VI. NOISE IN ELECTRON DEVICES

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A. HIGHER ORDER MOMENTS OF PHOTOELECTRON COUNTS

We report experimentally observed higher order moments of the photoelectron counts in a photomultiplier illuminated by a maser operating below but near threshold. The results are compared with theory. The photoelectron counts occurring in a set of predetermined intervals of length T were recorded on tape and then fed to a computer. The second, third, and fourth order moments of the photoelectron counts as functions of the observation interval, T, were evaluated. The higher order moments contain information about the higher order correlation functions of the power P(t) incident upon the photomultiplier.

The probability w(K) of receiving exactly K photoelectrons within a time interval extending from t = 0 to t = T is[1]

$$w(K) = \left[a \int_0^T P(t) \, dt \right]^K \frac{e^{-a \int_0^T P(t) \, dt}}{K!} \tag{1}$$

where a is the proportionality factor

$$a = \frac{\eta}{h\nu}, \tag{2}$$

and η is the quantum efficiency of the photosurface. We may now evaluate the factorial moments of the photoelectron count. We have

$$\overline{K(K-1) \cdots (K-\ell+1)} = a^\ell \left[\int_0^T P(t) \, dt \right]^\ell = a^\ell \ell! \int_0^T dt_\ell \cdots \int_0^{t_2} dt_1 \, \overline{P(t_1) \cdots P(t_\ell)}. \tag{3}$$

This expression is obtained by changing the ℓth power of the time integral into an ℓ-fold time-ordered integral. Equation 3 can be applied to various assumed time dependences of the power emitted by a laser. Comparison of the experimental results with the theoretical expressions thus obtained provides checks on the correctness of the assumed model. We assume a correlation function for the amplitude of the electric field of the incident laser light when the laser is operated below threshold in a single mode

$$\overline{a(t) \, a(t+\tau)} = \overline{a^2} \, e^{-\omega_o \tau/2}. \tag{4}$$

Such an expression is valid when the laser line is Lorentzian. If we use expression (4) in (3) applied to the second-order factorial moment, we find

$$\frac{\overline{K(K-1)} - \overline{K}^2}{\overline{K}} = \frac{a\overline{P}}{\omega_o} 2\left[1 - \frac{1}{\omega_o T}\left(1 - e^{-\omega_o T}\right)\right].$$ (5)

When operating a laser below threshold, one finds in general that more than one single mode contributes to the output power. Under the assumptions that all but one of the output modes of the laser are broad-band and that the single narrow-band mode contributes a fraction w to the output power, we obtain, instead of Eq. 5, the following expression for the normalized second-order factorial moment:

$$\frac{\overline{K(K-1)} - \overline{K}^2}{\overline{K}} = w^2 \frac{a\overline{P}}{\omega_o} 2\left[1 - \frac{1}{\omega_o T}\left(1 - e^{-\omega_o T}\right)\right].$$ (6)

For a Gaussian distribution of the amplitude of the laser signal below threshold, all higher order correlation functions of the power can be related to the correlation function of the amplitude. When this is done, we obtain for the normalized third-order factorial moment

$$\frac{\overline{K(K-1)(K-2)} - \overline{K}^3}{\overline{K}^2} = \frac{a\overline{P}}{\omega_o} 3! \left\{ w^2\left(1 - \frac{1}{\omega_o T} + \frac{1}{\omega_o T} e^{-\omega_o T}\right)\right.$$
$$\left. + 2w^3\left[\frac{1}{\omega_o T} e^{-\omega_o T} + \frac{1}{\omega_o T} - \frac{2}{(\omega_o T)^2}\left(1 - e^{-\omega_o T}\right)\right]\right\}$$ (7)

and for the normalized fourth-order factorial moment

$$\frac{\overline{K(K-1)(K-2)(K-3)} - \overline{K}^4}{\overline{K}^3} = \frac{a\overline{P}}{\omega_o} 4! \left\{ w^2\left(\frac{1}{2} - \frac{1}{2\omega_o T} + \frac{e^{-\omega_o T}}{2\omega_o T}\right)\right.$$

$$+ 2w^3\left[\frac{1}{\omega_o T} - \frac{2}{(\omega_o T)^2} + e^{-\omega_o T}\left(\frac{2}{(\omega_o T)^2} + \frac{1}{\omega_o T}\right)\right] + w^4\left[\frac{1}{2\omega_o T} + \frac{4}{(\omega_o T)^2}\right.$$

$$\left.\left. - \frac{14}{(\omega_o T)^3} + e^{-\omega_o T}\left(\frac{11}{(\omega_o T)^2} + \frac{2}{\omega_o T} + \frac{13}{(\omega_o T)^3}\right) + \frac{e^{-2\omega_o T}}{(\omega_o T)^3}\right]\right\}.$$ (8)

In Fig. VI-1 Eqs. 6-8 have been plotted for the values $a\overline{P} = 4.3 \times 10^6$, $\omega_o = 0.77 \times 10^4$, and w = 0.44. Experimental points are shown for comparison. The values of $a\overline{P}$ were determined from the observed photoelectron count (since aP is the rate of photoelectron emission in the photomultiplier). The value of ω_o was determined from a spectral measurement that was run simultaneously with the photoelectron counting experiment. The value of w was obtained by matching Eq. 6 to the experimentally observed second-order
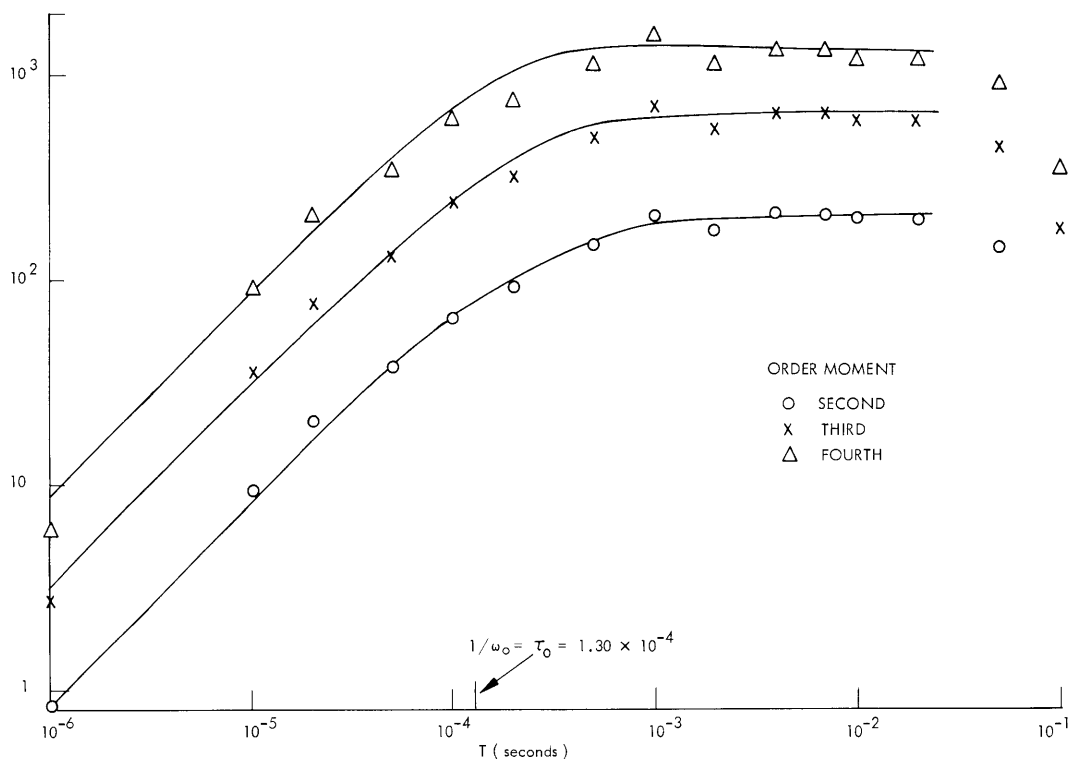
Fig. VI-1. Comparison of theoretical and experimental results.

factorial moment. By using these values, the theoretical curves for the third and fourth moments were evaluated. The check of these theoretical curves with the experimentally observed points is gratifying.

The experiments were performed at Lincoln Laboratory, M. I. T., jointly with Mr. C. Freed.

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References

1. L. Mandel, "Fluctuations of Photon Beams and their Correlations," Proc. Phys. Soc. (London) 72, 1037-1048 (1958).