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ABSTRACT

This paper argues that, for large air pollution sources, it may be better to set emission standards which differ for days with differing ventilation conditions instead of the common method of setting a single emission standard. A highly simplified mathematical model is treated to illustrate how such dynamic standards may be set and how their advantages may be quantified. A more realistic model, utilizing results from the theory of optimization over Markov chains, is similarly analyzed.

INTRODUCTION

One of the basic problems in the implementation of air pollution control programs is the translation of air quality standards into emission standards. Air quality standards specify how often various pollutant concentrations can be permitted to occur. Air quality standards specify pollutant levels which have negligible or no effect on the health of men, animals and plants. In the U.S., for example, the Environmental Protection Agency specifies that the daily average concentration of sulfur dioxide should not exceed 365 $\mu g/m^3$ more than once a year, and that the annual average concentration should be less than 80 $\mu gm/m^3$ [1]. In order to achieve a given air quality standard it is necessary to decide by how much emissions from the various sources will have to be reduced. For example, if a source is emitting 2000 lbs/day of SO_2 it may be possible to meet the required air quality by reducing emissions to 1000 lbs/day. The figure of 1000 lbs/day would then be the emission standard for that source. Emission standards thus specify how much pollutant an emitter will be allowed to release into the atmosphere.

The most widely used method of setting emission standards is to fix an upper limit on the quantity of emissions that may be released on any day. This kind of standard will be referred to as a static emission standard. In this paper, I propose to argue the merits of setting dynamic emission standards. These are emission standards which will meet the required air quality standards but which set down different emission levels for different meteorological conditions. By this approach, emission standards can be set so that when atmospheric conditions favor rapid dispersal of pollutants they will permit larger amounts of emission and when the atmosphere is stagnant it will only allow small amounts of emission. The setting of dynamic standards leads to greater efficiency in the utilization of air resources in that given air quality standards can be met at lower cost than static standards, or alternatively, better air quality can be achieved for a given cost. I do not mean to imply that dynamic standards are universally superior to static standards. They are considerably more complex to set and to administer. However, I feel that they have much to offer in controlling large sources of pollution such as power stations, kraft paper mills and municipal incinerators. For space heating of homes and offices and for automobiles, static standards are, perhaps, the only feasible means of control.

In the next few sections I indicate how operations research techniques may be used in setting dynamic emission standards and in quantifying the advantages of dynamic over static emission control policies.

A SIMPLE MODEL

In this section, I will describe and analyze a simple model for air pollution control which, though it is quite crude, does bring into focus the principal elements involved in setting dynamic emission standards.

I shall take as my starting point a model, suggested by Marcus [2]. This model, based on a 'box' model for diffusion, relates concentrations of a pollutant on two successive days as follows:

$$C(t) = (C(t-1) + e(t))\, v(t) \qquad (1)$$

where $C(t)$, $C(t-1)$ are the concentrations of the pollutant on days t and $t-1$ respectively

$e(t)$ is the amount of pollutant emitted on day t per unit volume of the diffusion 'box.'

$v(t)$ is the 'ventilation factor' for day t.

For our simple model we will assume that $v(t)$ can take on values of either 0 or 1 (that is, days either have perfect ventilation or complete stagnation). Further, let us suppose that $\{v(t)\}$ forms a homogenous Markov process such that

$$\text{pr } (v(t) = 0 \mid v(t-1) = 0) = p_0, \ \text{pr } (v(t) = 1 \mid v(t-1) = 0) = 1 - p_o$$

$$\text{pr } (v(t) = 1 \mid v(t-1) = 1) = p_1, \ \text{pr } (v(t) = 0 \mid v(t-1) = 1) = 1 - p_1$$

Let us suppose that we wish to set two levels of emission, level 1 corresponds to an emission of 1 unit of pollutant/day and level 2 corresponds to an emission of 2 units of pollutant/day. Let us suppose level 2 corresponds to a cost of \$ 0/day (no abatement) whereas level 1 corresponds to a cost of \$ d/day due to abatement (fuel switching, curtailed production value, etc.).

We have a single air quality standard to meet, namely that a given level of concentration C_o should not be exceeded more often than 100α % of days. We wish, in other words, to set a dynamic emission standard such that

$$\text{pr (a randomly chosen day has concentration} > C_o) \leqslant \alpha \qquad (2)$$

Clearly if we can meet (2) using level 2 only, then the optimal standard is to specify this level for all days. This would be the uninteresting case where no abatement is ever required. Let us suppose, to avoid triviality, that (2) cannot be satisfied in this way.

We shall determine the optimal dynamic policy in the class of policies of the following type:

Use level 1 on days when concentration reaches or exceeds a level k, otherwise use level 2 as the emission standard.

Now $k < C_o$ otherwise we will not be able to meet requirement (2). We wish to find what should be the optimal k from k = 0, 2, 4, ... C_o (assume C_o is even).

If we select a day at random from a long time period, the probability that we obtain a day which has concentration >0 is the probability that V=1 on that day.

From the theory of Markov chains we know that this probability is given by the steady state probability of being in ventilation state 1. Let π_0, π_1 be the steady state probabilities of being in ventilation states 0 and 1 respectively. π_0 and π_1 are given by

$$p_0 \pi_0 + (1-p_1)\pi_1 = \pi_0$$

and $$\pi_0 + \pi_1 = 1$$

Solving, we obtain $\pi_0 = \dfrac{1-p_1}{2-p_o-p_1}$ and $\pi_1 = \dfrac{1-p_0}{2-p_0-p_1}$

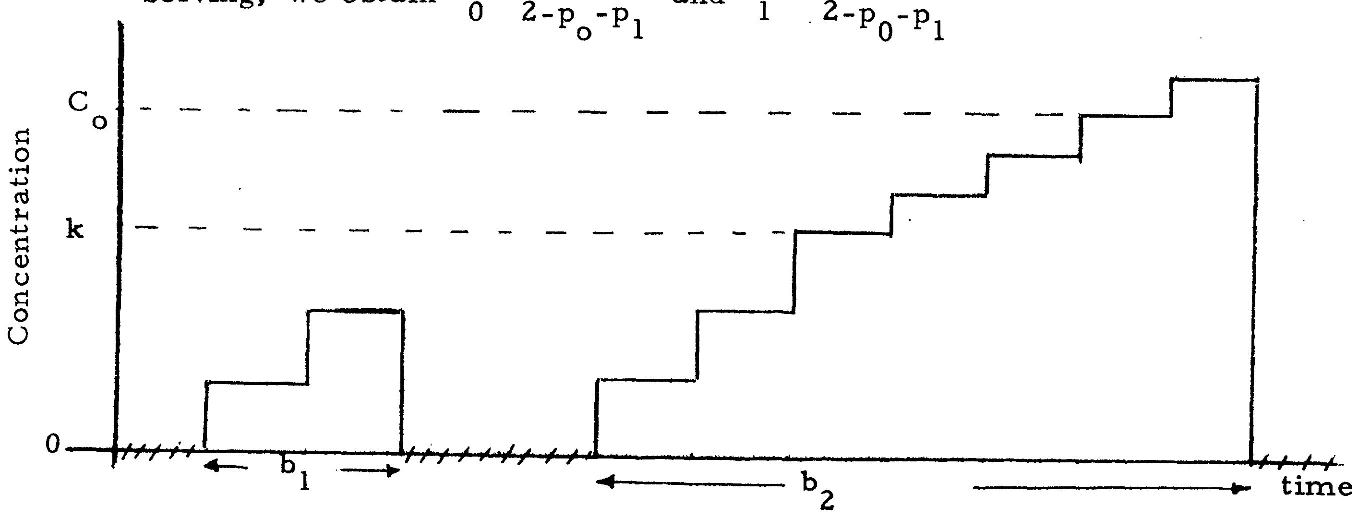

Figure 1

In other words pr (a randomly selected day belongs to some stepped block of the type shown in Fig. 1) = π_1

$$= \frac{1-p_0}{2-p_0-p_1} \qquad (3$$

Now, pr (random day belongs to a 'block' of base - length b | it belongs to some block)

$$= \frac{b \times \text{pr (random day belongs to a block of base-length b)}}{\sum_{k=1}^{\infty} k \text{ pr (random day belongs to a block of base-length k)}}$$

$$= \frac{b \quad p_1^{\,b-1} (1-p_1)}{E \text{ (base of blocks)}} = \frac{b \ p_1^{\,b-1} (1-p_1)}{1/(1-p_1)} = b \ p_1^{\,b-1} (1-p_1)^2 \qquad b = 1, 2, 3 \ldots \qquad (4)$$

Also,

pr (random day has concentration $> C_o$ | it belongs to a block of base-length b)

$$= \frac{1}{b} \left(b - C_o + \frac{k}{2}\right) \quad \text{for } b \geq C_o - \frac{k}{2}$$

$$= 0 \qquad \text{otherwise} \qquad (5)$$

Combining (3), (4) and (5) we obtain

pr (random day has concentration $> C_o$)

$$= \sum_{b=C_o-\frac{k}{2}}^{\infty} \frac{1}{b} \left(b - C_o + \frac{k}{2}\right) b \ p_1^{\,b-1} (1-p_1)^2 \ \frac{(1-p_0)}{(2-p_0-p_1)}$$

$$= \frac{(1-p_0) \ p_1^{\,C_o - k/2}}{(2-p_0-p_1)}$$

Clearly to minimize cost we should arrange to choose k as large as possible consistent with satisfying requirement (2) i.e., k must be the largest even integer for which

$$\frac{(1-p_0)\, p_1^{C_o - \frac{k}{2}}}{(2 - p_0 - p_1)} \leqslant \alpha$$

$$\text{or} \quad \frac{(1-p_0)\, p_1^{C_o}}{\alpha\,(2-p_0-p_1)} \leqslant p_1^{k/2} \tag{6}$$

Since the log function in monotone increasing

$$\log(1-p_o) + C_o \log p_1 - \log \alpha - \log(2-p_0 - p_1) \leqslant \frac{k}{2} \log p_1$$

so that $\frac{k}{2}$ must be the largest integer ($k \leqslant C_o$) such that

$$\frac{k}{2} \leqslant \frac{\log \dfrac{(1-p_0)}{(2-p_0-p_1)\alpha}}{\log p_1} + C_o$$

or more simply

$$\frac{k}{2} \leqslant \frac{\log(\pi_1/\alpha)}{\log p_1} + C_o \tag{7}$$

We have thus obtained the optimal dynamic emission policy that will meet air quality standards at lowest cost. (Note that the actual value of d plays no role in the analysis thus far.)

We shall next obtain the average cost per day of the optimal dynamic policy for comparison with the static policy.

pr (Random day has concentration $\geqslant$ k | it belongs to a block of base-length b)

$$= \frac{b - \frac{k}{2} + 1}{b} \quad \text{for } b \geqslant k/2$$

$$= 0 \qquad \text{otherwise}$$

pr (Random day has concentration $\geq$ k)

$$= \sum_{b=\frac{k}{2}}^{\infty} \left\{\frac{2b-(k-2)}{2b}\right\} \; b\, p_1^{\,b-1} \; (1-p_1)^2 \frac{(1-p_0)}{(2-p_0-p_1)}$$

$$= \pi_1 \, p_1^{\,k/2\,-1} \tag{8}$$

For optimal k given by (6),

$$E\,(\text{cost}) = \frac{d\,\pi_1}{p_1} \cdot \frac{\pi_1 p_1^{\,C_o}}{a} = \frac{d\pi_1^{\,2}\; p_1^{\,C_o-1}}{a} \tag{9}$$

Let us now develop for purposes of comparison the cost for a static emissions policy.

Let $1 \leq f \leq 2$ be the static emission standard per day.

Using analysis similar to that used above we obtain

$$\text{pr (Random day has concentration} > C_o) = \pi_1 \; p_1^{\left[\frac{C_o}{f}\right]}$$

where the notation [x] indicates the largest integer $\leq$ x.

To minimize cost we will choose the largest value of f such that $\pi_1 p_1^{\left[\frac{C_o}{f}\right]} \leq a$.

$$\text{or} \quad \left[\frac{C_o}{f}\right] \geq \frac{\log a/\pi_1}{\log p_1} \tag{10}$$

Let $\phi(f)$ be the cost (\$/day) of restricting the emission per day to a value of f. (Note $\phi(2) = 0$ and $\phi(1) = d$.)

Then the expected cost for the static policy will be $\phi(f^*)$ where f^* is the solution to (10).

Computation of the average costs for dynamic and static emission control strategies for a hypothetical example are shown below.

Let $d = 1$, $p_0 = p_1 = 1/2$, $\alpha = \frac{1}{256}$, $C_o = 8$

Let ϕ (f) be linear

Then we find $\pi_0 = \pi_1 = 1/2$

$$E\,(\text{cost dynamic}) = \frac{(\frac{1}{4})\,(\frac{1}{2})^{8-1}}{1/256} = 1/2$$

$$\text{Also,} \quad \left[\frac{C_o}{f}\right] \geq \frac{\log \frac{1/256}{1/2}}{\log 1/2} = 7 \qquad f^* = \frac{8}{7}$$

$$E\,(\text{cost static}) = 1\,(2 - \tfrac{8}{7}) = \frac{6}{7}$$

Ratio dynamic to static cost $= \frac{7}{12}$

A MORE COMPLEX MODEL

The model discussed in the previous section needs to be expanded to make it more realistic. In this section we will consider a model which is expanded in order to: (a) incorporate ventilation states other than total and zero ventilation, (b) have air quality standards specified by both a constraint of type (2) and an average annual concentration constraint, (c) more than two levels of emission standards.

Again we use as our starting point relation (1)*. We shall assume that there are n levels of ventilation $v_1, v_2, \ldots, v_n$ ($0 \leq v_j \leq 1$, $j = 1, 2, n$). Let there be q levels of emission to be incorporated in the dynamic standard $e_1, e_2, \ldots e_q$ (with $e_1 < e_2, \ldots < e_q$). Let us also restrict concentrations to $C_1, C_2, \ldots C_m$.

*In fact, any relation of the type $C(t) = f(C(t-1), u(t), v(t))$ where f is an arbitrary function can be treated with the analysis outlined here.

We shall assume that the sequence $\{v(t)\}$ forms a homogenous Markov chain on the n states $v_1, v_2, \ldots v_n$. Let $q(j|L) = pr\ \{v(t) = j\ |v(t-1) = L\}$. From relation (1) using discretization we can compute

$p(i, j|k, L, d) - pr\ (C(t) = i,\ v(t) = j\ |C(t-1) = k,\ v(t-1) = L \text{ and } e(t) = e_d)$.

Now we have an induced Markov chain on the states $(C(t), v(t))$. Let us denote the states of this Markov chain by s_{ij}, $i = 1, 2, \ldots m$, $j = 1, 2, \ldots n$. Thus s_{ij} state corresponding to concentration level C_i and ventilation level v_j.

Suppose we have as our air quality standard both requirement (2) and the requirement that the annual average concentration should be less than β units.

Let k_d be the cost of operating one day at the emission level e_d.

Then the problem of setting dynamic emission standards so as to minimize costs while meeting air quality standards can be shown to be the following linear programming problem [3]:

$$\text{Min} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{d=1}^{q} k_d w_{ijd}$$

subject to

$$-\sum_{d=1}^{q} w_{ijd} + \sum_{k=1}^{m} \sum_{L=1}^{n} \sum_{d=1}^{q} p\,(i, j\,|\,k, L, d)\, w_{kLd} = 0 \qquad \text{for } i = 1, 2, \ldots, m \quad j = 1, 2, \ldots, n$$

$$\sum_{k=1}^{m} \sum_{L=1}^{n} \sum_{d=1}^{q} w_{kLd} = 1$$

$$\sum_{k \geq C_o} \sum_{L=1}^{n} \sum_{d=1}^{q} w_{kLd} \leq \alpha \qquad (11)$$

$$\sum_{k=1}^{m} \sum_{L=1}^{n} \sum_{d=1}^{q} C_k w_{kLd} \leq \beta \tag{12}$$

$$w_{kLd} \geq 0 \qquad \begin{array}{l} k=1, 2, \ldots, m \\ L=1, 2, \ldots, n \\ d=1, 2, \ldots, q \end{array}$$

where w_{ijd} are the decision variables which are solved for in the linear program. Physically w_{ijd} is the probability of setting emission level at e_d and being in state s_{ij}. Note that in general the solution will give us a randomized strategy as optimal. That is to say, our solution will say that when in state s_{ij} the emission level for the optimal dynamic strategy should be e_d for a certain percentage of the time (not necessarily 0 or 100%).

An important advantage of this approach is that the shadow prices on rows (11) and (12) explicitly give us the marginal costs of altering the air quality restrictions. Thus, it is possible to investigate the economic impact of raising or lowering the air quality standards.

To illustrate this, a small hypothetical model was solved on a computer.

For this example: $q = 2$, $m = 6$, $n = 4$

$v_1 = 0$, $v_2 = 0.1$, $v_3 = 0.2$, $v_4 = 0.4$

$C_1 = 0$, $C_2 = 2$, $C_3 = 4$, $C_4 = 6$, $C_5 = 8$, $C_6 = 10$

$e_1 = 1$, $e_2 = 4$, $k_1 = 1$, $k_2 = 0$

and the transition matrix $q(j|i)$ is given by the element in the i^{th} row and j^{th} column of the following matrix:

$$\begin{bmatrix} 0.5 & 0.3 & 0.2 & 0.0 \\ 0.2 & 0.5 & 0.2 & 0.1 \\ 0.0 & 0.3 & 0.5 & 0.2 \\ 0.0 & 0.2 & 0.3 & 0.5 \end{bmatrix}$$

The air quality requirements were given by $C_o = 6$, $\alpha = 0.02$, $\beta = 3.0$.

Table I below shows the results of parametric runs on the model and illustrates the information these give on the cost effects of tightening or relaxing air quality standards.

Table I

Case No.	α	β	Minimum Cost	Shadow Prices on α row	Shadow Prices on β row
1	0.020	3.0	.1634	1.61	0
2	0.011	3.0	.1779	1.61	0
3	0.005	3.0	.1875	1.61	0
4	0.02	1.2	.1634	1.61	0
5	0.02	0.6	.3095	0	.625

To obtain the cost for the static emission policy for comparison with the dynamic, we can use the following procedure.

1. Set d=q
2. Solve $\sum_k \sum_L \Pi_{kL} p(i, j \quad k, L, d) = \Pi_{ij}$

$$\sum_k \sum_L \Pi_{kL} = 1$$

for Π_{ij} = steady state probability of being in S_{ij} under static emission level e_d.

3. If $\sum_{k \geq C_o} \sum_{L} \Pi_{kL} \leq \alpha$ and $\sum\sum \Pi_{kL} C_k \leq \beta$ then quit. The value of d obtained is the optimal for a static policy. Otherwise reduce d by 1 unit and repeat step 2.

For our example problem the cost for the static policy is 1.

CONCLUSION

In this paper I have suggested that dynamic standards are more efficient than static standards for control of large pollutant sources. I have analyzed a simple model which provides some insight into the nature of dynamic standards and a more realistic model which would help in optimally setting dynamic standards. These models will also aid quantitative estimation of the advantages to be gained from dynamic over static control policies.

One aspect of dynamic strategies which has implications for research and development of anti-pollution devices is worth noting. The dynamic strategy may render feasible techniques and apparatus for pollution control which have high operating costs. In such a case, if one computed the cost for a static emission standard it may be too exhorbitant to impose on the polluter. However, the dynamic strategy may require only intermittent use of the expensive pollution-control equipment and hence, may reduce costs to an acceptable magnitude. Thus, research efforts could be justifiably directed towards developing pollution control technology even if it has high operating costs.

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