XIX. PROCESS ANALYSIS AND SYNTHESIS**

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## RESEARCH OBJECTIVES

For the past several years the aim of this research has been to seek theoretical descriptions of some of the subjective aspects of communication. Experimental verification of the plausibility of theoretical results is also being pursued.

The theory of groups and the generalized concepts of symmetry are being used in an attempt to describe certain aspects of perception and recognition. It is assumed that the structure of such processes can be described by a set of elements and by a set of rules that operate on these elements.

The processes now being investigated include perception of brightness, color, depth, and form. Handwriting is also being studied by methods aimed at extending the work of Dernier van der Gon. A rudimentary simulator of the handwriting process has been constructed, and an improved version is being completed. Work in the immediate future will comprise the use of this simulator to test the validity of the principles of simulation and to attempt to describe some of the principles involved in the recognition of handwriting. Work is also being done on the subjective structure of music.

Mathematical work includes the investigation of the complete subgroup structure of the symmetric group of degree five, the theory of uniformization of curves and the theory of moments in connection with the algebraic domain.

A tabulation of the extended Lommel functions is being prepared, together with graphical representations. These results are not directly related to the investigations cited above, but are based on previous research.
M. V. Cerrillo

## A. HANDWRITING SIMULATION

This project has its origin in a paper published by Dernier van der Gon and others, ${ }^{1}$ in which a very simple system for the simulation of fast cursive handwriting was discussed. The present research seeks to extend the results reported by van der Gon. The van der Gon system is based on four postulates that are assumed to hold for fast cursive handwriting. (i) Fast handwriting is done without instantaneous position feedback. (ii) Writing movements are caused by two independent groups of muscles. Each group governs movements to and fro in one direction. The directions of these two groups are more or less perpendicular. (iii) These two pairs of antagonistic groups of muscles alternately apply forces to hand and pencil. The hand-pencil combination can be considered as a mass with viscous friction. (iv) After its onset, the force rises to a certain fixed value and remains more or less constant. The duration of application of this force is related to the magnitude of the writing movement. Thus different lengths of
*This work was supported in part by the National Science Foundation (Grant G-16526), the National Institutes of Health (Grant MH-04737-03), and the National Aeronautics and Space Administration (Grant NsG-496).
strokes in uninterrupted handwritten characters are caused by a different time of application, and not by a different magnitude of force. Van der Gon reported that an


Fig. XIX-1. Block diagram of simulator.
electromechanical device based upon these principles gave an oscillographic output that could be adjusted to match a given signature quite closely.

Figure XIX-1 is a block diagram of van der Gon's simulator. The rectangular wave generators simulate the force waveforms in accordance with postulate (iv). They share a common trigger, but are otherwise independent. They have provision for adjustment of the position in time of all transitions of the wave. Each rectangular wave generator is followed by a clipped integrator which simulates the nonzero rise-time of the muscles. The positive and negative clipping levels on the integrator can be adjusted independently, but there is no provision for providing different amplitudes of the segments of the wave. These in turn are followed by second-order linear systems in accordance with postulate (iii). This second-order system is represented by the differential equation

$$
\begin{equation*}
m_{x} \frac{d^{2} x}{d t}+k_{x} \frac{d x}{d t}=f_{x}(t) \tag{1}
\end{equation*}
$$

where $x$ is the displacement in one of the independent directions, $f x$ is the force acting in this direction, and $m_{x}$ and $k_{x}$ are the associated mass and friction constants, respectively. There is a similar equation that is assumed for $y$-displacement. The output of the entire system is displayed on an oscilloscope with one of the independent channels controlling vertical deflection, and the other channel controlling horizontal deflection.

The first step in the present investigation was to build an all-electronic simulator having essentially the same block diagram as Fig. XIX-1. The only change in our system is that the clipped integrator is replaced by a first-order lag network, giving the
"force" an exponential rather than linear rise.
This device was used to generate simulated handwriting. A typical example is shown in Fig. XIX-2b. Figure XIX-2a is the original handwriting that the simulated output of


Fig. XIX-2. (a) Original sample.
(b) Simulated sample.
(c) Simulated sample with tail of "y" completed.

Fig. XIX-2b is supposed to match. It should be emphasized here that the matching procedure as it now stands is simply one of trial and error; the transitions in the rectangular waves are adjusted to give the best possible match between the output of the simulator and the sample.

The simulation is quite successful except for the tail of the "y". It was found that the last stroke on the tail of the "y" could not be made to have the same shape as the sample without changing the rest of the work so that it did not agree with the sample. Figure XIX-2c presents an attempt to compromise between matching the " $y$ " and matching the "le". It is easy to see that this sample fails quite badly, particularly on the tail of the "y". Figure XIX-3 shows the rectangular waves and the linearsystem outputs pertinent to the sample of Fig. XIX-2b.

The example shown in Fig. XIX-3 indicates that the principles of the simulator, though compatible with the production of handwritinglike outputs, are not completely compatible with an arbitrary sample of cursive script. By constructing a system that
provides for independent variation of the amplitude of the segments of the "force" waveform, as well as for their duration (violates postulate (iv)), it was found that the "y"


Fig. XIX-3. System outputs for the sample in Fig. XIX-2b.
in the example above could be matched quite well. The result appears in Fig. XIX-4. Only the "y" appears because the new rectangular wave generator as it existed when the sample shown in Fig. XIX-4 was made had only enough sections to generate approximately one letter at a time. A complete rectangular wave generator permitting variable amplitude and duration of individual "force" segments has been designed and is under construction at the present time.

The simulation of errors commonly made

Fig. XIX-4. Improved simulation of "y" in Fig. XIX-2a. in handwriting is interesting. Examples using the word "in" appear in Fig. XIX-5. In these two samples, the only difference between them is the positions of the transitions of the rectangular waves. The linear systems are the same in both samples.

The discussion above opens the question of modifying van der Gon's postulate (iv). Upon completion of the above-mentioned rectangular wave generator, simulations will be carried out to explore this question further. Also, there does not seem to be any reason to assume that the independent directions of the force are perpendicular, that they are constant over a sequence of several words or that the angle between them and the writing-line remains constant. Equipment is now being designed for a system to allow simulation of these cases.

$\qquad$ VERTICAL SIGNAL
HORIZONTAL SIGNAL

WAVE
(a)

(b)

Fig. XIX-5. Two simulations of the word "in" obtained by varying only the transitions in the rectangular waves. Time scales for vertical and horizontal signals differ from those for the rectangular waves.

Work is also progressing on a system to analyze actual handwriting in terms of the model assumed in this simulation.
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## References

1. J. J. Dernier van der Gon, J. Ph. Thuring, and J. Strackee, A Handwriting Simulator, Phys. Med. Biol. 6, 407-414 (January 1962).
