IX. NOISE IN ELECTRON DEVICES*

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A. NOISE FIGURE FOR NEGATIVE SOURCE RESISTANCE

When parametric amplifiers and Esaki diode amplifiers are used as one stage in a cascade of twoport amplifiers (Fig. IX-1), it often happens that the stage following such an amplifier "looks" into a source impedance (the output impedance of the preceding stage) with a negative real part. For brevity, we shall call a twoport that exhibits an output impedance with a negative real part "negative-resistance twoport." The noise figure for a twoport connected to a source impedance with a negative real part has not been defined by the Institute of Radio Engineers. The question arises how one should go about either measuring or computing the noise figure of the entire system, or for any one of the stages.

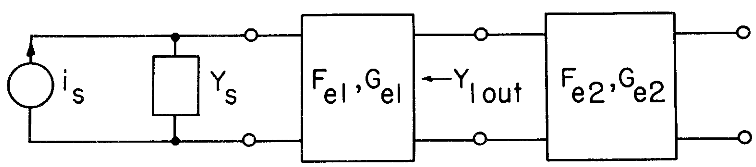

Fig. IX-1. Cascade of two amplifiers.

The measurement of the over-all noise figure of the system does not usually encounter any difficulty because the source (usually an antenna) impedance connected to the input of a receiving system has a positive real part.

When computing the over-all noise figure of a system one may resort to two equivalent methods:

(a) One may compute the noise figure without separating the system into successive amplifying stages.

(b) One may extend the IRE noise-figure definition to the case of negative-source resistance,[1] compute the noise figure for each stage, and evaluate the over-all noise figure from an appropriate cascading formula.

Both approaches are equivalent if one is interested in the over-all noise figure (or noise temperature) of the system. When one is interested in measuring or computing

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noise parameters for the individual negative-resistance twoport in a cascade it is convenient to use formulas from which the noise parameters of this twoport may be evaluated in a systematic way. The noise at a particular frequency of any twoport, whether a negative-resistance twoport or not, is completely described in terms of four parameters. All of these may be obtained by noise-figure measurements on the twoport for various source impedances. In practice, one measures the noise figure of the entire system, consisting of the negative-resistance twoport used as the first stage, followed by higher stages, and one must be able to extract from the over-all noise-figure measurement the noise figure of the first stage. This can be done conveniently by using the cascading formula, provided that one uses the noise-figure definition that is consistent with this formula.[1] The cascading formula for the two stages in cascade is

$$F_e = F_{e1} + \frac{F_{e2} - 1}{G_{e1}} \tag{1}$$

Here, the F_e's are the noise figures defined on the basis of exchangeable power,[1] and G_{e1} is the exchangeable gain of the first stage. In order to find F_{e1} from a measurement of F_e, one must know G_{e1} and $(F_{e2}-1)$ for the same value of source impedance as that which drives the second stage in the cascade. If this impedance has a negative real part, one must overcome the practical problem of determining experimentally $F_{e2} - 1$ for negative-source impedance, a rather difficult and inconvenient measurement to perform directly. Furthermore, G_{e1} cannot be obtained from a gain (G_e) measurement of the over-all system unless G_{e2} (the exchangeable gain of the second stage) is known for a negative-source resistance. The purpose of this letter is to show how one can determine the second term in Eq. 1 from signal measurements on the first stage, and from noise measurements on the second stage by using only source admittances with a positive real part. For this purpose, we rewrite the second term in a form containing terms that are more conveniently measured.

$$\frac{F_{e2} - 1}{G_{e1}} = \frac{(F_{e2}-1)\, G_{1\,out}}{G_{e1} G_{1\,out}}, \tag{2}$$

where $G_{1\,out}$ is the output conductance of the first stage that acts as the source conductance of the second stage.

$$G_{1\,out} = G_{s2} \tag{3}$$

It is well known[2] that the conventional (IRE definition of the) noise figure of a twoport can be expressed as a function of the source admittance $Y_s = G_s + jB_s$ for $G_s > 0$ as follows:

$$F = F_o + \frac{R_n}{G_s}\left[(G_s-G_o)^2 + (B_s-B_o)^2\right]. \tag{4}$$

The quantities F_o, R_n, G_o, and B_o are the four noise parameters that describe the noise of the twoport (the second stage). The extended definition[1] of the noise figure, F_e, preserves the analytic form of F in terms of G_s and B_s for negative values of G_s. If one plots

$$H = (F_e-1)\, G_s = (F_o-1)\, G_s + R_n\left[(G_s-G_o)^2 + (B_s-B_o)^2\right]$$

as a function of G_s one finds it to be a parabola with a minimum

$$H_{min} = G_o(F_o-1) = \frac{(F_o-1)^2}{4R_n} + R_n(B_s-B_o)^2$$

at

$$G_s = G_o = \frac{F_o - 1}{2R_n}.$$

It can be shown that $H_{min} > 0$ in all cases. Therefore, a plot of H against G_s has the appearance of Fig. IX-2. A measurement of H for positive G_s may be extrapolated to negative values of G_s. One simple way of doing this is to plot

$$H - H_{min} \text{ vs } x = \left(G_s - G_o + \frac{F_o - 1}{R_n}\right)^2$$

This plot is a straight line, a fact that can serve as a check on the experimental accuracy of the measurements with positive values of G_s, as well as a simple means of extrapolating H to negative values of G_s. The measurement of $G_{1\ out}$ is straightforward. The experimentally obtained value of $G_{1\ out}$ is used as the source conductance on the

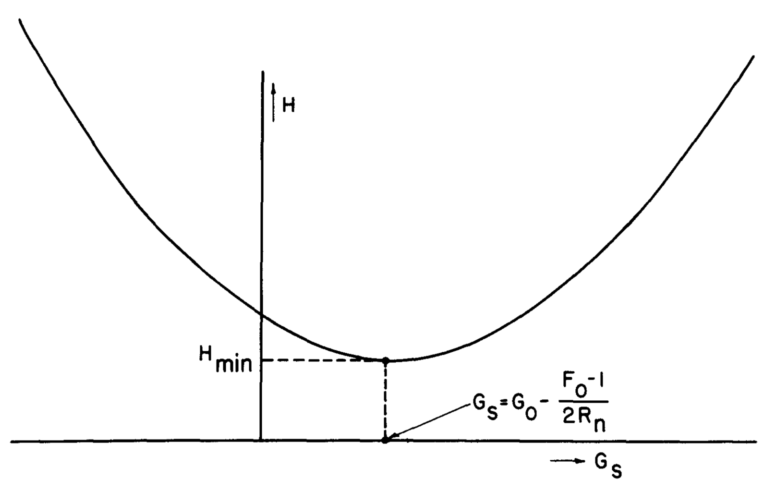

Fig. IX-2. Plot of H versus G_s.

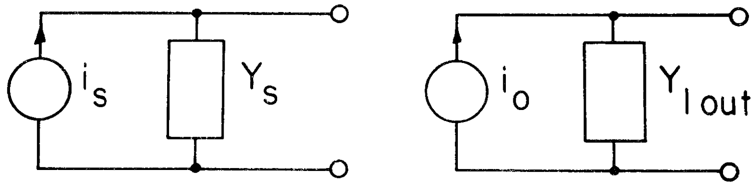

Fig. IX-3. Norton's equivalents of source and output of the first stage.

previously obtained plot to read off the value of $H = (F-1)\,G_s$ for the second stage.

The value of $G_{e1}G_{1\,out}$ may be obtained from a transducer gain (G_T) measurement on the first stage. If we use Norton's equivalents of the source on the one hand, and the output port of the first stage connected to the source on the other hand (Fig. IX-3), for the transducer gain of the first stage, we have

$$G_{T_1} = \frac{\dfrac{\overline{i_o^2}\,G_L}{(Y_{1\,out}+Y_L)^2}}{\dfrac{\overline{i_s^2}}{4G_s}} = \frac{\overline{i_o^2}}{\overline{i_s^2}}\,\frac{4G_LG_s}{(Y_{1\,out}+Y_L)^2}$$

with $G_L = \mathrm{Re}\,(Y_L)$, where Y_L is the admittance of the load connected to the output of the first stage. The exchangeable gain is

$$G_{e1} = \frac{\overline{i_o^2}}{\overline{i_s^2}}\,\frac{G_s}{G_{1\,out}}.$$

Therefore

$$G_{e1}G_{1\,out} = G_{T_1}\,\frac{(Y_{1\,out}+Y_L)^2}{4G_L}.$$

Thus one may obtain $G_{e1}G_{1\,out}$ from measurements of the transducer gain of the first stage, of the load admittance Y_L, and of the output admittance of the first stage. With these measurements one may determine $(F_{e2}-1)/G_{e1}$ as it appears in the cascading formula. $(F_{e1}-1)$ may thus be evaluated.

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References

1. H. A. Haus and R. B. Adler, An extension of the noise figure definition, Letter to the Editor, Proc. IRE 45, 690 (1957).

2. A. G. Th. Becking, H. Groendijk and K. S. Knol, The noise factor of 4 terminal networks, Philips. Res. Rep., vol. 10, pp. 349-357, 1955, and IRE Subcommittee 7.9 on Noise, Representation of noise in linear twoports, Proc. IRE 48, 69-74 (1960).