The Mathematics of Climate Modeling

(Review of the Mathematics of Climate Analysis)

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Lecture Contents

1. What is climate modeling?
2. What variables do we consider?
3. The equations for modeling climate
4. Solving the equations: discretization
5. Spatial grid, time step and stability
6. Computation time and parameterization
7. Assessing model uncertainty
8. 2-D and 1-D models

1. What is climate modeling?

Like models in any other field… climate models:

- Are an abstraction and a simplification of reality.
- They try to capture just the essential processes and to predict the effects of changes and interactions.

But because of the complexity of the earth’s climate system, building any model that can simulate the changing climate to the satisfaction of policy makers is a difficult task and an ongoing subject of research.

Such a model has to answer what happens to temperature, precipitation, humidity, wind speed and direction, clouds, and ice all around the globe over long time scales as a result of radiative forcing due to changing amounts of greenhouse gases and aerosols.

2. What variables are needed ?

Different aspects of climate, from cloud cover to snowfall can be calculated by calculating the values of the following variables through time. For the atmosphere:

Var.	*Description*	*units*
T	Temperature	°C or °K
u	East-west wind speed (E-W component of wind vector)	m/s
v	North-south wind speed (N-S component of wind vector)	m/s
w	Vertical wind speed (vertical component of wind vector	m/s
P	Pressure	mb or atm
ρ	Mass density	g/cm^3, kg/m^3
χ	Mixing ratio (ratio of number of molecules of a certain chemical to total # of molecules)… including water vapor and other greenhouse gases	%, ppm, ppb
ρ_{water}	Liquid water content (in clouds, fog)	g/m^3
ρ_{ice}	Ice (in clouds)	g/m^3

3. The Equations

Models of the earth's climate are based on laws of physics:

- conservation of energy
- Conservation of momentum
- Conservation of mass
- Ideal gas law

We can express the changes in the variables by a "continuity equation":

$$\left(\frac{\partial \Phi}{\partial t}\right) = \frac{d\Phi}{dt} - \frac{\partial}{\partial x}(u\Phi) - \frac{\partial}{\partial y}(v\Phi) - \frac{\partial}{\partial z}(w\Phi)$$

The total change (rate of accumulation) of Φ in the box

Actual production or destruction of Φ within the box

Change in Φ due to loss to downstream boxes or arrival of Φ from an upstream box (called advection or convection)

Examples

Φ	Production and Loss $\left(\frac{d\Phi}{dt}\right)$	Advection
Mass (ρ)	0 (mass is conserved absolutely)	Winds or currents, convection, turbulence
Atmospheric Gases (pollution, water vapor, etc)	Emissions, chemical reactions, absorption by particles and clouds, condensation and evaporation (for water vapor), deposition to land or ocean surface	Winds, convection, turbulence
Atmospheric Temperature (Joules / cubic meter)	Latent heat releases (during condensation and freezing of water), chemical reactions, radioactive decay, radiative cooling, radiative absorption	Winds, turbulence, conduction, radiation, falling rain, etc.
Global Avg Climatic Temperature	Exchange with hard earth (volcanos, rock formation, etc)	Sunlight (in) and thermal radiation (out)
Clouds	Evaporation and condensation of water vapor, rain that falls to the surface	Winds, convection, tubulence

Basic Equations of the Atmosphere and Oceans in 3-D

Mass Continuity

$$\frac{\partial\rho}{\partial t} = -\frac{d(u\rho)}{dx} - \frac{d(v\rho)}{dy} - \frac{d(w\rho)}{dz}$$

Equations of Motion (momentum continuity)

$$\frac{\partial u}{\partial t} = -\frac{d(uu)}{dx} - \frac{d(vu)}{dy} - \frac{d(wu)}{dz} + \ldots$$

$$\frac{\partial v}{\partial t} = -\frac{d(uv)}{dx} - \frac{d(vv)}{dy} - \frac{d(wv)}{dz} + \ldots$$

$$\frac{\partial w}{\partial t} = -\frac{d(uw)}{dx} - \frac{d(vw)}{dy} - \frac{d(ww)}{dz} + \ldots$$

+ pressure gradient

+ Coriolis force

+ gravity

+ friction

Thermodynamic Equation (energy continuity)

$$\frac{\partial T}{\partial t} = -\frac{d(uT)}{dx} - \frac{d(vT)}{dy} - \frac{d(wT)}{dz} + \frac{1}{c_v}\left(J - p\frac{D(1/\rho)}{Dt}\right)$$

J : radiation, conduction, latent heat release, etc

D(1/ρ) / Dt : conversion between thermal and mechanical energy in fluid system

Chemical Continuity Equation

$$\frac{\partial\chi}{\partial t} = -\frac{d(u\chi)}{dx} - \frac{d(v\chi)}{dy} - \frac{d(w\chi)}{dz}$$

+ Chemical Production

- Chemical Loss

4. Solving the equations

- To model the change in climate through time: We want to solve for the values of the variables described by these equations over time. i.e. to integrate the set of differential equations.
- Essentially we have seven (or more) variables described by the same number of equations that describe change with respect to time. (T,p, ρ, u, v, w, water, etc.). So we should be able to solve for the values of the variables through time…

- BUT these equations cannot be cannot solved analytically; there is no closed form solution.
- So need to use numerics: discretize in time and space…

5.1 The spatial grid

We divide the earth's atmosphere into a finite number of boxes (grid cells).

Assume that each variable has the same value throughout the box.

Write a budget for each each box, defining the changes within the box, and the flows between the boxes.

McGuffie, K. and Henderson-Sellers, A., *A Climate Modeling Primer*. New York: John Wiley & Sons, 1997.

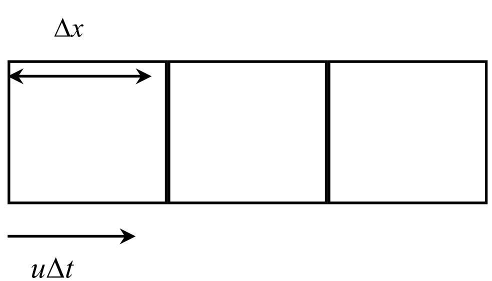
5.2 Time stepping and stability

Time is also treated in discrete units. The values within the boxes are recomputed at specific time intervals, adding and subtracting changes due to processes that have occurred within the box, and flows into and out of the box.

Time intervals depend on the size of the boxes:

<u>General Rule for stability: the CFL condition</u>

$$\frac{u\Delta t}{\Delta x} \leq 1$$



Intuitively don't want to transport more than a grid cell over a time step.

Eg. In atmosphere max u = 100m/s; grid spacing = 300 km;
Constraint: Δt < 3000 seconds (less than 1 hour)

6.1 Demand on Computing Resources

Total Computation Time:

For example, for a 2.8° x 2.8 ° degree atmospheric model

How Many Grid Cells?

128	Longitudes
64	Latitudes
* 18	Vertical Levels

~ 150,000 Grid Cells

What Happens at each Grid Cell?

10	Variables
* 100	Computations Each

~ 1,000 Computations per Grid Cell per Time Step

How Many Time Steps Per Year?

24+	Time Steps per Day
365	Days per Year

~ 10,000 Time Steps per Year

$$150{,}000\ \text{(Grid Cells)} * 1{,}000\ \frac{\text{Computations}}{\text{(Grid Cell) (Time Step)}} * 10{,}000\ \frac{\text{Time Steps}}{\text{Year}} \approx 1.5\ \text{Trillion}\ \frac{\text{Calculations}}{\text{Year}}$$

With a 1 GHz machine, a 1 year simulation takes about three hours

And, remember, this is just about the simplest possible model and we generally want to run the model for decades or centuries…

6. 2 Parameterizations

For global climate models, grid cells are typically hundreds of miles across and often there are thirty vertical layers for the atmosphere.

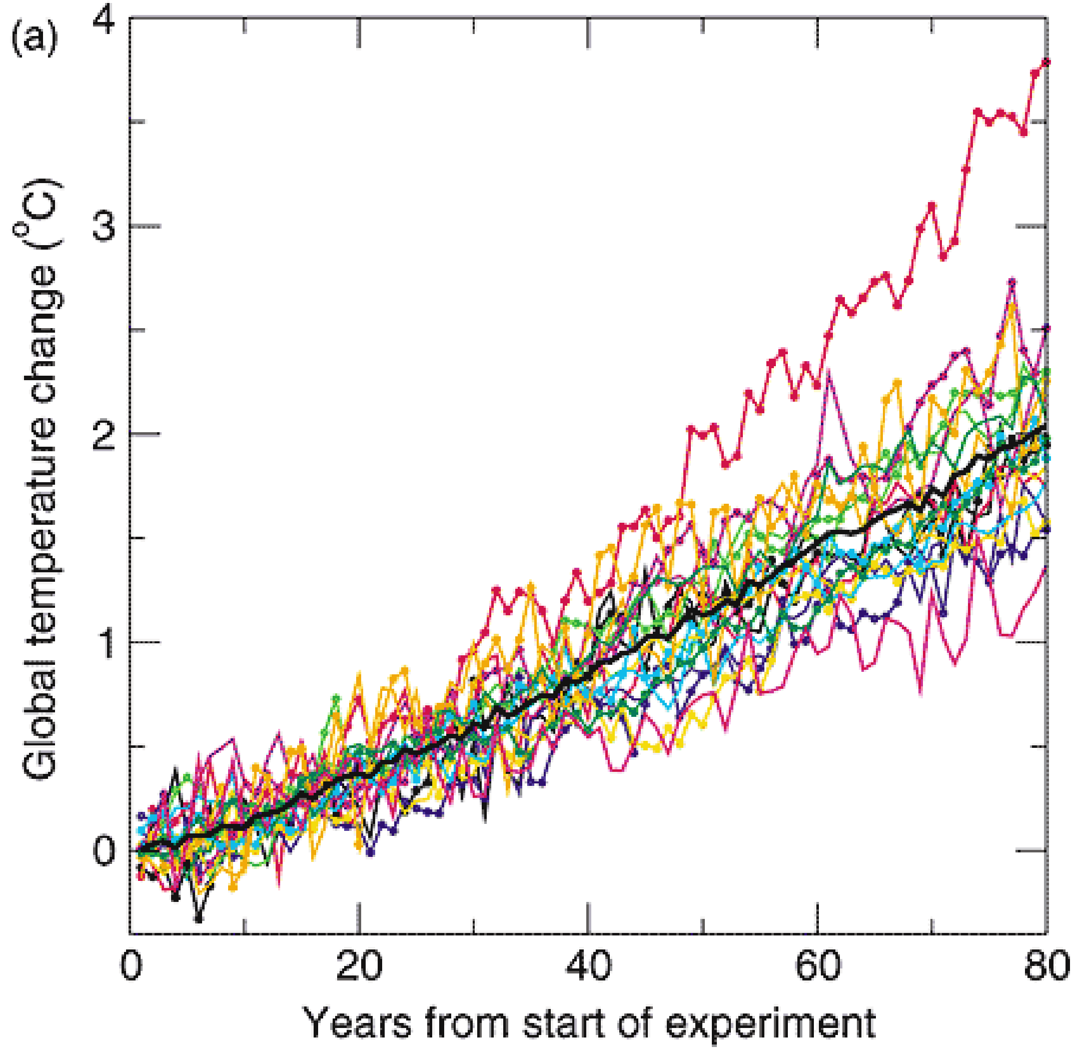
Many processes happen at smaller scales and must be approximately included (a.k.a., parameterized), including:

- Convection
- Cloud Cover
- Ice Cover: sea and land (glaciers)
- Snow Cover
- Rainfall
- Emissions of Pollutants
- River Runoff into Oceans
- “Eddy Fluxes”
- Sharp weather fronts
- “Gravity Waves”
- Mountains
- Cities (heat islands, emissions, etc)

7. Assessing Model Uncertainty

- How do we get a range of uncertainty for these very expensive 3-D Models?
- One way is to run each available model once and pretend that the variation between the models represents the variability between future climates

The 2nd Climate Model Inter-comparison Project did this:



The time evolution of the globally averaged temperature change relative to the control run of the CMIP2 simulations (Unit: °C) for a scenario in which CO_2 concentrations compound at 1% per year

Courtesy of Environment Canada

8.1 Two dimensional models

Another approach to uncertainty is to reduce the number of calculations, and the time it takes for the model to run.

If it runs quickly enough, you can vary the values of the unknown input parameters to represent how uncertain you believe them to be. This will give you an range of possible outputs. (Generally, this approach requires thousands of individual model runs.)

MIT's approach to reducing computational expense is to represent the climate in two dimensions.

Uncertainties are then generated from a single model and are based on physical uncertainty, not disagreement between models.

8.2 One dimensional modeling: The Energy Balance Model

Fundamentally, climate models consider what happens to solar energy from the time it enters the Earth's atmosphere until the time it exits.

The simplest model of this ignores motions of the ocean and atmosphere and almost everything else. It only considers the temperature at the surface and at one or several atmospheric layers.

At each level, we may write a budget equation: all of the energy must be accounted for. And all of the energy that enters the climate system must leave (this is an equilibrium model). You will have a chance to work through the mathematics of this in your first problem set.

1. Solar Radiation from the Sun passes through the atmosphere to the surface, where some is absorbed and some is reflected

2. Energy is emitted from the surface thermally (also convectively in a slightly more sophisticated version)

3. The lower atmospheric layer absorbs some of the energy from below (2) and re-emits it thermally, half up and half down

4. The upper atmospheric layer absorbs some of the energy from both 2. and 3. and re-emits it thermally, also half up and half down.