V. NOISE IN ELECTRON DEVICES*

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A. PHOTON STATISTICS OF OPTICAL MASER OUTPUT

The measurement of the output-power spectra of narrow-band optical-maser oscillators or amplifiers is currently under way. Information on the power spectra may be obtained from the statistics of the output photon number. These, in turn, may be inferred from the count of photoelectrons from a photosurface exposed to the optical-maser beam. We shall first present the classical theory of such an experiment, and then summarize the preliminary experimental results.

We assume that the probability of producing a photo-electron current pulse within the time interval dt is $\alpha P(t)\,dt$; $P(t)$ is the power in the incident light (in general, a statistical quantity), and α is a factor incorporating the photoefficiency of the phototube. The probability of obtaining exactly K counts in an interval T follows the Poisson law for nonstationary processes.

$$p(K) = \frac{n^K}{K!} e^{-n}, \tag{1}$$

$$n = \int_0^T \alpha P(t)\,dt. \tag{2}$$

If P(t) is a statistical variable, an average has to be taken with respect to its probability distribution. We shall indicate this last average by a bar.

$$\overline{n_T} = \sum_{K=1}^{\infty} p(K)\,K = \int_0^T \alpha\,\overline{P(t)}\,dt = \alpha T \overline{P} \tag{3}$$

and

$$\overline{n_T^2} = \sum_{K=1}^{\infty} \overline{K^2 p(K)} = \overline{n_T} + \overline{\left(\int_0^T \alpha P(t)\,dt\right)^2} . \tag{4}$$

The last integral can be evaluated easily in two limits; for $T \ll \tau_o$, and $T \gg \tau_o$, where τ_o is the correlation time of P(t) which is of the order of the inverse bandwidth of the maser. (For optical-maser amplifiers with highly reflecting mirrors this time will in general not be shorter than 10 μsec. For an oscillator it will be much longer than

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that.) In the limit $T \ll \tau_o$,

$$\overline{\delta n_T^2} = \overline{n_T^2} - \overline{n}_T^{\,2} = \alpha T\overline{P} + \alpha^2 T^2 \overline{P^2(0)} - \alpha^2 T^2 \overline{P}^2. \tag{5}$$

For a Gaussian amplitude distribution we have[1]

$$\overline{P^2} = 3\overline{P}^2 \tag{6}$$

and thus

$$\overline{\delta n_T^2} = \overline{n}_T(1+2\overline{n}_T). \tag{7}$$

This result applies for a linearly polarized light output. If the light is unpolarized, the mean-square fluctuations are additive, and we have

$$\overline{\delta n_T^2} = \overline{n}_T(1+\overline{n}_T). \tag{8}$$

For a sinusoid, this is

$$\overline{\delta n_T^2} = \overline{n}_T. \tag{9}$$

Thus, for a photoelectron count $\overline{n}_T \gg 1$ within a time $T \ll \tau_o$, there is a great difference between the fluctuations observed for a Gaussian signal and that of a sinusoid (or near sinusoid).

In the limit $T \gg \tau_o$, we may approximate

$$\left[\int_0^T P(t)\, dt\right]^2 \cong T \int_0^T \overline{P(t)\ P(t+\tau)}\, dt. \tag{10}$$

For a Gaussian process[2]

$$\overline{P(t)\ P(t+\tau)} = \overline{P}^2 + 2R^2(\tau), \tag{11}$$

where $R^2(\tau)$ is the square of the amplitude autocorrelation function that is related to the spectrum, and thus to the line shape, by a Fourier transform. We obtain for linearly polarized light (compare Purcell[3])

$$\overline{\delta n_T^2} \cong \overline{n}_T\left(1 + \overline{n}_T \frac{\tau_o}{T}\right). \tag{12}$$

Here, we have set

$$\tau_o = \frac{2\int_{-\infty}^{+\infty} R^2(\tau)\, d\tau}{\overline{P}^2}. \tag{13}$$

The limiting value of $\overline{\delta n_T^2}/\bar{n}_T$ attained for large T, if the photon flux is held constant (so that $\bar{n}_T$ is proportional to T) gives the correlation time τ_o.

We have made preliminary measurements on an optical maser oscillator of 6328 Å center frequency and 1-mw output, using a phototube (EMI/9558) and a high-speed counter with a maximum counting rate of 10 counts/μsec. The light was attenuated before reaching the multiplier and the low spikes were rejected in the count. The shortest counting interval used was 100 μsec, with $\bar{n}_T = 5.5$. If the maser signal had been Gaussian, we should have seen the large $\overline{\delta n_T^2}$ corresponding to Eqs. 7 or 8. This was not observed. We found $\overline{\delta n_T^2} = \beta \bar{n}_T$, for $T = 10^{-4}$ to 10 sec in decade steps, where β was a number varying in magnitude between 0.5 and 12, the large deviation from unity occurring for relatively long observation times T and large average photon counts. The deviations of β from unity may be due in part to the statistics of the photomultiplication. We believe that the experiment has shown that the signal is not Gaussian, exhibiting power fluctuations much smaller than those corresponding to a Gaussian signal of a bandwidth that one may reasonably expect for a maser oscillator. The experiments are continuing. The experiments are undertaken jointly with C. Freed and R. J. Carbone of Lincoln Laboratory, M.I.T., and J. McDonald, a senior in the Department of Electrical Engineering, M.I.T.

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References

1. W. B. Davenport, Jr. and W. L. Root, Random Signals and Noise (McGraw-Hill Book Company, New York, 1958), p. 254.

2. Ibid, p. 255.

3. E. M. Purcell, The question of correlation between photons in coherent light rays, Nature 178, 1447 (December 29, 1956).