THE MERIDIONAL TRANSFER OF KINETIC ENERGY

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ABSTRACT

The meridional transport of large-scale eddy kinetic energy between 17.5°N and 77.5°N is studied for the year 1951. Assuming the 500-mb surface to be a representative level of the troposphere, the magnitude, direction and convergence of the flux of kinetic energy is computed from the geostrophic winds. Results show poleward transport occurring to about 70°N, with a maximum yearly mean between 37.5°N and 42.5°N.

Upon further analysis of the kinetic energy transport term, it is found that, in particular north of 52.5°N, the total flux is accounted for by the contribution of the net meridional perturbation transport of perturbed kinetic energy. When compared to values for the advection of latent heat and of sensible heat in the atmosphere, it is seen that the advection of kinetic energy by large-scale eddies is one of two orders of magnitude less. However, in the expression for the time rate of change of meridional eddy kinetic energy within a latitude belt, the transport term is found to be the same order of magnitude as estimates of $\int uv$ measured over the same region.

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TABLE OF CONTENTS

LIST OF TABLES AND FIGURES

I. INTRODUCTION

In the mathematical expression for the time rate of change of kinetic energy in the atmosphere (Starr, 1951) it is shown that the horizontal kinetic energy in a fixed region not including the whole atmosphere varies as a result of:

1. A production of new kinetic energy within the region.
2. A dissipation of kinetic energy within the region through friction.
3. The performance of work by pressure forces and horizontal velocity components at the boundary of the region.
4. Advection of kinetic energy across the boundary of the region.

It will be the purpose of this paper to investigate the nature of the last of the above terms - specifically, the meridional advection of horizontal kinetic energy in the troposphere - with particular regard to the magnitude, direction and convergence of the flux term.

In general, other than to refer to what already has been accomplished (Kao, 1954; Benton, 1957; Mintz, 1958), there is no a priori way to determine the direction of the flux of kinetic energy, as there would be in the case of total energy.

Assumptions concerning the magnitude of the advection term have been made, and it is generally accepted that in the middle and higher latitudes the rate of meridional transfer of heat energy is of higher order of magnitude than of kinetic energy. Starr (1951) in his discussion of the flux of total energy in the atmosphere neglects the latter term. He points out that a transfer of kinetic energy of horizontal motion across a symmetrical polar cap is due to the advection of existing kinetic energy across the boundary and through the work done by pressure forces at the boundary. Through the use of the equation of state, the second term becomes at least one order of magnitude greater than the first $(\rho c_p T v > \rho c^2 v)$, since the microscopic motions of the atmosphere are larger than the gross atmospheric ones.

To formulate the problem mathematically, a general expression for the time rate of change of horizontal kinetic energy is first developed, and the advection term isolated for further analysis. Ideally, the contributions at many levels should be averaged to obtain a mean value for the horizontal flux of kinetic energy in the troposphere. However, at this point 500 mb is assumed to represent a level where this flux has a mean value. This enables us to utilize geostrophic data for the year 1951 to compute, as a first approximation, mean seasonal and yearly meridional kinetic energy transport terms for the troposphere. The magnitude of the results herein obtained will be lower than if a higher single level had been chosen where much stronger winds prevail (i.e. the 200-mb surface). Kao (1954) using geostrophic winds found that for January, 1949, the intense meridional transport of kinetic energy occurred between 100 and 300 mb with the greatest northward flux of kinetic energy, and its greatest meridional convergence and divergence, occurring in the vicinity of the westerly jet center. Thus it may be anticipated that the large contribution of the levels above 500 mb to the total kinetic energy will cause results obtained from vertical integration of data at many levels to be somewhat greater than those of this study.

A second objective of this paper is to examine the component terms involved in the meridional transport of kinetic energy to determine those which contribute most to the transport. The results of this analysis will be shown to differ appreciably from assumptions previously made by Kao (1954).

II. APPROACH TO THE PROBLEM

A. Mathematical Statement of the Problem

For the sake of completeness and for the convenience of the reader, the mathematical expression for the time rate of change of kinetic energy in the atmosphere as developed by White and Saltzman (1956) is repeated below. The horizontal equation of motion in isobaric coordinates may be written as

$$\frac{d\vec{V}}{dt} - f\vec{k} \times \vec{V} = -\nabla\Phi - \vec{F} \qquad (1)$$

where $\vec{V}$ is the horizontal wind vector, f is the Coriolis parameter, $\vec{k}$ is the unit vector in the vertical $\Phi = gz$ is the geopotential of an isobaric surface, ∇ is the 2-D del-operator in a pressure surface, and $\vec{F}$ is the vector frictional force per unit mass.

Upon scaler-multiplying (1) by $\vec{V}$, we have

$$\vec{V}\cdot\frac{d\vec{V}}{dt} + \vec{V}\cdot f\vec{k} \times \vec{V} = -\vec{V}\cdot\nabla\Phi - \vec{V}\cdot\vec{F} \qquad (2)$$

Expanding the total derivative and letting $E = \frac{\vec{V}^2}{2}$ we obtain

$$\frac{\partial E}{\partial t} + \vec{V}\cdot\nabla E + \omega\frac{\partial E}{\partial p} = -\vec{V}\cdot\nabla\Phi - D \qquad (3)$$

where $D = \vec{V} \cdot \vec{F}$. $\omega = \frac{dP}{dt}$. E is the kinetic energy of horizontal motion per unit mass, and D is the rate of frictional dissipation per unit mass.

The hydrostatic equation may be written as

$$\frac{\partial \Phi}{\partial P} = -\alpha \tag{4}$$

In isobaric coordinates the equation of continuity is

$$-\frac{\partial \omega}{\partial P} = + \nabla \cdot \vec{V} \tag{5}$$

Using (4) and (5) Equation (3) may be written in the form

$$\frac{\partial E}{\partial t} = -\nabla \cdot (E + \Phi)\vec{V} - \frac{\partial}{\partial P}(E + \Phi)\omega - \omega\alpha - D \tag{6}$$

The first two terms on the right side of (6) are advective terms which vanish if the equation is integrated over the entire atmosphere leaving

$$\frac{d}{dt}\int_M E\,dm = -\int_M \omega\alpha\,dm - \int_M D\,dm \tag{7}$$

where $dm = \frac{1}{g}\,dx\,dy\,dP$

Thus in the entire atmosphere the kinetic energy of the horizontal wind may vary as a result of 1) a production term of the form $\omega\alpha$, and 2) frictional dissipation.

If we substitute spherical coordinates in (7)

$$dm = \left(\frac{a^2}{g}\right)\cos\phi \cdot d\lambda \cdot d\phi \cdot dP$$

and $$\nabla(\ ) = \left[\frac{1}{a\cos\phi}\frac{\partial(\ )}{\partial\lambda} + \frac{1}{a}\frac{\partial(\ )}{\partial\phi}\right]$$

Integrating (7) between any two latitude circles where P_0 is the surface pressure, we obtain

$$\frac{d}{dt}\int_0^{P_0}\int_0^{2\pi}\int_{\phi_1}^{\phi_2} E\,dm = -\int_0^{P_0}\int_0^{2\pi}\int_{\phi_1}^{\phi_2}\frac{a}{g}\left[\frac{\partial}{\partial\lambda}(E+\Phi)u\right]d\lambda\,d\phi\,dP \quad (8)$$

$$-\int_0^{P_0}\int_0^{2\pi}\int_{\phi_1}^{\phi_2}\left[\frac{\partial}{\partial\phi}(E+\Phi)v\right]\frac{a}{g}\cos\phi\,d\lambda\,d\phi\,dP$$

$$-\int_0^{P_0}\int_0^{2\pi}\int_{\phi_1}^{\phi_2}\left[\frac{\partial}{\partial P}(E+\Phi)\omega\right]\frac{a^2}{g}\cos\phi\,d\lambda\,d\phi\,dP$$

$$-\int_0^{P_0}\int_0^{2\pi}\int_{\phi_1}^{\phi_2}\omega\alpha\,dm - \int_0^{P_0}\int_0^{2\pi}\int_{\phi_1}^{\phi_2} D\,dm$$

Considering the rhs of (8), upon integration around a latitude circle, the first term vanishes. Integration over the entire atmosphere will cause the third term to be zero, since ω is zero at the top and, for all practical purposes, at the bottom of the atmosphere.

Integrating the second term with respect to λ and noting that $\overline{(\ )} = \frac{1}{2\pi}\int_0^{2\pi}(\ )\,d\lambda$, we obtain:

$$\frac{d}{dt}\int_0^{P_0}\int_0^{2\pi}\int_{\phi_1}^{\phi_2} E\,dm = -\frac{2\pi a}{g}\int_0^{P_0}\int_{\phi_1}^{\phi_2}\frac{\partial}{\partial\phi}\left[\overline{(E+\Phi)v}\cos\phi\right]d\phi\,dP \quad (9)$$

$$-\int_0^{P_0}\int_{\phi_1}^{\phi_2}\int_0^{2\pi}\omega\alpha\,dm - \int_0^{P_0}\int_0^{2\pi}\int_{\phi_1}^{\phi_2} D\,dm$$

Now $$\overline{(E+\Phi)v} = \overline{Ev} + \overline{v\Phi}$$

$$\overline{(E+\Phi)v} = \bar{E}\bar{v} + \overline{E'v'} + \bar{v}\,\bar{\Phi} + \overline{v'\Phi'} \tag{10}$$

But for geostrophic winds $\bar{v} = 0$ and $\overline{\Phi' v'} = 0$

Therefore

$$\overline{(E+\Phi)v} = \overline{E'v'} \tag{11}$$

Substituting (11) into (9) we have

$$\frac{d}{dt}\int_0^{p_0}\int_0^{2\pi}\int_{\phi_1}^{\phi_2} E\,dm = -\frac{2\pi a}{g}\int_0^{p_0}\int_{\phi_1}^{\phi_2}\frac{d}{d\phi}\left[\overline{E'v'}\cos\phi\right]d\phi\,dP - \int_0^{p_0}\int_{\phi_1}^{\phi_2}\int_0^{2\pi}\omega\alpha\,dm - \int_0^{p_0}\int_{\phi_1}^{\phi_2}\int_0^{2\pi} D\,dm \tag{12}$$

The above equation states that the change of the kinetic energy of the horizontal geostrophic wind with time in a latitude belt is dependent on 1) advection of kinetic energy across the boundary of the region, 2) an effect resulting from the product of ω and specific volume, α, and 3) a frictional dissipation throughout the mass of the fluid. It should be noted that (12) does not contain any term expressing the performance of work by pressure forces at the boundary of the region as did the equivalent expression derived by Starr. The change to pressure coordinates has incorporated this contribution into the $\omega\alpha$ effect which, according to White and Saltzman (1956), actually represents a conversion of potential and internal energy associated with the rising of warm air masses and the sinking of cold air masses. Henceforth this study will concern itself only with the investigation of the first term, exploring in a quantitative fashion the magnitude, direction, and convergence of the meridional flux of horizontal kinetic energy. Upon integration of this term with respect to ϕ, we obtain

$$-\frac{2\pi a}{g}\int_{0}^{P_0}\left[\overline{E'v'}\cos\phi\right]_{\phi_1}^{\phi_2} dP \qquad (13)$$

Here we assume that the 500-mb level represents a layer typical of the troposphere. Thus (13) becomes

$$\left(\frac{2\pi a}{g}\right)\overline{E'v'}\cos\phi_1 - \left(\frac{2\pi a}{g}\right)\overline{E'v'}\cos\phi_2 \qquad (14)$$

where each term denotes the kinetic energy per unit volume that flows across the given meridian at the constant pressure surface. This then is the expression that is evaluated in the first part of this investigation.

Remembering the second objective of this study, let us now analyse further the nature of the component terms that make up the total kinetic energy flux. From the previous section

$$\overline{vE} = \overline{v\frac{(u^2+v^2)}{2}} \tag{15}$$

where again $\overline{(\;)} = \frac{1}{2\pi}\int_0^{2\pi}(\;)\,d\lambda$

The quantities u and v may be separated into a mean motion $(\bar{u}, \bar{v})$ and a perturbation motion (u', v') such that

$$u = \bar{u} + u' \quad ; \quad v = \bar{v} + v' \tag{16}$$

Thus (15) may be expressed

$$\overline{v\frac{(u^2+v^2)}{2}} = \overline{\frac{(\bar{v}+v')}{2}\left[\bar{u}^2 + 2\bar{u}u' + u'^2 + \bar{v}^2 + 2\bar{v}v' + v'^2\right]} \tag{17}$$

For geostrophic flow along any latitude circle (as we have seen before)

$$\bar{v} = 0 \quad ; \quad v_g' = v_g \quad ; \quad u_g = \bar{u}_g + u_g' \tag{18}$$

and (17) becomes

$$\overline{\frac{(u_g^2+v_g^2)}{2}v_g} = \left(\bar{u}_g\,\overline{u_g'v_g'}\right) + \overline{v_g'\frac{(u_g'^2+v_g'^2)}{2}} \tag{19}$$

But from (10) and (11)

$$\overline{Ev} = \overline{E'v'} = \overline{\frac{(u_g^2+v_g^2)}{2}v_g} \tag{20}$$

Therefore (19) becomes

$$\overline{E'v'} = \overline{v_g'\left(\frac{u_g'^2 + v_g'^2}{2}\right)} + \overline{u_g}\ \overline{u_g' v_g'} \qquad (21)$$

The first term on the rhs of (21) expresses the net meridional perturbation transport of perturbed kinetic energy, while the second term represents the meridional transport of kinetic energy associated with the mean zonal current and the northward transport of westerly momentum.

B. Available Data

The data used in this study were obtained by Dr. B. Saltzman and Dr. A. Fleisher of M.I.T. as part of the M.I.T. General Circulation Project. From 500-mb hemispheric charts for the year 1951, values of height, Z , were extracted for every five-degree latitude and ten-degree longitude intersections over the area between latitudes 15°N and 80°N. This procedure resulted in an evaluation of the geostrophic wind components (and consequently the kinetic energy transport value) at 36 points along each latitude circle. Having defined $E = \frac{u^2 + v^2}{2}$, u and v were obtained by using finite difference techniques according to the geostrophic wind equation. The product of $v_g\left(\frac{u_g^2 + v_g^2}{2}\right)$ at each point averaged around a latitude circle represents the expression $\overline{E'v'}$ in (14).

Consequently, 365 days of kinetic energy transport values were obtained across every five degrees of latitude from 17.5°N to 77.5°N.

As has been previously shown, since $\overline{E'v'}$ is computed from geostrophic wind data, this term does not include any flux of kinetic energy resulting from mean meridional cells. Instead, the data are sufficient only to evaluate the flux of energy due to the horizontal eddies. Moreover, since the wind components are averaged over ten-degrees intervals of longitude, only the flux due to large-scale horizontal eddies is represented.

C. Treatment of Data

Having obtained the data by the methods described in the previous section, the process of tabulation commenced. In each analysis the basic data consisted of 365 days of transport terms for each latitude from 17.5°N to 72.5°N for five-degree intervals. Monthly means and sample standard deviations were first obtained for these thirteen latitudes; then the monthly data combined into seasonal and, finally, yearly means and standard deviations.

Confidence limits were calculated for both seasonal and yearly averages and have been included with the tabulated results. These limits are defined as twice the standard error and indicate approximately the 95% confidence level. The formula used is $2E_m = \frac{2\sigma_s}{\sqrt{N}}$

where E_m is the standard error of the mean of any quantity, n. σ_s is the standard deviation of the sample, and N is the number of independent observations (in this case the number of days of data).

III. RESULTS

A. Discussion of Numerical Computations

As was anticipated, the winter season evidences the greatest horizontal flux of kinetic energy; the largest individual contributions being made in December and January. During the winter months (November–April) the flux of kinetic energy is poleward from 17.5°N across 67.5°N, thence reversing direction (Fig. 1). Zero total transport occurs at about 70°N. The range of maximum energy transport extends from 32.5°N to 47.5°N, within the region of poleward flux, with a maximum of 10.4×10^{17} ergs $sec^{-1} mb^{-1}$ between 37.5°N and 42.5°N.

Maximum energy transport for the summer months is about one-third as much as for winter (Fig. 2). July and August are the months of minimum kinetic energy transport. A broad region of maximum transport poleward is evident between 37.5°N and 52.5°N, displaced northward about five degrees from the corresponding area for winter. Maximum summer transport of 3.3×10^{17} ergs $sec^{-1} mb^{-1}$ is found at 42.5°N, similar to the winter situation, with a secondary max of 3.2×10^{17} ergs $sec^{-1} mb^{-1}$ at 52.5°N. A reversal of direction (equatorward) is again evident at 72.5°N where the transport term changes sign.

The yearly mean graph (Fig. 3) closely resembles those of the individual seasons, having the same general configuration and latitudes of poleward and equator directed transport. The maximum northward flux at 42.5°N has a value of 6.8 × 10^{17} ergs $sec^{-1}mb^{-1}$.

In all three of the above tabulations the averages for each latitude, inclusive of the data for 62.5°N, far exceed the confidence limits. This fact indicates that a large degree of reliability be given to these results.

The data have been arranged in Table 4 to show the mean convergence of the flux of kinetic energy of large-scale eddies into five-degree zonal rings. Divergence is evident south of 42.5°N, with a maximum value of 2.6 × 10^{17} ergs $sec^{-1}mb^{-1}$ between 27.5°N and 32.5°N. Above 42.5°N convergence is the rule except for the mean summer data which show a weak belt of divergence between 47.5°N and 52.5°N. A maximum value for convergence of 1.71 × 10^{17} ergs $sec^{-1}mb^{-1}$ is found in the zonal ring between 57.5°N and 62.5°N for the mean yearly data, while a secondary max of 1.66 × 10^{17} ergs $sec^{-1}mb^{-1}$ shows up between 57.5°N and 62.5°N.

The mean rate of meridional perturbation flux of perturbed kinetic energy, $v_g'\left(\frac{u_g'^2+v_g'^2}{2}\right)$, for seasonal and yearly data is found in Tables 7 - 9. The data is also plotted on Figs. 1 - 3 so that the total eddy kinetic energy transport may be more readily compared with this perturbation component. These graphs show the perturbation term to account for nearly all of the total kinetic energy transport above 57.5°N and more than half between 47.5°N and 57.5°N. Below 47.5°N its contribution was negligible for all practical purposes. It must be noted, however, that confidence limits north of 57.5°N were larger than the mean value computed.

In an attempt to subject the results to a finer analysis, individual graphs of the horizontal flux of eddy kinetic energy were drawn for each month. Only in January was it evident that periods of 3 to 4 days existed when maximum flux was displaced latitudinally with time. Otherwise the general tendency was for isopleths of constant flux to have a general north-south orientation with no appreciable tilt in the direction of the time axis. This means that, in general, any significant poleward or equator directed flux of kinetic energy occurs simultaneously over a wide belt over many degrees of latitude.

B. Comparison with Other Studies

Mintz (1958) and Benton (1957) utilizing the same data for January-February and July-August, 1949, have computed mean kinetic energy flux values due to horizontal eddies. These are summarized in Table 5 and Fig. 5. Benton's results, obtained from spectral analysis techniques, are for the 300-mb level, while Mintz's values are vertically integrated results from 1010 mb to 200 mb. In order to make a valid comparison of the above with the results of this study, only the same four months were used.

For the winter months Benton's computations at the 300-mb level are an order of magnitude higher than those of this study, a result not unexpected, but still surprisingly high. Although larger in general, Mintz's integrated values were of the same order of magnitude as the present study's. This fact is not unusual when one considers the very large contribution to the kinetic energy of the higher levels. For the summer months excellent agreement existed with Mintz, while Benton's results were about 3-4 times larger. It is of interest to examine the spectral distribution of the perturbation flux of perturbed kinetic energy. Results of computations for the year 1951 for two latitude circles (22.5°N and 72.5°N) are available through the courtesy of Dr. Saltzman, and the author is indebted to him for permission to include this data in the present study.

To provide a brief mathematical background, let $U(m,n)$ and $V(m,n)$ represent the complex spectral functions for the zonal and meridional components of the geostrophic wind (u and v respectively). Then the spectral function for T, the perturbation flux of perturbed kinetic energy, is

$$A(\phi, n) = X + Y \tag{22}$$

where

$$X(\phi,n) = \frac{\pi a \times 10^{3} \cos\phi}{g} \sum_{\substack{m=-15 \\ m\neq 0}}^{15} \left(V_{(-n,-m)} U_{(m)} U_{(n)} + V_{(n,-m)} U_{(m)} U_{(-n)} \right)$$

$$Y(\phi,n) = \frac{\pi a \times 10^{3} \cos\phi}{g} \sum_{\substack{m=-15 \\ m\neq 0}}^{15} \left[V_{(-n,-m)} V_{(m)} V_{(n)} + V_{(n,-m)} V_{(m)} V_{(-n)} \right] \tag{23}$$

$$U(\rho,\phi,P,t) = \frac{1}{2\pi} \int_0^{2\pi} u(\lambda,\phi,P,t)\, e^{-i\rho\lambda}\, d\lambda$$

$$V(\rho,\phi,P,t) = \frac{1}{2\pi} \int_0^{2\pi} v(\lambda,\phi,P,t)\, e^{-i\rho\lambda}\, d\lambda$$

Finally T may be represented by T_u and T_v, the zonal and mean meridional components respectively of this quantity

$$T = T_u + T_v = \sum_{n=1}^{15} (X + Y) \tag{24}$$

where

$$T_u = \sum_{n=1}^{15} X(\phi, n)$$
$$T_v = \sum_{n=1}^{15} Y(\phi, n) \tag{25}$$

Individual plots of the values of the spectral function $A(\phi, n)$ versus wave number (through wave number 15) for the seasonal and yearly data for 22.5°N and 72.5°N can be found in Figs. 7 through 9.

From Fig. 7 it is seen that the net perturbation transport of perturbed kinetic energy is southward across 72.5°N for both winter and summer data. Disturbances of wave number two clearly contribute most to this transport for each season. A secondary maximum of southward transport results from wave number four, while relatively strong mean northward flux is accounted for by disturbances of wave number three and five.

At 22.5°N (Fig. 8) the winter season accounts for a large net southward transport, particularly the contributions from wave numbers two and nine, despite a large poleward transport resulting from disturbances of wave number six. Except for a southward contribution occurring at wave number two, the net perturbation flux of perturbed kinetic energy for the summer months is northward, with maximum poleward values occurring at wave numbers three and six.

The yearly mean data presented in Fig. 9 clearly indicates that, largely due to the disturbance occurring at wave number two, the perturbation flux of perturbed kinetic energy is directed toward the equator for both latitudes. Across 22.5°N the mean value is $- 105.7 \times 10^{14}$ ergs $sec^{-1} mb^{-1}$, while across 72.5°N it is $- 154.8 \times 10^{14}$ ergs $sec^{-1} mb^{-1}$. (Compare with Table 9.) The large poleward winter season contribution from wave number six is still evident on the mean yearly graph of flux across 22.5°N.

IV. CONCLUSIONS

Utilizing 500-mb hemispheric data for the year 1951, the meridional flux of the kinetic energy of large-scale eddies has been computed across every five degrees of latitude from 17.5°N to 77.5°N. The magnitude of the averaged results varied from a maximum of 10.4×10^{17} ergs $sec^{-1} mb^{-1}$ at 42.5°N during the winter season, to a minimum of 0.047×10^{17} ergs $sec^{-1} mb^{-1}$ at 72.5°N, also during the winter. The yearly mean maximum of 6.8×10^{17} ergs $sec^{-1} mb^{-1}$ occurred at 42.5°N

The flux was directed poleward to about 70°N, thence equatorward for both yearly and seasonal mean data. Divergence of horizontal kinetic energy out of five degrees zonal rings extended up to 40°N with a maximum yearly mean divergence of 2.7×10^{17} ergs $sec^{-1} mb^{-1}$ occurring at 30°N. Convergence took place north of 45°N, with a yearly mean maximum of about 1.7×10^{17} ergs $sec^{-1} mb^{-1}$ existing near 50°N and 60°N.

Kao (1954) stated that the meridional flux of kinetic energy could be approximated by the product of the mean velocity of the zonal current and the rate of northward transport of westerly momentum (see Equation 21). This investigation has shown his assumption to be generally valid south of 42.5°N. Elsewhere over the region studied, and in particular north of 52.5°N, the total flux is accounted for by the contribution of the net meridional perturbation transport of perturbed kinetic energy. In conjunction with the above finding, an investigation by Saltzman into the spectral distribution of the perturbation flux term has shown mean seasonal and yearly transport to be directed southward for 22.5°N and 72.5°N, dominated by disturbances of wave number two.

When compared to the values in Table 6 for the flux of total energy in the atmosphere as obtained by White and Starr (1954), it is seen that the advection of kinetic energy by large-scale eddies is one to two orders of magnitude less than the individual contributions from the advection of latent heat and of sensible heat. Thus the original assumption made by Starr is justified.

Saltzman and Fleisher (1959) have estimated the eddy contribution to the rate of conversion of available potential energy into kinetic energy for February, 1959. Their value of 2.873 ergs $sec^{-1}cm^{-2}mb^{-1}$ agree in order of magnitude with estimates of frictional dissipation made by Brunt (1941), as well as with a value of approximately 1 erg $sec^{-1}cm^{-2}mb^{-1}$ for the rate of northward transport of eddy kinetic energy across 30°N as obtained from the January-February mean data of this study. Thus although negligible in the case of total energy transport, the meridional transport of eddy kinetic energy is important to the net rate of change of kinetic energy within a limited region. In addition if contributions to the transport term arising from $\overline{E}\,\overline{v}$ and $\overline{v}\,\overline{\Phi}$ expressions are negligible or in the same direction as those of the $\overline{E'v'}$ term evaluated herein, it may be concluded from Table 4 that the effect resulting from the $\omega\alpha$ integral of Equation (12) is positive south of 42.5°N.

APPENDIX A

Tabulated Data

Table 1. Mean rate of meridional flux of kinetic energy at 500 mb for November–April 1951.
Units: 10^{14} ergs sec^{-1} mb^{-1}

Latitude (degrees)	Mean	$2\sigma \div \sqrt{N}$ N = 181	σ = S.D.
17.5	1259	± 424	2853
22.5	2627	± 710	4774
27.5	5782	±1042	7010
32.5	9982	±2734	18,391
37.5	10,392	±2026	13,625
42.5	10,427	±2124	14,286
47.5	9679	±2171	14,604
52.5	6084	±2020	13,585
57.5	3824	±1825	12,277
62.5	1334	±1414	9510
67.5	309	± 819	5508
72.5	− 47	± 611	4111
77.5	− 274	± 367	2449

Table 2. Mean rate of meridional flux of kinetic energy at 500 mb for May--October 1951.

Units: 10^{14} ergs $\sec^{-1}$ mb^{-1}

Latitude (degrees)	Mean	$2\sigma \div \sqrt{N}$ N = 184	σ = S.D.
17.5	220	± 209	1419
22.5	588	± 223	1515
27.5	910	± 279	1895
32.5	2179	± 509	3451
37.5	3039	± 678	4598
42.5	3308	± 811	5501
47.5	3070	± 858	5821
52.5	3208	±1090	7392
57.5	2472	± 917	6219
62.5	1629	± 706	4786
67.5	489	± 498	3375
72.5	-145	± 405	2745
77.5	-369	± 271	1840

Table 3. Mean rate of total meridional flux of kinetic energy at 500 mb for the year 1951.

Units: 10^{14} ergs mb^{-1} sec^{-1}

Latitude (degrees)	Mean	$2\sigma \div \sqrt{N}$ N = 365	σ = S.D.
17.5	735	± 273	2611
22.5	1809	± 385	3674
27.5	3326	± 593	5667
32.5	6048	±1439	13,746
37.5	6685	±1142	10,910
42.5	6828	±1190	11,364
47.5	6347	±1211	11,565
52.5	4634	±1152	11,006
57.5	3142	±1019	9731
62.5	1483	± 786	7511
67.5	400	± 477	4560
72.5	-96	± 366	3490
77.5	-272	± 227	2166

Table 4. Mean convergence of the flux of total kinetic energy into 5° zonal rings at 500 mb for 1951.

\+ means convergence

\- means divergence

Units: 10^{14} ergs mb^{-1} sec^{-1}

Zonal Belt (degrees)	Winter 1951	Summer 1951	Year 1951
17.5 - 22.5	-1366	- 368	- 864
22.5 - 27.5	-2155	- 323	-1727
27.5 - 32.5	-4200	-1260	-2722
32.5 - 37.5	- 410	- 860	- 637
37.5 - 42.5	- 35	- 209	- 153
42.5 - 47.5	+ 748	+ 238	+ 491
47.5 - 52.5	+3896	- 138	+1713
52.5 - 57.5	+2260	+ 736	+1492
57.5 - 62.5	+2490	+ 843	+1656
62.5 - 67.5	+1025	+1140	+1083
67.5 - 72.5	+ 634	+ 356	+ 496
72.5 - 77.5	+ 127	+ 224	+ 186

Table 5. Mean rate of meridional flux of eddy kinetic energy according to various sources.

Units: 10^{14} ergs sec^{-1} mb^{-1}

(Minus sign indicates equatorward advection)

January-February Data				July-August Data			
Latitude (degrees N)	Mintz (1010-200 mb)	Benton (300 mb)	Gottuso (500 mb)	Lat.	Mintz	Benton	Gottuso
77.5			- 105	77.5			- 520
75.0	- 0.0			75.0	- 600		
72.5			- 46	72.5			- 54
70.0	700	1548		70.0	- 400	-2051	
67.5			- 20	67.5			582
65.0	2800			65.0	700		
62.5			56	62.5			1147
60.0	7400	18749		60.0	1500	4394	
57.5			3874	57.5			810
55.0	10300			55.0	1200		
52.5			7610	52.5			1333
50.0	10400	27286		50.0	1200	2846	
47.5			12537	47.5			1962
45.0	9700			45.0	1800		
42.5			11591	42.5			1916
40.0	11700	35824		40.0	1800	4394	
37.5			9242	37.5			1407
35.0	15500			35.0	1500		
32.5			4580	32.5			781
30.0	16600	55995		30.0	800	2176	
27.5			6014	27.5			135
25.0	12000			25.0	200		
22.5			3328	22.5			483
20.0	5300	16489		20.0	100	0	
17.5			1468	17.5			136

Table 6. Total eddy kinetic energy versus flux of kinetic energy due to large-scale eddies.

Units: 10^{14} cal sec^{-1}

	After Starr and White (1950)			Gottuso (1951)
Latitude (degrees N)	Total Eddy			Flux of kinetic energy
	Sensible	Latent	Total	
72.5				− 0.002
70.0	3.3	0.8	4.1	
67.5				0.01
57.5				0.08
55.0	5.8	2.3	8.1	
52.5				0.11
42.5	5.6	2.7	8.3	0.16
32.5				0.14
31.0	2.8	3.1	5.9	

Note: Data by Starr and White neglect the advection of kinetic energy due to large-scale eddies.

Table 7. Mean rate of meridional perturbation flux of perturbed kinetic energy at 500 mb for May–October 1951. Units: 10^{14} ergs sec^{-1} mb^{-1}

Latitude (degrees)	Mean	$2\sigma \div \sqrt{N}$, N = 184	σ = S.D.
17.5	275	± 207	1404
22.5	364	± 215	1456
27.5	187	± 181	1226
32.5	287	± 295	2000
37.5	264	± 441	2995
42.5	226	± 616	4179
47.5	904	± 629	4269
52.5	2160	± 844	5724
57.5	2383	± 804	5452
62.5	1826	± 621	4211
67.5	666	± 433	2935
72.5	-131	± 298	2019
77.5	-247	± 231	1565

Table 8. Mean rate of meridional perturbation flux of perturbed kinetic energy at 500 mb for November–April 1951. Units: 10^{14} ergs sec^{-1} mb^{-1}

Latitude (degrees)	Mean	$2\sigma \div \sqrt{N}$ N = 181	σ = S.D.
17.5	70	± 316	2125
22.5	-378	± 382	2572
27.5	-335	± 572	3848
32.5	511	±1120	7480
37.5	1419	±1392	9360
42.5	2991	±1646	11073
47.5	4931	±1698	11422
52.5	5030	±1771	11915
57.5	3735	±1638	11018
62.5	1333	±1181	7942
67.5	408	± 644	4334
72.5	-184	± 473	3181
77.5	-184	± 319	2144

Table 9. Mean rate of meridional perturbation flux of perturbed kinetic energy at 500 mb for the year 1951.

Units: 10^{14} ergs sec^{-1} mb^{-1}

Latitude (degrees)	Mean	$2\sigma + \sqrt{N}$ $N = 365$	σ = S.D.
17.5	173	± 189	1801
22.5	-103	± 224	2138
27.5	- 72	± 299	2856
32.5	388	± 571	5471
37.5	837	± 732	6994
42.5	1597	± 886	8467
47.5	2937	± 924	8821
52.5	3583	± 988	9433
57.5	3053	± 910	8697
62.5	1431	± 664	6342
67.5	538	± 387	3698
72.5	-157	± 279	2660
77.5	-266	± 197	1877

APPENDIX B

Graphical Data

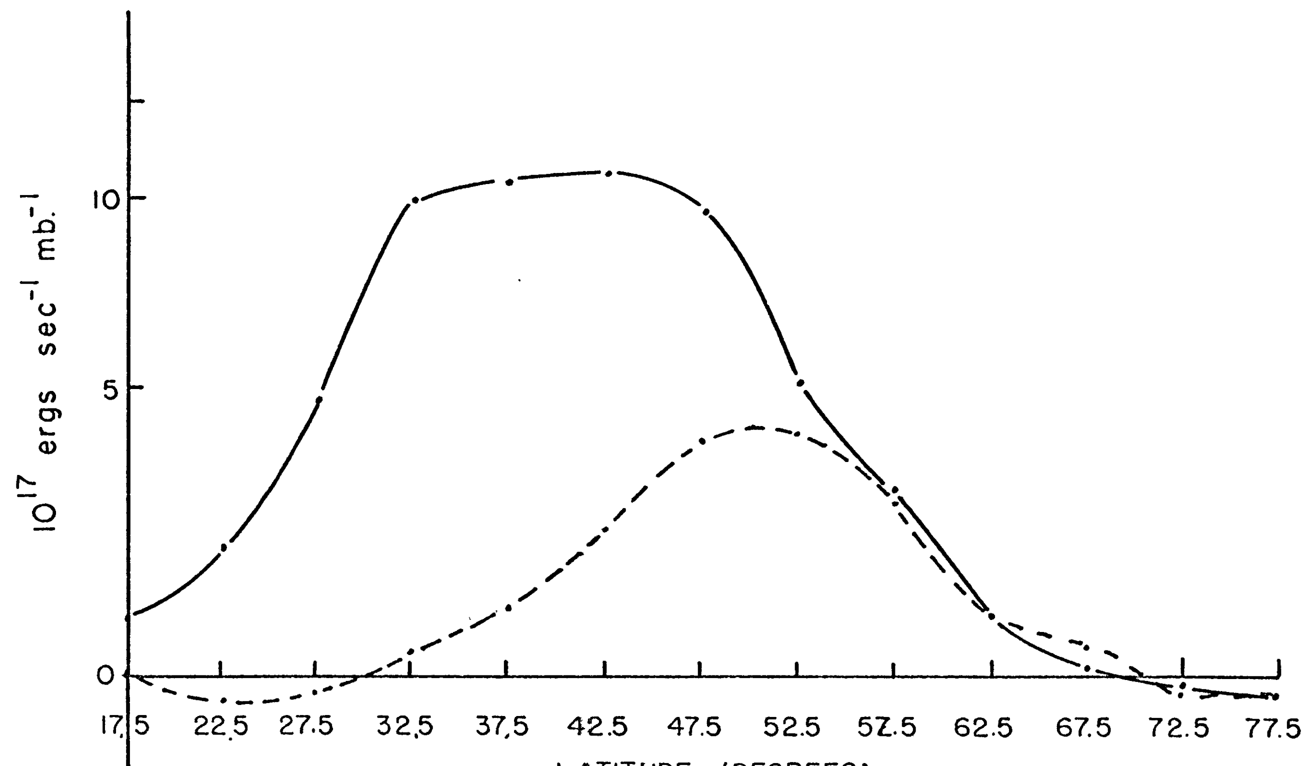

Fig. 1. Mean Rate of Meridional Flux of Kinetic Energy (solid curve), and its Net Perturbation Component (dashed curve), at 500mb for November – April, 1951

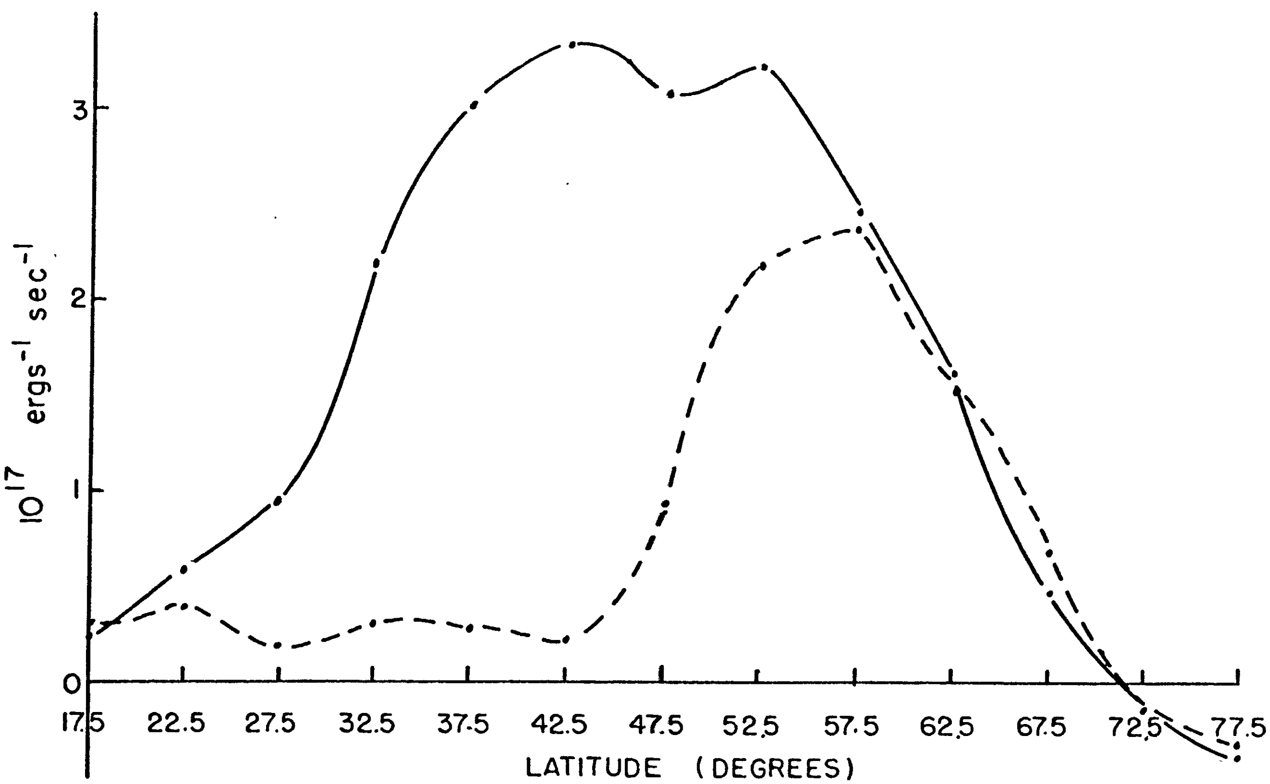

Fig. 2. Mean Rate of Meridional Flux of Kinetic Energy (solid curve), and its Net Perturbation Component (dashed curve), at 500 mb, for May - October, 1951.

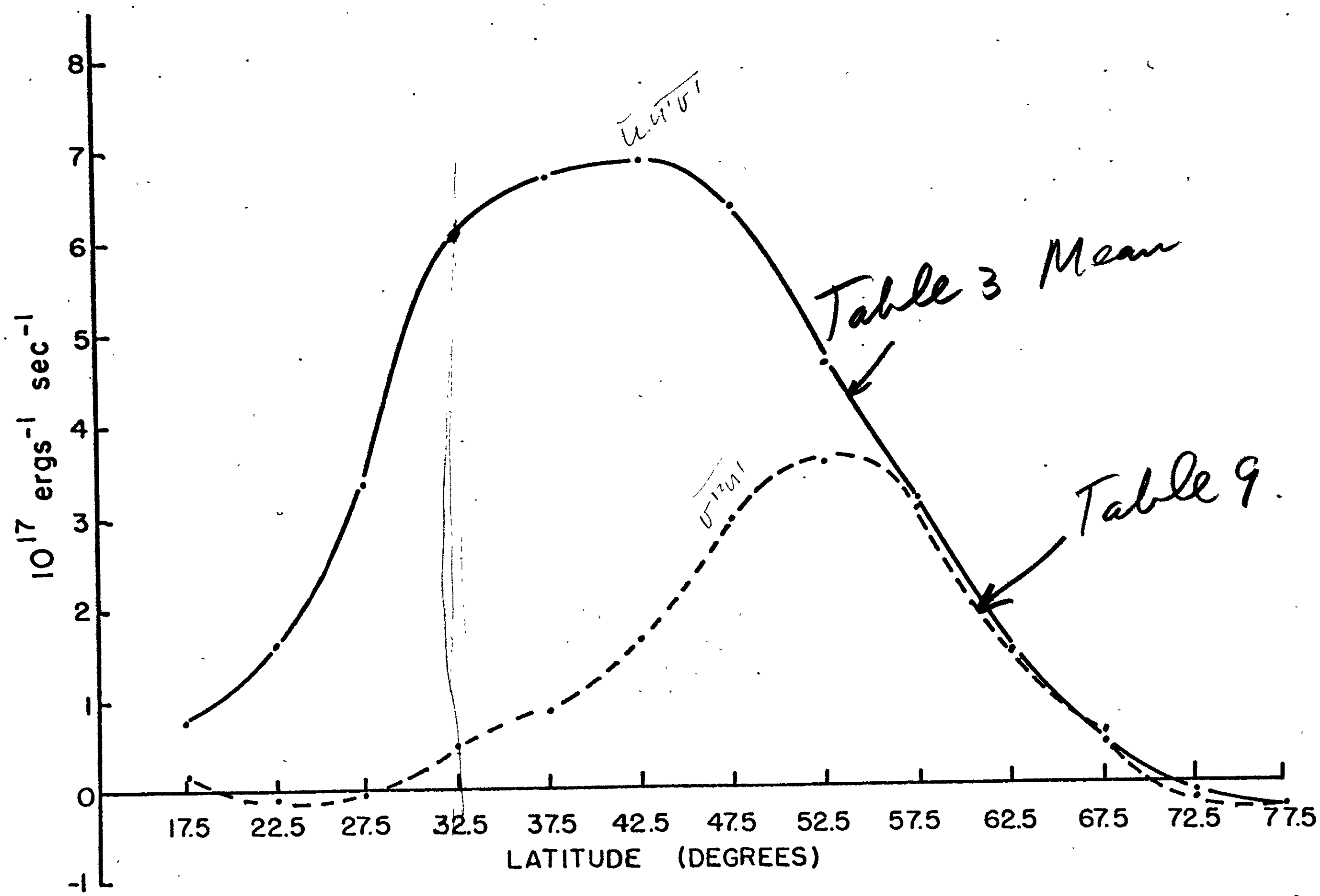

Fig. 3. Mean Rate of Total Meridional Flux of Kinetic Energy (solid curve), and its Net Perturbation Component (dashed curve), at 500mb for the Year 1951.

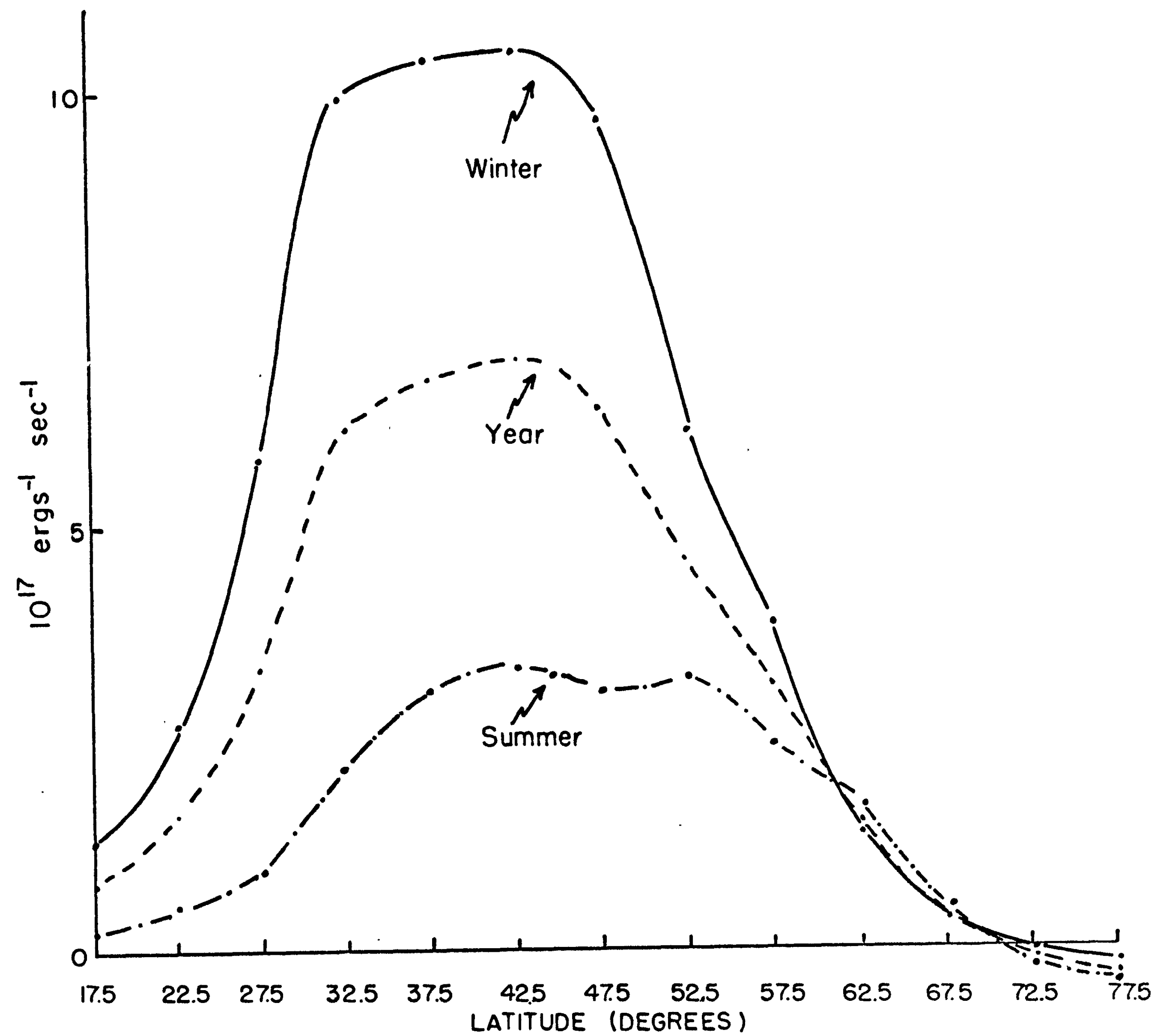

Fig. 4. Mean Rate of Total Meridional Flux of Kinetic Energy at 500mb for 1951.

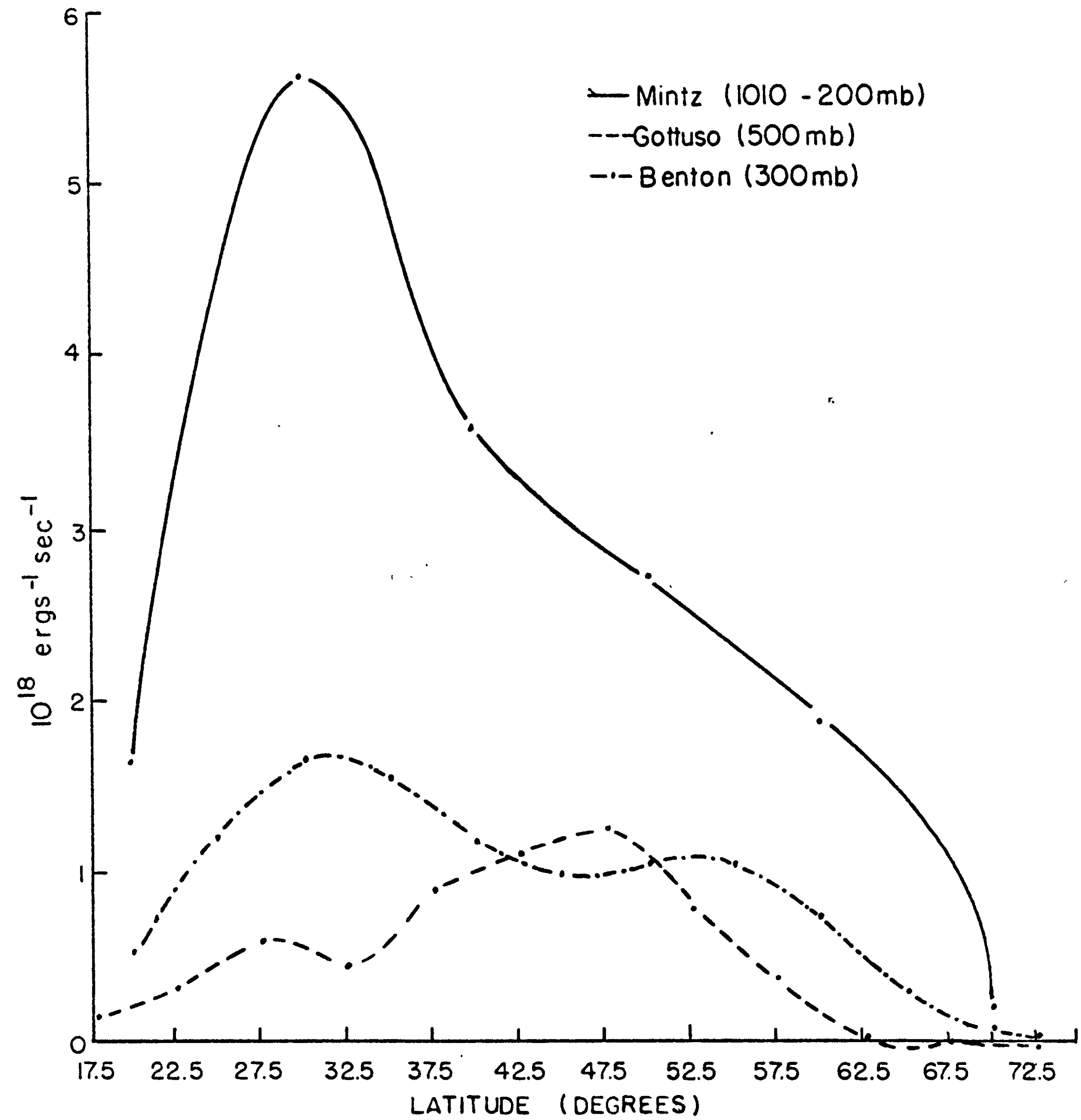

Fig. 5. Mean Rate of Meridional Flux of Eddy Kinetic Energy According to Various Sources.

January — February Data

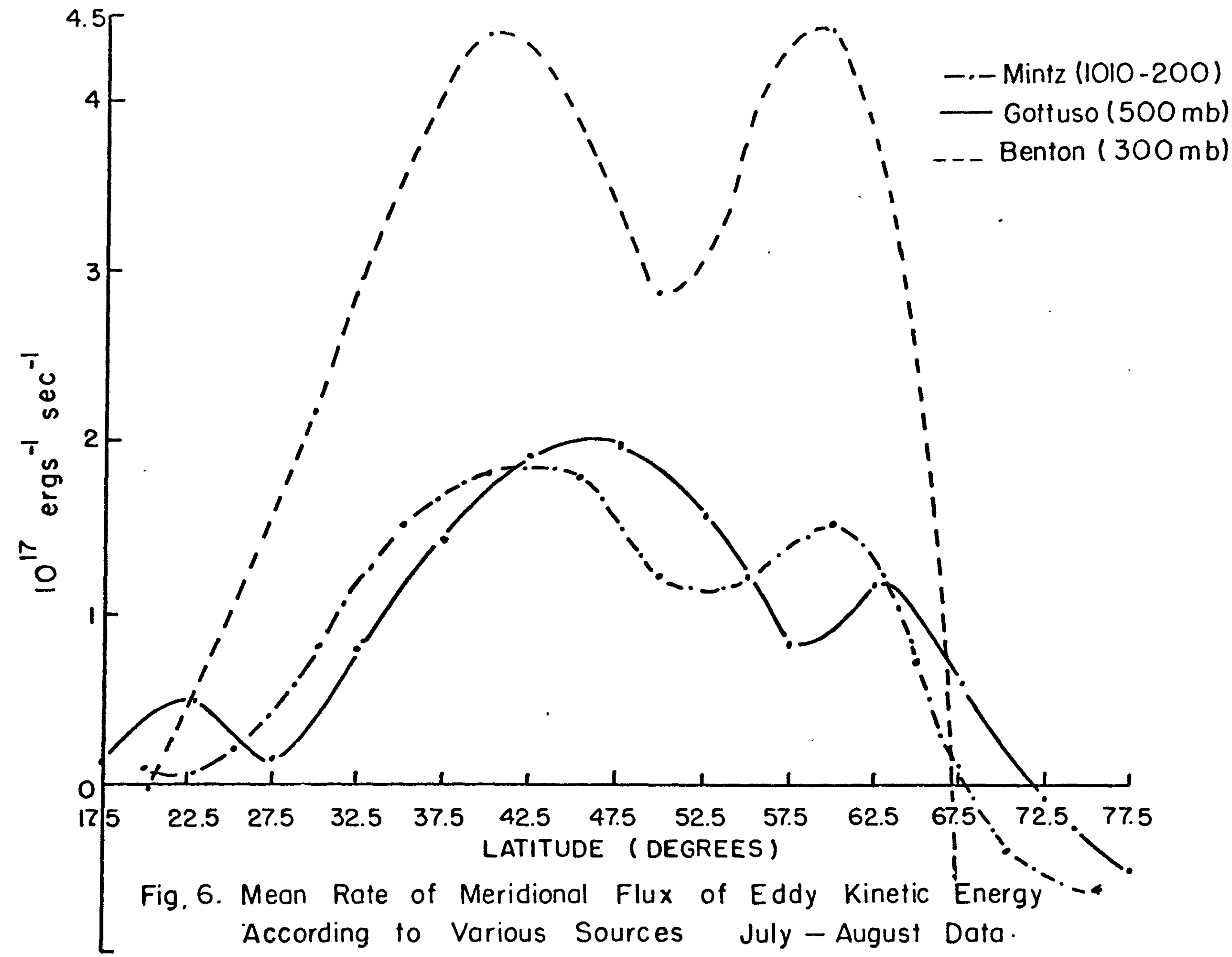

Fig. 6. Mean Rate of Meridional Flux of Eddy Kinetic Energy According to Various Sources July — August Data.

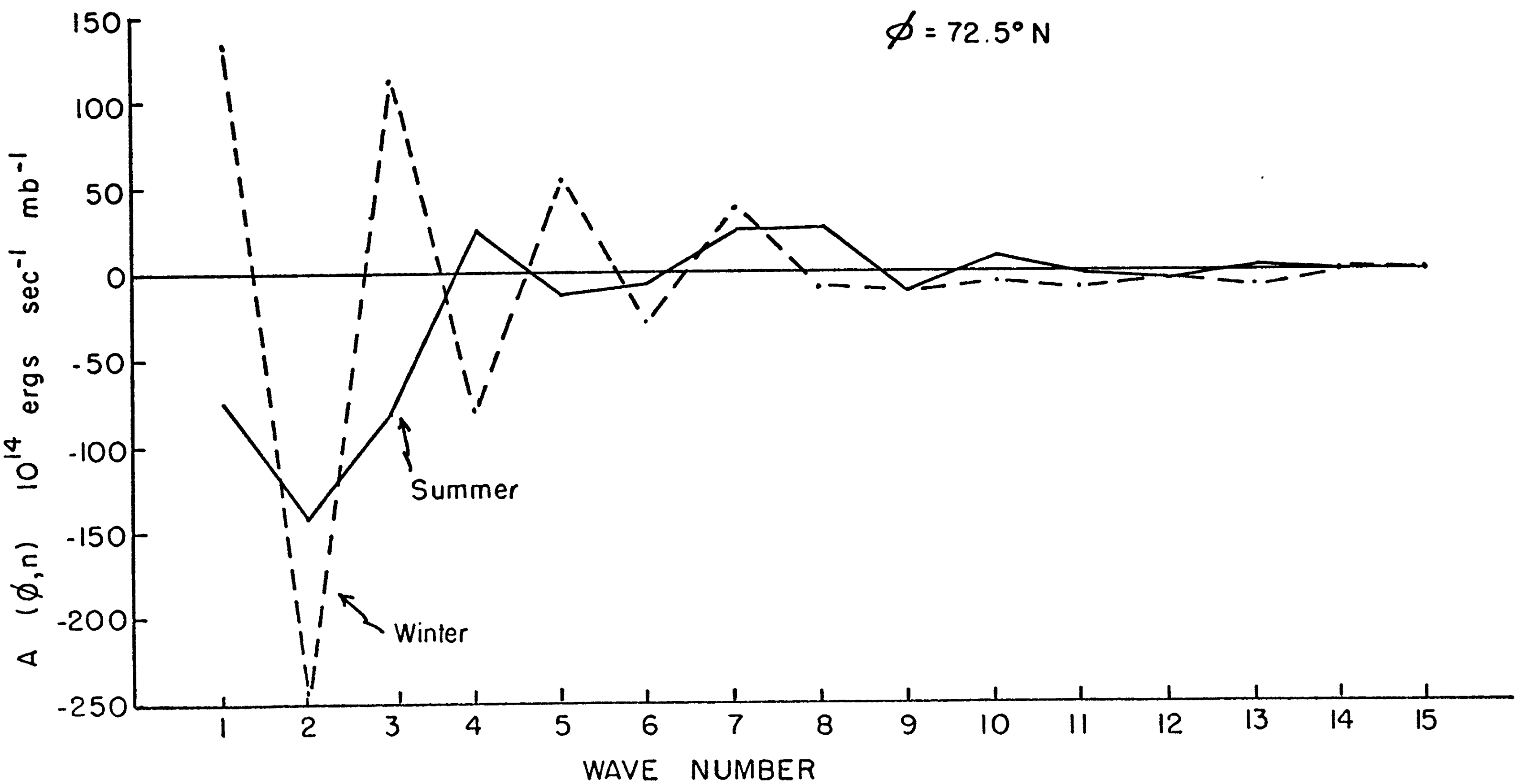

Fig. 7. Mean Seasonal Spectra of 500 mb Eddy Flux of Eddy Kinetic Energy Across 72.5° N.

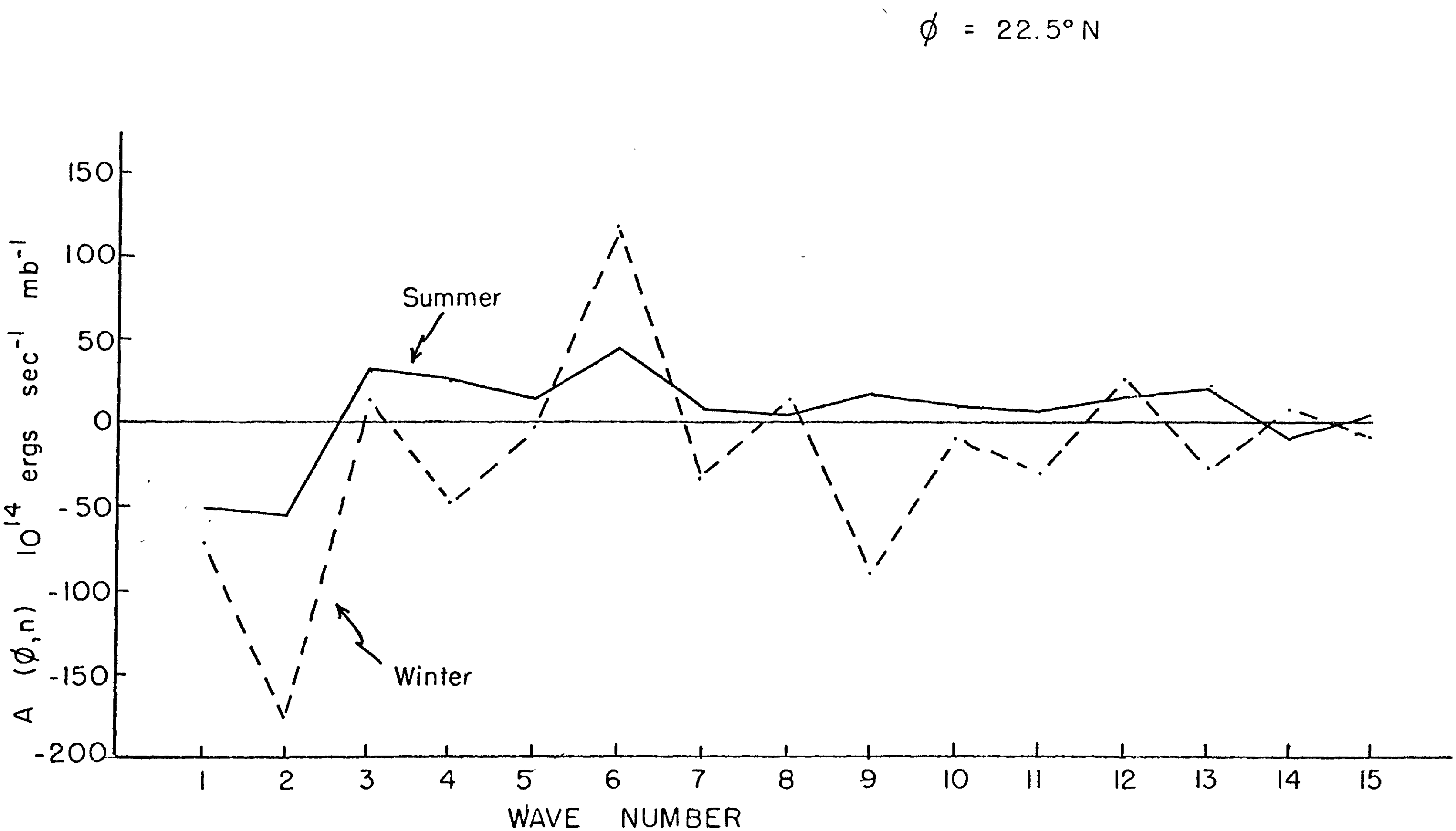

Fig. 8. Mean Seasonal Spectra of 500mb Eddy Flux of Eddy Kinetic Energy Across 22.5° N.

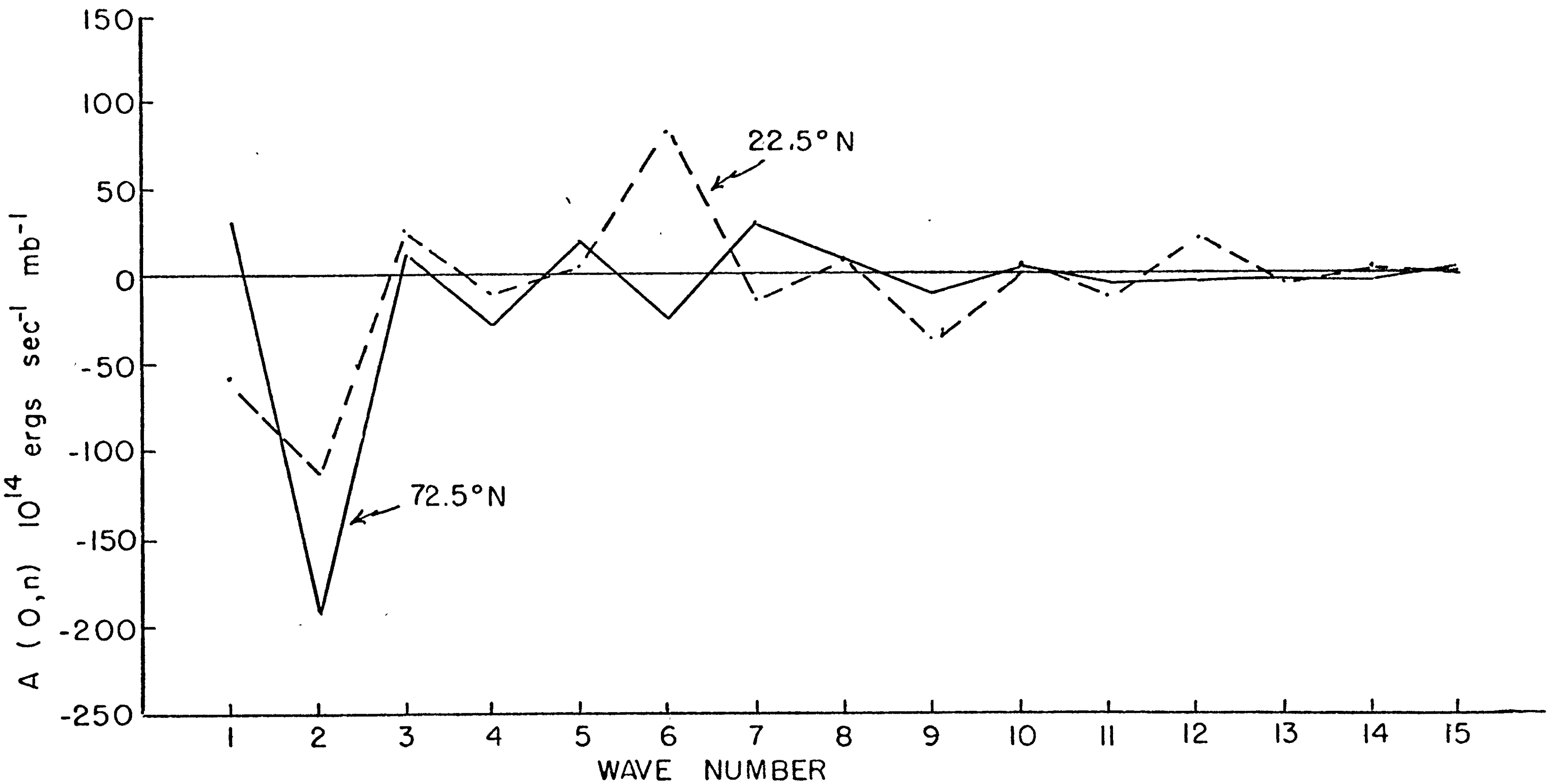

Fig. 9. Mean Yearly (1951) Spectra of 500mb Eddy Flux of Eddy Kinetic Energy Across 22.5° N and 72.5° N.

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