Indeterminate Structures

Architecture 4.440

Outline

- Introduction
- Static Indeterminacy
- Support Conditions
- Degrees of Static Indeterminacy
- Design Considerations
- Conclusions

Forces in the Legs of a Stool

Three-Legged Stool

Statically determinate

One solution for the axial force in each leg

Why?

3 unknowns

3 equations of equilibrium

Uneven floor has no effect

Four-Legged Stool

Statically indeterminate

A four legged table on an uneven surface will rock back and forth

Why?

It is *hyperstatic:*

4 unknowns

3 equations of equilibrium

Four-Legged Stool

Infinite solutions exist

Depends on unknowable support conditions

A four legged table on an uneven surface will rock back and forth

The forces in each leg are constantly changing

Fundamental difference between hyperstatic and static structures

Forces in the Leg of a Stool

Statically determinate

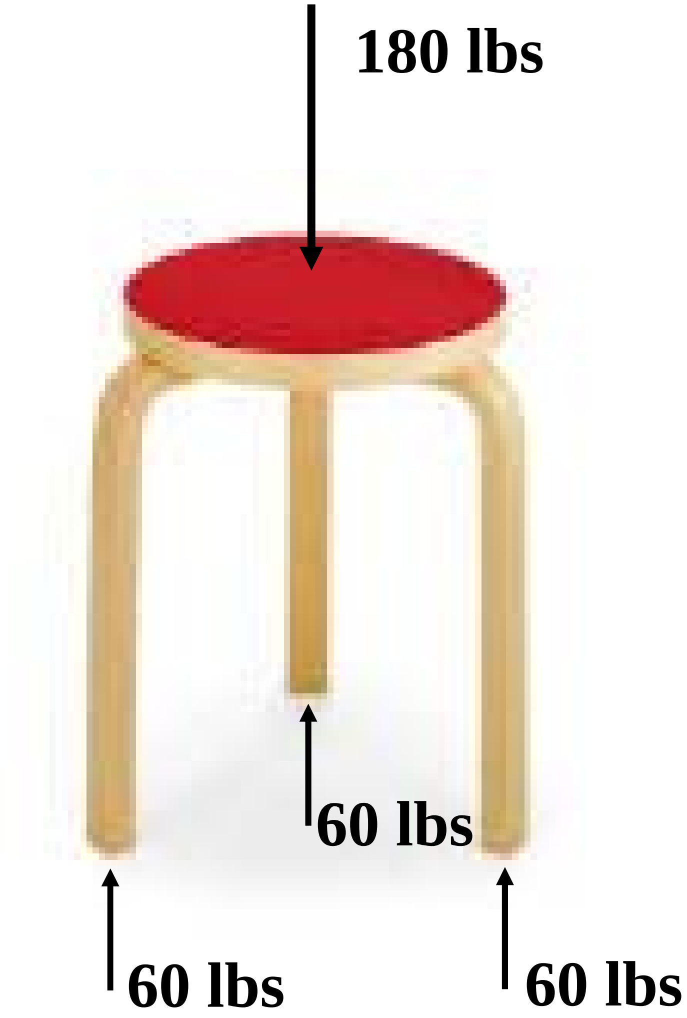
Statically Indeterminate (hyperstatic)

Three-Legged Stool

Design for a person weighing 180 pounds

→ 60 pounds/leg

Regardless of uneven floor



Collapse of a Three-Legged Stool

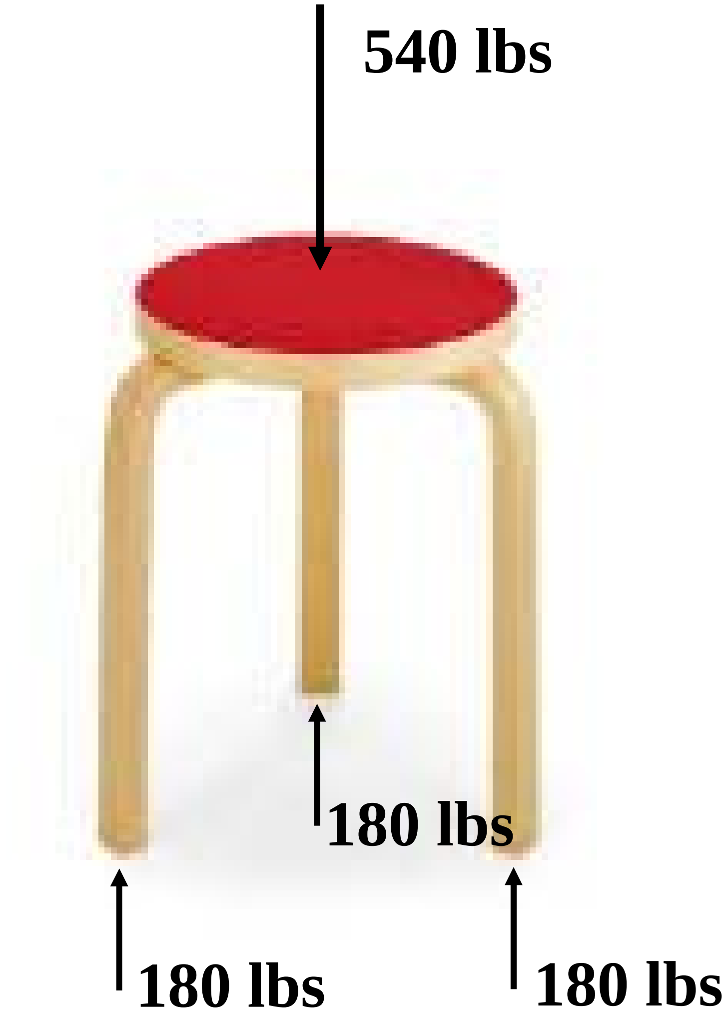
Design for a person weighing 180 pounds

If the safety factor is 3:

$P_{cr} = 3(60) = 180$ lbs

And each leg would be designed to fail at a load of 180 pounds

The stool would carry a total load of 540 pounds

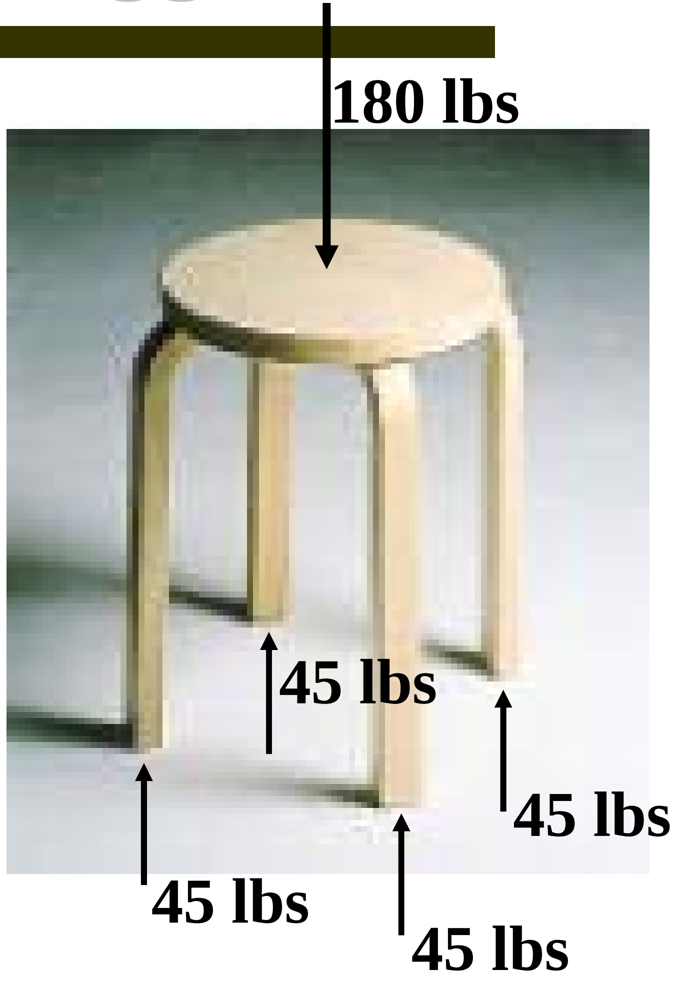


Elastic Solution for 4-Legged Stool

Design for a person weighing 180 pounds

→ 45 pounds/leg

But if one leg does not touch the floor…



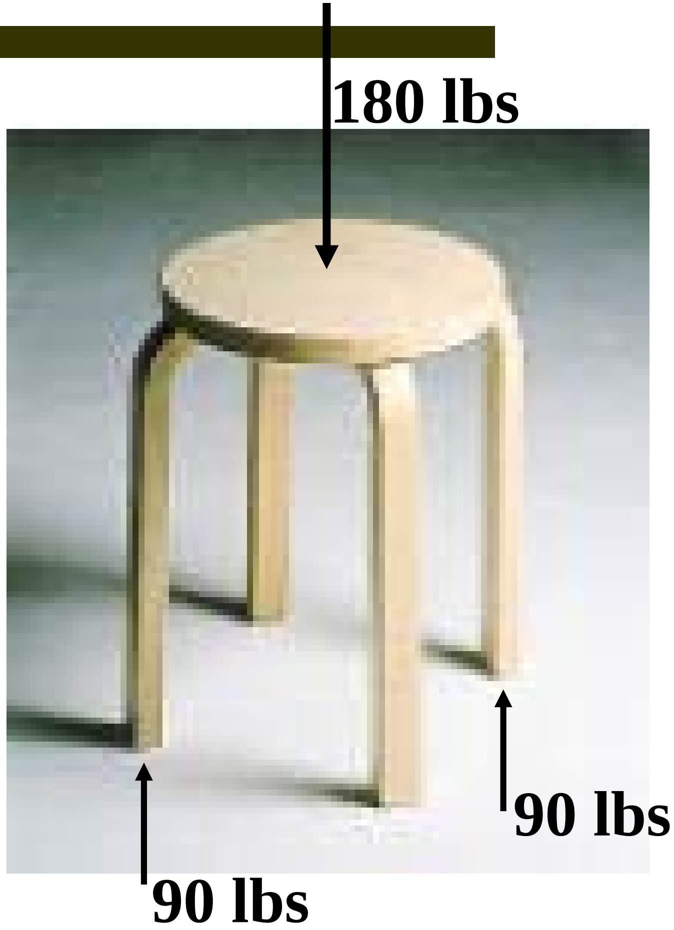
Four-Legged Stool

If one leg doesn't touch the floor, the force in it is zero.

If one leg is zero, then the opposite leg is also zero by moment equilibrium.

The two remaining legs carry all of the load:

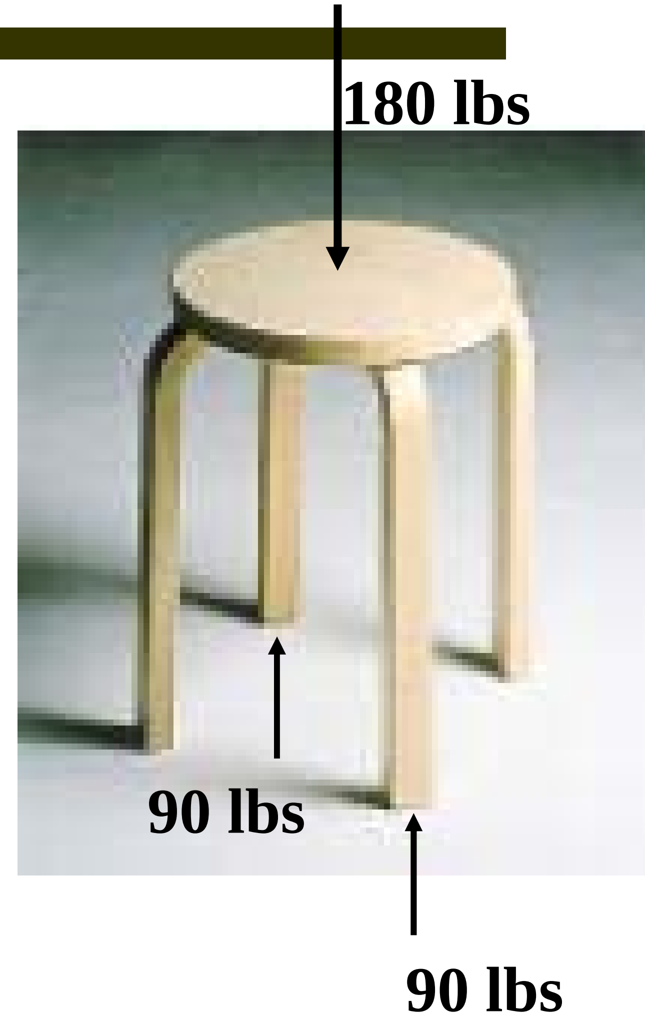
→ <u>90 pounds/leg</u>



Four-Legged Stool

Therefore…

All four legs must be *designed* to carry the 90 pounds (since any two legs could be loaded)



Four-Legged Stool

If the elastic solution is accepted, with a load in each leg of 45 pounds, then assuming a safety factor of 3 gives:

$$P_{cr} = 3(45 \text{ lbs}) = \underline{135 \text{ lbs}}$$

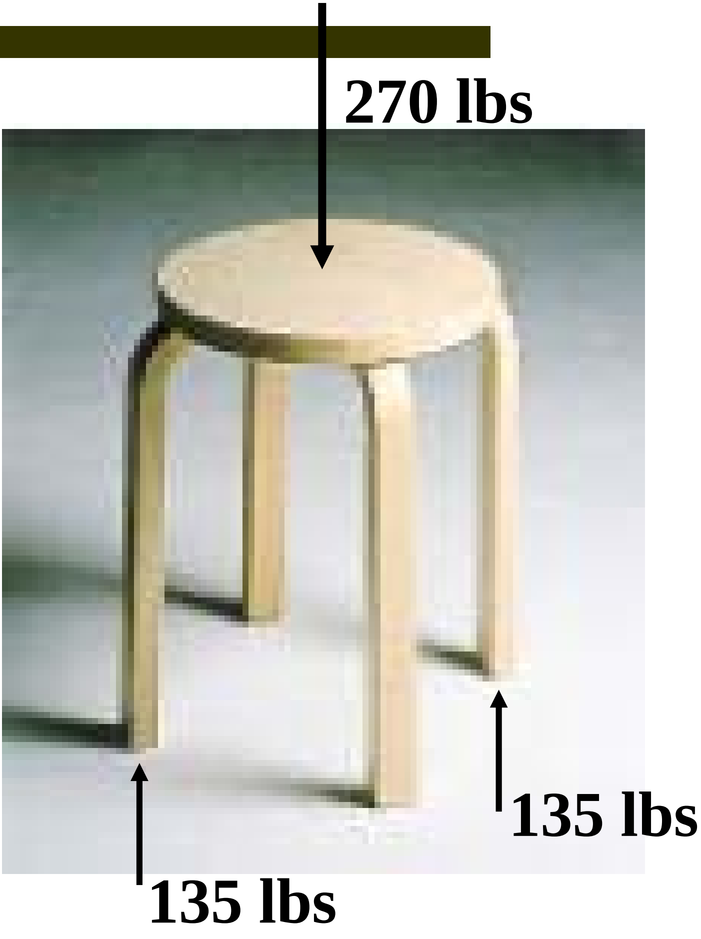
And each leg would be designed to fail at a load of 135 pounds

Four-Legged Stool

Now imagine the load is increased to cause failure

When load is 270 lbs, the two legs will begin to fail

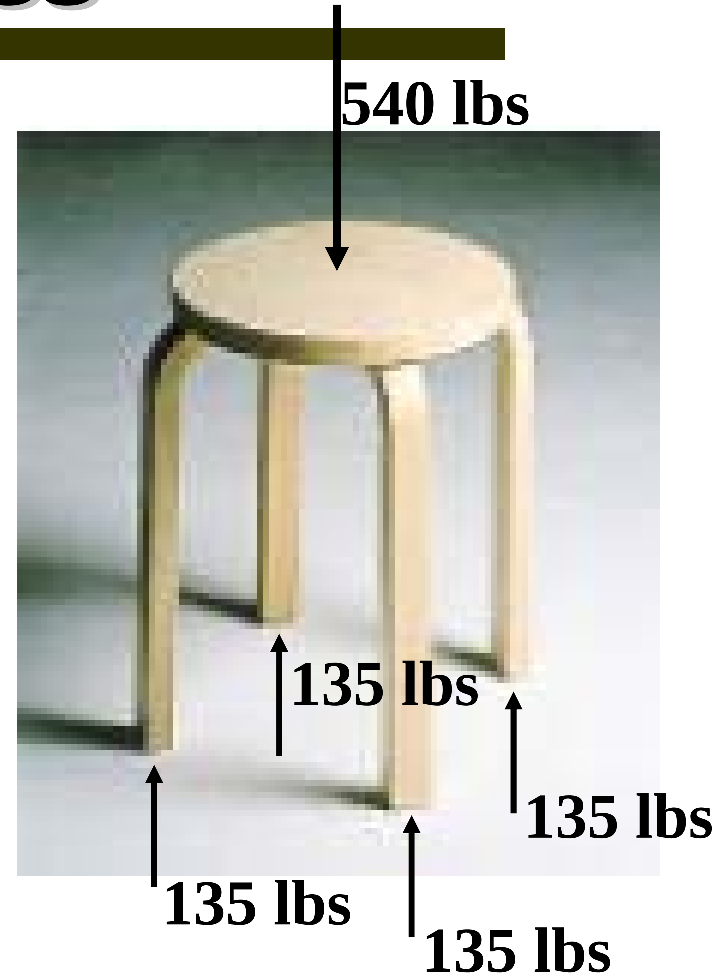
As they “squash,” the other two legs will start to carry load also



Collapse of a 4-Legged Stool

At final collapse state, all four legs carry 135 pounds and the stool carries 540 pounds.

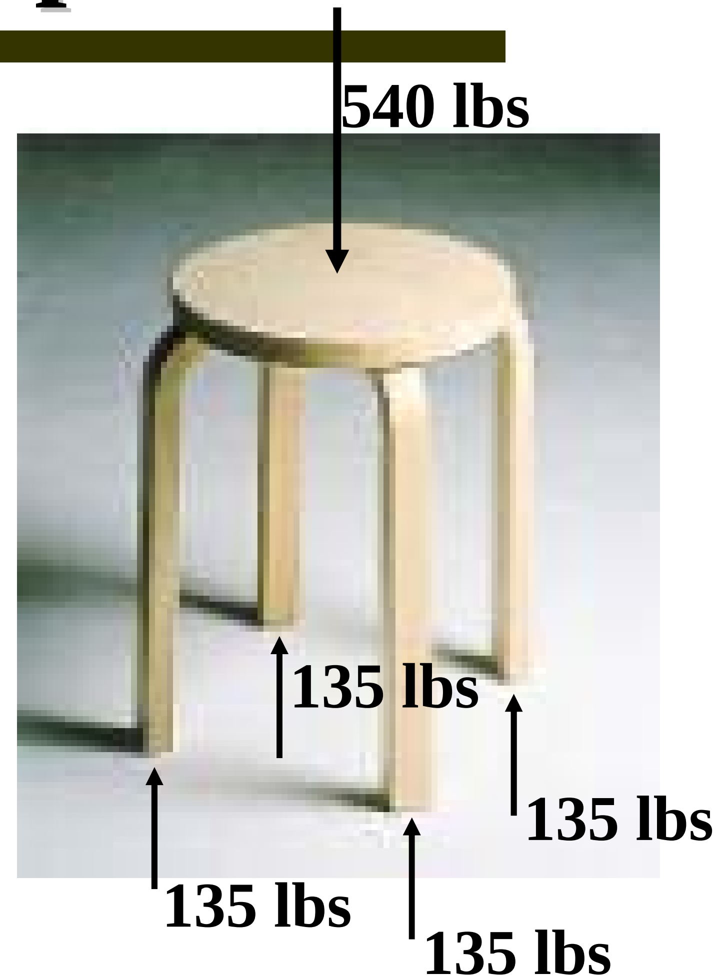
This occurs only if the structure is ductile (ie, if the legs can “squash”)



Ductile Collapse

So small imperfections do not matter, as long as the structural elements are ductile

The forces in a hyperstatic structure cannot be known exactly, but this is not important as long as we can predict the collapse state

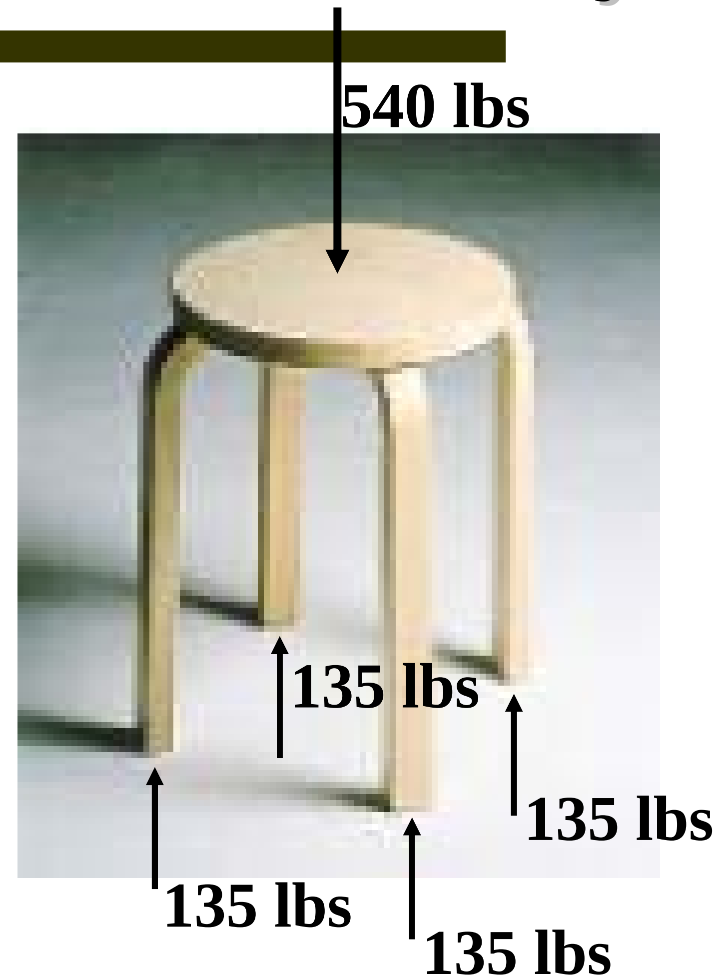


Lower Bound Theorem of Plasticity

If you can find one possible set of forces, then the structure can find a possible set of forces

It does not have to be correct, as long as the structure has capacity for displacements (ductility)

For indeterminate structures, we cannot be certain of the internal state of the forces



Examples of Statically Determinate Structures

- Unstressed by support movements or temperature changes

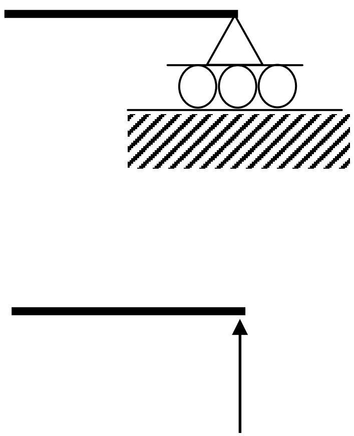
  - Three-legged stool
  - Simply supported beam
  - Cantilever beam
  - Three-hinged arch

Simply Supported Bridge

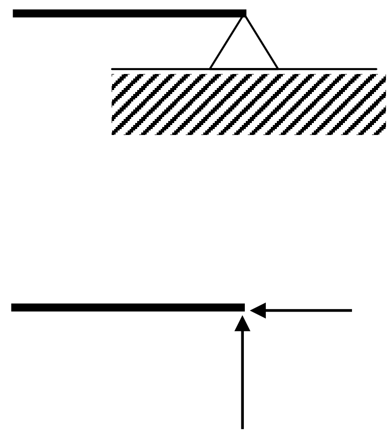
Can adjust to support movements and temperature changes

Support Conditions

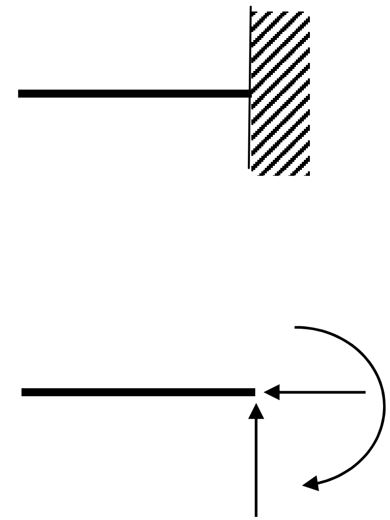
Roller



Pin (hinge)



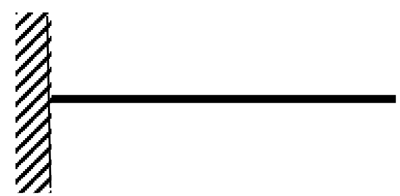
Fixed



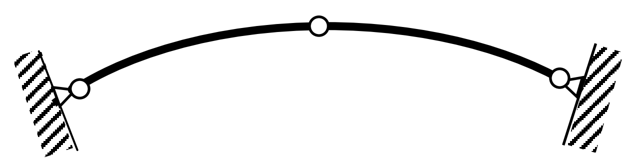
Statically Determinate Structures

- Simply supported beam

- Cantilever beam



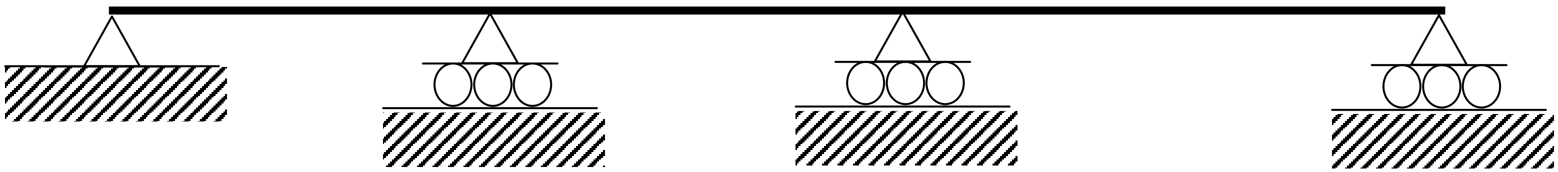
- Three-hinged arch



Simply Supported Beam

Statically Determinate	Indeterminate (hyperstatic)
Simply supported beam	Continuous beam
Cantilever beam	Propped cantilever beam
Three-hinged arch	Fixed end arch
Three-hinged frame	Rigid frame

Continuous Beam

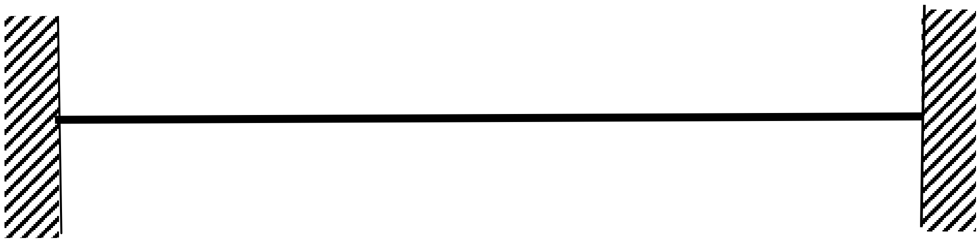


- How many unnecessary supports?
- What is the "degree of static indeterminacy"?

Pin-Ended Beam

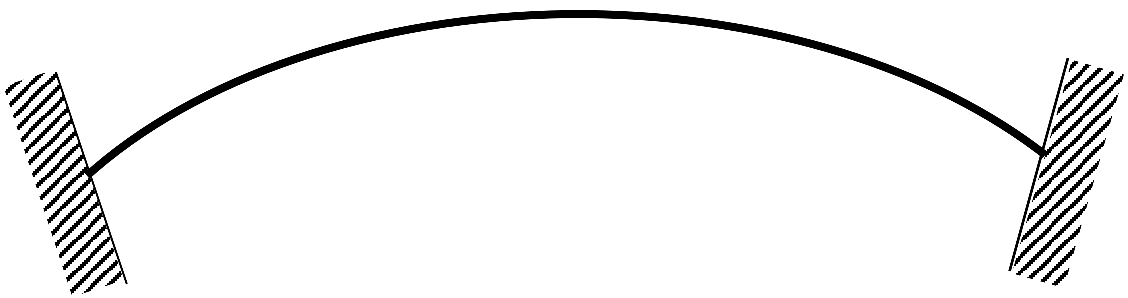
- Will temperature changes cause forces in the beam?
- How many unnecessary supports?
- What is the “degree of static indeterminacy”?

Fixed-End Beam



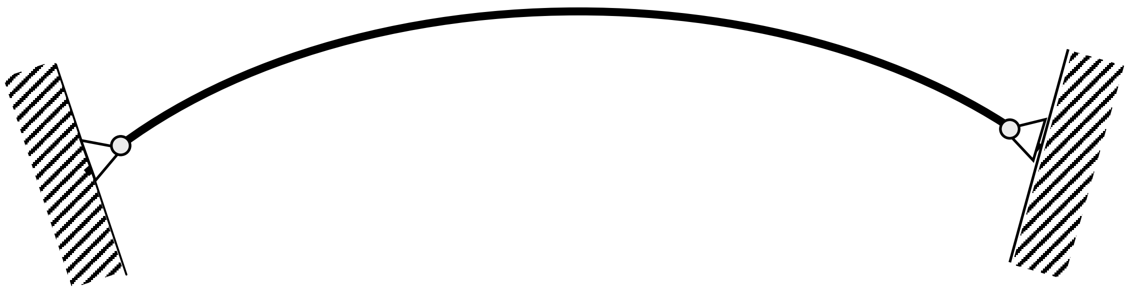
- Will temperature changes cause forces in the beam?
- How many unnecessary supports?
- What is the “degree of static indeterminacy”?

Fixed-End Arch



- Will temperature changes or support movements cause forces in the arch?
- How would you make this structure statically determinate?
- What is the “degree of static indeterminacy”?

Two-Hinged Arch



- Will temperature changes or support movements cause forces in the arch?
- How would you make this structure statically determinate?
- What is the “degree of static indeterminacy”?

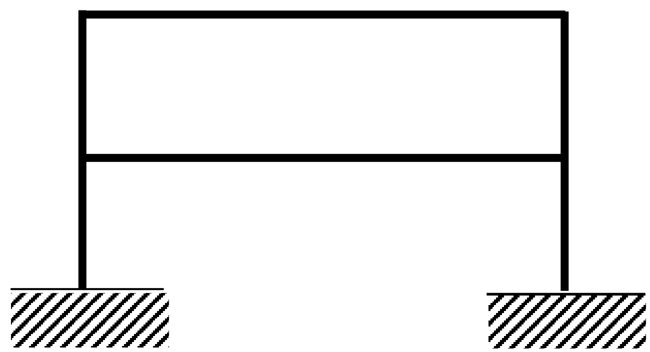
Pinned Frame

- Will temperature changes or support movements cause forces in the frame?
- How would you make this structure statically determinate?
- What is the “degree of static indeterminacy”?

Fixed Frame

- **Will temperature changes or support movements cause forces in the frame?**
- **How would you make this structure statically determinate?**
- **What is the "degree of static indeterminacy"?**

Fixed Frame



- Will temperature changes or support movements cause forces in the frame?
- How would you make this structure statically determinate?
- What is the “degree of static indeterminacy”?

How to find forces in statically indeterminate structures

- **Approximate "hand" calculations**
  - **Make simplifying assumptions**
- **Computer: Finite Element Methods**
  - **Solve for internal forces based on relative stiffness of each element and many other assumptions (elastic analysis)**
- **Analyze limiting cases to determine one possible state of internal forces**

Finite Element Analysis

- Divide structure into a “mesh” of finite elements
- Solves for internal forces based on relative stiffness of each element

Finite Element Analysis

But can’t account for imperfections in supports and construction

Like a four-legged stool, it is impossible to know the exact forces

Finite element analysis is more sophisticated, but is not necessarily better

Design Considerations

- Statically indeterminate structures offer greater redundancy, i.e. more possible load paths

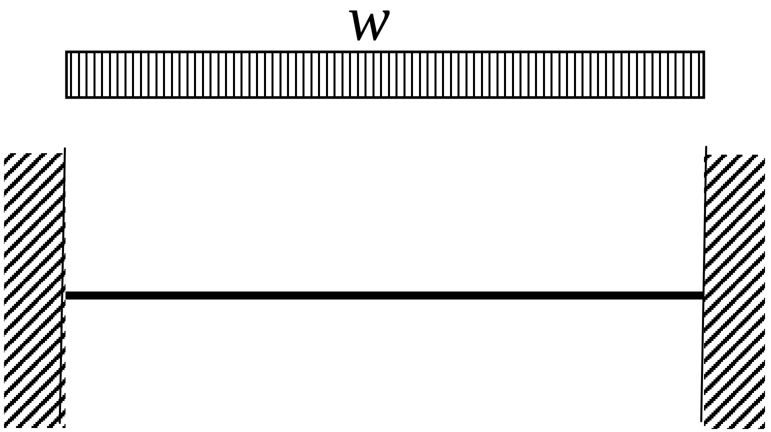
- But are less clear in their structural action
  - More complicated to design and assess
  - May be more difficult to repair

Static Indeterminacy

- **For a given set of applied loads, any possible equilibrium state is acceptable (internal forces in the legs of the stool)**
- **Find extreme equilibrium cases by "releasing" the extra supports (i.e., assume two legs don't touch the ground)**
- **You can choose any internal equilibrium state as long as buckling does not occur (lower bound theorem)**

Statically Indeterminate Beams

- **What is the moment diagram for this beam under a uniform load, *w*?**



Statically Indeterminate Beams

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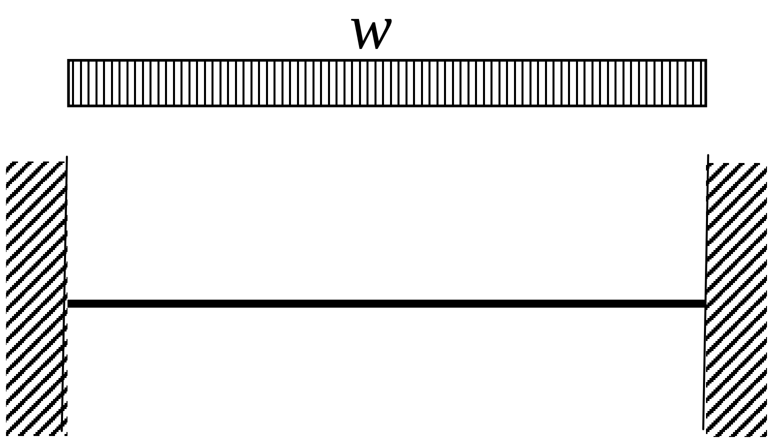
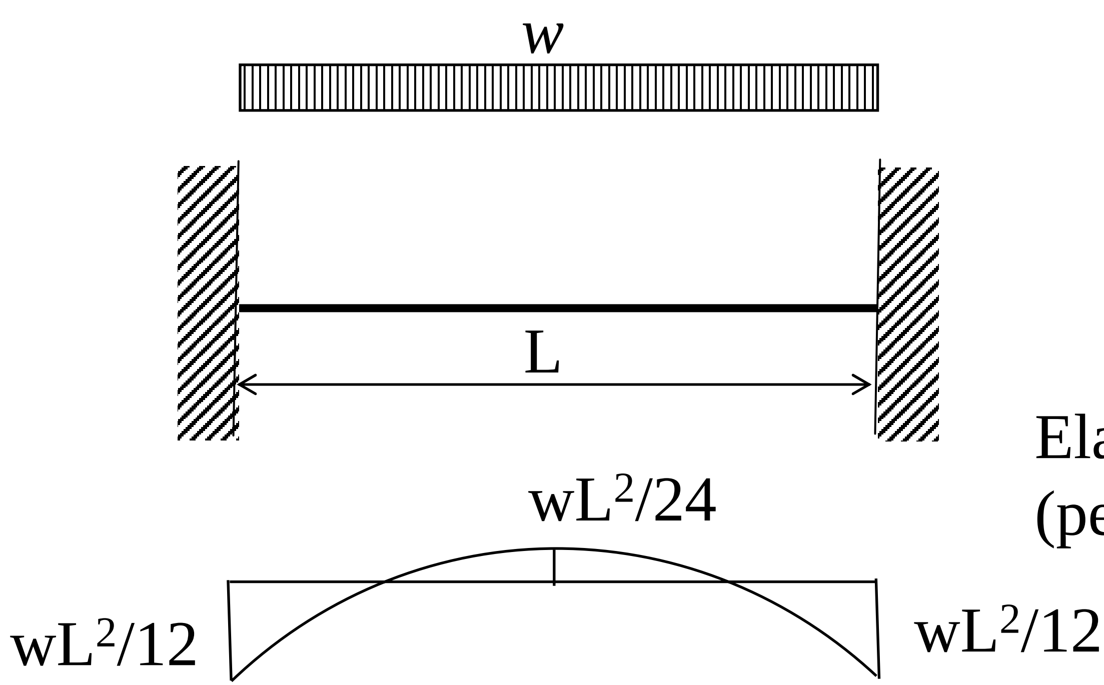


- **Is there one answer?**

Statically Indeterminate Beams

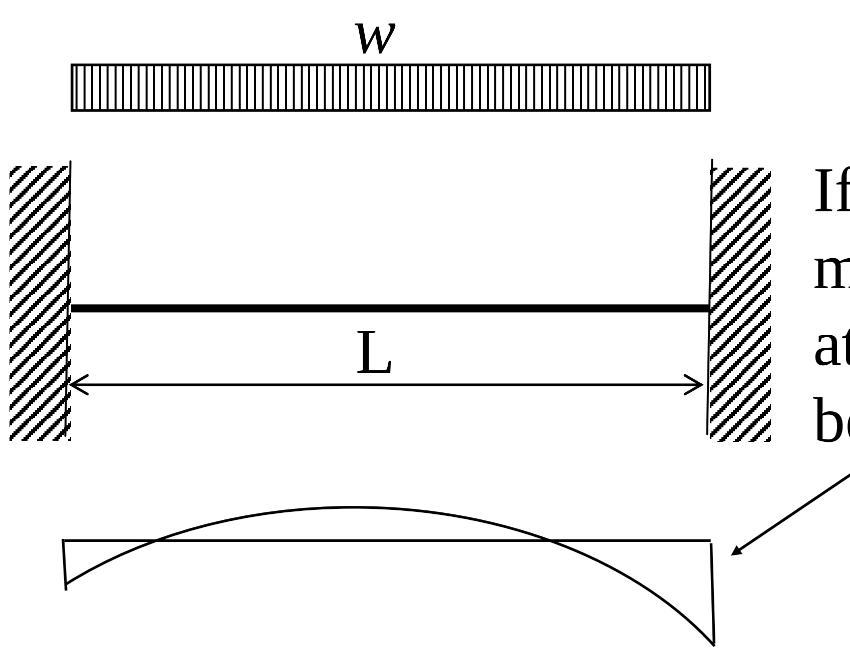
- **What is the moment diagram for this beam under a uniform load, *w*?**



Elastic solution
(perfect world)

Statically Indeterminate Beams

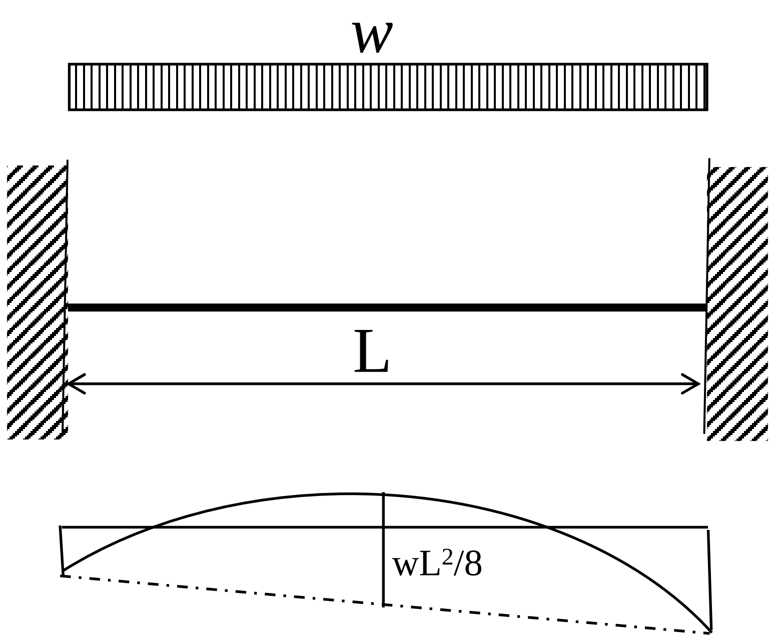
- **But what did we learn from the 4-legged stool?**



If this support is more rigid, it will attract more of a bending moment.

Statically Indeterminate Beams

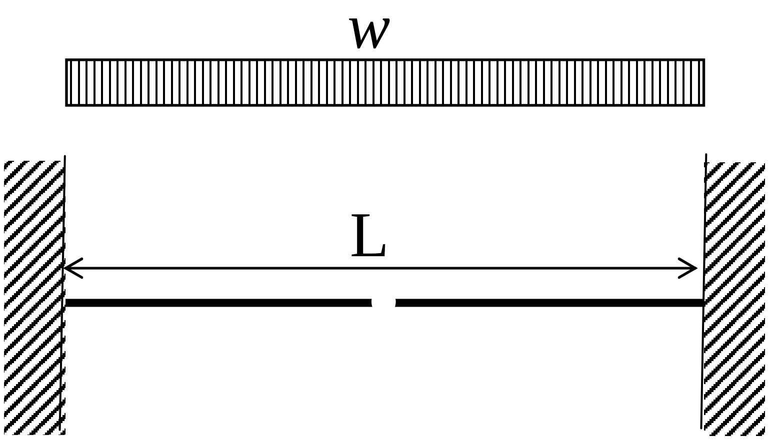
- **The difference between the midspan moment and the "closing line" is always $wL^2/8$ due to a <u>uniform load</u>.**



If this support is more rigid, it will attract more of a bending moment.

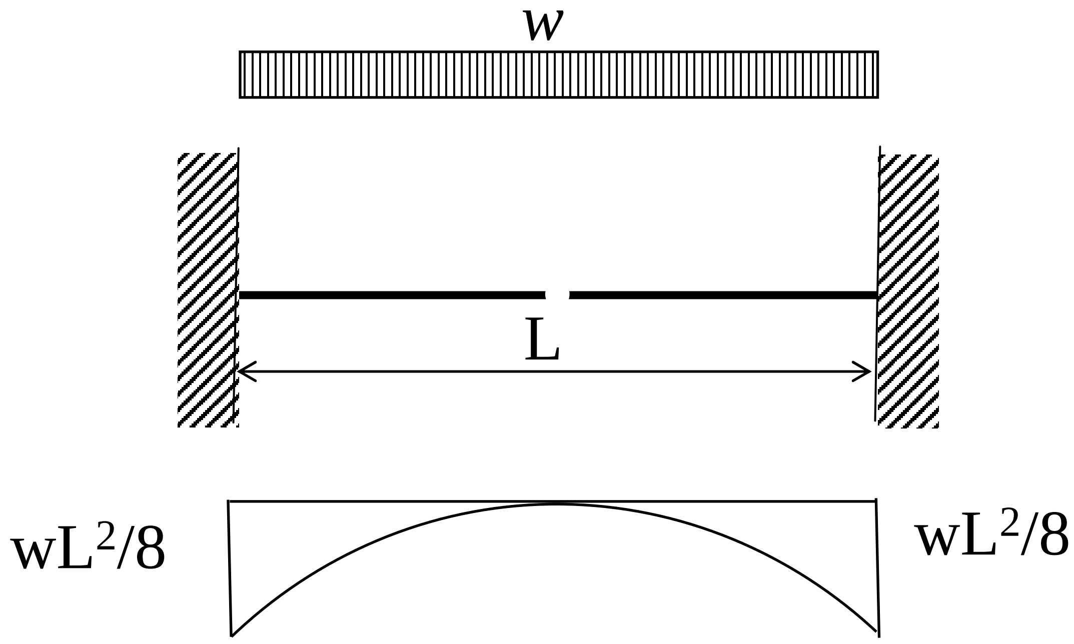
Statically Indeterminate Beams

- **What is the moment diagram for this beam under a uniform load, *w*, if we make a cut at midspan?**



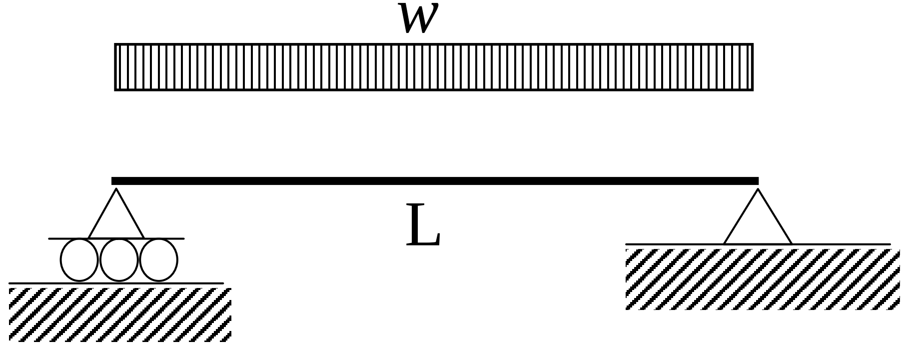
Statically Indeterminate Beams

- **What is the moment diagram for this beam under a uniform load, *w*, if we make a cut at centerspan?**



Statically Indeterminate Beams

- **What is the moment diagram for this beam under a uniform load, *w*, if it is simply supported?**



Statically Indeterminate Beams

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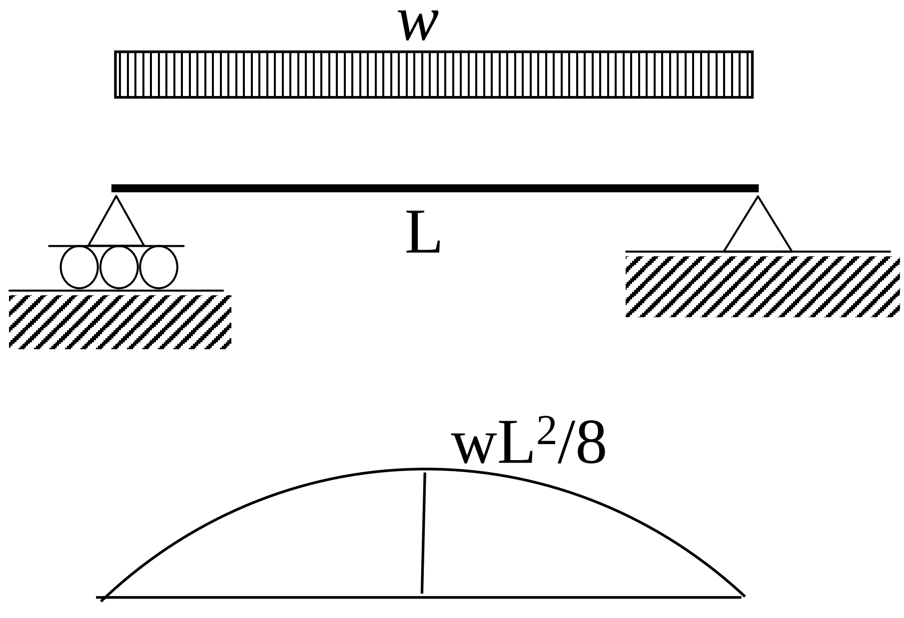
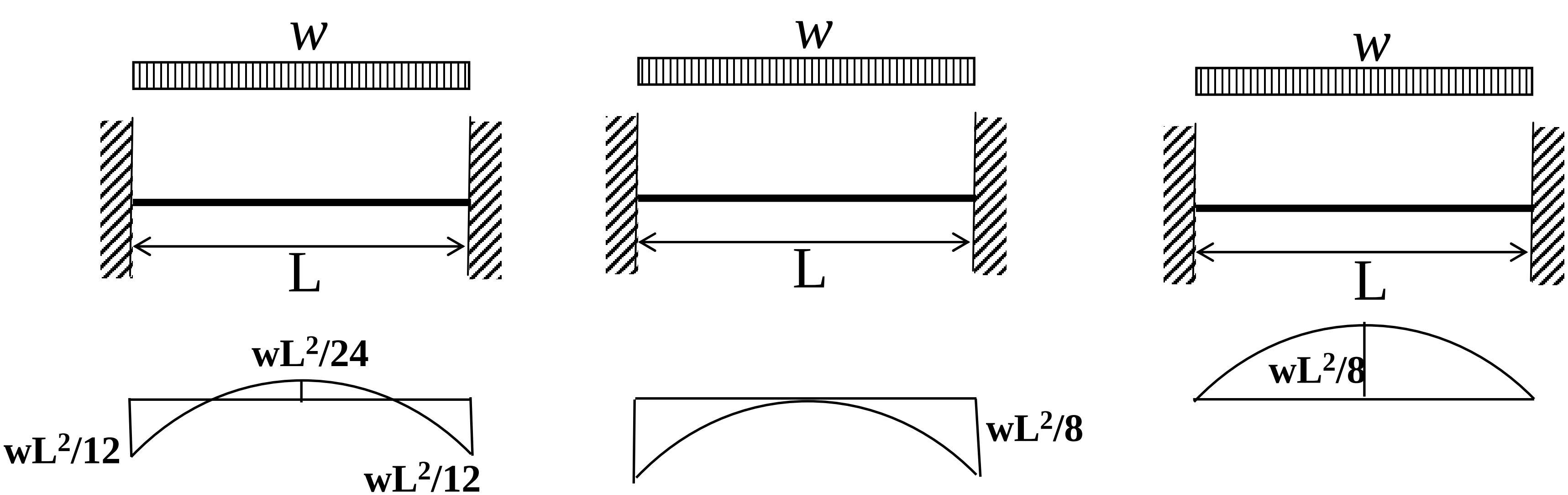


Statically Indeterminate Beams

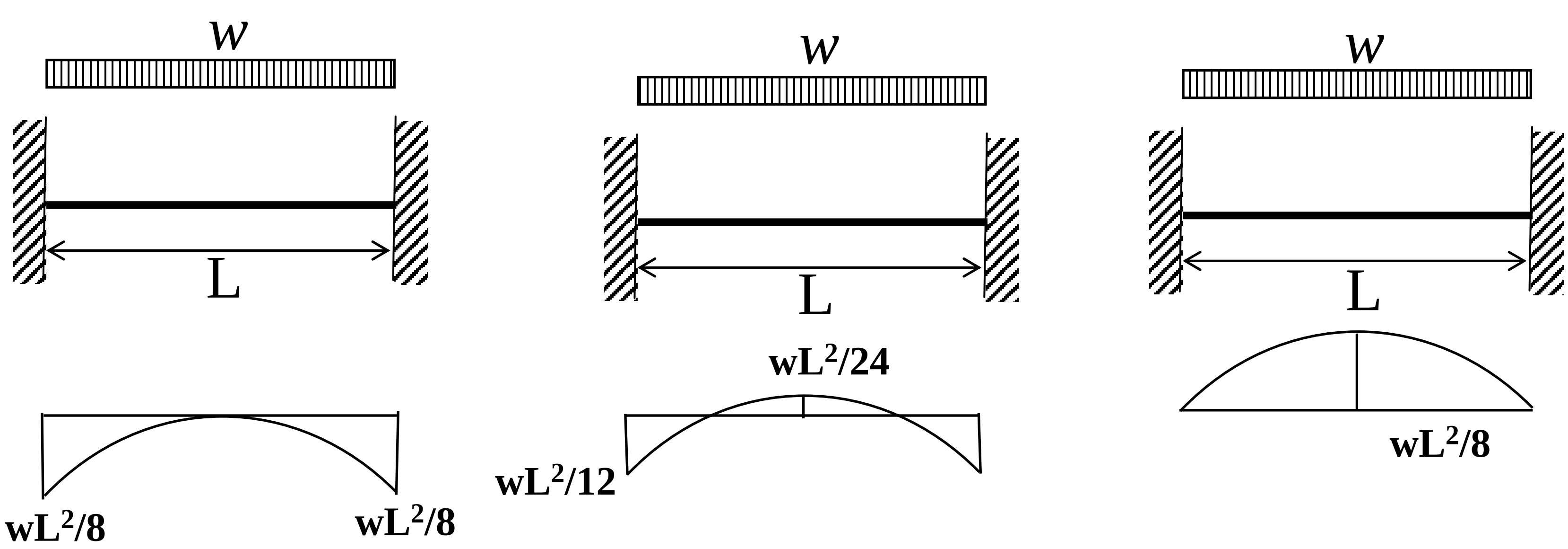
- **Various possible bending moment configurations for a beam under uniform load**



- **Moment diagram shifts up and down as the supports change their degree of fixity**

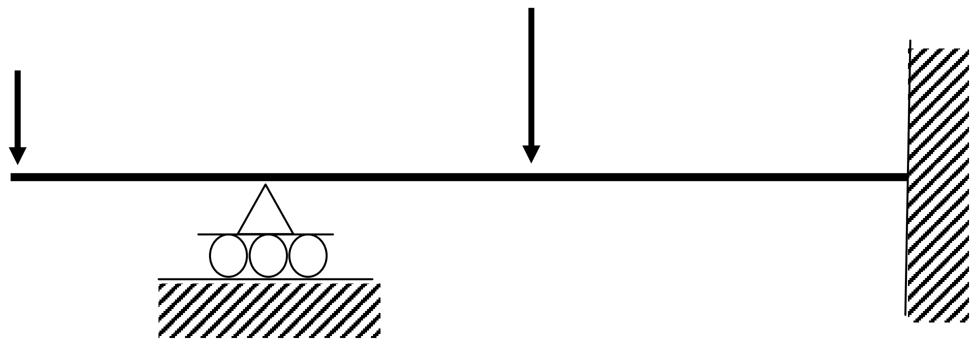
Statically Indeterminate Beams

- Which is correct? All of them!
- As a designer, you choose the function by choosing the form
- Shape the structure to reflect the load acting on it
- Articulate the role of each structural connection



Statically Indeterminate Beams

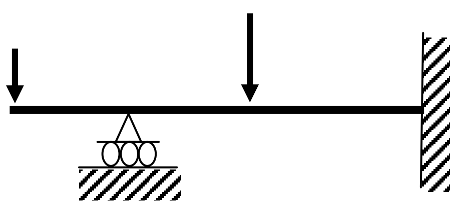
- **What is the moment diagram for this beam under two point loads?**



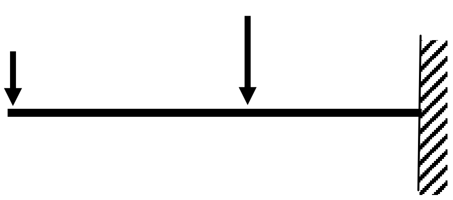
Statically Indeterminate Beams

Release unknown reactions until the structure becomes statically determinate.

Draw moment diagram for statically determinate structure.



Remove roller support to make it a cantilever beam



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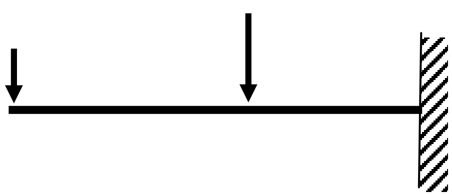

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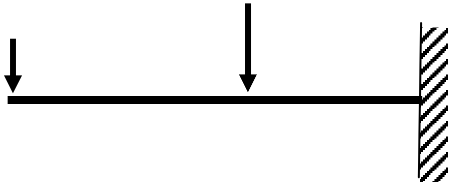

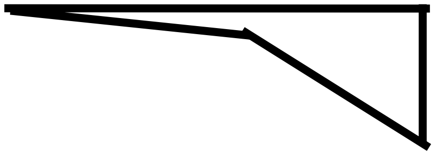

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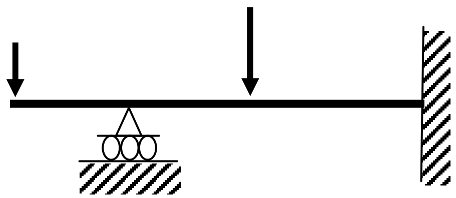
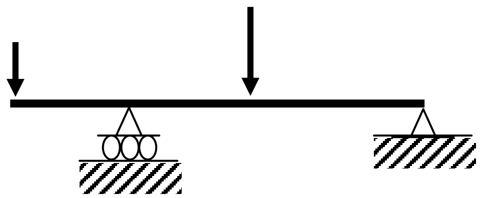


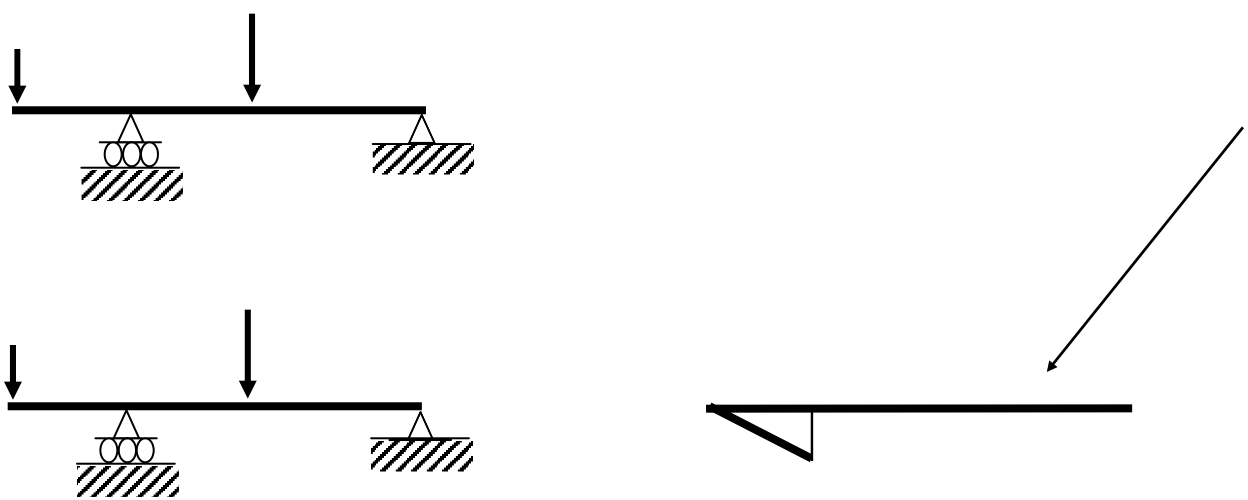
Remove fixed support to make it a simply-supported beam.



Statically Indeterminate Beams

Release unknown reactions until the structure becomes statically determinate.

Draw moment diagram for statically determinate structure.

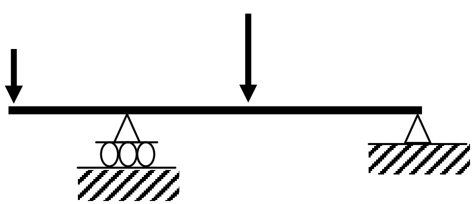


What shape is the moment diagram here?

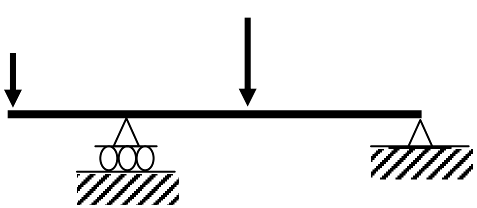
Statically Indeterminate Beams

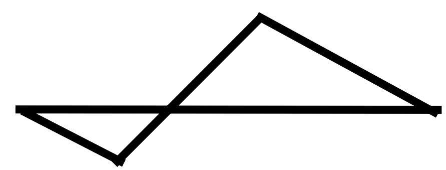
Release unknown reactions until the structure becomes statically determinate.

Draw moment diagram for statically determinate structure.

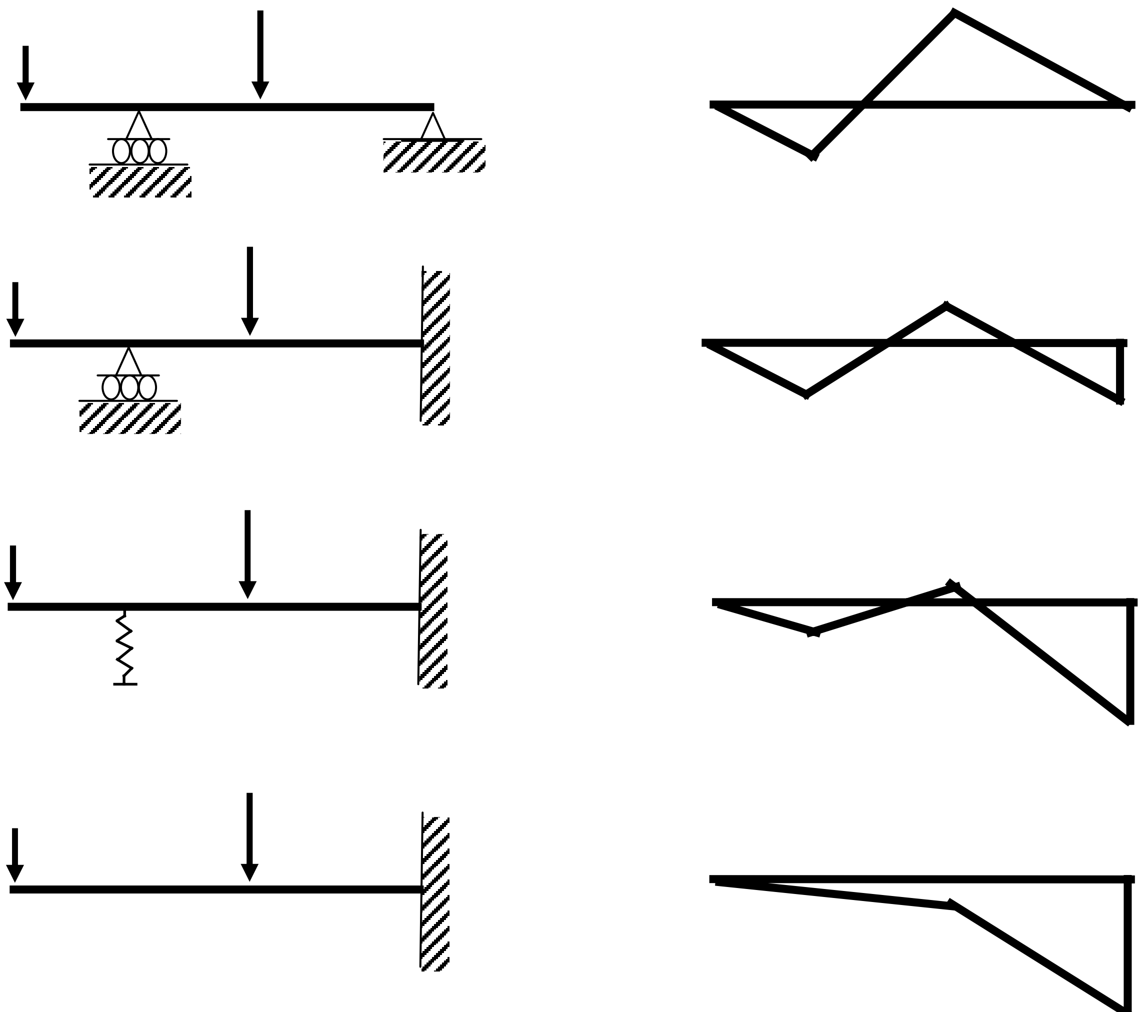


A: The shape of the hanging cable





Statically Indeterminate Beams



Simply-supported

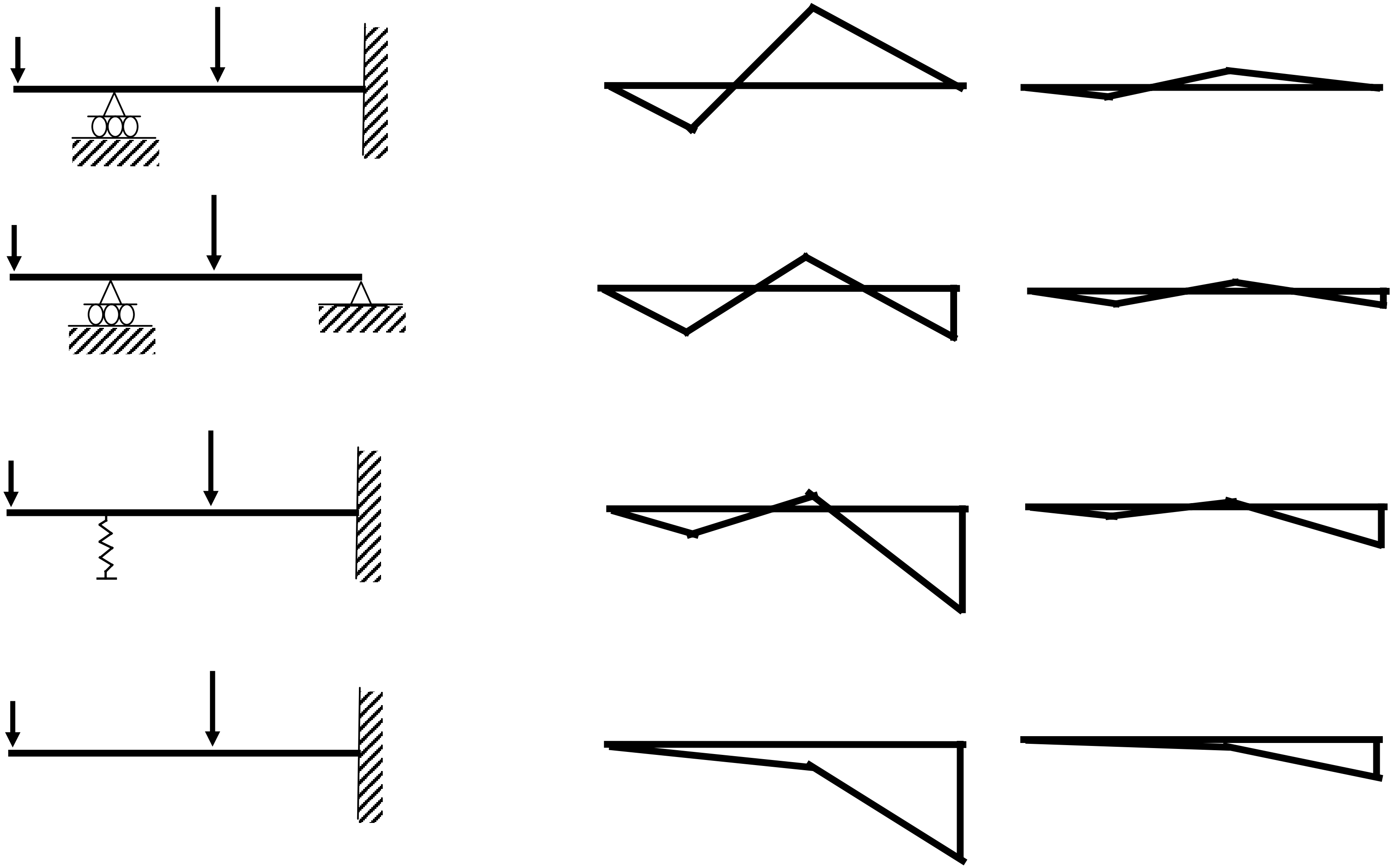
Indeterminate

Indeterminate

Cantilever

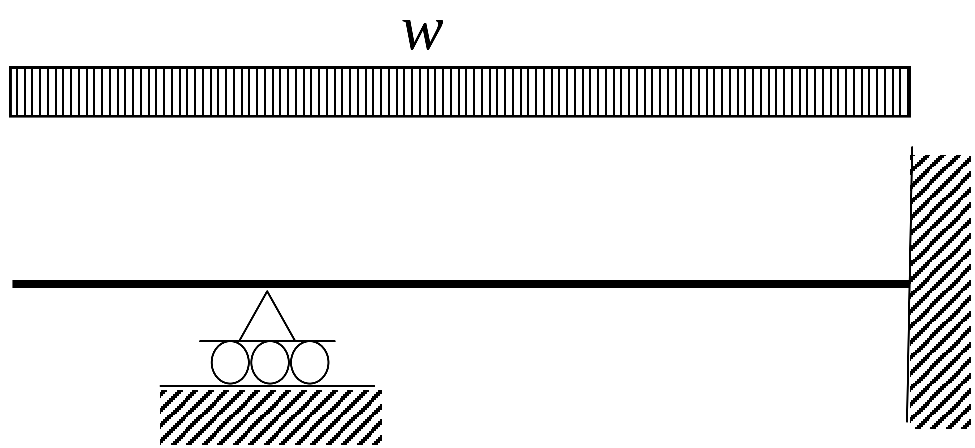
Again, moment diagram shifts up and down

Statically Indeterminate Beams



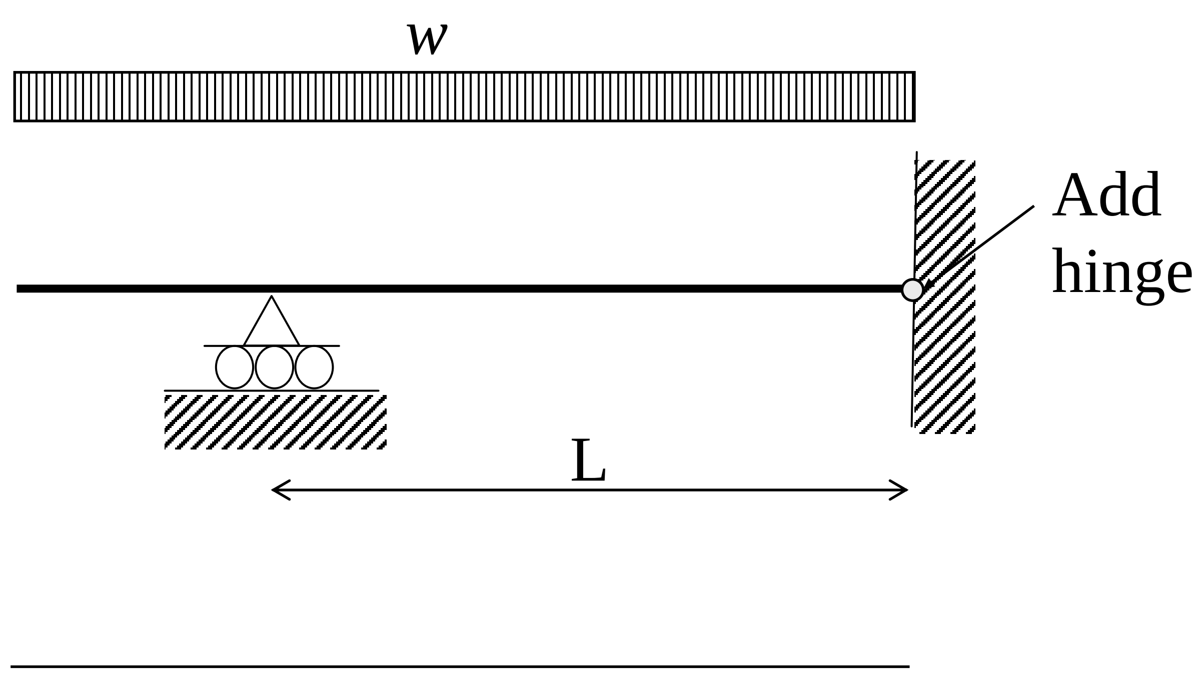
Statically Indeterminate Beams

- **What is the moment diagram for this beam under a uniform load, *w*?**



Statically Indeterminate Beams

- **Release the right hand support by adding a hinge**



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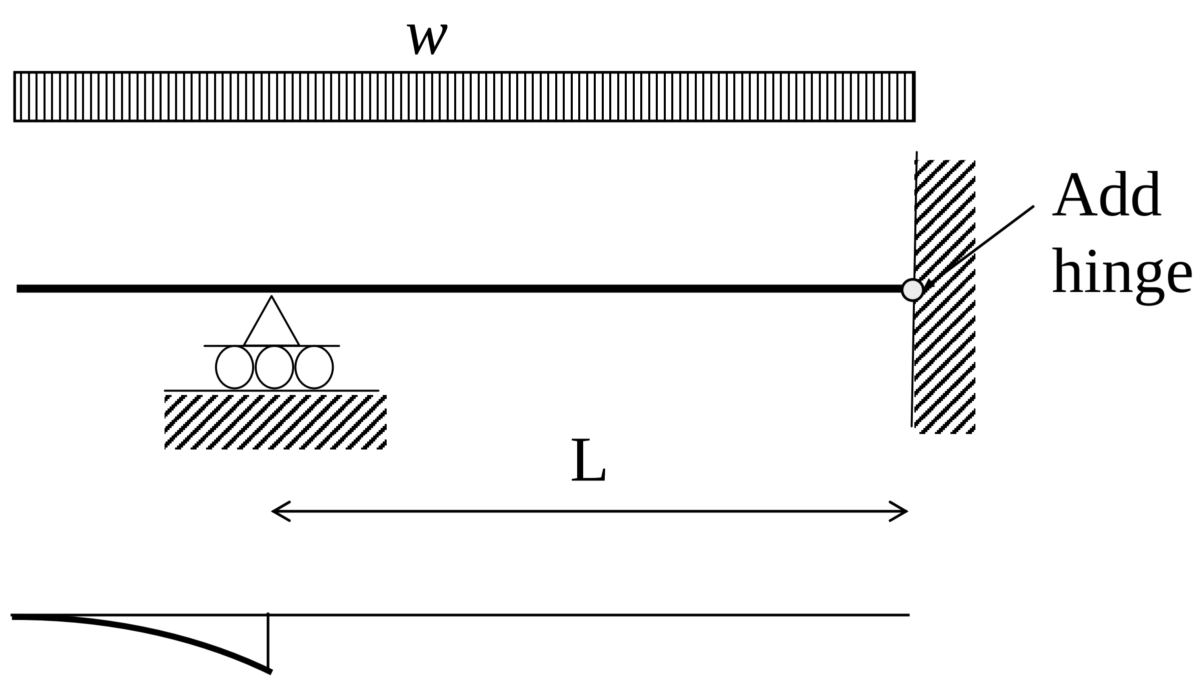

Statically Indeterminate Beams

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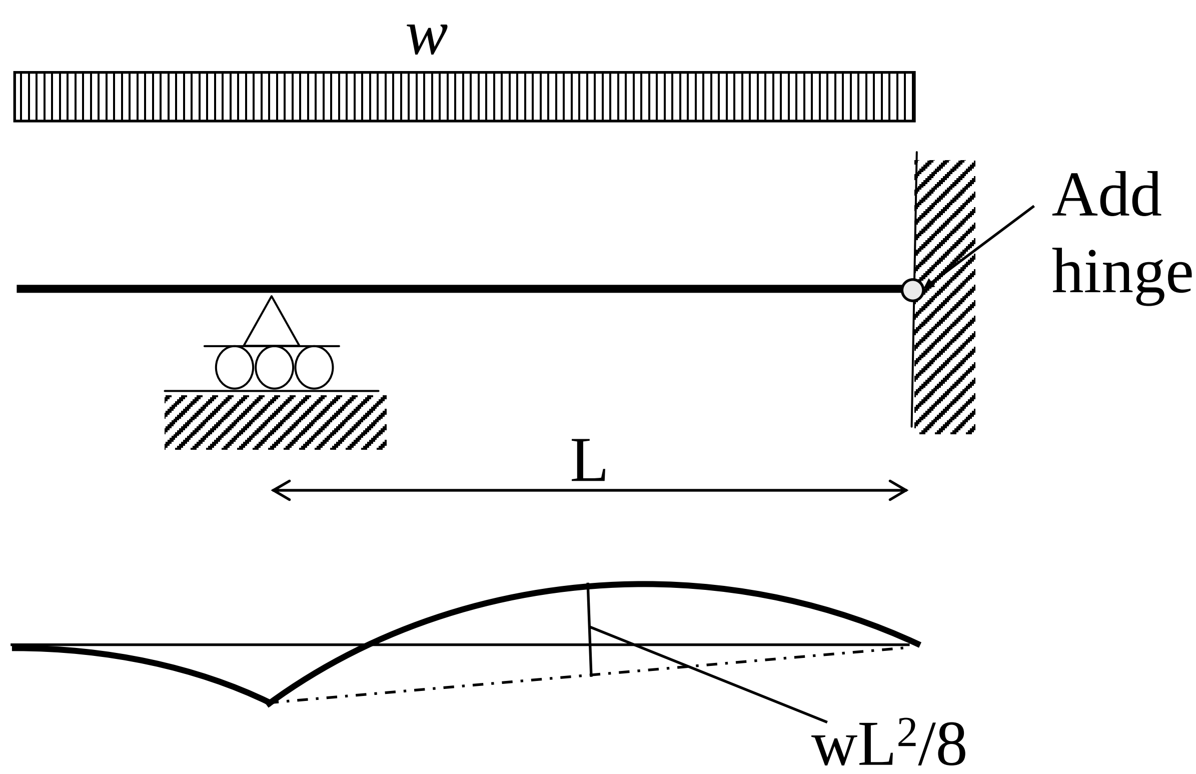
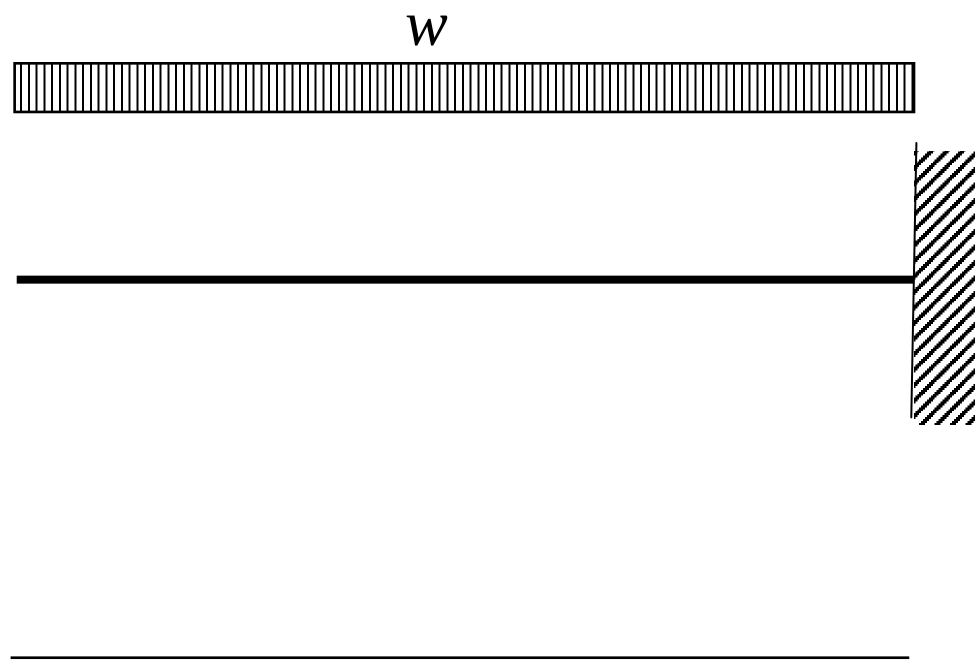


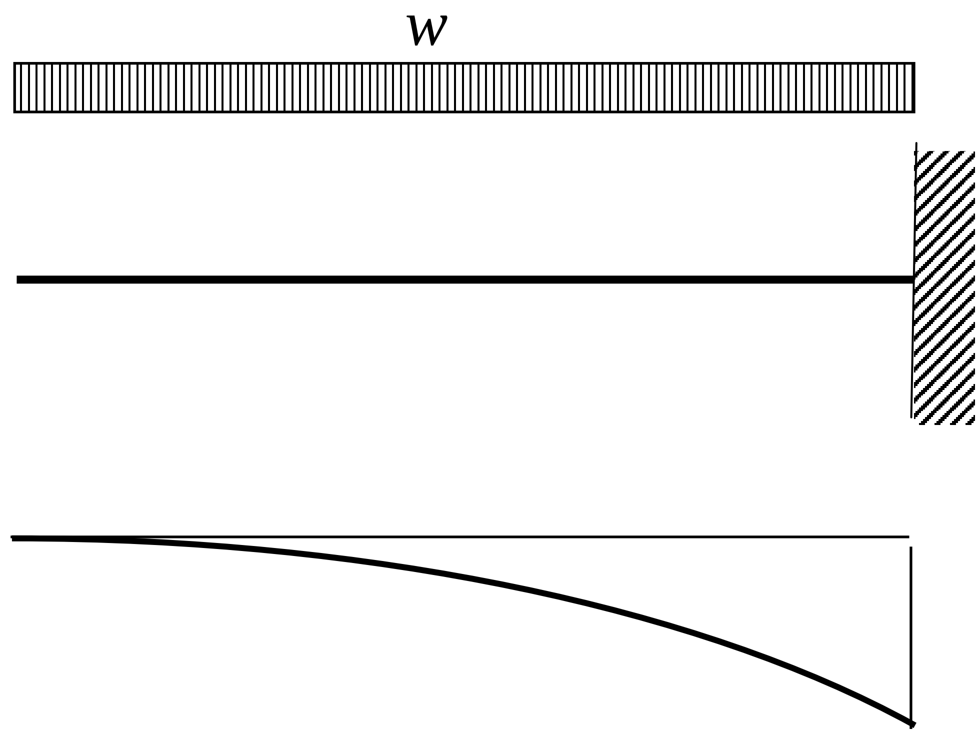
Statically Indeterminate Beams

- **Make statically determinate by removing the roller support**



Statically Indeterminate Beams

- **What is the moment diagram for this beam under a uniform load, *w*?**



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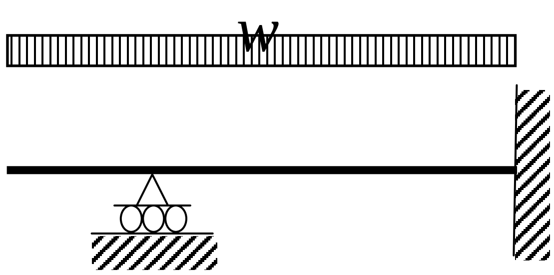
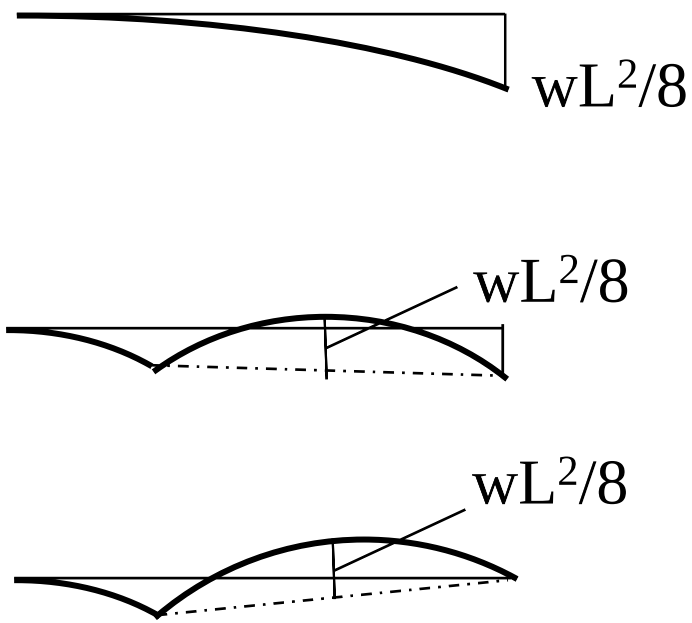
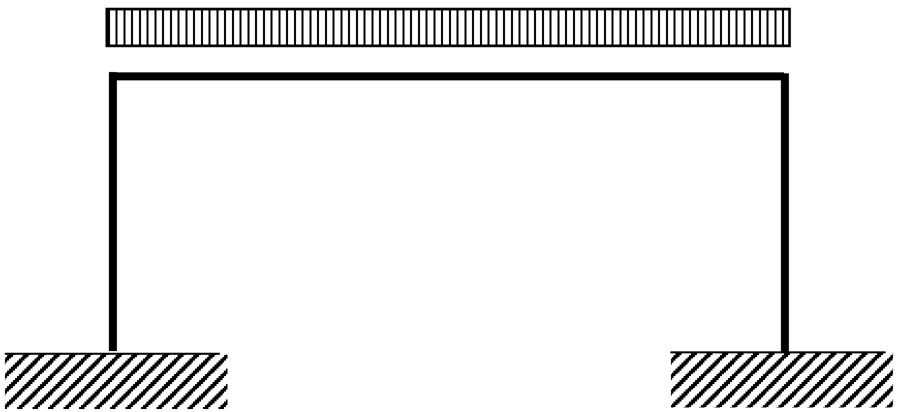




Conclusions

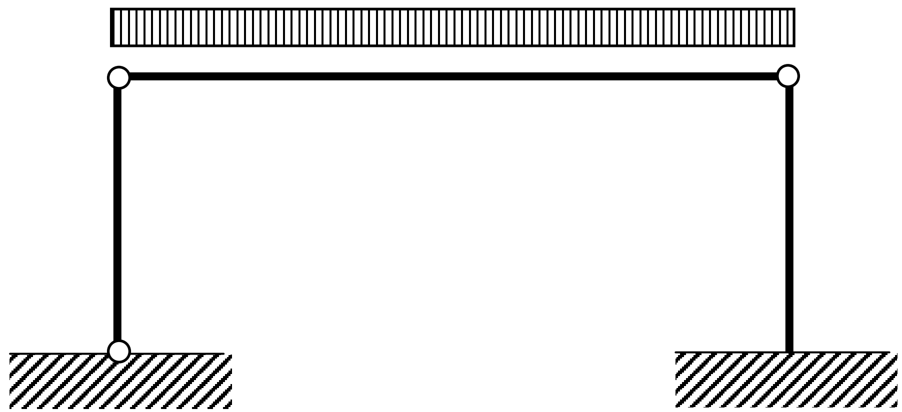
- **You choose the function by choosing the form → function follows form**
- **For a given loading, the moment diagram simply moves *up and down* as you change the support conditions**
- **Must prevent buckling**

Fixed Frame Under Uniform Load



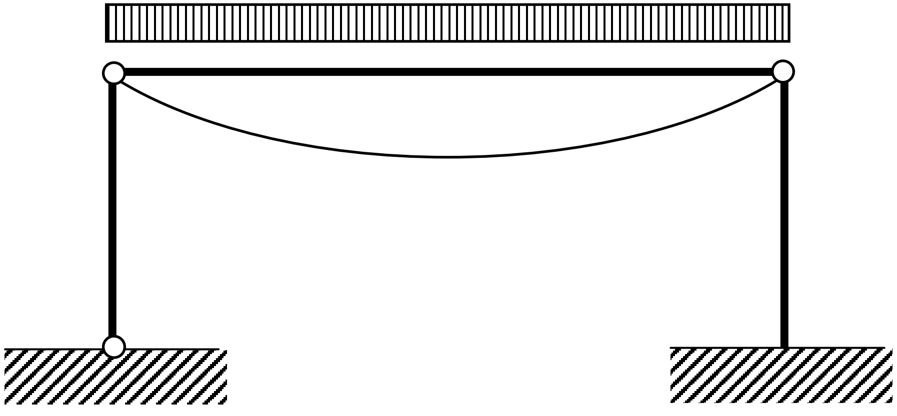
- **Propose three possible moment diagrams for this frame**

Fixed Frame Under Uniform Load



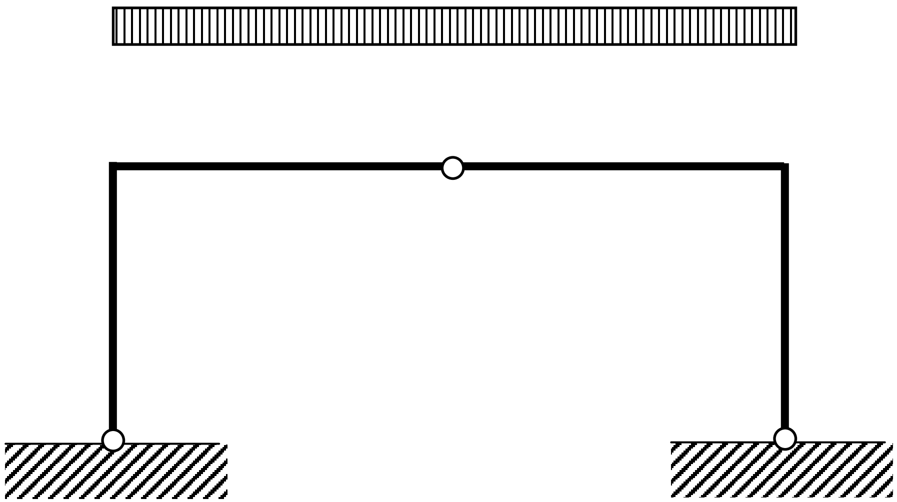
- Simply-supported beam on posts

Fixed Frame Under Uniform Load



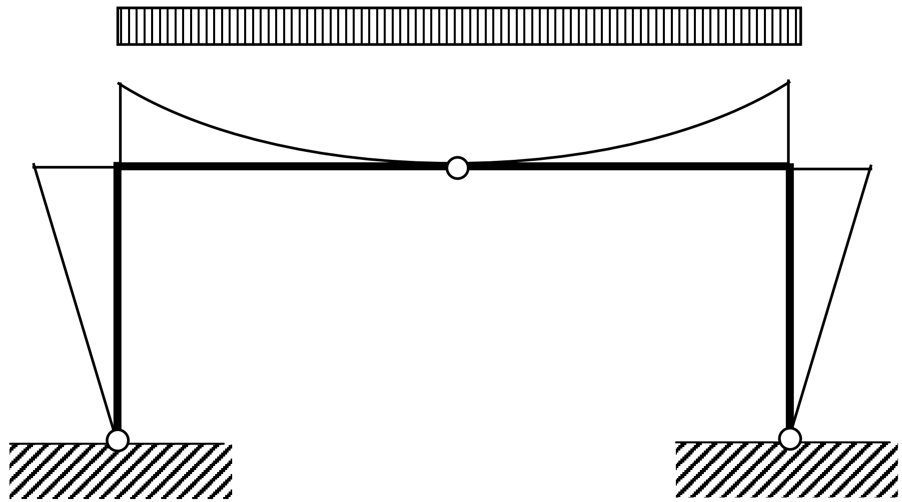
- **Simply-supported beam on posts**

Fixed Frame Under Uniform Load



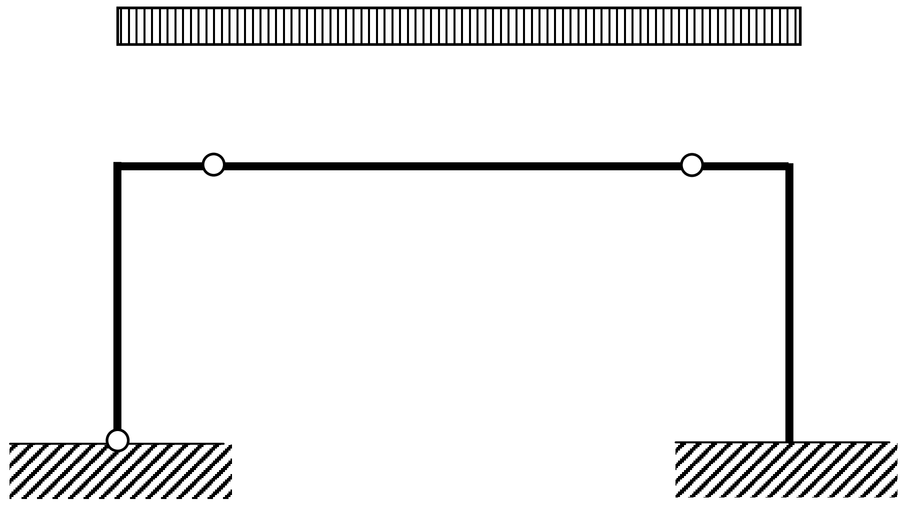
- **Three-hinged frame**

Fixed Frame Under Uniform Load



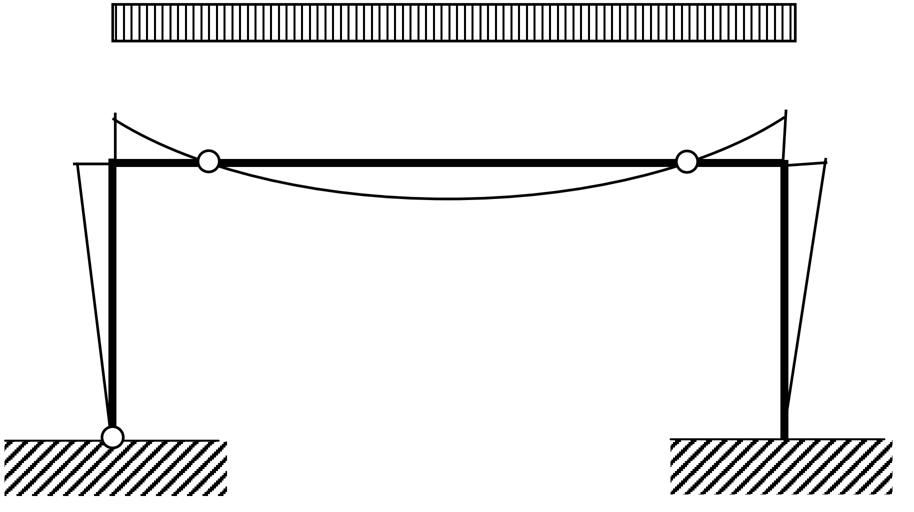
- **Three-hinged frame**

Fixed Frame Under Uniform Load



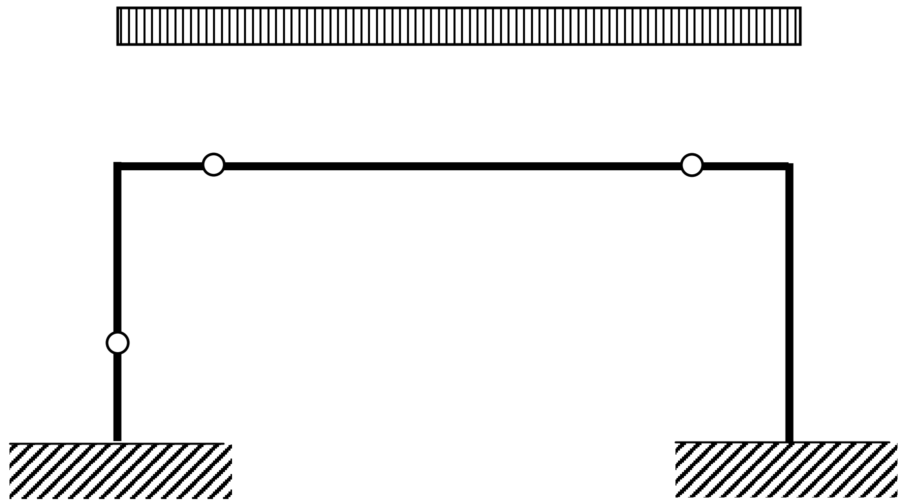
- Alternative three-hinged frame

Fixed Frame Under Uniform Load



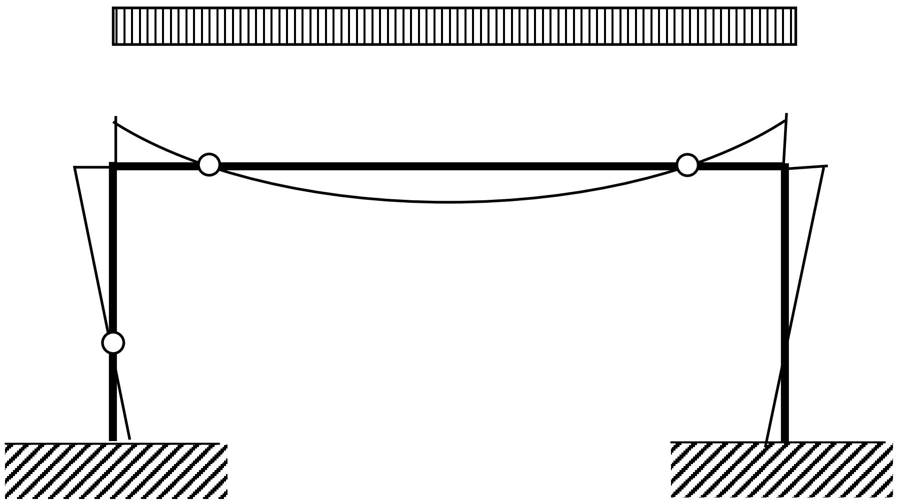
- Alternative three-hinged frame

Fixed Frame Under Uniform Load



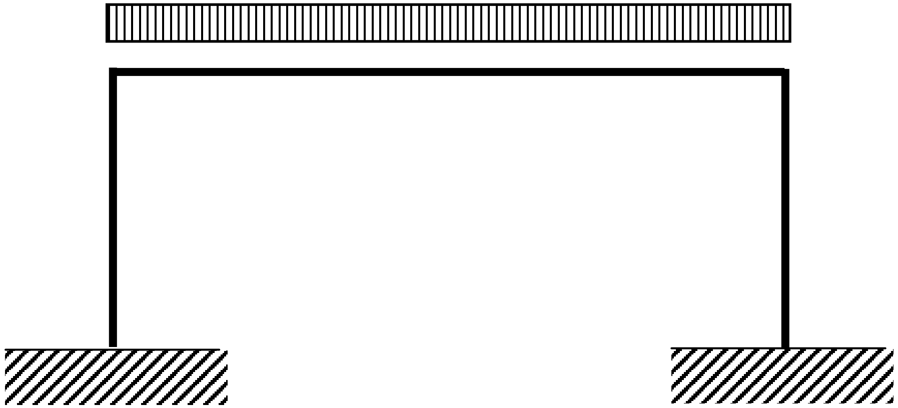
- **Alternative three-hinged frame**

Fixed Frame Under Uniform Load



- **Alternative three-hinged frame**

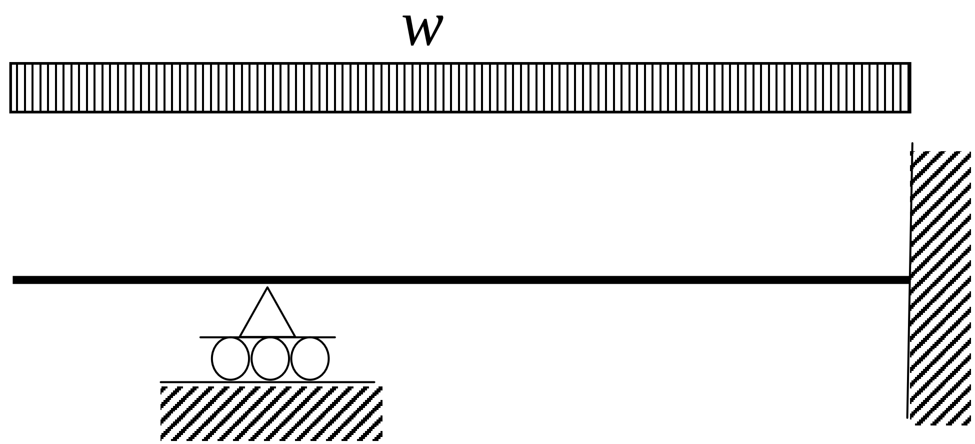
Fixed Frame Under Uniform Load



- **What type of structural forms would work for this load case?**

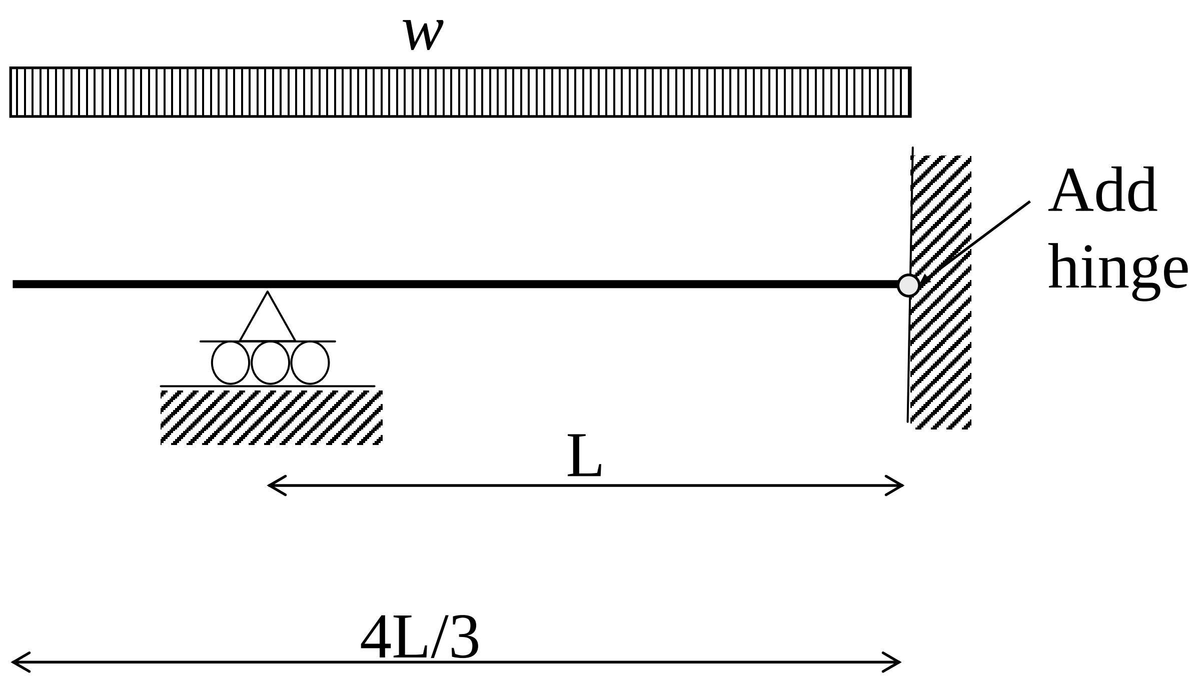
Statically Indeterminate Beams

- **What is the moment diagram for this beam under a uniform load, *w*?**



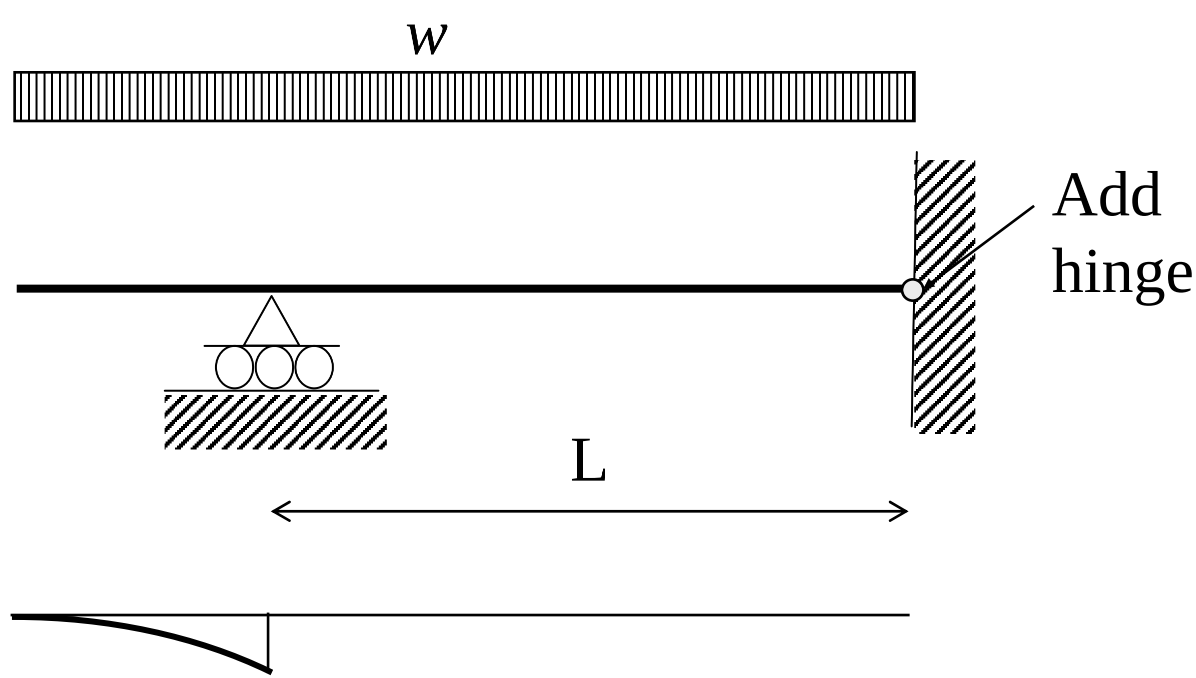
Statically Indeterminate Beams

- **Release the right hand support by adding a hinge**



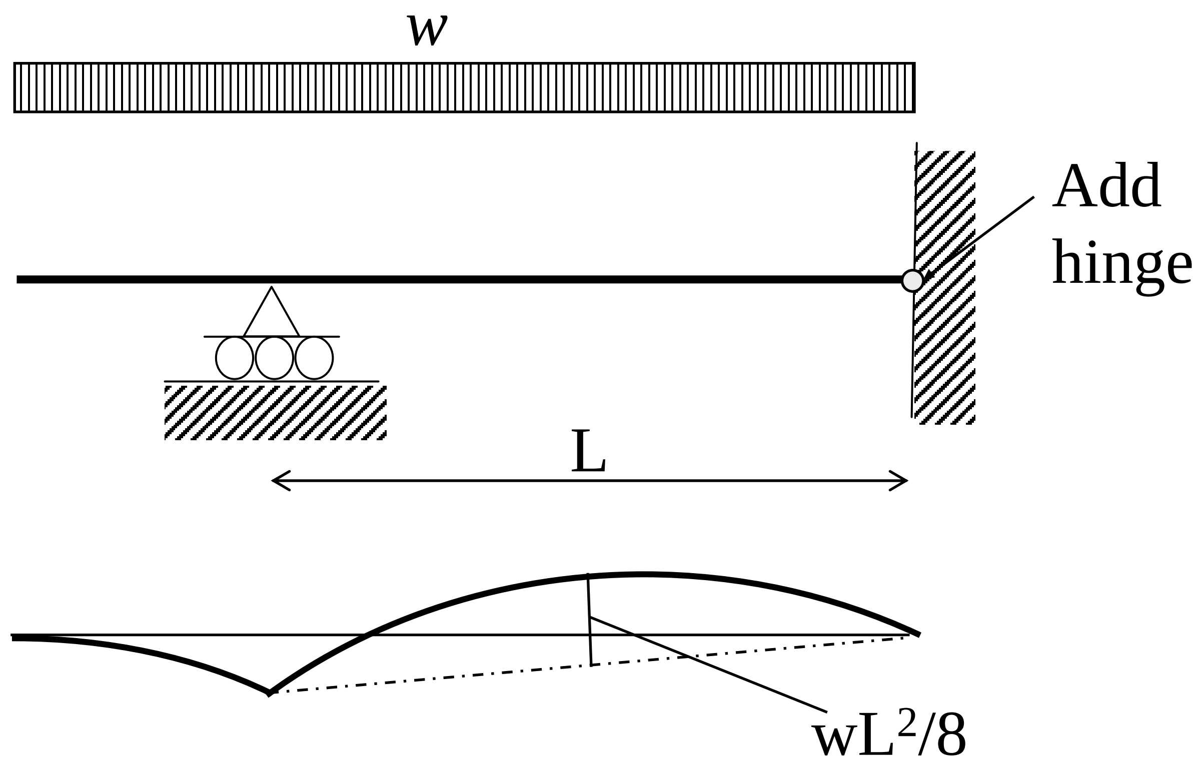
Statically Indeterminate Beams

- **Release the right hand support by adding a hinge**



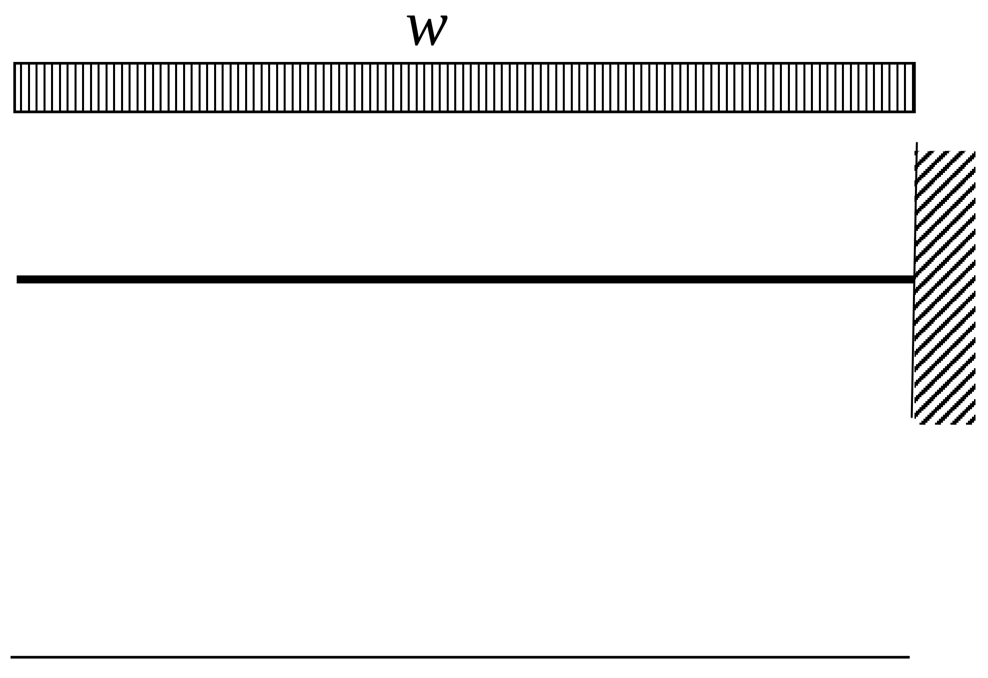
Statically Indeterminate Beams

- **Release the right hand support by adding a hinge**



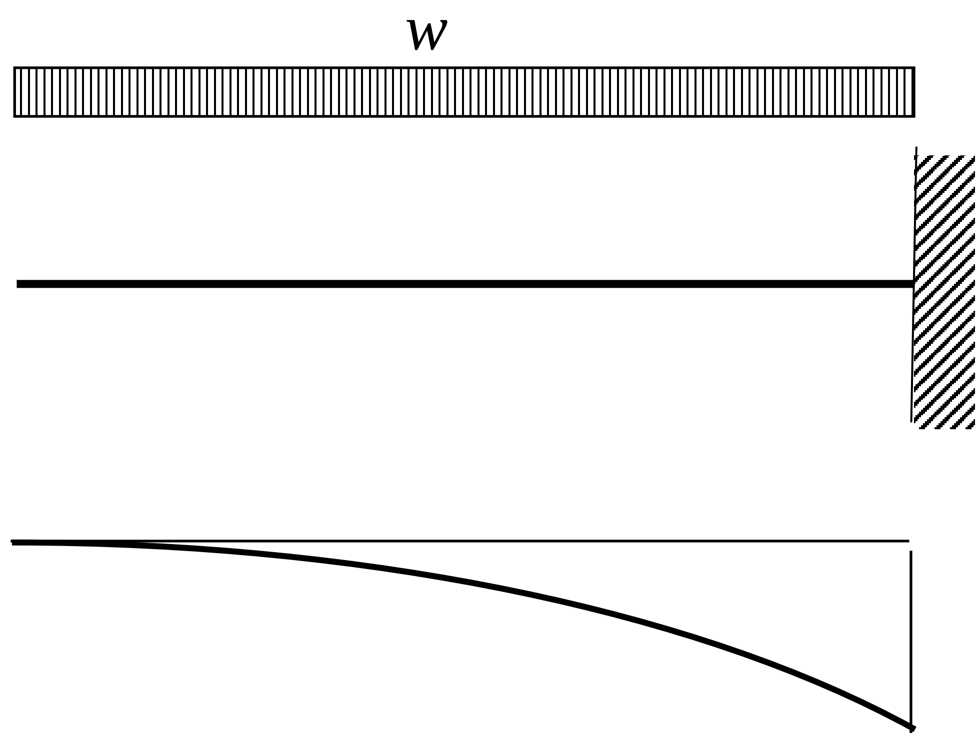
Statically Indeterminate Beams

- **Make statically determinate by removing the roller support**



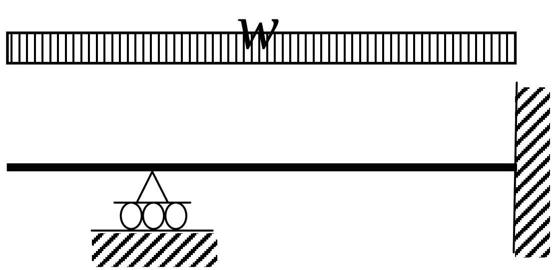
Statically Indeterminate Beams

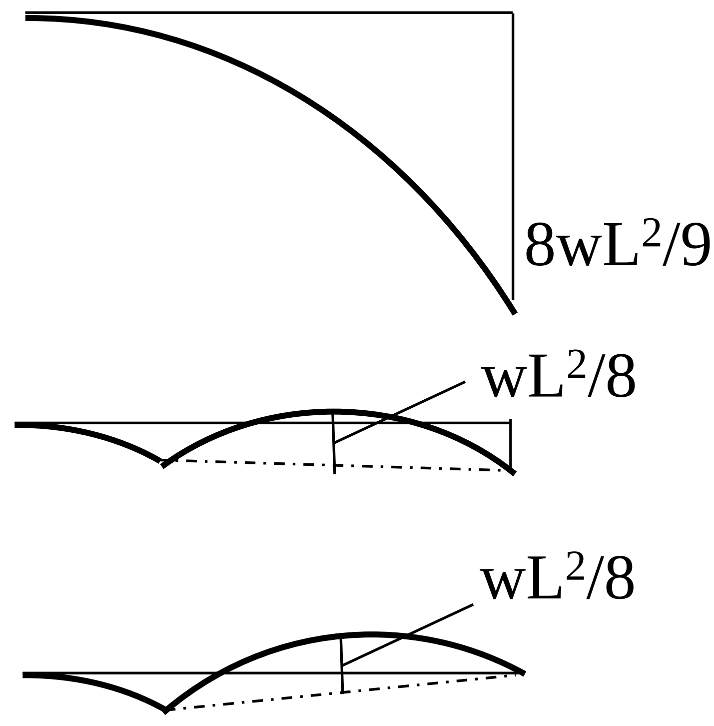
- **What is the moment diagram for this beam under a uniform load, *w*?**



Statically Indeterminate Beams

- **What is the moment diagram for this beam under a uniform load, *w*?**





Review: Indeterminate Structures

- For a given loading on a beam, the moment diagram simply moves *up and down* as you change the support conditions

- You choose the function by choosing the form → function follows form

- Must prevent buckling (think of three-legged stool example)