

IV. NUCLEAR MAGNETIC RESONANCE AND HYPERFINE STRUCTURE

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A. DIELECTRIC SUSCEPTIBILITY TENSOR FOR AN ATOMIC VAPOR

The dielectric susceptibility of an atomic vapor in the presence of a magnetic field can be expressed as a 3×3 tensor. All of the properties of the medium relevant to the propagation of electromagnetic waves through the vapor, such as absorption, dispersion, Faraday effect, and so forth, can be obtained from the tensor elements. The imaginary parts of the tensor elements can be obtained from the absorption coefficients for right and left circularly polarized light traveling parallel to the field, and for linearly polarized light propagating perpendicular to the applied field. The absorption coefficients can be calculated from known atomic oscillator strengths and the level structure of the ground and excited atomic states. The tensor analogs of the scalar Kramers-Kronig dispersion relations can then be used to calculate the remaining components of the tensor.

The dispersing medium exhibits an axial symmetry about the direction of the applied magnetic field. In this case the susceptibility tensor must be invariant to rotations about the symmetry axis or field direction. This permits reduction of the tensor to the general form:

$$X = \begin{bmatrix} X_{11} & -X_{21} & 0 \\ X_{21} & X_{11} & 0 \\ 0 & 0 & X_{33} \end{bmatrix}. \quad (1)$$

There are, therefore, only three independent components. For purposes of calculation, the tensor is rewritten as

$$X = \begin{bmatrix} \beta-1 & -ia & 0 \\ ia & \beta-1 & 0 \\ 0 & 0 & \gamma-1 \end{bmatrix} \quad \begin{aligned} a &= a' - ia'' \\ \beta &= \beta' - i\beta'' \\ \gamma &= \gamma' - i\gamma''. \end{aligned} \quad (2)$$

Maxwell's equations can then be written in the form

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$$\begin{bmatrix} 0 & -\frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & -\frac{\partial}{\partial x} \\ -\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = -i\omega\mu \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & -\frac{\partial}{\partial x} \\ -\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = i\omega\epsilon_0 \begin{bmatrix} \beta & -ia & 0 \\ ia & \beta & 0 \\ 0 & 0 & \gamma \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}.$$

If we assume that the magnetic field is parallel to z , the propagation vector parallel to z , and the field independent of x and y , the formulas yield the following pairs of wave equations.

$$\left[\frac{\partial^2}{\partial z^2} + \omega^2 \mu \epsilon_0 (\beta - a) \right] \begin{bmatrix} E_+ \\ H_+ \end{bmatrix} = 0 \quad E_{\pm} = \frac{1}{\sqrt{2}} (E_x \pm iE_y) \quad (4)$$

$$\left[\frac{\partial^2}{\partial z^2} + \omega^2 \mu \epsilon_0 (\beta + a) \right] \begin{bmatrix} E_- \\ H_- \end{bmatrix} = 0.$$

The effective dielectric permeabilities for the two circularly polarized waves are:

$$\epsilon_+ = \epsilon_0 (\beta - a) \quad (5)$$

$$\epsilon_- = \epsilon_0 (\beta + a).$$

If we assume that the magnetic field is parallel to z , the propagation along x , and the light linearly polarized along z , we obtain the wave equation

$$\left[\frac{\partial^2}{\partial x^2} + \omega^2 \mu \epsilon_0 \gamma \right] E_z = 0.$$

The effective permeability for a wave propagating along x , linearly polarized parallel to H is

$$\epsilon_{\pi} = \epsilon_0 \gamma. \quad (7)$$

The tensor elements may now be expressed in terms of the permeabilities for the

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several polarizations of waves. The imaginary parts of the permeabilities correspond to the absorption coefficients for the medium. These may be taken as experimentally known and are sufficient to determine the susceptibility tensor.

The absorption coefficients for the three arrangements of propagation vector, \vec{k} , polarization, and field for the mercury $^1S_0 - ^3P_1$ transition are:

For right circularly polarized light, H parallel to k,

$$k_+ = k_0 e^{-(\omega + \Delta\omega)^2}$$

For left circularly polarized light, H parallel to k,

$$k_- = k_0 e^{-(\omega - \Delta\omega)^2}$$

For linearly polarized light parallel to H, H perpendicular to k,

$$k_\pi = k_0 e^{-\omega^2}$$

The corresponding elements are

$$\alpha'' = \frac{1}{2\epsilon_0} (\epsilon_+'' - \epsilon_-'') = \frac{k_0 \chi}{2} \left[e^{-(\omega + \Delta\omega)^2} - e^{-(\omega - \Delta\omega)^2} \right] \quad (8)$$

$$\beta'' = \frac{1}{2\epsilon_0} (\epsilon_+'' + \epsilon_-'') = \frac{k_0 \chi}{2} \left[e^{-(\omega + \Delta\omega)^2} + e^{-(\omega - \Delta\omega)^2} \right]$$

$$\gamma'' = \epsilon_\pi / \epsilon_0 = k_0 \chi e^{-\omega^2}.$$

Here, use has been made of the relations

$$k_+ = \frac{\epsilon_+''}{\epsilon_0 \chi}$$

$$k_- = \frac{\epsilon_-''}{\epsilon_0 \chi}$$

$$k_\pi = \frac{\epsilon_\pi''}{\epsilon_0 \chi}.$$

The Kramers-Kronig relations connecting the real and imaginary parts of the tensor elements are:

$$u_{ik}(\Omega) = -\frac{2}{\pi} P \int_0^\infty \frac{\omega v_{ik}(\omega)}{\omega^2 - \Omega^2} d\omega \quad (9a)$$

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$$v_{ik}(\Omega) = \frac{2\Omega}{\pi} P \int_0^{\infty} \frac{u_{ik}(\omega)}{\omega^2 - \Omega^2} d\omega. \quad (9b)$$

Here, $X_{ik} = u_{ik} + iv_{ik}$, and use may be made of (1) and (2) to obtain the following relations:

$$\begin{aligned} u_{11} &= \beta' - 1 & u_{21} &= \alpha'' & u_{33} &= \gamma' - 1 \\ v_{11} &= -\beta'' & v_{21} &= \alpha' & v_{33} &= -\gamma'' \end{aligned}$$

The real parts of the susceptibility tensor, then, are:

$$\begin{aligned} \alpha' &= \frac{2\omega}{\pi} P \int_0^{\infty} \frac{\alpha''(\omega')}{\omega'^2 - \omega^2} d\omega' = -\frac{k_0 \chi}{2\sqrt{\pi}} [F(\omega + \Delta\omega) - F(\omega - \Delta\omega)] \\ \beta' - 1 &= +\frac{2}{\pi} P \int_0^{\infty} \frac{\omega \beta''(\omega')}{\omega'^2 - \omega^2} d\omega' = -\frac{k_0 \chi}{2\sqrt{\pi}} [F(\omega + \Delta\omega) + F(\omega - \Delta\omega)] \\ \gamma' - 1 &= +\frac{2}{\pi} P \int_0^{\infty} \frac{\omega \gamma''(\omega')}{\omega'^2 - \omega^2} d\omega' = -\frac{k_0 \chi}{\sqrt{\pi}} F(\omega). \end{aligned}$$

The complete susceptibility tensor therefore is defined as

$$\begin{aligned} X_{11} &= \frac{k_0 \chi}{2\sqrt{\pi}} \left[-F(\omega + \Delta\omega) - F(\omega - \Delta\omega) - i\sqrt{\pi} \left(e^{-(\omega + \Delta\omega)^2} + e^{-(\omega - \Delta\omega)^2} \right) \right] \\ X_{21} &= \frac{k_0 \chi}{2\sqrt{\pi}} \left[\sqrt{\pi} \left(e^{-(\omega - \Delta\omega)^2} - e^{-(\omega + \Delta\omega)^2} \right) - i(F(\omega + \Delta\omega) - F(\omega - \Delta\omega)) \right] \\ X_{33} &= \frac{k_0 \chi}{\sqrt{\pi}} \left[-F(\omega) - i\sqrt{\pi} e^{-\omega^2} \right]. \end{aligned}$$

Table IV-1 gives the parameters describing the medium for various arrangements of field, polarization, and direction of propagation.

For small values of $\Delta\omega$, the expressions given in Table IV-1 may be written in terms of derivatives. Given $\frac{\partial F(\omega)}{\partial \omega} = 1 - 2\omega F(\omega)$, we can write the magneto rotation and the magneto double refraction coefficients for a length ℓ of dispersing medium as

$$\begin{aligned} X_{\ell} &\approx \frac{-k_0 \ell}{\sqrt{\pi}} \frac{\partial F}{\partial \omega} \Delta\omega = \frac{-k_0 \ell}{\sqrt{\pi}} [1 - 2\omega F(\omega)] \Delta\omega \\ \Phi_{\ell} &\approx \frac{-k_0 \ell}{2\sqrt{\pi}} \frac{\partial^2 F}{\partial \omega^2} (\Delta\omega)^2 = +\frac{k_0 \ell}{\sqrt{\pi}} [\omega + F(\omega)(1 - 2\omega^2)] (\Delta\omega)^2. \end{aligned}$$

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Table IV-1. Optical parameters.

Field Orientations	Absorption	Refractive Index
H parallel to k Right circularly polarized light	$k_+ = k_0 e^{-(\omega+\Delta\omega)^2}$	$n_+ - 1 = \frac{k_0 \chi}{\sqrt{\pi}} F(\omega+\Delta\omega)$
Left circularly polarized light	$k_- = k_0 e^{-(\omega-\Delta\omega)^2}$	$n_- - 1 = \frac{k_0 \chi}{\sqrt{\pi}} F(\omega-\Delta\omega)$
H perpendicular to k E parallel to H	$k_\pi = k_0 e^{-\omega^2}$	$n_\pi - 1 = \frac{-k_0 \chi}{\sqrt{\pi}} F(\omega)$
E perpendicular to H	$k_\sigma = \frac{k_0}{2} [e^{-(\omega+\Delta\omega)^2} + e^{-(\omega-\Delta\omega)^2}]$	$n_\sigma - 1 = \frac{-k_0 \chi}{2\sqrt{\pi}} [F(\omega+\Delta\omega) + F(\omega-\Delta\omega)]$

Magneto Rotation

$$\psi_\ell = \frac{(n_+ - n_-)\ell}{2\chi} = \frac{k_0 \ell}{2\sqrt{\pi}} [F(\omega+\Delta\omega) - F(\omega-\Delta\omega)]$$

Voigt Effect

$$\Phi_\ell = \frac{(n_\sigma - n_\pi)\ell}{\chi} = \frac{-k_0 \ell}{2\sqrt{\pi}} [F(\omega+\Delta\omega) + F(\omega-\Delta\omega) - 2F(\omega)]$$

The variation in the absorption coefficients for a small excited-state splitting, $\Delta\omega$, is given by

$$\delta k_\sigma \approx \frac{k_0}{2} \frac{\partial^2 e^{-\omega^2}}{\partial \omega^2} (\Delta\omega)^2 = k_0 e^{-\omega^2} (2\omega^2 - 1) (\Delta\omega)^2$$

$$\therefore \delta k_\pi = 0.$$

The only other case of interest is the absorption coefficient for linearly polarized light propagating along the field direction. The variation of this absorption coefficient with small changes in $\Delta\omega$ will be the same as that for k_σ . This can be shown by treating the linearly polarized light as a sum of right and left circularly polarized beams and by using the absorption coefficients given above for right and left circularly polarized light.

The derivation of the tensor form of the Kramers-Kronig relations is given by

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Gourary,¹ and the parameter k_o is a reciprocal penetration depth as defined by Mitchell and Zemansky.² The parameter $\omega = 2\sqrt{\ln 2} (\nu - \nu_o) / \Delta\nu_D$, where $\Delta\nu_D$ is the Doppler width. The splitting of the excited state is characterized by

$$\Delta\omega = 2\sqrt{\ln 2} g_2 m \mu_B H / h \Delta\nu_D,$$

and

$$F(\omega) = e^{-\omega^2} \int_0^\omega e^{y^2} dy.$$

This calculation was made for a $^3P_1 - ^1S_0$ transition. The approach may be generalized to include transitions between states with different multiplicities by rewriting the absorption coefficient and proceeding in essentially the same manner as above.

For a transition involving a ground state with levels denoted by m , and an excited state with levels denoted by m' , the absorption coefficient would be

$$k = \sum_{m, m'} k_{mm'}$$

$$k_{mm'} = k_o H_{mm'} e^{-(\omega + \Delta\omega_{mm'})^2}$$

$$\Delta\omega = (2\sqrt{\ln 2} / \Delta\nu_D) [(g_1 m + g_2 m') \mu_B H / h].$$

Here, g_1 is the g -factor for the ground state, and g_2 is the g -factor for the excited state. $H_{mm'}$ contains the angular dependence of the transition probability and is essentially the square of a Clebsch-Gordan coefficient.

R. L. Fork

References

1. B. Gourary, J. Appl. Phys. 28, 283 (1957).
2. A. Mitchell and M. Zemansky, Resonance Radiation and Excited Atoms (Cambridge University Press, London, 1934).

B. HYPERFINE STRUCTURE AND ISOTOPE SHIFT IN RADIOACTIVE CADMIUM

We have completed our measurements¹ of the isotope shift between radioactive Cd^{109} and Cd^{113} . The value of the shift in the $3261\text{-}\overset{\circ}{\text{A}}$ transition is $0.033 \pm 0.004 \text{ cm}^{-1}$ with the sign of the volume-dependent isotope shift. A well-blazed, high-resolution grating,² 10-inches wide, mounted in the 36-foot multiple wavelength mirror monochromator³

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Table IV-2. Relative isotope shifts in cadmium. These represent the variations in $\langle R^2 \rangle$ of the nuclear charge distribution. After correcting for the normal mass effect, the shifts were normalized to the shift between the even isotopes 110-112. The increasing nuclear polarizability as one gets into the collective region (here, larger N) is apparent. The staggering parameter $\gamma = \text{odd pair I. S.}/1/2$ (even pair I. S.).

Isotope pair	Relative isotope shift	Staggering parameter γ
108-110	1.00	0.06
108-109	0.03	
110-112	1.00	0.18
110-111	0.09	
112-114	0.93	0.28
112-113	0.13	

was used for these spectroscopic measurements. After correcting the isotope-shift value of Cd^{109} for the normal mass effect, and using the available results for the shifts in the stable isotopes,⁴ we obtained the relative shifts given in Table IV-2. It can be observed that the "odd-even staggering" increases with decreasing neutron number; that is, as we proceed toward the shell-model region we again appear to have a manifestation of the decreasing nuclear polarizability as a magic number (N=50) is approached. We have previously observed such a trend in mercury isotopes.⁵ The error limits on the stable-isotope results are considerably smaller than for the radioactive experiment and are a small fraction of the actual shifts. The former results were obtained from the Cd II 4416 Å line for which the shifts are much larger than for Cd 3261 Å. Therefore, in the observations we can rely only on the actual value of the Cd^{109} shift. More precise experiments will have to await the development of cooled hollow-cathode light sources that are suitable to work with the small quantities of material that are useful, at present, in electrodeless lamps.

H. H. Stroke

References

1. R. J. Hull and H. H. Stroke, Hyperfine structure and isotope shift in radio-cadmium, Quarterly Progress Report No. 62, Research Laboratory of Electronics, M.I.T., July 15, 1961, pp. 108-109.
2. G. W. Stroke, J. Opt. Soc. Am. 51, 1321 (1961).
3. H. H. Stroke and K. K. Y. Li, J. Opt. Soc. Am. 51, 678 (1961).
4. P. Brix and A. Steudel, Z. Physik 128, 260 (1950); H. G. Kuhn and S. A. Ramsden, Proc. Roy. Soc. (London) A 237, 485 (1956).
5. W. J. Tomlinson III and H. H. Stroke, Phys. Rev. Letters 8, 436 (1962).

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C. LEVEL CROSSINGS IN MERCURY 195 AND 195*

Zeeman level crossings¹ have been observed, and the magnetic fields at which these crossings occur have been measured in Hg¹⁹⁵ (9.5-hour halflife) and its isomer Hg^{195*} (40-hour halflife). Our apparatus is similar to that used by Hirsch^{2, 3} to measure the hyperfine structure of Hg¹⁹⁷ and Hg^{197*}. The principal changes in the apparatus have been in the use of a Harvey-Wells 12-inch magnet (Model L-128) to improve homogeneity and hence the precision of the measurements, and a new more compact magneto-optic scanning apparatus⁴ that provides a variable-frequency source of resonance radiation

Table IV-3. Summary of preliminary results.

Isotope	Description of crossing levels (low field representation)	Approximate crossing field (gauss)	Measured proton resonance frequency at crossing (kc)
Hg ¹⁹⁵ (I=1/2)	F = 3/2, m _F = -3/2 and F = 1/2, m _F = 1/2	7603	32369.79 ± .66
Hg ^{195*}	F = 15/2, m _F = 15/2 and F = 13/2, m _F = 11/2	7968	33925.86 ± .84

in the neighborhood of 2537 Å. Searching for the crossings was greatly facilitated by the spectroscopic data on these isotopes obtained by Tomlinson and Stroke.⁵ The isotopes were produced by the reaction Au¹⁹⁷(p, 3n) Hg¹⁹⁵ in the Harvard Cyclotron by using the procedure of Tomlinson and Stroke for the spectroscopic runs.

Preliminary results for the level crossings in the ³P₁ state observed, thus far, are summarized in Table IV-3. Errors are two standard deviations plus an estimate of the systematic error in measuring the field.

Using the position of the Hg¹⁹⁵ crossing and Hirsch's value⁶ for g_J = 1.4861 ± .0003, I calculated the magnetic dipole interaction constant for the ³P₁ state to be A₁₉₅ = 15810.82 ± .2 mc. From the spectroscopic value of the electric quadrupole interaction constant B⁵ and the one measured level crossing in Hg^{195*}, I obtained A_{195*} = -2368.081 ± .17 mc. Indicated errors in A are 2σ, and represent random errors (including the effect of the uncertainty in B for Hg^{195*}).

An effort was made to see if the Hg¹⁹⁵ crossing field would be shifted by changes in

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source lamp power, scanning field (that is, lamp frequency) or by reversing the field in the Harvey-Wells splitting magnet. No significant shifts were observed, but the possibility of error resulting from changes in field homogeneity if the Harvey-Wells magnet is not cycled several times through a hysteresis loop after the field is suddenly reversed is still to be investigated.

The accuracy of the value for A_{195^*} could be improved by a factor of 10, or more, by a different level-crossing measurement in Hg^{195^*} (since $B \neq 0$). The intensity and approximate position of the second most intense level crossing in this isotope have been calculated by using computer programs run on the IBM 7090 computer at the Computation Center, M.I.T. The intensity change that is due to scattering by the two crossing levels is approximately 48 per cent; but when the incoherent light scattered by the other Zeeman levels lying within a Doppler width of the crossing point is included, it is found that the change in total scattered intensity at the crossing is less than 0.1 per cent. This small change may, nonetheless, be observable by simultaneous application of the techniques of field modulation, lock-in detection, and comparison of the light scattered through two different angles. A feasibility test for the proposed second crossing experiment is in progress. This will consist in looking for the very weak level crossings in stable Hg^{201} by using a natural mercury cell (that is, nonenriched sample). If this test is unsuccessful, a second measurement in Hg^{195^*} will be attempted by means of a standard double-resonance experiment.

W. W. Smith

References

1. F. D. Colegrove, P. A. Franken, R. R. Lewis, and R. H. Sands, *Phys. Rev. Letters* 3, 420 (1959).
2. H. R. Hirsch and C. V. Stager, *J. Opt. Soc. Am.* 50, 1052 (1960).
3. H. R. Hirsch, Hyperfine structure in the 3P_1 level of the twenty-four hour isomer of mercury 197, *J. Opt. Soc. Am.* 51, 1192-1202 (1961); Technical Report 372, Research Laboratory of Electronics, M.I.T., October 15, 1961.
4. W. W. Smith, A compact magneto-optic scanning apparatus, *Quarterly Progress Report No. 63*, Research Laboratory of Electronics, M.I.T., October 15, 1961, pp. 83-85.
5. W. J. Tomlinson III and H. H. Stroke, *Bull. Am. Phys. Soc.* 7, 350 (1962).
6. H. R. Hirsch, Ph.D. Thesis, Department of Physics, M.I.T., 1960.

D. THE DETAILED ANALYSIS OF A PLASMA

In 1956, an attempt was made to describe in detail the energy flows and particle flows in one of the best-known plasmas, the low-pressure mercury discharge in the presence of a millimeter, or so, of rare gas.¹ Recent observations by Verweij² have made

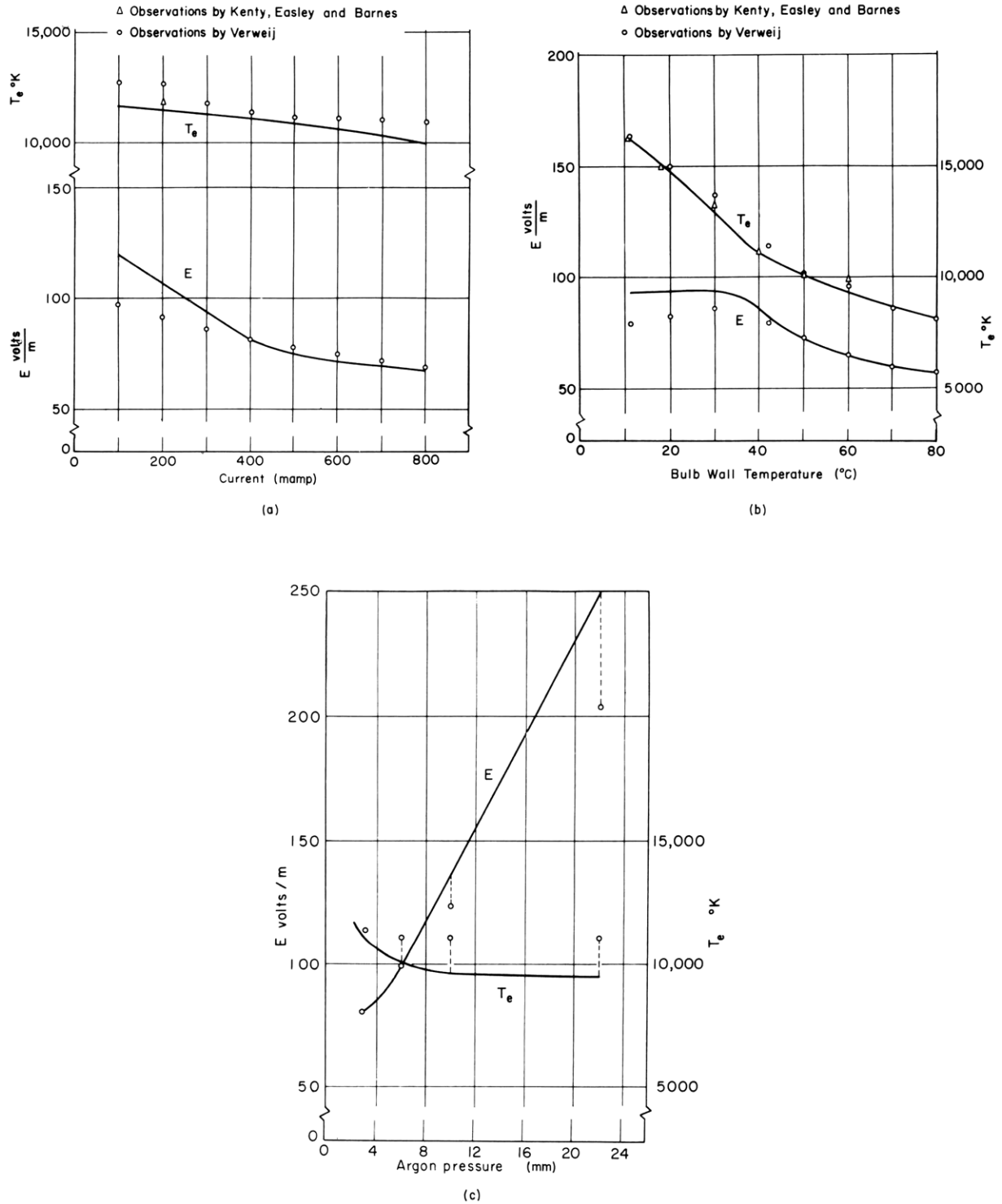


Fig. IV-1. Plots of experimental and computed electron temperature and axial field in a mercury-argon discharge in a 1.5-in. diameter tube as the lamp current, bulb-wall temperature and argon pressure are separately varied around the following standard values: lamp current, 400 ma; bulb-wall temperature, 42°C; argon pressure, 1 mm.

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possible a more detailed comparison between theory and experiment. This has been reported in detail in a lecture recently given by the writer at Cornell University.³ The chief results of this comparison are shown in Fig. IV-1. They concern the electron temperature T_e and electric field E in a mercury discharge in a tube approximately 1.5 inches in diameter with a vapor pressure corresponding to a bulb-wall temperature of 42° C, an argon pressure of 1 mm, and a lamp current of 400 ma as each of the last four variables are changed separately around the given values. Verweij shows that the observed electron mobility can be quite well accounted for by known electronic collision cross sections. A study of the existing data indicates that the chief uncertainty of the analysis lies in the radial diffusion of resonance radiation.

Another study of this discharge, with more justifiable assumptions regarding the excited states of mercury, is being undertaken. Also, an analysis by J. F. Waymouth⁴ of the theory of Langmuir-probe measurements when the probe dimensions are comparable to the electron mean-free path indicates the need for a review of certain aspects of the existing data.

F. Bitter

References

1. J. F. Waymouth and F. Bitter, J. Appl. Phys. 27, 122 (1956); F. Bitter and J. F. Waymouth, J. Opt. Soc. Am. 46, 882 (1956).
2. W. Verweij, Physica 25, 980 (1959).
3. To be published by the Material Sciences Center of Cornell University.
4. This work will be published as Technical Report 406, Research Laboratory of Electronics, M.I.T.; see also Section IV-F.

E. THE MAGNETIC MOMENT OF MERCURY 197 BY MEANS OF OPTICAL PUMPING

A Ph.D. thesis with this title was presented to the Department of Physics, M.I.T., June 1962, by the writer. The final numerical values were also reported in Quarterly Progress Report No. 65, page 39.

W. T. Walter

F. PERTURBATION OF A PLASMA BY A PROBE

One of the fundamental conditions to be satisfied before Langmuir-probe characteristics can be interpreted to yield data on plasma density and temperature is that probe size be much smaller than mean-free paths of plasma particles. Then and only then are the values of electron density and plasma potential the

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same as those in the absence of the probe.

In several recent high-caliber probe measurements that have been published, this condition has been violated to a greater or lesser degree. Moreover, in the presence of a magnetic field, when the effective mean-free path becomes comparable to the cyclotron radius it is especially difficult to meet this requirement.

A study has been made of this problem, starting with the hypothesis that the sheath equations still apply, but that the plasma density and potential at the sheath boundary are perturbed from their values in the absence of the probe by depletion resulting from the probe current. In the plasma outside the sheath boundary there is a density and potential gradient to supply, by ambipolar diffusion to the sheath boundary, the currents of electrons and ions extracted across the sheath by the probe.

The problem has been considered for a spherical probe in the absence of a magnetic field, and for a plane disc probe normal to a magnetic field. The results for both cases can be expressed as follows:

$$n_p = \frac{n_o}{1 + \beta Q}$$

$$\Delta V = - \left[V_e - \frac{Q_e (V_e + V_i)}{Q} \right] \ln(1 + \beta Q),$$

where n_o is the plasma density in the absence of the probe, n_p the plasma density at the probe sheath boundary, V_e and V_i the voltage equivalents of electron and ion temperatures, ΔV is the perturbation of plasma potential, and β is the magnetic field factor $1 + \mu_e \mu_i B^2$, with μ_e and μ_i the electron and ion mobilities. Here, Q is given by

$$Q = Q_e + Q_i$$

$$Q_e = \frac{r_p}{\mu_e (V_e + V_i)} \sqrt{\frac{e V_{es}}{2\pi m_e}} \exp(V_s/V_{es}) \quad V_s < 0$$

$$= \frac{r_p}{\mu_e (V_e + V_i)} \sqrt{\frac{e V_{es}}{2\pi m_e}} \quad V_s > 0$$

$$Q_i = \frac{r_p}{\mu_i (V_e + V_i)} \sqrt{\frac{e V_{is}}{2\pi m_i}} \quad V_s < 0$$

$$= \frac{r_p}{\mu_i (V_e + V_i)} \sqrt{\frac{e V_{is}}{2\pi m_i}} \exp(-V_s/V_{is}) \quad V_s > 0.$$

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Here, V_s is the potential across the sheath, assumed to be positive when the probe is positive with respect to the plasma at the sheath boundary; V_{es} and V_{is} are electron and ion temperatures (in volts) at the sheath boundary. Under conditions for which this analysis is valid ($Q \gg 1$), these are the same as the respective temperatures in the plasma. The radius of the spherical or disc probe is r_p .

A report is in preparation which gives the derivation of these equations in considerable detail, as well as calculations for the perturbed probe current-voltage characteristics to be expected for various values of Q .

J. F. Waymouth

