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THE VELOCITY BEHAVIOR
OF A GROWING CRACK

by

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ABSTRACT

Following N. F. Mott, the principle of conservation of energy is applied to a crack propagating in an infinite plate of vanishing thickness with a uniform tension applied at infinity. A refinement of mathematical technique over that used by Mott yields $\dot{c} = V_T(1-\frac{c_o}{c})$ where $\dot{c}$ is the velocity of the crack; V_T the terminal velocity; c_o the initial crack length; and c the crack length. The predicted velocity behavior is compared to data taken on sheets of polymethylmethacrylate. The comparison of the data with the theory is inconclusive because surface energy per unit crack length was not constant and because of other effects such as plastic flow.

It is concluded that evaluation of velocity behavior predicted by Mott's theory must await careful velocity studies on a brittle material such as glass. General design considerations of such an experiment are discussed.

Thesis Supervisor: W. F. Brace
Title: Assistant Professor of Geology

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None of the above are to be held accountable in any way for errors in mathematics or conclusions drawn.

THE VELOCITY BEHAVIOR OF A GROWING CRACK

I. Introduction

1. Statement of the Problem

The problem investigated in this paper is that of theoretically predicting the behavior of the velocity of a propogating crack in a brittle material from the time of onset of rupture until terminal velocity is approached. The crack will be considered to grow from an originally static one at an interior region of an infinitely thin plate, infinite in the other two directions. A uniform tensile stress is applied at infinity normal to the long axis of the crack. The tensile stress is increased until the crack becomes unstable and starts to grow in its own plane. The stress is held fixed at this critical value during propogation of the crack. The crack is approximated by an ellipse with a very small minor axis. See Fig. 1.

2. Previous Work

Some work has been done on predicting the terminal velocity of rupture. The agreement of the predicted terminal velocities with those found experimentally is apparently not conclusive according to some authorities,

Figure 1 <u>Plan View of a Griffith Crack</u>

Approximated by an ellipse with a very small minor axis, the crack grows in the positive and negative x-directions due to the action of the uniform stress σ, applied at infinity.

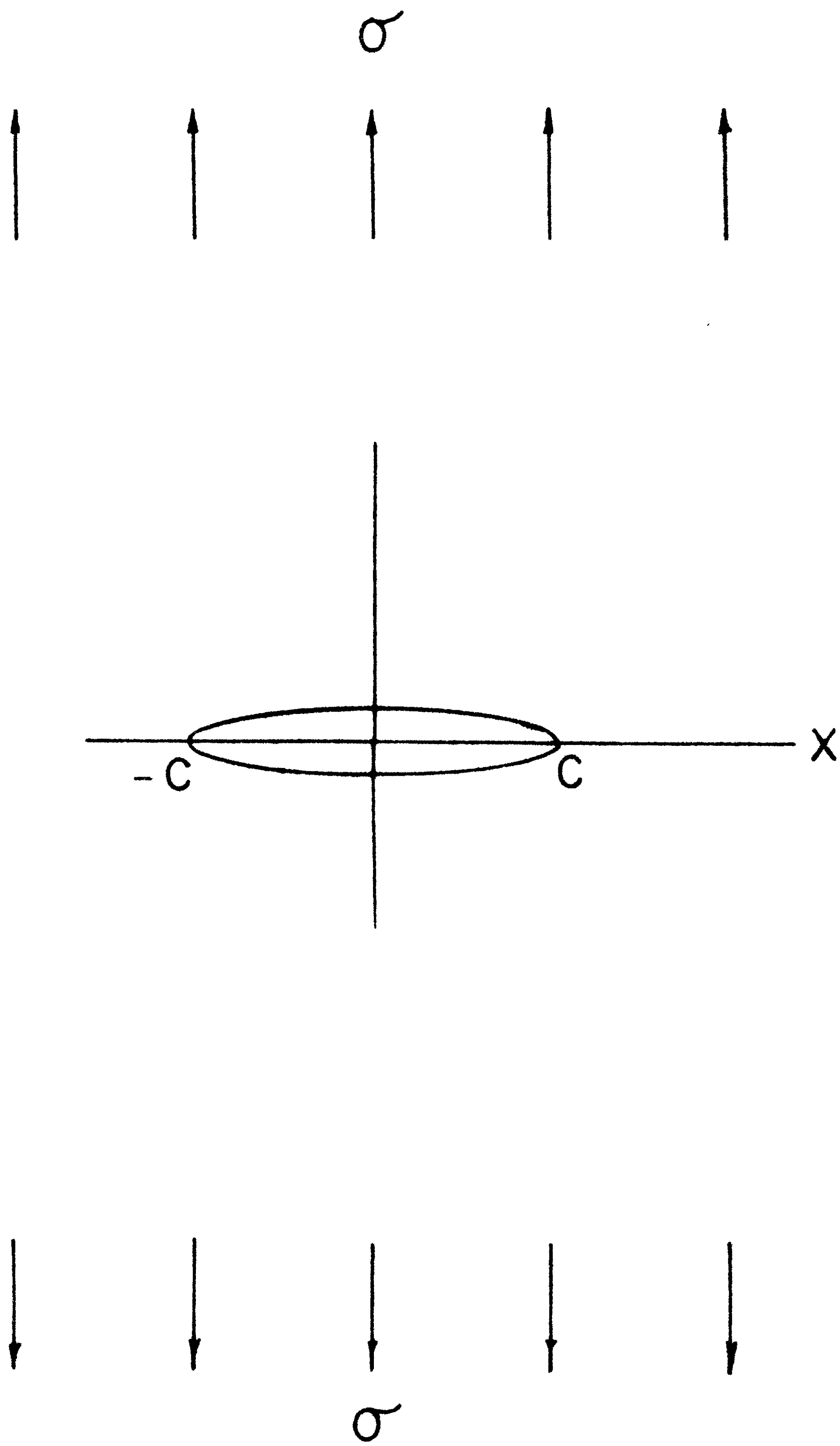

FIG 1

e.g., Schardin[1]. Schardin feels that terminal crack velocity is a function of an effective free surface energy and is not simply related to other parameters. In support of this, he points to the evident lack of success of other proposed prediction theories.

Stroh[2] feels that a propogating crack is similar to the phenomenon of Rayleigh waves and hence inertial effects would determine terminal velocity.

Yoffe[3] has calculated the stress field about a moving elliptical hole of constant length in a simple two-dimensional geometry. She tabulates the stress field at the head of the hole as a function of velocity and shows that the direction across which maximum tensile stress occurs branches noticeably at approximately 0.5 the speed of transverse waves in the material. Cracks are observed to branch at roughly this velocity; if the branching velocity is necessarily the limiting one, then the limiting velocity would be controlled by inertial effects.

Irwin[4] concludes that inertial effects control the limiting velocity and states that Irwin and Wells in a personal communication calculated that terminal velocity should be 0.5 the speed of transverse waves. Irwin presents a brief table of data showing excellent correlation with properly performed experiments on various materials.

Mott[5] demonstrates on theoretical grounds that terminal velocity should be equal to a constant times the square root of Young's modulus divided by the square root of the density. Schardin presents a plot of this quantity versus terminal velocity for several types of glass. The data points substantially deviate from a straight line. The developments of this paper are a direct extension of Mott's work in which he applied the principle of conservation of energy to a moving crack of the simple two-dimensional geometry which we are considering here. Lacking precise information as to the manner in which terminal velocities were measured in the glasses, the apparent disagreement with Mott's theory is attributed to experiments inadvertantly performed incorrectly relevant to the criterion presented in Irwin and discussed in this paper.

Mott and later Roberts and Wells[6] were primarily concerned with the terminal velocity of cracks while apparently overlooking that Mott's approach will also yield the behavior of the velocity from zero to terminal velocity. The following study is directed toward prediction of the entire velocity behavior.

II. Analysis of the Behavior of a Growing Crack

1. Review of Mott's Approach

Consider Fig. 1. The ellipse represents a crack in an infinitely thin plate growing equally in the positive and negative x-directions. σ represents a uniform plane tension applied at infinity. Mott applied the conservation of energy to the growing crack. This can be expressed as the sum of three terms F+S+K=constant. The terms represent the following three energies associated with the crack:

S= the stored elastic energy in the plate in the presence of the crack minus the stored elastic energy in the plate in the absence of the crack with the same uniform stress at infinity in both cases.

F= free surface energy of the crack.

K= kinetic energy of the time varying displacement field associated with the moving crack.

When properly evaluated, the kinetic energy term, K, accounts for elastic disturbances radiating energy from the region of the crack.

If the half length of the crack is c as shown in Fig. 1, and T is the free surface energy for unit area, then the free surface energy, F, is given by F=4cT.

Evaluation of terms S and K depend upon the stress field about a moving crack. It is appropriate to note that Yoffe's expression cannot be used in the evaluation of these terms. We require integration of the stress and displacement fields over all space where the approximation of substituting a moving elliptical hole of constant length for a crack is no longer valid. In

In order to evaluate terms S and K we must use the expressions for the fields associated with a standing crack due to Inglis[7] primarily because these are all that are available. Roberts and Wells, and Irwin offer brief discussions of the validity of using the standing crack expressions; however, the most convincing demonstration to date of the validity of using Inglis's expressions is a series of photographs of the photoelastic effect associated with a moving crack presented by Wells and Post[8]. The photographs show that the dynamic stress field and static stress field do not noticeably differ significantly. Assuming that the static crack representations are valid, we have Mott's expressions for S and K.

$$S = -\pi\sigma^2 c^2/E \quad \text{and} \quad K = 0.5k\eta c^2 \dot{c}^2 \sigma^2/E^2$$

where E = Young's modulus, k = a numerical factor less than 1 determined by Roberts and Wells, η = density and $\dot{c}$ = the time derivative of c, i.e., the velocity of propagation of the crack.

The resulting equation for the conservation of energy in a growing crack is

$$\frac{1}{2}k\eta c^2 \dot{c}^2 \frac{\sigma^2}{E^2} - \pi\sigma^2 \frac{c^2}{E} + 4cT = \text{constant} \tag{1}$$

Mott then differentiated with respect to c to remove the unknown constant. Because he was primarily investigating terminal velocity, Mott allowed $\frac{\partial \dot{c}}{\partial c}$ to equal zero to facillitate the analysis, which then leads to

$$\dot{c} = \sqrt{\frac{2\pi}{k}} \sqrt{\frac{E}{\eta}} \sqrt{\left(1 - \frac{c_o}{c}\right)}$$

where for 4T we have $4T = 2\pi\sigma^2 c_o/E$ from Griffith's[9] rupture criterion and c_o is the initial crack length.

2. The Contribution of Roberts and Wells

The value of the unknown numerical constant k was not determined by Mott. Mott presented an integral formulation of k which Roberts and Wells evaluated for Poisson's ratio equal to 0.25. When k is so evaluated its value is such that $\sqrt{2\pi/k} = 0.38$.

Hence, Mott's theory as evaluated by Roberts and Wells yields

$$\dot{c} = 0.38\sqrt{E/\eta}\ \sqrt{(1-c_o/c)}\ . \qquad (2)$$

3. A Refinement of Mathematical Technique

The approximation used by Mott and later Roberts and Wells of setting $\frac{\partial \dot{c}}{\partial c} = 0$ is only valid when $\dot{c}$ is very near terminal velocity. Eq. (2) shows that $\frac{\partial \dot{c}}{\partial c} \neq 0$ in defiance of the assumption.

Eq. (1) may be solved rigorously without appealing to the approximation above. The solution of Eq. (1) so obtained is correct within the framework assumptions implied in Eq. (1). Let us replace 4T by the Griffith criterion in Eq. (1) and call the constant U_o. We then have

$$\frac{1}{2}k\eta c^2 \dot{c}^2 \frac{\sigma^2}{E^2} - \pi\sigma^2 \frac{c^2}{E} + 2\pi\sigma^2 c_o \frac{c}{E} = U_o\ . \qquad (3)$$

Roberts and Wells point out that a boundary condition for the velocity, $\dot{c}$, is

$$c = c_o \quad \text{then } \dot{c} = 0.$$

Applying this boundary condition to Eq. (3) yields

$$0 - \pi\sigma^2 \frac{c_o^2}{E} + 2\pi\sigma^2 \frac{c_o^2}{E} = U_o$$

$$\text{and} \qquad U_o = \pi\sigma^2 \frac{c_o^2}{E} \; . \qquad (4)$$

Substituting the value of U_o shown in Eq. (4) we have the Mott energy balance equation of a growing Griffith crack.

$$\frac{1}{2} k\eta c^2 \dot{c}^2 \frac{\sigma^2}{E^2} - \pi\sigma^2 \frac{c^2}{E} + 2\pi\sigma^2 c_o \frac{c}{E} = \pi\sigma^2 \frac{c_o^2}{E} \qquad (5)$$

Eq. (5) may be solved algebraically for $\dot{c}$.

$$\dot{c}^2 = \frac{2\pi}{k} \frac{E}{\eta} \left(1 - \frac{2c_o}{c} + \frac{c_o^2}{c^2}\right)$$

$$\dot{c} = \sqrt{2\pi E/k\eta} \left(1 - \frac{c_o}{c}\right) \qquad (6)$$

Eq. (6) is the prediction of the velocity behavior according to Mott's theory. The behavior predicted by Eq. (6) differs from that found by using the approximation $\frac{\partial \dot{c}}{\partial c} = 0$, in that the term $(1 - \frac{c_o}{c})$ in Eq. (6) appears under a square root sign in Eq. (2).

4. The Resulting Expression for the Growing Griffith Crack

As the crack grows without limit c_o/c will approach zero and the terminal velocity will be $\sqrt{2\pi E/k\eta}$. Let us call V_T the terminal velocity.

$$V_T = \sqrt{2\pi E/k\eta} \text{ and}$$

$$\dot{c} = V_T \left(1 - \frac{c_o}{c}\right)$$

$$= V_T - V_T c_o \,(1/c) \qquad (7)$$

When $\dot{c}$ is plotted against $1/c$ a straight line results[10] as shown in Fig. 2. The velocity axis intercept of the line is V_T and the reciprocal crack length axis intercept is c_o^{-1} . This plot when applied to velocity measurements provides a convenient check of the theory.

We conclude that the proper solution of Mott's energy equation predicts that crack velocity should make an hyperbolic asymptotic approach to terminal velocity. Fig. 3 is a plot of $\dot{c}$ versus c from Eq. (7) showing the predicted velocity curve.

5. Assumptions Implied in the Analysis

In addition to the assumed validity of standing crack stress and displacement fields when applied to a moving crack the following conditions must be met by the physical situation described by the mathematics of the above analysis:

a. The surface energy per unit length of the crack is constant.

b. No work is added at the boundaries during propagation of the crack. This corresponds to constant strain at infinity rather than constant stress but is a valid assumption for an infinite plate as discussed by Roberts and Wells.

Figure 2 <u>Theoretically Predicted Velocity Plotted Against Inverse Crack Length</u>

The velocity ($\dot{c}$) axis intercept is the terminal velocity ($V_T = \sqrt{2\pi/k}\ \sqrt{E/\eta}$ while the inverse crack length axis intercept is the reciprocal length of the initiating crack c_o. $\dot{c} = V_T - V_T c_o (1/c)$.

Figure 3 <u>Theoretically Predicted Velocity Plotted Against Crack Length</u>

The velocity is given by $\dot{c} = V_T(1-c_o/c)$. The velocity curve makes an asymptotic approach to V_T.

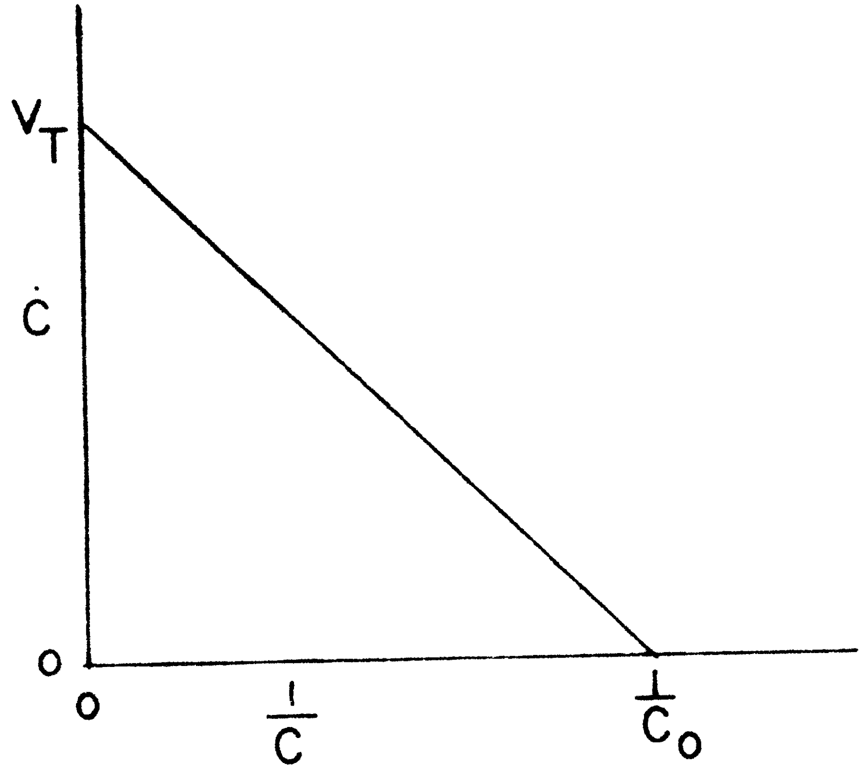

FIG 2

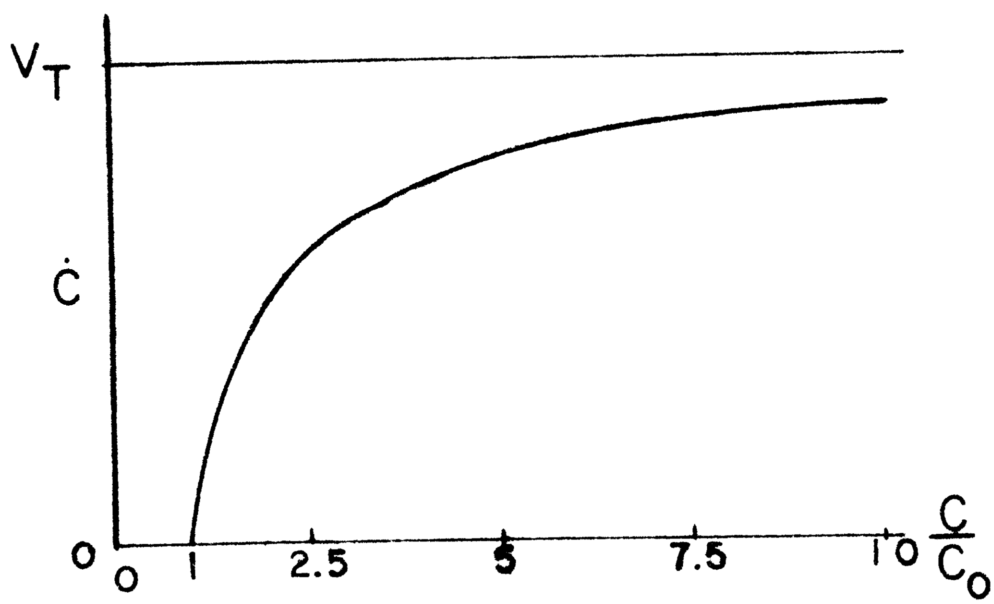

FIG 3

infinite plate as discussed by Roberts and Wells.

c. The elastic parameters of the material are constant during propagation. In particular this includes Young's modulus.

d. The fracture is brittle; i.e., no plastic flow.

e. The plate is infinite.

The effects of a physical situation not complying with some of the above conditions are discussed in the following section.

III. Comparison of Theory with Measurements of Crack Velocities in Polymethylmethacrylate

1. Description of the Experiments

In order to test the theory developed above, measurements of crack velocity in polymethylmethacrylate were made available by Professor E. Orowan from studies of Mr. J. Neimark to be published elsewhere.

Rupture was initiated in sheets measuring roughly 6 ft. by 2 ft. by 1/4 inch at a sawed notch a fraction of a centimeter to two centimeters long cut halfway along one of the six foot sides. Tension was applied at the ends by means of a configuration of clamps suspended on knife edges. When rupture occurred, the passage of the crack across the face of the sheet was observed photographically on a cathode ray oscilloscope as the crack broke evenly spaced lines of printed circuit paint placed perpendicularly to the path of the crack. From these data velocity of the crack versus crack. From these data velocity of the crack versus distance from the edge of the sheet was obtained.

2. Description of the Data

Six of these velocity versus distance curves are presented here in Fig. 4. The designation of the specimens are the original ones tentatively used by Professor Orowan.

Specimens 10P and 11P apparently approach a terminal velocity within the width of the plate. 10P and 11P branched between 35 and 40 centimeters, the total width of the plate being 63 centimeters. The surface of the crack near the originating cracks in 10P and 11P are very smooth and become progressively rougher, until just before forking the surface is so roughened that small (up to two centimeters in length) branching cracks occur. It is probably a good assumption that the roughening implies an increase in effective surface energy per unit crack length. After branching, one of the cracks becomes progressively smoother and the other maintains a relatively rough surface. Mott's theory is clearly not applicable after branching.

The other specimens in Fig. 4 exhibit a much smaller change in surface character during propagation. By the time the approaching edge of the plate is reached these samples have only a slightly frosted crack surface such as is found in samples 10P and 11P at a much earlier part of the crack.

Figure 4 Velocity Versus Crack Length in Polymethylmethacrylate

These are some of Professor Orowan's data, presented here for comparison to theory. Samples 10P and 11P branched where the curves end. Measurements were performed by Mr. Neimark on sheets 63 centimeters in width.

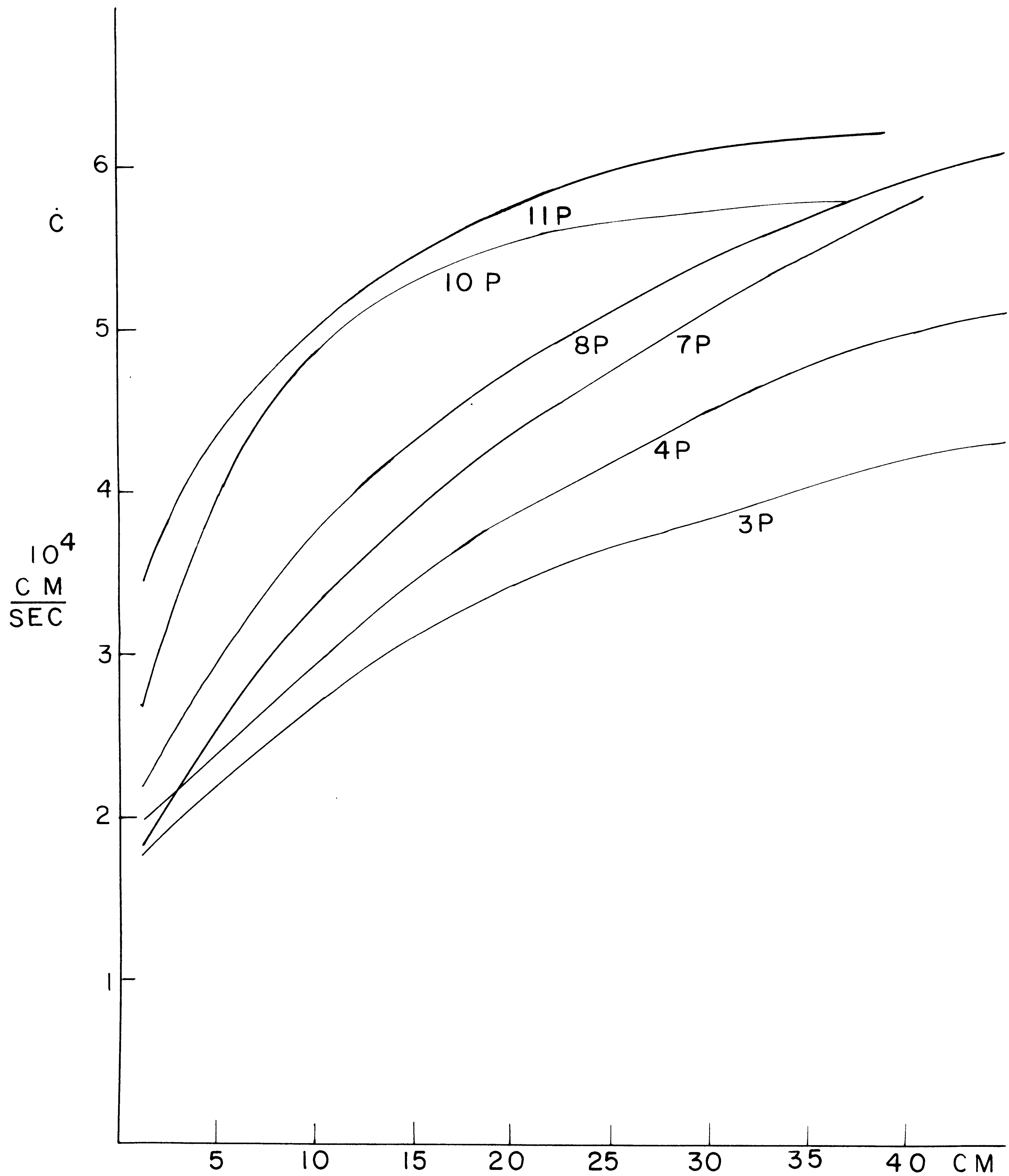

FIG 4

Figure 5 and Figure 6

Comparison of 10P and 11P With Theory

The cracks branched where the curves end. Theory predicts that when effective surface energy per unit crack length increases with crack length, as it does in the specimens, the velocity when plotted against reciprocal crack length should not be straight lines as it is over a considerable range in these curves. A combination of effects causes a straight line plot here.

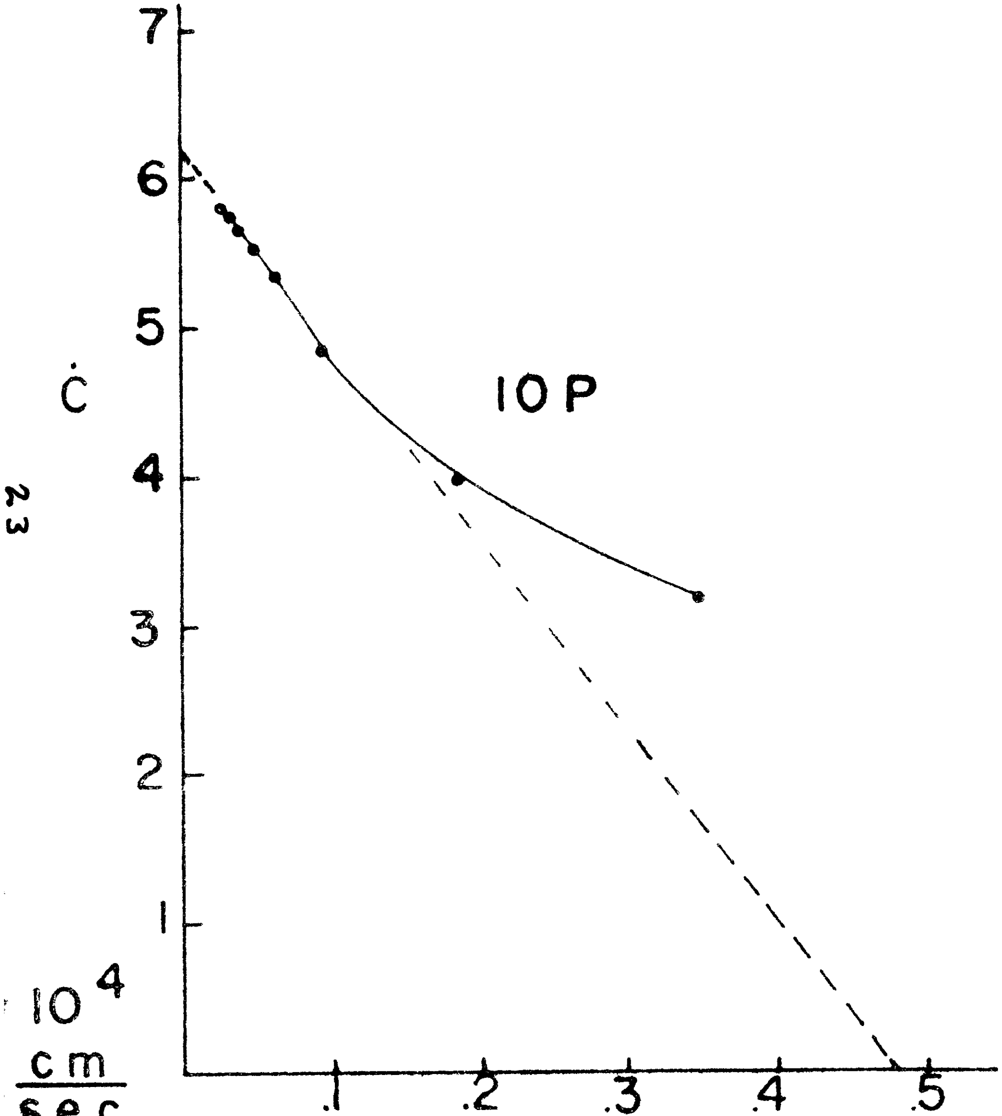

FIG 5

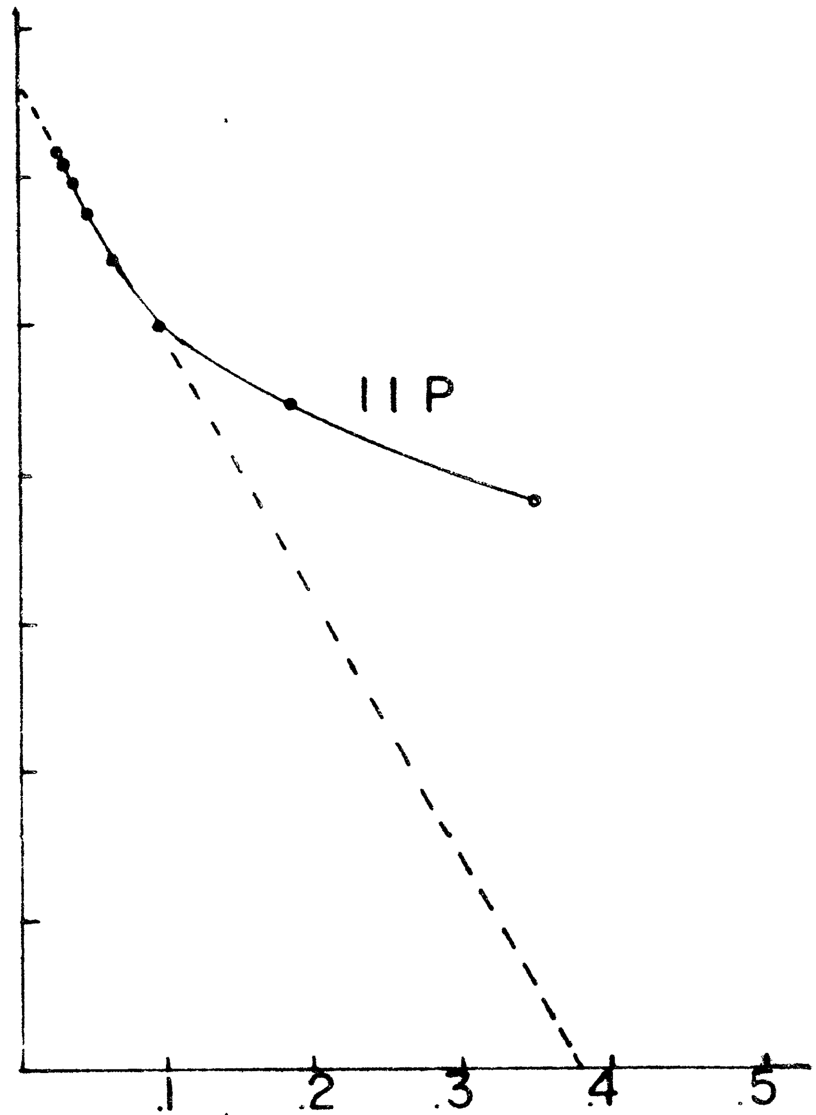

FIG 6

Figure 7 Plot of Velocity Versus Reciprocal Crack Length for Two Specimens

Note that the curves are not straight lines over any range contrary to the prediction of simple theory. The cracks did not branch within the width of the sheets.

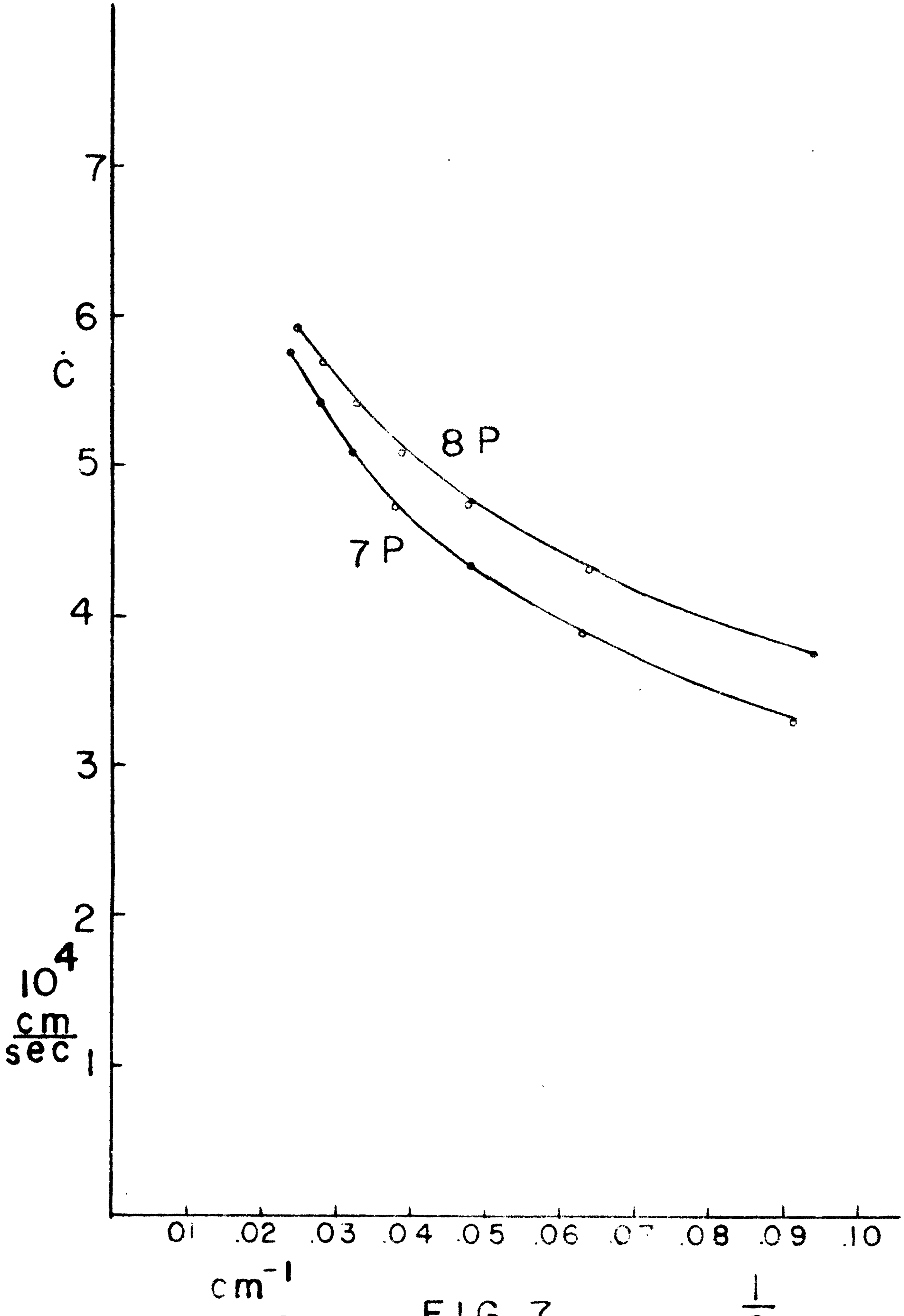

FIG 7

Figure 8 Plot of Velocity Versus Reciprocal Crack Length for Two Specimens

Note that the curves are not straight lines over any range contrary to the prediction of simple theory. The cracks did not branch within the width of the sheets.

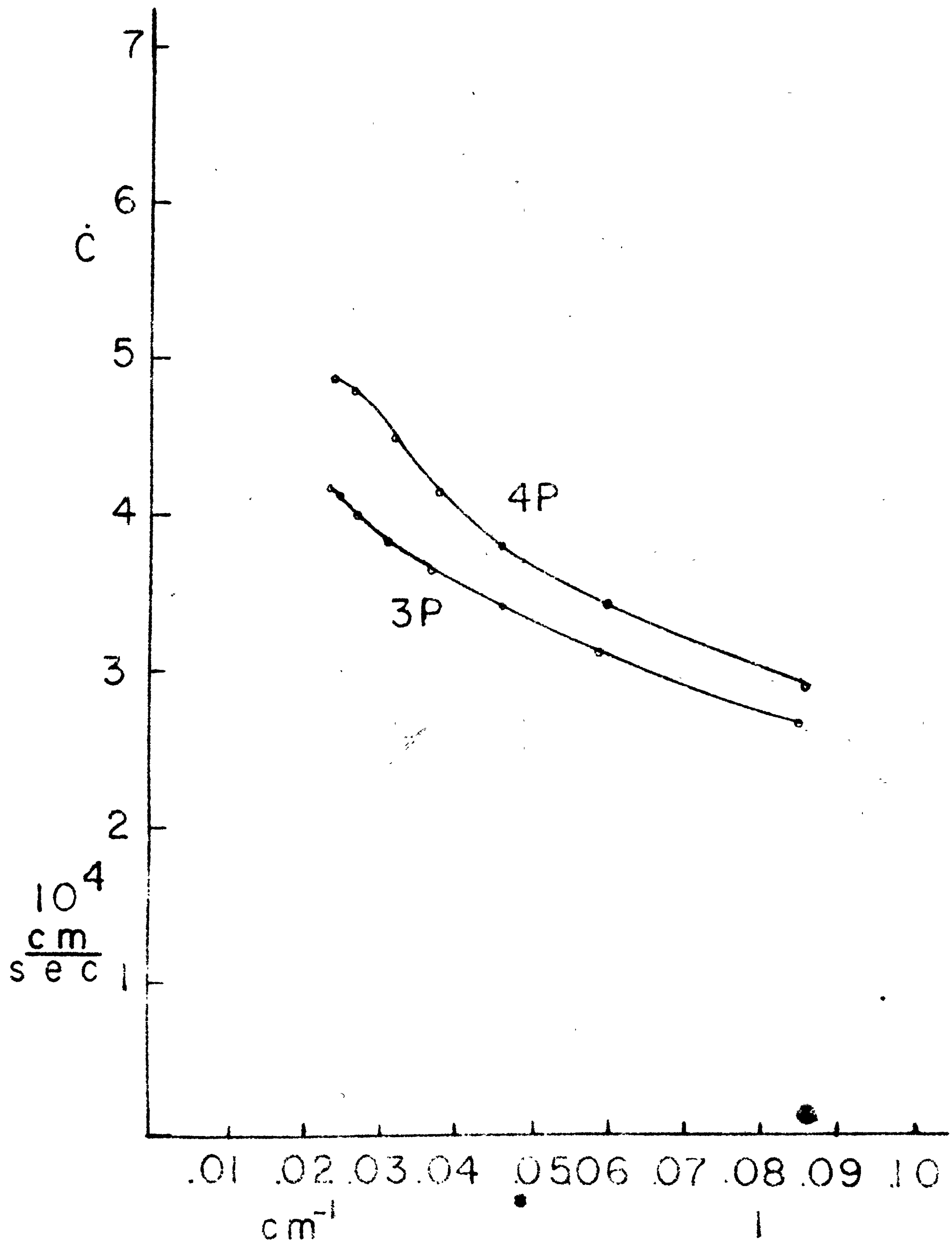

FIG 8

on the character of the surfaces when the studies are published. A brief discussion of some reasons for roughening may be found in Irwin.

3. Comparison with Theory

The comparison with theory is best done by plotting velocity versus inverse crack length. Figs. 4 and 5 show such plots for specimens 10P and 11P. The small circles are not data points but are points used to redraft the velocity curves. The points for transcribence were selected at 5 centimeter intervals.

The theoretically predicted curve would be a straight line between c_o and $V_T = 0.38\sqrt{E/\eta}$. c_o in this case would be the length of the sawed notch plus a small crack that grew from the notch while the specimen was brought up to load. The value of V_T obtained from Young's modulus could not be determined because Young's modulus varied according to strain rate, etc. Hence the best determination of V_T according to Mott's theory is obtained by extrapolating the straight line regions of the curves which lie approximately bwtween 10 eentimeters and 35 to 40 centimeters. When the extrapolation is done as in Fig. 5 and 6 we see that the terminal velocities lie between 5×10^4 and 6×10^4 cm/sec.

The crack velocities never reached this value of V_T but branched first. Branching might have occurred too soon because of the influence of the approaching edge. At the point of branching the cracks were 2/3 of the way across the sheets and it is to be expected that the approaching boundary would have some influence on the stresses near the crack. Whether or not the edge caused branching cannot be answered here; however, we can say that if the edge was not influential in branching then the terminal velocity as predicted by Mott's theory was not reached.

Since Yoffe's calculations show that a continually increasing branching of the stress field at the head of the crack would occur at crack velocities below V_T, the question arises whether or not the terminal velocity predicted by Mott's theory will ever be reached or will branching occur instead. The approach to V_T is hyperbolic. Therefore, barring infinite crack lengths, to say that V_T is "reached" must involve some approximation. The theory states that when the crack length is $100c_o$, the velocity is $\dot{c} = 0.99V_T$, when $c = 1000c_o$, $\dot{c} = 0.999V_T$. Hence V_T can never be "reached" before branching occurs or the edge of the plate is reached. At the present time it cannot be said at what fraction of V_T branching will occur.

If the extrapolated value of V_T is taken as that predicted by Mott's theory, then it will be seen that in Fig. 5 and 6 the measured velocities are everywhere less than those predicted. The straight line portions of 10P and 11P behave as if the cracks originated at an effective c_o approximately 5 times the c_o observed.

10P and 11P behave as if the cracks originated at an effective c_o approximately 5 times the c_o observed. This discrepency between predicted and observed velocity is what would result if energy were somehow removed from the region of the crack. This removal could be accounted for by a changing surface energy per unit length of the crack or by other types of losses. These losses could be due to radiative losses of elastic energy to other regions of the plate or plastic flow in the region of the crack.

Stroh feels that the radiative losses would possibly be greatest during the early stages of spreading when presumably acceleration is greatest. Radiative losses will be assumed small in order to simplify the analysis.

The loss of energy due to plastic deformation is probably important in these measurements because polymethylmethacrylate does exhibit plastic deformation for slow strain rates. No quantitative analysis of the effect can be given because the amount of flow is uncertain and more so the work involved in the deformation. The qualitative effect would be to make measured velocities less than those predicted. In every specimen this is the case. See Fig. 5, 6, 7, 8.

4. The Effect of Changing Surface Energy Per Unit Crack Length

Curves 10P and 11P plot as straight lines over much of their range in spite of the noticeable change in the effective surface energy per unit crack length. This effect may be evaluated as follows.

$T(c)$ is surface energy per unit length as a function of c. In the specimens above, $T(c)$ was visibly an increasing function of c, that $T(c) < T(c+a)$ where "a" is a positive length. The surface energy term $F = 4cT$ that appears in Eq. (1) is obtained from the integral

$$F = 4\int_0^c T(c)dc = 4cT \text{ when } T \text{ is constant.}$$

Designate T_o as the value of T from $c = 0$ to $c = c_o$. Then let $T(c) = N(c)T_o$. From Griffith criterion we have

$$T(c) = N(c)\frac{\pi\sigma^2 c_o}{2E} \quad \text{and} \quad F = 2\pi\sigma^2\frac{c_o}{E}\int_{c_o}^{c} N(c)dc + 2\pi\sigma^2\frac{c_o^2}{E}$$

$$\text{or } F = \frac{2\pi\sigma^2 c_o}{E}\left[\int_{c_o}^{c} N(c)dc + c_o\right].$$

Substituting in Eq. (5) and solving for $\dot{c}$ yields

$$\dot{c} = V_T^2\left[1 + \frac{c_o^2}{c^2} - \frac{2c_o}{c^2}\left(\int_{c_o}^{c} N(c)dc + c_o\right)\right]. \qquad (8)$$

$N(c)$ is always > 1, hence for $c >> c_o$ we have

$$\dot{c} = V_T^2\left[1 - 2\frac{c_o}{c}(1/c)\int_{c_o}^{c} N(c)dc\right]. \qquad (9)$$

The result from Eq. (5) when $c >> c_o$ is

$$\dot{c} = V_T^2\left[1 - 2\frac{c_o}{c}\right]$$

Eq. (8) may be used to evaluate a changing surface energy per unit crack length if $T(c) = N(c)T_0$ is known. Eq.(9) shows that no marked continual increase in free surface energy will still yield a hyperbola for $\dot{c}$.

Specimens 10P and 11P are hyperbolic over much of their range despite the noticeable change in effective surface energy with crack length. This means that other effects are also present and neither these curves, nor the other, fit the simple theory of Mott derived for brittle material.

5. Conclusions to be Drawn from Comparison with Experiment

We conclude that specimens 10P and 11P fit the curve predicted by Mott's theory over a considerable range but when a formulation is made of the effect of changing surface energy, such as is observed, the curves no longer fit the theory and other effects, probably pastic flow, are important also.

The measured velocities of the other four specimens when plotted against reciprocal crack length form a curved line over their entire range rather than a straight one as theory predicts.

None of the specimens fit Mott's theory. This could be due to effects that are properties of plastic materials only or due to effects from which brittle materials would suffer, too.

Effects that would cause brittle substances as early stages of the crack growth

(b) edge effects when working with cracks initiated from an edge

(c) addition of work at the ends under tension during propagation of the crack.

(d) dynamic unloading of the sample so that it is not moving in the same stress field that it would move in if a uniform tension was applied at infinity on an infinitely long plate.

In the above list (b) refers to edge effects of the plate perpendicular to the direction of the crack while (c) and (d) refer to the parallel edges.

Effects that presumably would be important in plastic materials only are:

(a) changing surface energy of the crack as it grows

(b) plastic deformation particularly in the region of the crack.

The effect in (a) may be a result of (b)[11].

We tentatively conclude that primarily due to plastic flow, plastic materials should require a greater ratio of crack length to initiating crack length before terminal velocity is approached or branching occurs than would brittle materials.

IV Suggested Future Experiments

A searching test of Mott's theory must await careful velocity measurements, such as those furnished by Professor Orowan, performed on a brittle substance.

Glass is suggested. The surface energy of a crack in glass does not noticeably change by important factors as the crack growsnnor would plastic flow be expected.

In order to eliminate the effect of the edge at the beginning of rupture it would be best to start from a crack in the middle of a large plate. However, the edge effect is probably very small after the crack is a few centimeters long. This could be evaluated by experiment.

Whether rupture is initiated at an edge or in the middle, the plate should be of sufficient width that by the time the crack has grown to 50 or 100 times the length of the initiating crack no more than half the plate should be traversed by the crack. Hence initiating crack lengths should be kept as short as possible.

If elastic disturbances radiated by the moving crack are important, the edge to which the uniform tension is applied will have to be far enough away from the crack so that stress waves will not be reflected by these edges and returned to the region of the crack before terminal velocity is approached and branching occurs.

If it is felt after experiment that stress waves reflected from the edge that the crack is approaching could be causing deviations from the theoretical velocity values, then the plate will have to be made wider or the size of the initiating crack further reduced.

Design of velocity measurements to conform to the above conditions require an estimate of accumulated time as a function of the ratio c/c_o. The accumulated time may be obtained from a first time integral of the velocity expression.

$$\frac{dc}{dt} = V_T\left(1-\frac{c_o}{c}\right)$$

$$\int_0^t dt = \frac{1}{V_T} \times \int_{c_o}^{c} \frac{c\,dc}{c-c_o}$$

The integral is singular at $c= c_o$. This is an expression of the fact that the crack is in unstable equilibrium at $c = c_o$, hence the crack may stay at length c_o for an infinite amount of time or no amount of time. Accumulated time may be calculated between the length c and $c_o(1+\delta)$ where δ is any positive number greater than 1 and $c>c_o(1+\delta)$. That is, after the crack has started to move, the time between any two points may be calculated.

When the integral is so formulated we have

t [accumulated between $c=c_o(1+\delta)$ and $c>c_o(1+\delta)$] =

$$\frac{1}{V_T} \int_{c_o(1+\delta)}^{c} \frac{c\,dc}{c-c_o}$$

The integration yields

$$t = \frac{c_o}{V_T}\left[(1+\delta)(n-1)+\ln\left[\frac{n(1+\delta)-1}{\delta}\right] \right]$$

$$\text{where } n^{-1} = \frac{c_o(1+\delta)}{c}$$

If $\delta \ll 1$ and $n \gg 1$ then

$$t = \frac{c_o}{V_T}\left[n + \ln \frac{n}{\delta}\right], \quad n \text{ large}, \ \delta \text{ small}.$$

It is not possible to state how much time expires between c_o and $c_o(1+\delta)$. We should like to have an estimate of this time because presumably stress waves are emitted by the crack as soon as it starts to grow from c_o. The only simple way out of this difficulty is to assume some average velocity when calculating the time to grow from c_o. The above equations can be used for calculating the time between observations of the cracks as a function of crack length.

Placement of conducting strips could be arranged so that approximately equal time intervals would appear on an oscilliscope or timing between camera exposures could be adjusted so that the crack had moved approximately equal distances between exposures, thus obtaining maximum resolution in both cases.

FOOTNOTES

1. Schardin, H.; "Velocity Effects in Fracture," paper No. 12, CONFERENCE OF FRACTURE, April 12-14, 1959, Swampscott, Mass.

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