COMMUNICATION SCIENCES

AND

ENGINEERING

XXI. STATISTICAL COMMUNICATION THEORY*

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RESEARCH OBJECTIVES AND SUMMARY OF RESEARCH

This group is interested in various aspects of statistical communication theory. Research in progress and work proposed for the near future are summarized as follows.

1. Nonlinear Theory Based on Functional Techniques

Studies in nonlinear theory based on the functional techniques of Wiener will continue. Some of the current problems include: the relationship between nonlinear differential equations and the functional representation; methods for the simplification of the functional representation for a nonlinear system; methods of synthesis; the approach to nonlinear oscillating systems; and applications to important engineering problems.

2. Statistics of Switching-Time Jitter

A model has been developed for switching-time jitter in a tunnel diode threshold-crossing detector. This model satisfactorily relates the statistics of the jitter to the load resistance, the slope of the input ramp, and the tunnel-diode characteristics. This study will next proceed with an investigation of the switching-time jitter of transistor switching circuits as flip-flops and Schmidt triggers. The object of this study is

(a) To obtain models for switching-time jitter in various solid-state switching circuits.

(b) To provide optimum circuit designs to minimize the switching-time jitter for a given device.

(c) To answer basic questions regarding the attainable noise figure of amplifiers operating in the switching mode.

3. High-Efficiency Realization for Amplitude-Modulated Transmitters

A study is under way to investigate the feasibility of low-frequency amplitude-modulated transmitters operating entirely in the switching mode. Pulse logic will eliminate the conventional modulator at a considerable increase in efficiency.

4. Model for Noise in Magnetic Tape

A study continues to develop a model for noise in the process of magnetic tape recording. It is expected to yield a model that is capable of satisfactorily

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explaining modulation noise.

5. Theoretical Investigation of the Two-State Modulation Systems

A study is planned of the performance of two-state modulation systems in terms of the parameters of their block diagrams. Objectives of the study are to obtain simple design relations and, possibly, to indicate modifications of the systems which will enhance certain performance characteristics, such as distortion in an amplifier or error in a power conversion system.

6. Recording and Reproduction of Sound

A study of the relative effects of the normal mode structure of rooms and loudspeakers on the reproduction of sound is nearing completion. The results are being prepared for publication. The study effort has now shifted focus to problems associated with the recording of sound. Spatial problems and problems associated with the combined effect of reverberation during both recording and reproduction are now under investigation.

7. Continuous Waveform Estimation

Theoretical and experimental studies in the area of estimating continuous waveforms are being continued. In the context of the analog modulation problem, performance of various suboptimum and optimum demodulators is being analyzed. Problems such as acquisition in the presence of noise and the effect of different message spectra are of interest.

8. Adaptive Systems

Adaptive systems are being studied in both the communication and radar contexts. The performance of various decision-directed schemes is being studied both theoretically and by simulation. The related problems of measurement of randomly time-variant channels are also of interest.

9. Feedback Structures

In many cases, considerable insight can be gained into both nonlinear and linear estimation problems by exploiting the Markov structure of the process. The implications of this technique are being studied. Related problems such as noise observation matrices are also being studied.

10. Space-Time Processing

The extension of the detection or estimation problem from the scalar to the vector case is formally trivial; however, many new issues arise in the area of combined space and time processing. These have particular application to both the seismic and sonar problem. Work in these areas continues.

11. Nonlinear Filtering of Convolved Signals.

During the past year homomorphic system theory has been applied to problems in nonlinear filtering. The primary result of this work is concerned with the optimization of nonlinear filters within an arbitrary class of homomorphic systems. It has been shown that a necessary and sufficient condition for a homomorphic system to be optimum is that the linear portion in the canonic representation for the class be optimum under a mean-square or integral-square error criterion.

At present, one of the most promising areas of application for this approach to nonlinear filtering is in the filtering of convolved signals. Problems of this class arise typically in the detection of timing of echoes, or more generally signal detection and separation in a reverberation environment. Also, the processing of speech waveforms, particularly the extraction of the glottal waveform, require the separation of convolved signals. It is interesting to note that the most recent and promising techniques, referred to as the "cepstral technique" in echo timing and pitch extraction, employ a cascade of operations identical to the canonic form for homomorphic filters that are suggested for this class of problems, although these techniques were developed from a different point of view. Consideration of homomorphic filters for this class of problems suggests some possible improvements over the cepstral technique. These techniques will be investigated more in detail.

12. Optimum Quantization

Exact expressions for the quantization error as a function of the quantizer parameters, the error-weighting function, and the amplitude probability density of the quantizer-input signal have been derived. An algorithm based on these expressions, which permits us to determine the specific values of the quantizer parameters that define the optimum quantizer (with respect to some particular error-weighting function), has been developed. This algorithm is valid for both convex and nonconvex error-weighting functions. Both the expression for the error and the algorithm have been extended to the case in which the quantizer-input signal is a message signal contaminated by a noise signal.

During the past year studies have concentrated on three particular areas.

(a) Theoretical investigation of the operation of linear prefiltering and postfiltering on quantization.

(b) Subjective evaluation of speech quantization when the number of quantizer levels is small and there are requirements of high intelligibility and naturalness.

(c) Theoretical investigation of the autocorrelation and power density spectrum functions of the quantizer-output signals.

Preliminary results in each of these areas have been obtained and reported. In continuing the work in these areas during the coming year, emphasis will be placed on speech quantization. For speech signals, the preliminary results indicate the need to raise questions of a subjective nature and for a more detailed analysis of the quantizer-output signal.

In addition to these areas of research, investigations will be made of the properties of quantizers which are optimum for quantizer inputs consisting of message signals contaminated by noise. Emphasis will be placed on discrete message signals. For this case, the quantizer can be regarded as a nonlinear zero-memory filter or as a decision-making device that indicates an estimate of the transmitted signal. The results here will be compared with those from classical detection theory.

Y. W. Lee

A. WORK COMPLETED

1. A SUBJECTIVE STUDY OF OPTIMUM QUANTIZATION

This study has been completed by Thomas H. Nyman. In September 1965, he submitted the results to the Department of Electrical Engineering, M. I. T., as a thesis in partial fulfillment of the requirements for the degree of Master of Science.

J. D. Bruce

B. NONLINEAR FILTERING OF CONVOLVED SIGNALS

In 1963, Bogert, Healy, and Tukey[1] discussed an approach to the detection and timing of echoes. Later Noll[2] successfully applied these techniques to the detection of pitch in speech waveforms. In brief, Bogert and his co-workers considered a signal z(t) consisting of another signal y(t) and its echo,

$$z(t) = y(t) + a \cdot y(t-T).$$

The objective of the processing which these authors propose is to determine the time T of the echo. If we let $\Phi_{zz}(\omega)$ denote the power spectrum (or energy spectrum if z(t) is aperiodic) of z(t), and $\Phi_{yy}(\omega)$ denote the spectrum of y(t), it follows that

$$\Phi_{zz}(\omega) = \Phi_{yy}(\omega)\left[1 + a^2 + 2a\cos\omega T\right] \tag{1}$$

or, if we take the logarithm of both sides of Eq. 1, we have

$$\log\left[\Phi_{zz}(\omega)\right] = \log\left[\Phi_{yy}(\omega)\right] + \log\left[1 + a^2 + 2a\cos\omega T\right]. \tag{2}$$

If a is sufficiently small, then (2) can be rewritten approximately as

$$\log\left[\Phi_{zz}(\omega)\right] \cong \log\left[\Phi_{yy}(\omega)\right] + 2a\cos\omega T. \tag{3}$$

This expression consists of the sum of two terms, one of which is periodic (in ω) and whose period we wish to determine. Hence, by taking the logarithm of both sides of (1), Bogert et al. transformed the original problem into one involving the sum of a periodic and nonperiodic function, thereby relating it to one of the well-known problems in linear filtering, although in this case the desired signal is periodic in frequency. Because of the duality of the time and frequency domains, however, this poses no particular problems, and we apply operations that would normally be applied in the frequency domain for the linear filtering of a signal that is periodic in frequency. Hence, a linear time-invariant filter, for the problem posed by Eq. 3, would correspond to a linear frequency-invariant filter. Thus, Bogert et al. approach this problem by thinking of the log-power spectrum, $\Phi_{zz}(\omega)$, as a time series and then extracting the periodic component by conventional means. Specifically, if (3) represented a time series, we might choose to highpass-filter and observe the frequency at which a peak in the spectrum occurred. Analogously, then, in this problem we may pass the log spectrum through a "long-pass lifter" (the term which they have chosen for the dual of a highpass filter) and observe the time at which a peak

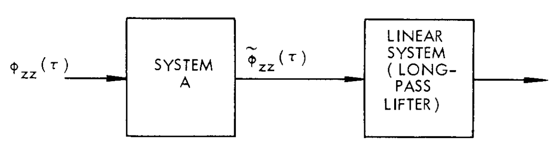

Fig. XXI-1. System for determination of the liftered-log-power spectrum.

occurs in the transform of the "liftered-log-power spectrum." To emphasize the duality, and because these operations are unconventional with respect to some of the more standard types of signal processing, the transform of the log-power spectrum is referred to as the "cepstrum" of the original time function and is expressed as a function of "quefrency." The processing that they propose is summarized in Fig. XXI-1. The input is taken to be the autocorrelation function of the signal z(t). The output of system A, $\tilde{\phi}_{zz}(\tau)$, is the cepstrum of z(t) and is related to $\phi_{zz}(\tau)$ in such a way that if $\Phi_{zz}(\omega)$ and $\tilde{\Phi}_{zz}(\omega)$ denote the transforms of $\phi_{zz}(\tau)$ and $\tilde{\phi}_{zz}(\tau)$, respectively, then

$$\tilde{\Phi}_{zz}(\omega) = \log \Phi_{zz}(\omega).$$

The linear system that they propose is a long-pass lifter which is the dual of a highpass filter. Thus, in the τ, or "quefrency," domain the long-pass lifter can be represented as in Fig. XXI-2.

In addition to the above-mentioned processing, Bogert et al. suggest a further step in the echo detection. The steps thus far proposed transform the problem into a

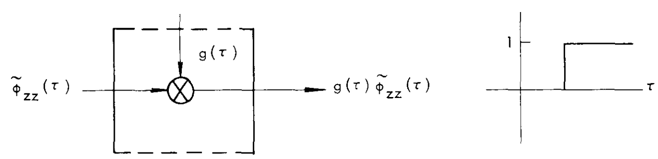

Fig. XXI-2. Long-pass lifter.

linear filtering problem. After filtering, they suggest the possibility of moving back toward the autocorrelation function by passing the output of the lifter through the inverse of system A, as shown in Fig. XXI-3; this results in the transform of the "delogged

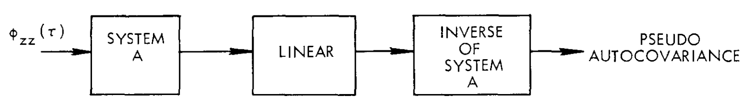

Fig. XXI-3. System for determination of the pseudo autocovariance.

liftered log spectrum," which, for brevity, they refer to as the "pseudo autocovariance." They comment, however, that "We must emphasize that this whole inquiry into pseudo-autocovariances is quite thoroughly empirical. We do not know, though it might not be hard to establish, whether it is reasonable to regard such a function of the original data as estimating something definite and reasonable. We are exploring with only heuristic guidance."

The purpose of this report is to attempt to relate this technique, which has been referred to as the cepstral technique, to homomorphic filtering.[3,4] This relation will offer a more formal framework for the cepstrum and the pseudo autocovariance and may suggest elaboration of the cepstral technique. Basically, the problem considered by Bogert et al. is concerned with the separation of convolved signals. Specifically, the autocorrelation function $\phi_{zz}(\tau)$ can be expressed as

$$\phi_{zz}(\tau) = \phi_{yy}(\tau) \otimes [(1+a^2)u_o(\tau)+au_o(\tau-T)+au_o(\tau+T)],$$

where $\otimes$ denotes convolution, and $u_o(\tau)$ is the unit impulse. To determine the echo timing, we wish to extract the term

$$(1+a^2)u_o(\tau) + au_o(\tau-T) + au_o(\tau+T).$$

Since convolution satisfies the algebraic postulates of vector addition, the separation of convolved signals falls within the class of homomorphic filtering problems. What I wish to demonstrate is that the systems of Fig. XXI-1 and Fig. XXI-3 are both homomorphic systems, the system in Fig. XXI-1 having convolution as the input operation and addition as the output operation, and that in Fig. XXI-3 having convolution as both the input and output operations.

It has been argued[3] that a necessary and sufficient condition that a system be homomorphic with some operation o as the input operation and some operation □ as the output operation is that it be decomposable into a cascade of three systems as shown in Fig. XXI-4. The system a_o is invertible and homomorphic, with o as the input operation. The system $\beta_\square^{-1}$ is invertible and homomorphic, with addition as the input operation and □ as the output operation. The system L is a linear system. Comparing this with the system shown in Fig. XXI-1, we see that to show that this system is homomorphic from convolution to addition, we need only show that system A is homomorphic from convolution to addition (system $\beta_\square^{-1}$ for this case is taken to be the identity system). To show that the system of Fig. XXI-3 is homomorphic from convolution to convolution we must again show that system A is homomorphic from convolution to addition. Because of the fact that the inverse of a homomorphic system is homomorphic, with the input and output operations reversed,[3] the inverse of system A will then also have the desired properties.

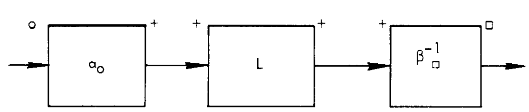

Fig. XXI-4. Canonic representation for homomorphic filters.

Let us argue that system A is homomorphic by considering it in the frequency domain, rather than in the time (or τ) domain. Since convolution in the time domain corresponds to multiplication in the frequency domain, and addition in the time domain corresponds to addition in the frequency domain, it then follows that system A represented in the frequency domain, must be homomorphic from multiplication to addition. It has been shown,[3] however, that the characteristic homomorphic system for multiplication is a logarithmic amplifier, which is identical to system A described in the frequency domain. Hence considered in the time domain, system A is homomorphic with convolution as the input operation and addition as the output operation. Therefore, the systems for the computation of the liftered cepstrum and the pseudo autocovariance are both homomorphic.

The general class of homomorphic filters for the separation of convolved signals are of the form shown in Fig. XXI-3, with members of the class differing only in the linear portion. The discussion above demonstrates that the system for determination of the pseudo autocovariance is a particular example of a homomorphic filter; however, this choice for the linear system does not necessarily result in the optimum filter. Furthermore, we may wish to impose other restrictions such as time invariance, realizability, and so forth. Because of the canonic form for homomorphic filters, however, any such restriction is reducible to considerations on the linear portion. In particular, the restrictions that I would like to discuss are: insensitivity to input amplitude; time invariance; and realizability.

Optimization of homomorphic filters has been discussed elsewhere,[4] and it has been shown that these filters could be considered as optimum if the linear portion is optimum under a mean-square or integral-square error criterion.

1. Insensitivity to Input Amplitude

Linear systems have the property that if the input is scaled up or down by some factor, then the output is scaled by the same factor. In designing a linear system for signal separation, then, we can essentially disregard the absolute amplitude of the incoming signal, and concentrate on the relative amplitude and shapes of the signals to be separated. With nonlinear systems in general, however, the effect of a change in input amplitude will affect more than just the amplitude of the output. It is interesting to note that just as for a linear system a scaling of the input results in a scaling of the output, for a homomorphic system the combination of an input with a scalar by means of the rule for scalar multiplication associated with the input operation results in a combination of the corresponding output with the same scalar, according to the rule for scalar multiplication associated with the output operation. For example, in the class of systems under consideration here, if $g(t)$ is the output for some input $f(t)$, then the output that is due to $f(t)$ convolved with itself n times (where n is not necessarily an integer) is $g(t)$ convolved with itself n times. This is analogous to the situation for linear systems, whereby if $g(t)$ is the output for some input $f(t)$, then the output that is due to $f(t)$ added to itself n times is $g(t)$ added to itself n times. We may refer to the result of adding an input to itself n times as a change in the linear amplitude of the signal by a factor of n and the result of convolving an input with itself n times as a change in the convolutional amplitude of the signal by a factor of n. Whereas the performance of a linear filter can be considered to be insensitive to linear amplitude but sensitive to convolutional amplitude, a homomorphic filter for the separation of convolved signals is, in general, insensitive to convolutional amplitude but sensitive to linear amplitude. But, by imposing the necessary restrictions on the linear portion of the canonic representation, this class of filters can be made insensitive to linear amplitude. Specifically, let us consider the canonic form for this class, represented in the frequency domain, as shown in Fig. XXI-5.

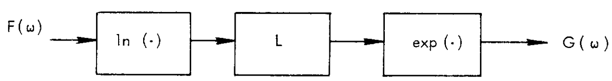

Fig. XXI-5. Canonic representation for homomorphic filters used for the separation of convolved signals.

Since linear amplitude in the time domain corresponds to linear amplitude in the frequency domain (that is, scaling the amplitude of a time function by some factor scales the spectrum by the same factor), we can ask that the system of Fig. XXI-5 be insensitive to linear amplitude, that is, if the input is changed by a scale factor, then the output is changed by a scale factor, although both scale factors do not have to be identical. Let us consider an input $kF(\omega)$ to the system of Fig. XXI-5. Then the output $G_k(\omega)$ that is due to this input is

$$G_k(\omega) = \exp\{L[\ln kF(\omega)]\}$$

or

$$G_k(\omega) = \exp[L(\ln k) + L(\ln F(\omega))]$$

$$= G(\omega) \exp[L(\ln k)],$$

where $G(\omega)$ is the response to $F(\omega)$. If it is required that, for any k, $G_k(\omega)$ is proportional to $G(\omega)$, then we must require

$$\exp[L(\ln k)] = \text{constant}$$

or

$$L(\ln k) = \text{constant}.$$

Since k is an arbitrary constant, we require that the response of the linear system to a constant spectrum be constant. In the time domain, then, the impulse response of the linear system must be an impulse, possibly, of different area. If the linear portion were restricted to be time-invariant, it must then be an amplifier of constant gain.

2. Time Invariance

To determine the appropriate restriction on the linear system such that the system of Fig. XXI-5 be time-invariant, let g(t) represent the response to an excitation f(t). Then we require that g(t-T) be the response to the excitation f(t-T), for any T. Hence, if ϕ represents the system transformation, then we require[5]

$$\phi[f(t) \otimes u_o(t-T)] = \phi[f(t)] \otimes u_o(t-T)$$

or

$$\phi[f(t)] \otimes \phi[u_o(t-T)] = \phi[f(t)] \otimes u_o(t-T).$$

Hence,

$$\phi[u_o(t-T)] = u_o(t-T)$$

for all T. If $\alpha(\cdot)$ denotes the transformation characterizing the first system in the canonic representation, so that $\alpha^{-1}(\cdot)$ is the transformation characterizing the last system, then

$$\alpha^{-1}L\alpha[u_o(t-T)] = u_o(t-T)$$

or

$$L\alpha[u_o(t-T)] = \alpha[u_o(t-T)].$$

But the transform of $u_o(t-T)$ is $e^{-j\omega T}$; consequently,

$$\alpha[u_o(t-T)] = -Tu_1(t),$$

where $u_1(t)$ is the derivative of $u_o(t)$. Therefore, we require

$$L[-Tu_o(t)] = -Tu_1(t)$$

or, since L is linear,

$$L[u_1(t)] = u_1(t).$$

Hence, the linear system L must satisfy the condition that its response to a unit doublet (that is, the derivative of a unit impulse) is a unit doublet. If L were itself restricted to be time-invariant, this would then imply that it is the identity system. Consequently, the only choice for ϕ such that both ϕ and L are time-invariant is the identity transformation. Except for this case, then, if ϕ is to be time-invariant, L cannot be. As an example of the application of the necessary and sufficient condition for the time invariance of this class of filters, consider the system proposed by Bogert et al. for the determination of the pseudo autocovariance. Since the linear portion is a long-pass lifter, ideally the function h(t) is zero until some time T_o, after which it is unity. The response of the linear system to a unit doublet $u_1(t)$ is h(t) $u_1(t)$ which, because of the choice of h(t), is zero. Consequently, the system is not time-invariant. On the other hand, if the linear system were, for some reason, chosen to be a short-pass lifter, so that h(t) is unity until some time T_o, after which it is zero, then the over-all system would be time-invariant.

The point of view expressed by Bogert et al., in which they treat the output of system A as though it were a time series, suggests an interesting, and possibly useful, notion regarding the selection of the linear filter. If we consider the log spectrum to be a function of frequency, then the linear filter that they select is frequency-invariant, that is, a translation of the input spectrum produces a corresponding translation in the output spectrum. If we pursue the duality between the time and frequency domains, it is straightforward to argue that whereas a linear time-invariant filter is represented in the frequency domain by the multiplication of the input spectrum by some function H(ω), a linear frequency-invariant filter is represented in the time domain by the multiplication of the input time function by another function of time, h(t). Thus, if we restricted the linear portion to be frequency-invariant, it would have the form shown in Fig. XXI-6. Note that the restriction that the linear system be frequency-invariant is equivalent to the restriction that the over-all system be frequency-invariant, because of the form of system A and its inverse.

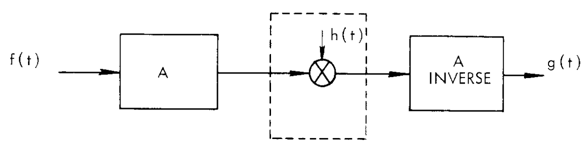

Fig. XXI-6. Canonic form when the class of filters is restricted to be frequency-invariant.

3. Realizability

In general, we cannot expect that the signal processing depicted by a system of the form of Fig. XXI-5 could be carried out in real time, that is, that the output at any instant of time is independent of future values of the input. It is certainly true that we could not carry out in real time the signal processing in the three steps corresponding to the three stages in the canonic representation, since both system A and its inverse are unrealizable. This was demonstrated in effect in the previous discussion where it was argued that the response of system A to a delayed impulse $u_o(t-T)$ is a doublet occurring at t = 0. It might be possible, however, that with certain restrictions on the linear system, the over-all system could be realizable, the inverse of system A "undoing" the unrealizability introduced by A (and possibly also by the linear system). There is certainly one case, although trivial, for which this is true, namely when the linear system is the identity system, in which case the output of the over-all system is equal to its input. Still, however, no necessary and sufficient conditions for realizability have been derived. One possible means of carrying out the desired signal processing in real time is to work, as Noll does, with the short-time spectrum. The implications of this with respect to the formalism presented here are not yet clear.

The class of problems to which filters discussed in this report is directed, is the separation of convolved signals. Admittedly, if one of two signals to be separated is known, a linear filter, whose impulse response is the inverse of the unwanted signal, can be used. This is analogous to the separation of two signals that have been added, by subtracting the unwanted signal from the sum. Hence, we are primarily interested in cases in which neither of the convolved signals is known exactly. This is clearly the case in the detection and timing of echoes, as discussed by Bogert et al. More generally, the recovery of a signal transmitted in a reverberation environment is a problem of general interest. Such situations arise, for example in sonar, in high-fidelity recording, in seismological studies. Multipath communication channels can also be modelled in terms of the convolution of the transmitted signal with noise; however, in this case, additive noise may also be an important factor, and its effect on the performance of convolutional filters must be investigated.

A. V. Oppenheim

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