFF(ILK)/.514
C3160 WRITE(6,34)HIGH(ILK),DDD(ILK),DVVM
3160  CONTINUE
      K=K-1
C     STEP 4    STORE HEIGHT AND PRESSUFE DATA
      DO 170 LL=1,J
         P(LL)=PRESS(LL)
         T(LL)=TE(LL)
170      Z(LL)=HITE(LL)*3.2825
      Z(J+1)=60000.
      T(J+1)=T(J)
C     STEP 5    COMBINE TEMPDATA AND SORT
      II=I+J
      IP1=I+1
      DO 510 M=IP1,II
         PRES(M)=PRESS(M-I)
         TEM(M)=TE(M-I)
510      DWPT(M)=DPT(M-I)
```

```
      DUMP=0
      IIM1=II-1
      DO 520 M=1,IIM1
      MP1=M+1
           DO 515 N=MP1,II
                IF (PRES(M).GT.PRES(N)) GO TO515
         IF(PRES(M).LT.PRES(N)) GO TO 514
         PRES(N)=0.
           IF(PRES(M).NE.0.) DUMP=DUMP+1
         GO TO 515
514             DUM=PRES(N)
                DUMM=TEM(N)
                DUMMY=DWPT(N)
                PRES(N)=PRES(M)
                TEM(N)=TEM(M)
                DWPT(N)=DWPT(M)
                PRES(M)=DUM
                TEM(M)=DUMM
                DWPT(M)=DUMMY
515             CONTINUE
520        CONTINUE
      PTS=II-DUMP
C     WRITE(6,51)
51    FORMAT(10X,'PRESSURE',5X,'TEMP',7X,'DWPT DEPRESSION')
      DO 530 KO=1,PTS
C530  WRITE(6,52)PRES(KO),TEM(KO),DWPT(KO)
530   CONTINUE
52    FORMAT(12X,F5.0,7X,F4.0,7X,F4.1)
C     STEP 6 CONVERT HITE-PRESS, COMBINE WIND DATA AND SORT
      JJ=J+K
           G=9.8
C     G IN METERS/SEC
           R=287
      JP1=J+1
      PRESS(JP1)=PRESS(1)
```

```
      DD(JP1)=DDD(1)
      FF(JP1)=FFF(1)
      JP2=J+2
      IF(K.LT.2) GO TO 6151
      DO 6150 M=JP2,JJ
           DO 6100 NN=1,JP1
6100       IF (HIGH(M-J).LT.Z(NN)) GO TO 115
115        NNN=NN-1
          PRESS(M)=PRESS(NNN)*EXP(-2*(G/R)*(HIGH(M-J)-Z(NNN))/(T(NNN)+
     +     T(NNN+1))/3.2825)
           DD(M)=DDD(M-J)
6150       FF(M)=FFF(M-J)
6151   CONTINUE
      DUMP=0
      JJM1=JJ-1
      DO 6170 M=1,JJM1
           MP1=M+1
           DO 6160 N=MP1,JJ
                IF (PRESS(M).GT.PRESS(N)) GO TO 6160
        IF(PRESS(M).LT.PRESS(N)) GO TO 6155
        PRESS(N)=0.
           IF(PRESS(M).NE.0.) DUMP=DUMP+1
        GO TO 6160
6155            CONTINUE
                DUM=PRESS(N)
                DUMM=DD(N)
                DUMMY=FF(N)
                PRESS(N)=PRESS(M)
                DD(N)=DD(M)
                FF(N)=FF(M)
                PRESS(M)=DUM
                DD(M)=DUMM
                FF(M)=DUMMY
6160            CONTINUE
6170       CONTINUE
```

```
      PTZ=JJ-DUMP
C     WRITE(6,61)
61    FORMAT(50X,'PRESSURE',3X,'DIRECTION',3X,'SPEED')
      DO 6180 KO=1,PTZ
C6180 WRITE(6,62)PRESS(KO),DD(KO),FF(KO)
6180  CONTINUE
62    FORMAT(52X,F5.0,7X,F4.0,6X,F4.0)
      RETURN
      END
```

```
      SUBROUTINE AVERAG (PRES,TEM,DWPT,PTS,PRESS,DD,FF,PTZ,ASBN,ASWS)
C       ***  SUBROUTINE AVERAG COMPUTES 5/0MB AVERAGES OF MIXING RATIO AND    ***
C       ***  COMPONENTS OF WIND NORMAL TO THE OPPOSITE SIDE OF THE TRIANGLE. ***
C       ***    PRES,TEM,DWPT ARE CONNECTED ARRAYS, CONTAINING PRESSURE IN MB,***
C       ***  TEMPERATURE IN KELVIN, AND RELATIVE HUMIDITY RESPECTIVELY.      ***
C       ***     PTS CONTAINS THE NUMBER OF LEVELS OF THERMODYNAMIC DATA FOR E***
C       ***  ACH STATION                                                      ***
```

```
C         ***    PRESS,DD,AND FF ARE CONNECTED ARRAYS CONTAINING PRESSURE IN MB***
C         ***   WIND DIRECTION IN DEGREES, AND WIND SPEED IN METERS PER SECOND. ***
C         ***    PTZ CONTAINS THE NUMBER OF LEVELS AT WHICH WIND DATA IS REPORT***
C         ***   ED.                                                             ***
C         ***   THE ORDER OF STORAGE OF INFORMATION IN THESE ARRAYS IS DESCRIBED***
C         ***    IN THE MAIN PROGRAM.                                           ***
C         ***   ASBN : AN ARRAY CONTAINING 50 MB AVERAGES OF NORMAL WINDS AT EAC***
C         ***   H VERTEX.                                                       ***
C         ***   ASWS : AN ARRAY CONTAINING CORRESPONDING MIXING RATIO AVERAGES  ***
C         ***   TANGENTIAL WIND COMPONENTS ARE ALSO COMPUTED FOR THE CALCULATION***
C         ***   OF VORTICITY.  THESE DATA ARE STORED IN ASBT                    ***
C         ***   COMMON/RCOOR/ IS DESCRIBED IN SUBROUTINE COOR.  THIS INFORMATION***
C         ***   IS USED TO DETERMINE THE NORMAL AND TANGENTIAL DIRECTIONS.      ***
C         ***                                                                   ***
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/TOPS/IPDUM(20)
      COMMON/SUMINT/INDEX
      COMMON/RCOOR/RLON(3),RLAT(3),ELEV(3),DIST(3),RHEAD(3),AREA
      COMMON/VORT/ASBT(20,4,4),VORTI(2,18)
      REAL*8 PT(50),PW(50),T(50),D(50),ANG(50),VEL(50),BT(1050)
      REAL*8 BD(1050),BNORM(1050),BTANG(1050),NORM(50),TANG(50),WS(1050)
      REAL*8 W(16),ASBN(20,4,4),ASWS(20,4,4)
      INTEGER P
      INTEGER PTS(1),PTZ(1)
      REAL PRES(8,50),TEM(8,50),DWPT(8,50),PRESS(8,50),DD(8,50),FF(8,50)
      INTEGER A,B,C,FL,LEVEL,LEVELS,U(16),V(16)
      IPTOP=IPDUM(INDEX)
      P=1.
      DO 700 IGL=1,1050
      BD(IGL)=0.
      BNORM(IGL)=0.
      BTANG(IGL)=0.
      WS(IGL)=0.
700   CONTINUE
      DO 702 IGL=1,20
```

```
      DO 702 JGL=1,4
      DO 702 KGL=1,4
      ASBN(IGL,JGL,KGL)=0.
      ASWS(IGL,JGL,KGL)=0.
      ASBT(IGL,JGL,KGL)=0.
702   CONTINUE
      Q=0
      U(1)=1
      U(2)=3
      U(3)=1
      U(4)=3
      U(5)=1
      U(6)=3
      V(1)=1
      V(2)=1
      V(3)=2
      V(4)=2
      V(5)=3
      V(6)=3
      W(1)=RHEAD(1)
      W(2)=RHEAD(1)
      W(3)=RHEAD(2)
      W(4)=RHEAD(2)
      W(5)=RHEAD(3)
      W(6)=RHEAD(3)
      LEVEL=20
      DO 79 L=1,6
      NUM=PTS(L)
      NUMB=PTZ(L)
      Q=Q+1
      A=U(Q)
      B=V(Q)
      AZI=W(Q)
      DO 110 I=1,NUM
      PT(I)=PRES(L,I)
```

```
      T(I)=TEM(L,I)
110   D(I)=DWPT(L,I)
      DO 120 I=1,NUM
      IF(D(I).LT.0) D(I)=20.
      D(I)=QSAT(T(I),PT(I))*D(I)/100.
120   CONTINUE
      DO 210 I=1,NUMB
      PW(I)=PRESS(L,I)
      ANG(I)=DD(L,I)
210   VEL(I)=FF(L,I)
      NUMM1=NUM-1
      DO 130 N=1,NUMM1
            NPTN1=PT(N+1)
            NPTN=PT(N)
            DO 140 I=NPTN1,NPTN
                  BT(I)=T(N+1)+(I-PT(N+1))*(-T(N+1)+T(N))/(PT(N)-PT(N+1))
                  IF (D(N+1).GE.0.) GO TO 125
                  BD(I)=-1.
                        IMAD=PT(N)
                        BD(IMAD)=D(N)
                  GO TO 140
125               BD(I)=D(N+1)+(I-PT(N+1))*(-D(N+1)+D(N))/(PT(N)-PT(N+1))
140               CONTINUE
130         CONTINUE
      DO 225 I=1,NUMB
            THETA=(AZI-ANG(I))*3.14159/180.
      NORM(I)=VEL(I)*DSIN(THETA)
      TANG(I)=VEL(I)*DCOS(THETA)
225         CONTINUE
      NUMB1=NUMB-1
      DO 230 M=1,NUMB1
      IF(PW(M+1).EQ.PW(M)) GO TO 230
            NPWM1=PW(M+1)
            NPWM=PW(M)
            DO 240 I=NPWM1,NPWM
```

```
                BNORM(I)=NORM(M+1)+(I-PW(M+1))*(-NORM(M+1)+
     +          NORM(M))/(PW(M)-PW(M+1))
                BTANG(I)=TANG(M+1)+(I-PW(M+1))*(-TANG(M+1)+
     +          TANG(M))/(PW(M)-PW(M+1))
240             CONTINUE
230        CONTINUE
      IPT1=PT(1)
      DO 300 I=IPTOP,IPT1.
           WS(I)=BD(I)
300   CONTINUE
      C=2
      DO 400 I=100,949,50
           SBTANG=0
           SBN=0
           SWS=0
           IF(I.LT.IPTOP) GO TO 411
           I49=I+49
           DO 410 J=I,I49
                SBTANG=SBTANG+BTANG(J)
                SBN=SBN+BNORM(J)
                SWS=SWS+WS(J)
410             CONTINUE
411        CONTINUE
           SBN=SBN/50.
           SBTANG=SBTANG/50.
           SWS=SWS/50.
           C=C+1
           ASBN(C,A,B)=SBN
           ASWS(C,A,B)=SWS
                                      ASBT(C,A,B)=SBTANG
           P=I+49
400        CONTINUE
      SBN=0
      SBTANG=0
      SWS=0
```

```
      IF(PT(1).LT.950.) GO TO 79
      IFpT1=PT(1)
      DO 600 J=950,IFPT1
            SBTANG=SBTANG+BTANG(J)
            SBN=SBN+BNORM(J)
            SWS=SWS+WS(I)
600         CONTINUE
      P=(PT(1)-950.)+1.
      SBN=SBN/P
      ASBN(LEVEL,A,B)=SBN
      SWS=SWS/P
      ASWS(LEVEL,A,B)=SWS
      SBTANG=SBTANG/P
                              ASBT(LEVEL,A,B)=SBTANG
      SBTANG=SBTANG/P
79    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DIVERG(ASBN,P1,P2,VCNY,QER,YTOT,E,G,I,SUM)
C       ***  SUBROUTINE DIVERG : CALCULATES DIVERGENCE IN ORDER TO          ***
C       ***  ESTABLISH THE PROPER VELOCITY CORRECTION TO BE USED IN        ***
C       ***  SUBROUTINE DIVEG2                                             ***
C       ***    ASBN - ARRAY CONTAINING NORMAL COMPONENTS OF WIND           ***
C       ***   P1 & P2 - ARRAYS CONTAINING SURFACE PRESSURES.IN P1 THE INITIAL***
C       ***  INDEX REFERS TO TIME, THE FINAL TO STATION                    ***
C       ***   VCNY - MASS FLUX DUE TO WIND AT GIVEN VERTEX AND LEVEL       ***
C       ***   Q - WEIGHTING FACTOR TO DETERMINE MEAN SURFACE PRESSURE IN TRIA***
C       ***  NGLE                                                          ***
C       ***   YTOT - LENGTH OF PERIMETER OF TRIANGLE IN METERS             ***
C       ***    E,G,I  : INDICES DETERMINED BY MAIN PROGRAM                 ***
C       ***   SUM - NET DIVERGENCE FOR EACH 50 MB LAYER                    ***
C       ***                                                                ***
C       ***                                                                ***
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/RCOOR/RLON(3),RLAT(3),ELEV(3),DIST(3),RHEAD(3),AREA
      REAL P1(2,4),P2(2,4)
      REAL*8 QER(1)
      REAL*8 ASBN(20,4,4),SUM(20),VCN(20,4),VCNY(20,4),Y(4)
      INTEGER A,B,E,F,G,H,P,Q(4),R,S,W(2)
      L=0
      YTOT=0.
      DO 10 INO=1,3
      Y(INO)=DIST(INO)
      YTOT=YTOT+Y(INO)
10    CONTINUE
      VC=0.
      DO 905 S=3,20
          SUM(S)=0.
905       CONTINUE
C                                              ***********************
C                                              THESE DIVIDE BY 2.
C                                              STATEMENTS ARE
C                                              NECESSARY IN THE
```

```
C                                                             BELLAMY TRIANGLE
C                                                             ALGORYTHM
C                                                             ***********************
      DO 1100 F=1,3
           DO 1200 S=3,19
                VN=ASBN(S,E,F)
                VCN(S,F)=VC+VN
                VCNY(S,F)=VCN(S,F)*Y(F)
                                                              VCNY(S,F)=VCNY(S,F)/2.
                SUM(S)=SUM(S)+VCNY(S,F)
1200            CONTINUE
           WF=(((P1(I,F)+P2(I,F))/2.)-950.)/50.
      QER(F)=WF*Y(F)/YTOT
           S=20
           VN=ASBN(S,E,F)
           VCN(20,F)=VN+VC
        VCNY(20,F)=VCN(20,F)*Y(F)*WF
                                                              VCNY(S,F)=VCNY(S,F)/2.
           SUM(S)=SUM(S)+VCNY(20,F)
1100       CONTINUE
C                                                             ***********************
      DO 1450 R=3,20
1450       CONTINUE
999   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DIVEG2(ASBN,ASWS,P1,P2,VC1,VC2,VCYY,STORE,FLUX)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/WATER/QWAT(2)
      COMMON/BAR/VXBAR(20,4),VYBAR(20,4)
      COMMON/VORT/ASBT(20,4,4),VORTI(2,18)
      COMMON/RCOOR/RLON(3),RLAT(3),ELEV(3),DIST(3),RHEAD(3),AREA
      COMMON/SUMINT/INDEX
      COMMON/SUMMA/      IDATS1(20,3),IDATS2(20,3),RAIN(20),DELS(20),
     ,DIVE(20),WATER(20,2),Q10(20,2),Q7(20,2),UOM100(20,2),
     ,UOM550(20,2),COM550(20,2),VELCOR(20,2)
      REAL IGE
      REAL FAC
      REAL*8 AQBAR(20,4,4)
      REAL*8 UXY(20,4)
      REAL*8 VORTIC(20)
      REAL*8 WIFF(3,4),WHIFF(3,20,4),STORE(2),FLUX(2)
      REAL*8 VXY(20,4),VCYY(2,20)
      REAL*8 ASWS(20,4,4),SVWYP(20),VWY(20,4),Y(4),AVWS(20,4,4)
      REAL*8 ASBN(20,4,4),VW(20,4),VWYP(20,4)
      REAL P1(2,4),P2(2,4)
      INTEGER A,B,D,E,F,G,H,P,Q(4),R,S,W,Z
      L=0
      DO 100 W=1,20
      DO 100 D=1,4
      ZSUM=0.
      DO 10 Z=1,4
10    ZSUM=ASWS(W,D,Z)+ZSUM
      DO 11 Z=1,4
11    AQBAR(W,D,Z)=ZSUM/3.
```

```
100   CONTINUE
      DO 101 W=3,20
         D=1
            DO 101 Z=1,3
      IGE=50*W-50
      FAC=(P1(1,Z)-IGE)/50.
      IF(FAC.GT.1.) FAC=1.
      IF(P1(1,Z).LT.IGE) FAC=0.
               ASBN(W,D,Z)=ASBN(W,D,Z)+VC1*FAC*F2(W,FAC)
101            CONTINUE
      DO 111 W=3,20
         D=3
            DO 111 Z=1,3
      IGE=50*W-50
      FAC=(P1(2,Z)-IGE)/50.
      IF(FAC.GT.1.) FAC=1.
      IF(P1(2,Z).LT.IGE) FAC=0.
               ASBN(W,D,Z)=ASBN(W,D,Z)+VC2*FAC*F2(W,FAC)
111            CONTINUE
      DO 112 W=3,20
         DO 112 D=1,3,2
            DO 112 Z=1,3
               AVWS(W,D,Z)=ASBN(W,D,Z)*AQBAR(W,D,Z)
112            CONTINUE
      CALL VEE(ASBN)
      Y(1)=DIST(1)
      Y(2)=DIST(2)
      Y(3)=DIST(3)
      E=-1
      G=0
      DO 999 I=1,2
      E=E+2
      G=G+2
      DO 905 S=3,20
         SVWYP(S)=0.
```

```
          VORTIC(S)=0.
905       CONTINUE
C                                                   ************************
C                                                   THESE DIVIDE BY 2.
C                                                   STATEMENTS ARE
C                                                   NECESSARY IN THE
C                                                   BELLAMY TRIANGLE
C                                                   ALGORYTHM
C                                                   ************************
      DO 1100 F=1,3
          DO 1200 S=3,19
              VXY(S,F)=ASBN(S,E,F)*Y(F)
                                                    VXY(S,F)=VXY(S,F)/2.
                          UXY(S,F)=ASBT(S,E,F)*Y(F)/2.
              VW(S,F)=AVWS(S,E,F)
              VWY(S,F)=VW(S,F)*Y(F)
              VWYP(S,F)=VWY(S,F)*50.
                                                    VWYP(S,F)=VWYP(S,F)/2.
              SVWYP(S)=SVWYP(S)+VWYP(S,F)
                         VORTIC(S)=VORTIC(S)+UXY(S,F)
1200          CONTINUE
          WF=(((P1(I,F)+P2(I,F))/2.)-950.)/50.
          WIFF(I,F)=WF
          S=20
          VXY(S,F)=ASBN(S,E,F)*Y(F)*WF
                                                    VXY(S,F)=VXY(S,F)/2.
                          UXY(S,F)=ASBT(S,E,F)*Y(F)/2.
          VW(S,F)=AVWS(S,E,F)
          VWY(S,F)=VW(S,F)*Y(F)*WF
          VWYP(S,F)=VWY(S,F)*50.
                                                    VWYP(S,F)=VWYP(S,F)/2.
          SVWYP(S)=SVWYP(S)+VWYP(S,F)
                         VORTIC(S)=VORTIC(S)+UXY(S,F)
1100      CONTINUE
C                                                   ************************
```

```
      DO 1170 S=3,20
           VCYY(I,S)=0.
           DO 1160 F=1,3
1160            VCYY(I,S)=VCYY(I,S)+VXY(S,F)
1170       CONTINUE
      SSVWYP=0.
      DO 1149 R=3,20
           SSVWYP=SSVWYP+SVWYP(R)
1149       CONTINUE
      FLUX(I)=SSVWYP
                        DO 4000 IQ=1,18
4000                    VORTI(I,IQ)=-VORTIC(21-IQ)*1.0D05/AREA
999   CONTINUE
      CALL DELQ(WF,ASWS,QWAT)
      DO 3000 IF=1,3,2
      DO 3000 IG=1,19
      DO 3000 IH=1,3
3000  WHIFF(IF,IG,IH)=1.
      DO 3001 IH=1,3
           WHIFF(1,20,IH)=WIFF(1,IH)
3001  WHIFF(3,20,IH)=WIFF(2,IH)
      DO 2000 D=1,3,2
           SSWS=0.
      SUMMID=0.
      SUMLOW=0.
           DO 2010 W=3,20
                SWS=0.
                DO 2020 Z=1,3
                SWS=SWS+ASWS(W,D,Z)*WHIFF(D,W,Z)
2020                 CONTINUE
                SWS=SWS/3.
                SSWS=SSWS+SWS
      IF(W.GE.15) SUMLOW=SUMLOW+SWS
      IF(W.LT.9) GO TO 2010
      IF(W.GE.15) GO TO 2010
```

```
      SUMMID=SUMMID+SWS
2010       CONTINUE
           SSWS=SSWS/18.
           STORE((D+1)/2)=SSWS
      Q10(INDEX,(D+1)/2)=SUMLOW/6.*1000.
      Q7(INDEX,(D+1)/2)=SUMMID/6.*1000.
2000       CONTINUE
      RETURN
      END
```

```
      SUBROUTINE COOR
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/RCOOR/RLON(3),RLAT(3),ELEV(3),DIST(3),RHEAD(3),AREA
      COMMON/ICOOR/ISTAT(3,3)
      DIMENSION COURSE(3)
      DATA PI/3.14159/
      RTODE(X)=X*45./ATAN(1.)
      DTORA(X)=X*ATAN(1.)/45.
      DO 100 I=1,3
100   READ(5,1)(ISTAT(I,J),J=1,3),RLON(I),RLAT(I),ELEV(I)
```

```
1       FORMAT(3A4,8X,F8.2,F8.2,F5.1)
        DO 200 I=1,3
        J=I+1
        IF(J.EQ.4) J=1
        ANUM=PI*(RLON(I)-RLON(J))
        ARG=PI/4.+DTORA(RLAT(J))/2.
        BRG=PI/4.+DTORA(RLAT(I))/2.
        ARG=DTAN(ARG)
        ARG=DLOG(ARG)
        BRG=DTAN(BRG)
        BRG=DLOG(BRG)
        DEN=180*(ARG-BRG)
        COURSE(I)=90.
        IF(DEN.EQ.0.) GO TO 105
        ARG=ANUM/DEN
        COURSE(I)=RTODE(DATAN(ARG))
105     CONTINUE
        IF(COURSE(I).EQ.90.) GO TO 110
        IF (COURSE(I).LT.0.) COURSE(I)=COURSE(I)+360.
        ARG=DTORA(COURSE(I))
        DIST(I)=60.*(RLAT(J)-RLAT(I))/DCOS(ARG)
        GO TO 200
110     ARG=DTORA(RLAT(I))
        DIST(I)=60.*(RLON(J)-RLON(I))*DCOS(ARG)
200     CONTINUE
        DO 220 I=1,3
        IF (DIST(I).GT.0.) GO TO 220
        DIST(I)=-DIST(I)
        COURSE(I)=COURSE(I)+180.
220     CONTINUE
        DO 230 I=1,3
        COURSE(I)=COURSE(I)-180.
        IF (COURSE(I).LT.0.) COURSE(I)=COURSE(I)+360.
230     CONTINUE
        DO 300 I=1,3
```

```
      J=I+1
      K=I+2
      IF(J.EQ.4) J=1
      IF(K.GT.3) K=K-3
      DELPHI=(RLON(J)+RLON(K))/2.-RLON(I)
      RLAM=(RLAT(J)+RLAT(K))/4.+RLAT(I)/2.
      RLAM=DTORA(RLAM)
      RHEAD(I)=COURSE(J)+DELPHI*DSIN(RLAM)
  300 CONTINUE
      AMAX=0.
      DO 310 I=1,3
      IF(DIST(I).LE.AMAX) GO TO 310
      AMAX=DIST(I)
      J=I
  310 CONTINUE
      K=J+1
      IF(K.EQ.4) K=1
      THETA=COURSE(J)+180.-COURSE(K)
      ARG=DTORA(THETA)
      AREA=DIST(J)*DIST(K)*DSIN(ARG)*1.714952E06
      TEMP=DIST(1)
      DIST(1)=DIST(2)*1852.
      DIST(2)=DIST(3)*1852.
      DIST(3)=TEMP   *1852.
               DO 400 I=1,3
               RHEAD(I)=RHEAD(I)-180.
               IF(RHEAD(I).LT.0.) RHEAD(I)=RHEAD(I)+360.
  400          CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SUMMAR
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON/SUM2/GRAD(20)
      COMMON/TOPS/IPTOP(20)
      COMMON/ICOOR/ISTAT(3,3)
      COMMON/SUMINT/INDEX
      COMMON/SUMMA/       IDATS1(20,3),IDATS2(20,3),RAIN(20),DELS(20),
     ,DIVE(20),WATER(20,2),Q10(20,2),Q7(20,2),UOM100(20,2),
     ,UOM550(20,2),COM550(20,2),VELCOR(20,2)
      WRITE(6,500)
500   FORMAT('1')
      WRITE(6,98)((ISTAT(I,J),J=1,3),I=1,3)
98    FORMAT(10X,3A4,',',3A4,',',3A4)
      WRITE(6,99)
      WRITE(6,1)
      WRITE(6,2)
      WRITE(6,3)
      WRITE(6,4)
4     FORMAT(1X,130('_'))
3     FORMAT(6X,'PERIOD',6X,'P-E',3X,'STORAGE',1X,1X,'V GRAD Q',1X,'|',
     3  'WATER',1X,'Q-LOW',1X,'Q-MID',2X,'100',3X,'550',3X,'550',3X,
     3  'COR.','|','|','WATER',1X,'Q-LOW',1X,'Q-MID',2X,'100',3X,'550',
     3  3X,'550',3X,'COR.','|',1X,'TOP')
```

```
2       FORMAT(33X,'_',8X,'|','PRECIP',13X,'OMEGA',1X,'OMEGA',1X,'OMEGA',1
       2X,'VEL.','|','|','PRECIP',13X,'OMEGA OMEGA OMEGA VEL.|')
1       FORMAT(33X,'Q DIV V',2X,'|',19X,'UNCORRECTED',11X,'||',19X,'UNCORR
       1ECTED',11X,'|')
99      FORMAT(33X,'_',9X,17('_'),'INITIAL',17('_'),2X,18('_'),'FINAL',18
       9('_'))
        INDX=INDEX-1
        DO 200 I=1,INDX
        WRITE(6,5)(IDATS1(I,J),J=1,3),DIVE(I)
5       FORMAT(4X,3A4,'|',18X,F6.3,1X,'|',42X,'|',42X,'|')
        WRITE(6,6)RAIN(I),DELS(I),          ((WATER(I,J),Q10(I,J),Q7(I,J)
       W   ,UOM100(I,J),UOM550(I,J),COM550(I,J),VELCOR(I,J)),J=1,2)
       W  ,IPTOP(I)
6       FORMAT(9X,'TO',5X,'|',F6.3,3X,F6.3,3X,3X,3X,2(1X,'|',1X,F4.2,2X,
       6  F4.1,2X,F4.1,3F6.0,1X,F5.2),' |',I3)
        WRITE(6,5)(IDATS2(I,J),J=1,3),GRAD(I)
        WRITE(6,4)
200     CONTINUE
        WRITE(6,7)
7       FORMAT(18X,'CM',7X,'CM',7X,'CM',6X,'CM',3X,'GM/KG',1X,'GM/KG',
       7  5X,'MB/12 HRS',6X,'M/S',4X,'CM',3X,'GM/KG',1X,'GM/KG',5X,
       7  'MB/12 HRS',6X,'M/S')
        WRITE(6,8)
        WRITE(6,8)
8       FORMAT(/)
        WRITE(6,9)
9       FORMAT(50X,'EXPLANATION')
        WRITE(6,8)
        WRITE(6,10)
        WRITE(6,8)
10      FORMAT(10X,'P-E - SUM OF PRECIPITATION MINUS EVAPORATION',/,10X,
       2  'STORAGE - CHANGE IN WATER VAPOR CONTENT OF VOLUME.  POSITIVE',
       3' VALUES REPRESENT DECREASE.',/,10X,'DIVERGENCE - SUM Q DIV V + V'
       4,' GRAD Q: MEAN FLUX OF WATER VAPOR IN SIDES OF VOLUME INTEGRATED'
       5,/,15X,'OVER 12 HOURS.  POSITIVE VALUES REPRESENT NET CONVERGENCE',
```

```
A/,10X,
6'PRECIP WATER - EQUIVALENT DEPTH OF WATER VAPOR IN VOLUME.',/,10X,
7'Q-LOW - MEAN MIXING RATIO IN SURFACE-700 MB LAYER,',/,10X,
8'Q-MID - MEAN MIXING RATIO IN 700-400 MB LAYER.',/,10X,
9'UNCORRECTER OMEGA 100 - OMEGA AT TOP OF VOLUME.  ',
-'(FROM MASS DIVERGENCE CALCULATIONS)',/,10X,
1'UNCORRECTED OMEGA 550 - OMEGA AT MIDDLE OF VOLUME.',/,10X,
2'OMEGA 550 - OMEGA AT MIDDLE OF VOLUME AFTER NORMAL COMPONENTS',
3' OF WIND CORRECTED TO SET OMEGA AT TOP MB EQUAL TO 0.',/,10X,
4'VEL. COR. - NECESSARY VELOCITY CORRECTION.  POSITIVE VALUES',
5' REPRESENT OUTWARD CORRECTION.',/,10X,
6'TOP - PRESSURE AT WHICH OMEGA SET EQUAL TO 0.')
 WRITE(6,500)
 RETURN
 END
```

```
SUBROUTINE DELQ(WF,ASWS,QWAT)
IMPLICIT REAL*8 (A-H,O-Z)
COMMON/RCOOR/RLON(3),RLAT(3),ELEV(3),DIST(3),RHEAD(3),AREA
COMMON/BAR/VXBAR(20,4),VYBAR(20,4)
REAL*8 WOF(20)
```

```
      REAL*8 QWAT(2)
      REAL*8 ASWS(20,4,4)
      DTORA(X)=X*ATAN(1.)/45.
      THETA=RHEAD(3)-RHEAD(2)+180.
      THETA=DTORA(THETA)
      DO 10 J=1,19
10    WOF(J)=1.
      WOF(20)=WF
      T3=DTORA(180.-RHEAD(2))
      LS=DIST(3)*DCOS(THETA)
      A=DCOS(T3)
      B=DSIN(T3)
      C=-B
      D=A
      DO 30 J=1,3,2
      L=J-(J-1)/2
      QWAT(L)=0.
      DO 20 I=3,20
      DQDXP=(ASWS(I,J,3)-ASWS(I,J,1))/DIST(2)
      QS=ASWS(I,J,1)+DQDXP*LS
      DQDYP=(ASWS(I,J,2)-QS)/(DIST(3)*DSIN(THETA))
      DQDX=A*DQDXP+B*DQDYP
      DQDY=C*DQDXP+D*DQDYP
      VDQ=VXBAR(I,J)*DQDX+VYBAR(I,J)*DQDY
      QWAT(L)=QWAT(L)+VDQ*WOF(I)*AREA*50.
20    CONTINUE
30    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE VEE(ASBN)
      IMPLICIT REAL*8 (A-H,O-Z)
      COMMON/BAR/VXBAR(20,4),VYBAR(20,4)
      COMMON/VORT/ASBT(20,4,4),VORTI(2,18)
      COMMON/RCOOR/RLON(3),RLAT(3),ELEV(3),DIST(3),RHEAD(3),AREA
      REAL*8 ASBN(20,4,4)
      DO 50 J=1,3,2
      DO 30 I=1,20
      XSUM=0.
      YSUM=0.
      DO 25 K=1,3
      RMAG=DSQRT(ASBN(I,J,K)**2+ASBT(I,J,K)**2)
      RTHE=3.14159/2.
      RTHE=DSIGN(RTHE,ASBN(I,J,K))
      IF(ASBT(I,J,K).EQ.0.) GO TO 20
      RTHE=DATAN(ASBN(I,J,K)/ASBT(I,J,K))
20    CONTINUE
      RTHE=RTHE-RHEAD(K)*3.14159/180.-180.
      XSUM=XSUM+RMAG*DSIN(RTHE)
      YSUM=YSUM+RMAG*DCOS(RTHE)
25    CONTINUE
      VXBAR(I,J)=XSUM/3.
      VYBAR(I,J)=YSUM/3.
30    CONTINUE
50    CONTINUE
      RETURN
      END
```

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